INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

SMR. 767 - 18

MINIWORKSHOP ON STRONG CORRELATIONS
AND QUANTUM CRITICAL PHENOMENA
(4 - 22 July 1994)

RENORMALIZATION GROUP, WARD IDENTITIES AND BOSONIZATION IN d DIMENSIONAL FERMI SYSTEMS

Walter METZNER
Institut für Theoretische Physik C
RWTH Aachen
Sommerfeldstrasse 26/28
5100 Aachen, Germany

These are preliminary lecture notes, intended only for distribution to participants.

- RG in $d=1$

  Fermi liquid fixed point

  Luttinger liquid fixed point

  Ward identities

- Dimensional crossover

  Short range interactions

  (Gauge fields)

- Ward identities vs. bosonization

- "Anderson instability"

  Phase shifts

  Orthogonality catastrophe

<u>Collaborators :</u> C. Castellani
C. Di Castro

RG in $d \geq 1$

$d = 1$: Sólyom '73 — scaling ansatz

$d \geq 1$: Benfatto, Gallavotti '90
Feldman, Trubowitz '90
Shankar '91
} Wilson

Partition function

$$Z = \int \mathcal{D}[\psi, \psi^*]\, e^{S[\psi,\psi^*] + \text{sources}}$$

Grassmann fields

$$\psi = \psi_{\vec{k}\omega} \quad , \quad \mathcal{D}[\psi,\psi^*] = \prod_{\vec{k}\omega} d\psi_{\vec{k}\omega}\, d\psi^*_{\vec{k}\omega}$$

$$-\infty < \omega < \infty \quad , \quad \vec{k} \in BZ \text{ or } |\vec{k}| < \Lambda_0$$

Action

$$S[\psi,\psi^*] = \int d^d k\, d\omega\, (i\omega - \xi_{\vec{k}})\, \psi^*_{\vec{k}\omega} \psi_{\vec{k}\omega}$$

$$+ \int g(1,2;1',2')\, \psi^*_{2'} \psi^*_{1'} \psi_2 \psi_1$$

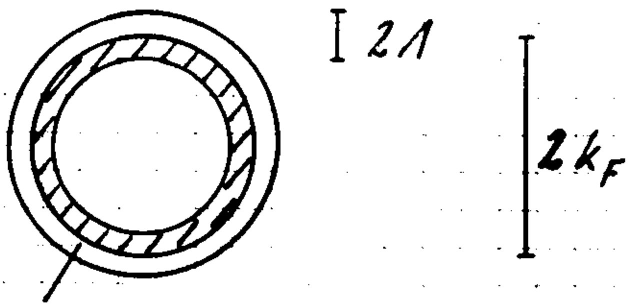
Mode elimination

Wanted: Excitations and correlation functions near Fermi surface, low energy physics

Decompose $\quad \Psi = \Psi_< + \Psi_> \qquad k_r = |\vec{k}| - k_F$

$$\underset{|k_r| < \Lambda}{\Psi_<} \qquad \underset{|k_r| > \Lambda}{\Psi_>}$$

Integrate out $\Psi_>$:



Cutoff

$\Lambda \ll k_F$

Fermi surface

$$Z = \int D[\Psi_<, \Psi_<^*] D[\Psi_>, \Psi_>^*] \, e^{S[\Psi, \Psi^*]} =$$

$$\int D[\Psi, \Psi^*] \, e^{S_\Lambda [\Psi_<, \Psi_<^*]}$$

"effective" action

S_Λ = quadratic term + many interactions

Rescale fields

$$\bar\psi_{\vec k\omega} = \psi_{\vec k\omega} / \sqrt{Z_\Lambda}$$

to get

$$\bar S_\Lambda = \int \Big(\overbrace{i\omega - v_F^\Lambda k_r}^{\text{marginal}} + \overset{\text{irrelevant}}{r^\Lambda(\vec k,\omega)}\Big)\, \bar\psi^*_{\vec k\omega}\, \bar\psi_{\vec k\omega}$$

$$+ \int \underset{\text{mixed}}{\bar g^\Lambda(\ldots)}\, \bar\psi^*\bar\psi^*\bar\psi\bar\psi + \underset{\text{irrelevant}}{\mathcal{O}(\bar\psi^6)}$$

where $\bar g^\Lambda = Z_\Lambda^2\, g^\Lambda$

"Irrelevant" terms negligible

for $\Lambda \ll k_F$

Don't rescale k_r !

(would screw up momentum conservation)

2 particle interactions

Expand

$$\bar{g}^{\Lambda}(k_1, k_2; k_1' k_2') =$$

$$\bar{g}^{\Lambda}(\vec{k}_{1F}, \vec{k}_{2F}; \vec{k}'_{1F}, \vec{k}'_{2F}) + \mathcal{O}(k_r, \omega)$$

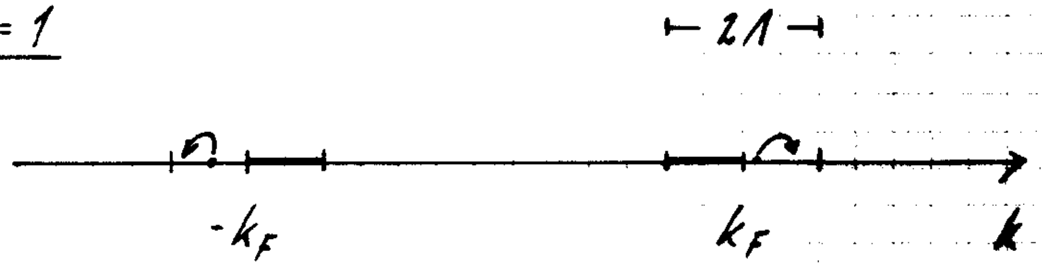
marginal (tree level)

irrelevant

but restricted phase space!

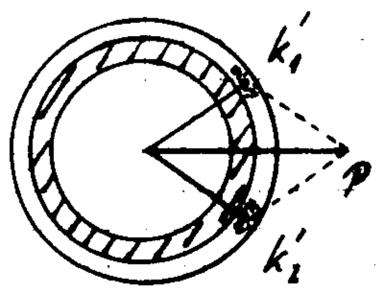
Phase space for scattering

$d = 1$



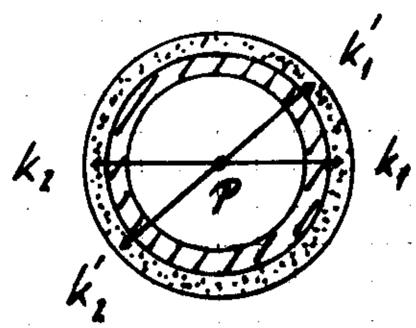
Finite density of scattering states

$d > 1$



$\measuredangle(1,2) \neq 0, \pi$

Vanishing density



$\measuredangle(1,2) = \pi$

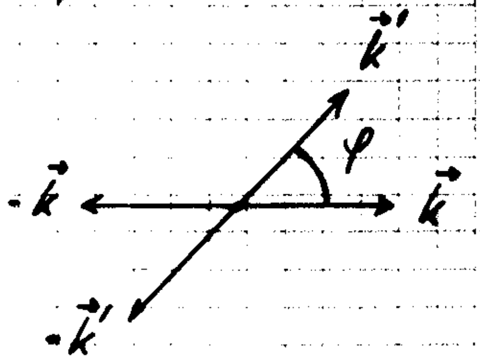
Finite density

$d > 1$: Cooper-instability

Fluctuations (1 loop) $\Rightarrow$

flow equation for

Cooper coupling $\bar{u}^\Lambda(\varphi)$



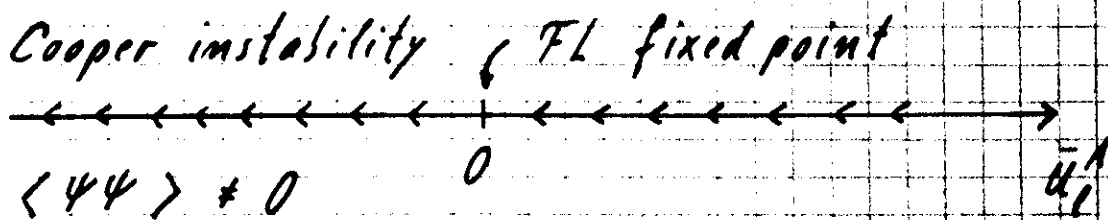
$$\frac{d\bar{u}^\Lambda(\varphi)}{d\ln\Lambda} = \int d\varphi' \, \bar{u}^\Lambda(\varphi-\varphi')\,\bar{u}^\Lambda(\varphi')$$

partial wave decomposition $\Rightarrow$

decoupled equations

$$\boxed{\frac{d\bar{u}_l^\Lambda}{d\ln\Lambda} = \left(\bar{u}_l^\Lambda\right)^2 , \quad l = 0,1,2,\ldots}$$

$\Rightarrow$ <u>Flow diagram:</u> $(\Lambda \to 0)$



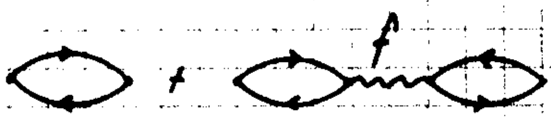
Fermi liquid fixed point

$$\left.\begin{array}{lcl} Z_\Lambda & \longrightarrow & Z \\ v_F^\Lambda & \longrightarrow & v_F^* \\ \bar{g}^\Lambda(\dots) & \longrightarrow & f(\theta) \end{array}\right\} \text{ for } \Lambda \to 0$$

- Residual scattering "irrelevant" for single particle properties

$$G = Z \longrightarrow = \frac{Z}{i\omega - v_F^* k_r}$$

- RPA exact for small $\vec{q}, \omega$ response

$\langle \rho\rho \rangle$ =  + ...

no Z! (charge conservation → Ward identity)

1D systems

Effective action: "g-ology" (Solyom)

4 marginal (tree level) couplings

Fluctuations (1 loop ...) $\Rightarrow$

backscattering $\bar{g}_1^{\Lambda}$ scales

2 cases:

$$0 \longleftarrow \bar{g}_1^{\Lambda} \longrightarrow -\infty$$

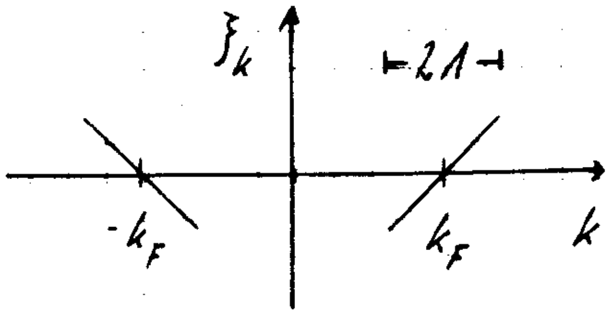
fixed point with higher symmetry: Luttinger liquid

spin gap

g-ology model

Solyom

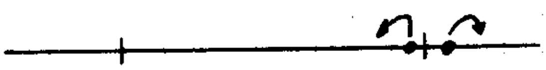
Kinetic energy:



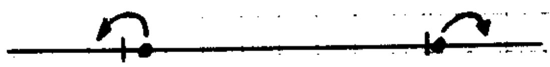
$$\xi_k = v_F^\Lambda k_r$$

Marginal interactions:

small momentum transfer

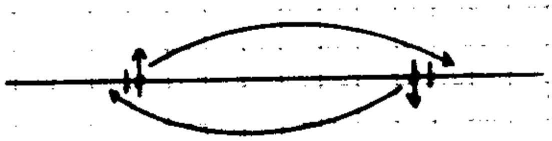


$\bar{g}_4^\Lambda$



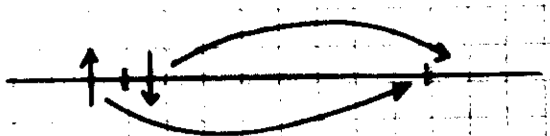
$\bar{g}_2^\Lambda$

backscatt.



$\bar{g}_1^\Lambda$

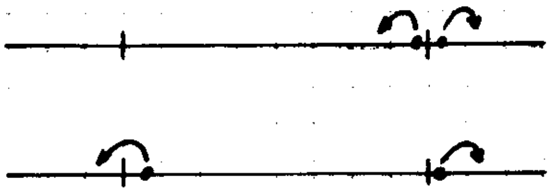
[Umklapp



$\bar{g}_3^\Lambda$]

Luttinger liquid fixed point

Only small q scattering survives:



$$\left.\begin{array}{l}\bar{g}_4^{\Lambda} \\[2ex] \bar{g}_2^{\Lambda}\end{array}\right\}\ \text{marginal to all orders}$$

"Luttinger model": completely solvable

(Lieb, Mattis '65; Dzyaloshinskii, Larkin '73;
Luther, Peschel '74; Haldane '81)

RPA density response

$Z_\Lambda \to 0$: anomalous scaling

Spin-charge separation

$$\boxed{\text{Asymptotic conservation law: Separate conservation of charge and spin on each Fermi point!}}$$

Luttinger liquid theory from conservation laws: W.M., C. Di Castro, PRB 47, 16107 (1993)

Ward identities

2 types of conservation laws:

i) exact, valid on all scales

ii) asymptotic, valid at low energy

Examples:

ad i) Global charge conservation $\rightarrow$

$$\partial_t \rho(\vec{r},t) + \vec{\nabla}\cdot\vec{j}(\vec{r},t) = 0 \quad \rightarrow$$

$$q_\mu \Gamma^\mu(p;q) = G^{-1}(p+q/2) - G^{-1}(p-q/2)$$

$\Rightarrow$ charge response not renorm. by Z, ...

ad ii) Conservation on each $\vec{k}_F$ $\rightarrow$

$$\vec{\Lambda}(p;q) \sim \vec{v}_p \Lambda^0(p;q)$$

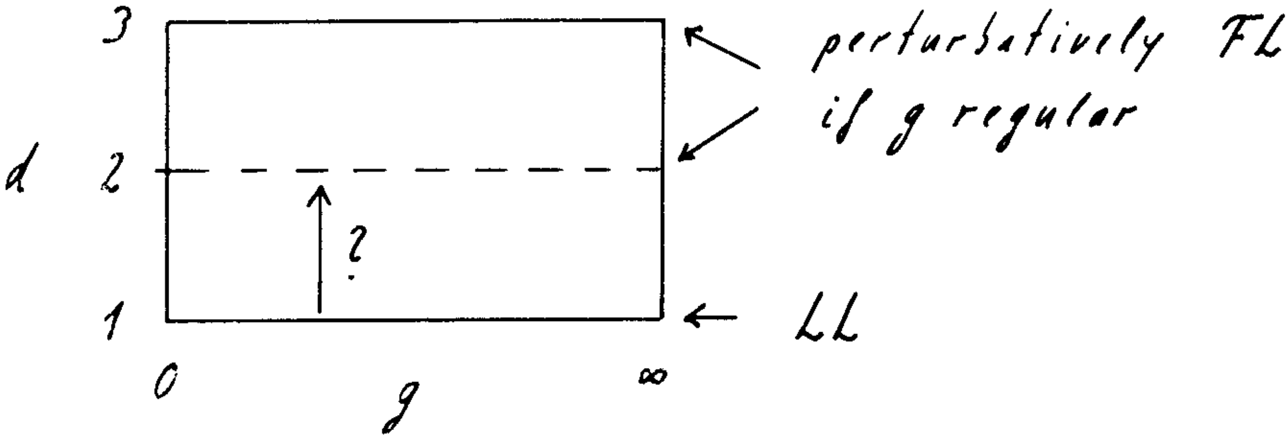
$\Rightarrow$ RPA response for small q, ω,

exact G for small k_r, ω, ...

Dimensional crossover

Castellani,
Di Castro, W.M.
PRL '94

Continue $d = 1, 2, 3, ..$ to $d \in \mathbb{R}$



- Critical dimensions ? $d_c^{FL} = 1$ or 2 ?

- Crossover from $d=1$ to $d=2$?

- Asymptotic Ward identities

Ueda, Rice '84 : $1+\epsilon$ expansion with g_1, g_2

$\Rightarrow$ FL or symmetry breaking (CDW, SDW, SC)

<u>Here :</u> $d-1$ not small, include $g_4 \hat{=} g_F(0)$

Perturbation theory for general d:

Integrals $\int_{\vec{k}} f(k, \vec{p}\cdot\vec{k}) =$

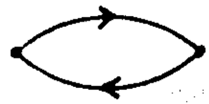
$$\frac{S_{d-1}}{(2\pi)^d} \int dk\, k^{d-1} \int_0^\pi d\vartheta \, \sin^{d-2}\vartheta \, f(k, pk\cos\vartheta)$$

$$\vartheta = \sphericalangle(\vec{p}, \vec{k})$$

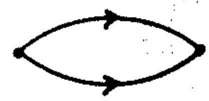
$$S_d = 2\pi^{d/2} / \Gamma(d/2)$$

Calculated:

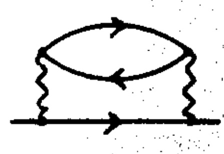
ph-bubble Π^{ph}



pp-bubble Π^{pp}



second order self-energy Σ



Results for self-energy:

- $\forall\, d \leq 2$, leading low energy contributions from angles $\vartheta \sim 0, \pi$.

- $\operatorname{Im} \Sigma(k_F, \omega) \propto \omega^d$ in $1 \leq d < 2$

$\Rightarrow$ $Z(\omega) = (1 - \partial\Sigma/\partial\omega)^{-1}$ finite $\forall\, d > 1$

- Contribution from $g_F(\vartheta)$, $\vartheta \sim 0$:

$$\operatorname{Im} \Sigma_F(p, \omega) \propto \omega^2 \, |\omega - \xi_p|^{d-2} \,, \quad d < 2$$

$\Rightarrow$ Splitting of QP-peak $\forall\, d < 2$

$\leftrightarrow$ "spin-charge separation" ?

$$\boxed{d_c^{FL} = 1 \text{ or } 2 \quad ?}$$

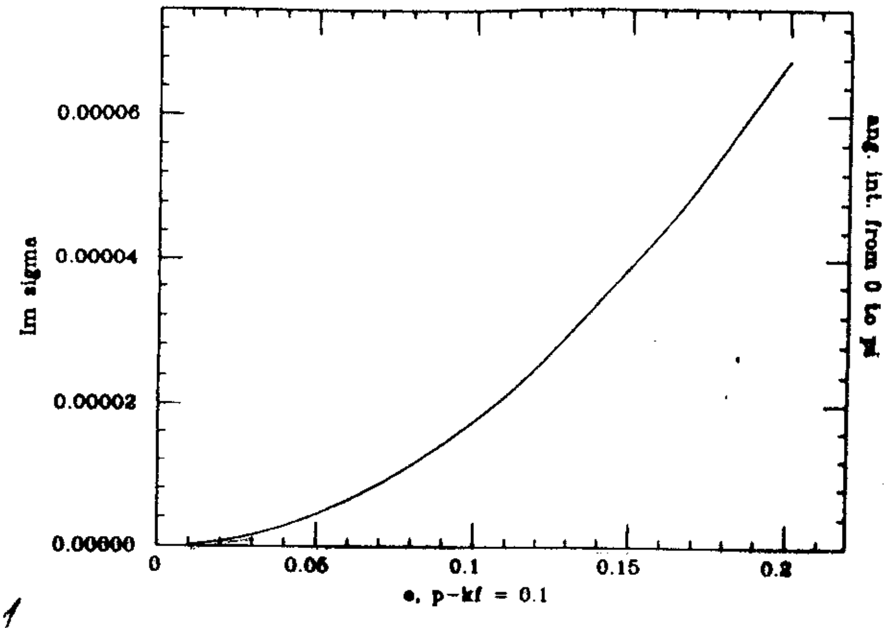
Try to sum all dominant contributions via Ward identities associated with asymptotic angle conservation.

$Im\ \Sigma$

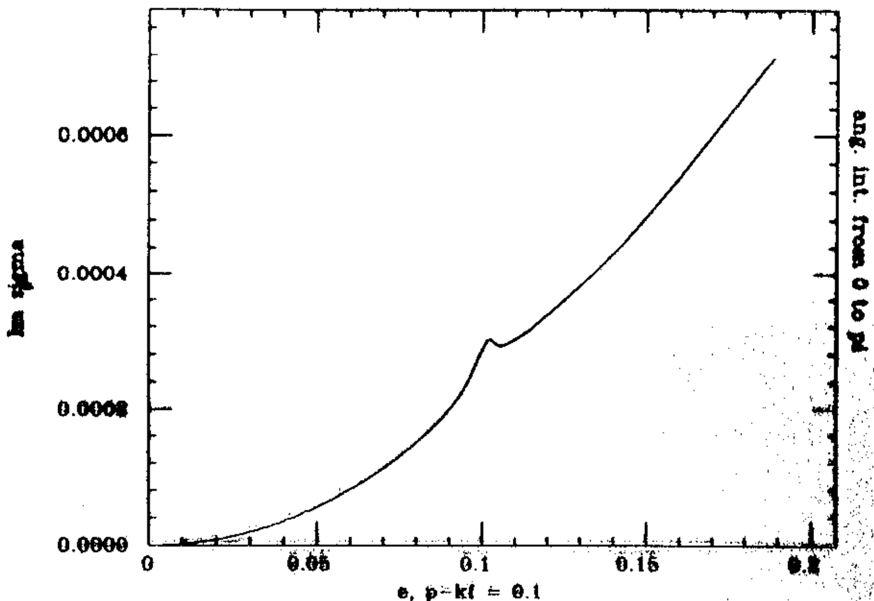
from

$$\frac{p-k_F}{k_F} = 0.1$$

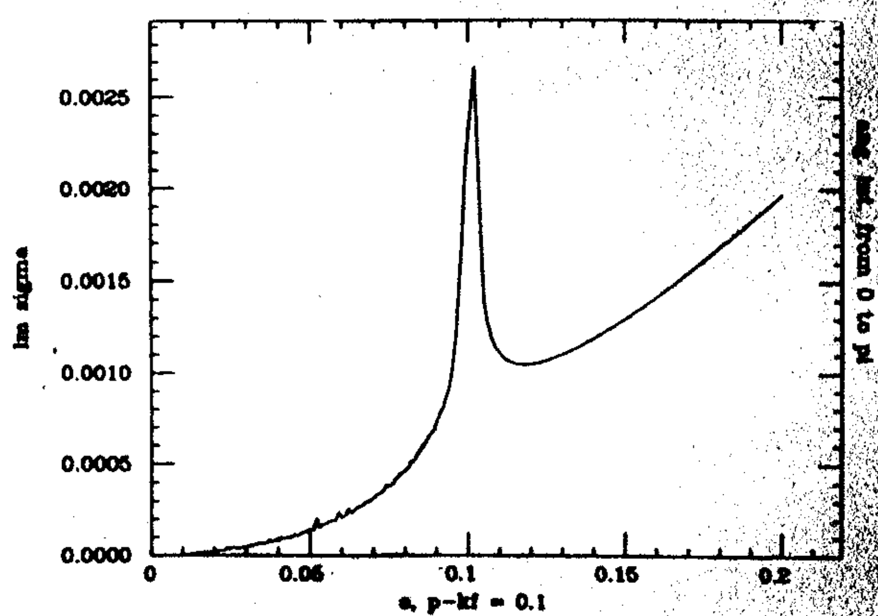
Im sigma(p,e), d = 3.



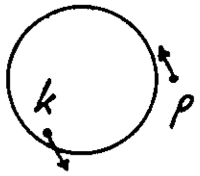
Im sigma(p,e), d = 2



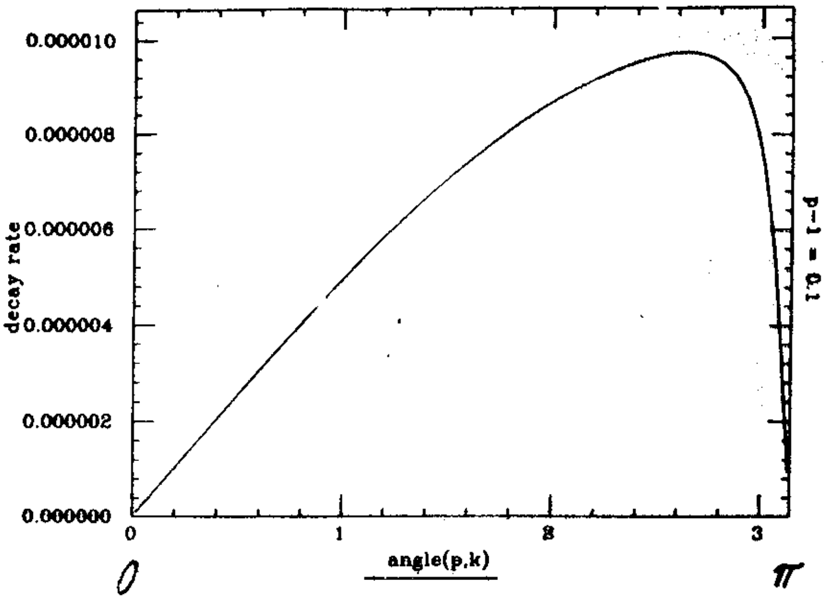
Im sigma(p,e), d = 1.5



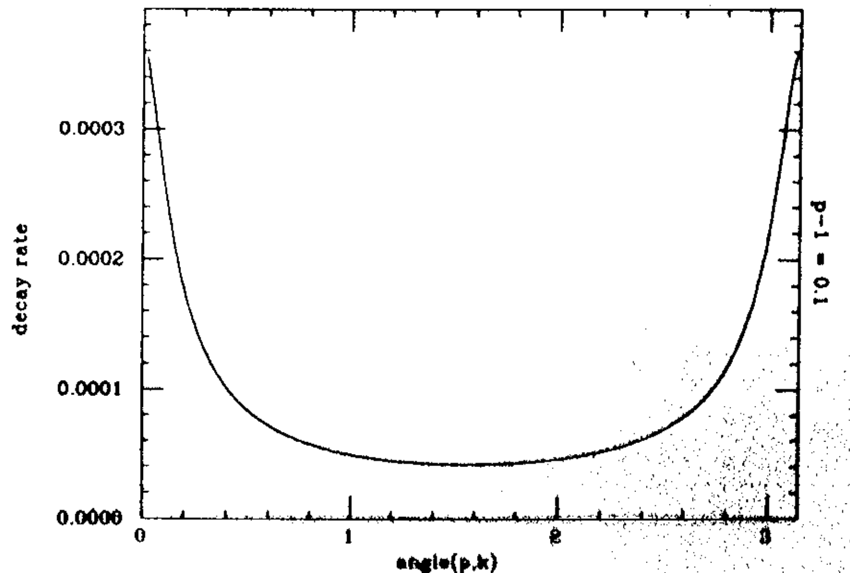
angular-
resolved
decay-
rate



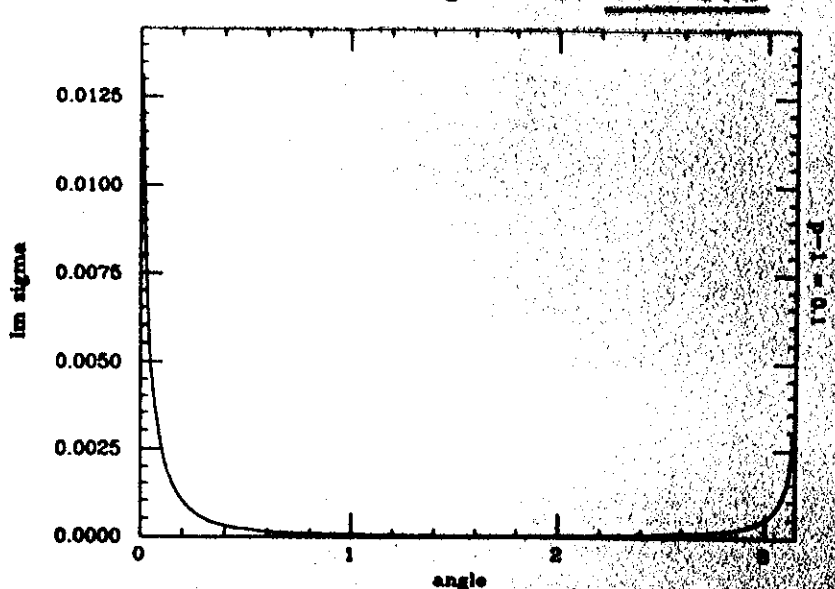
ang.res. decay rate, d = 3



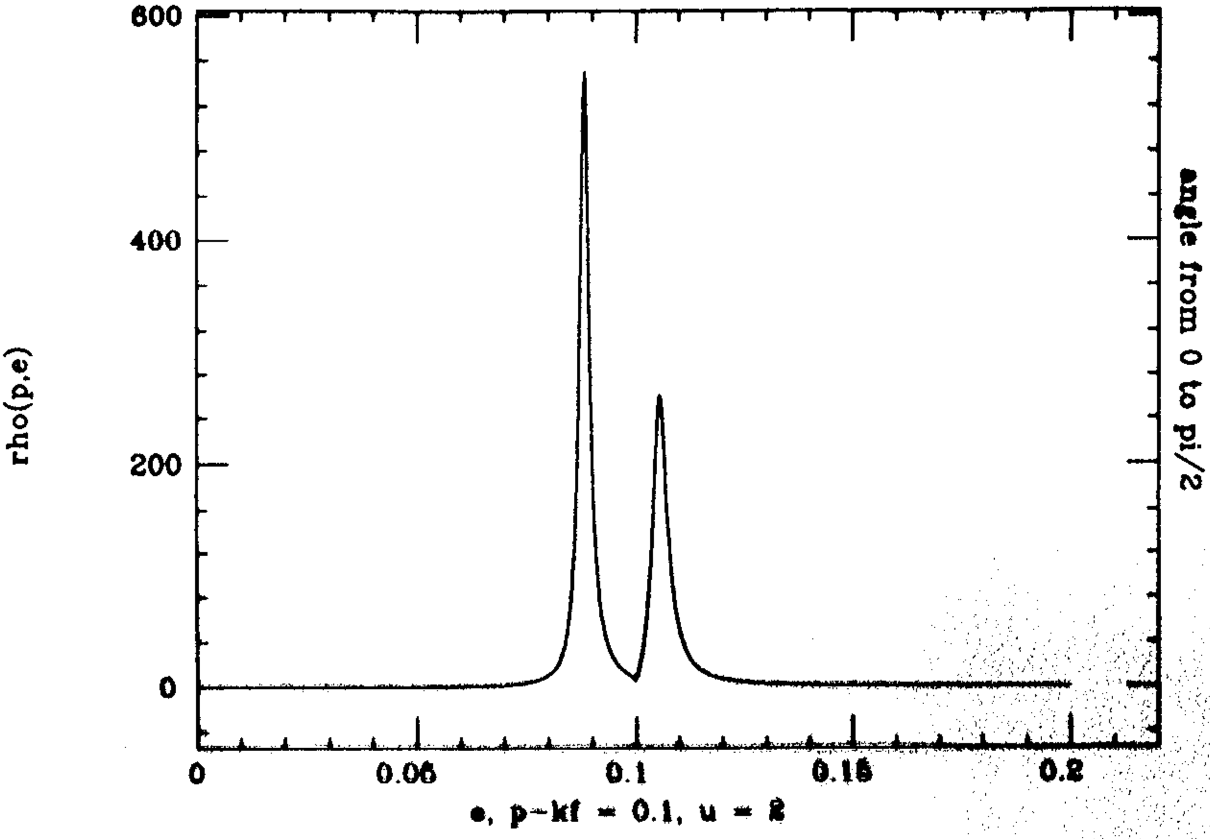
ang.res. decay rate, d = 2



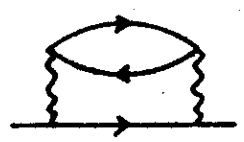
ang.res. decay rate, d = 1.5



Spectral fct., d = 1.5, lin. ek



Spectral function from

$\Sigma \sim$ 

Forward scattering model

$$H_0 = \sum_{\vec{k}} V_{\vec{k}}\, k_r\, a_\sigma^+(\vec{k})\, a_\sigma(\vec{k})$$

$$H_I = \frac{1}{V} \sum_{\sigma\sigma'} \sum_{\vec{k},\vec{k}',\vec{q}} g_{\vec{k}\vec{k}'}^{\sigma\sigma'}(\vec{q})\, a_\sigma^+(\vec{k}+\vec{q})\, a_\sigma(\vec{k})\, a_{\sigma'}^+(\vec{k}'-\vec{q})\, a_{\sigma'}(\vec{k}')$$

$$k_r = |\vec{k}| - k_F$$

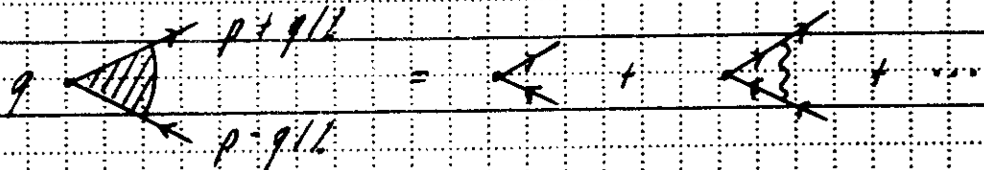
$$\boxed{g_{\vec{k}\vec{k}'}^{\sigma\sigma'}(\vec{q}) \neq 0 \quad \text{only for} \quad |\vec{q}| < \Lambda \ll k_F}$$

Here (for simplicity):

no dependence on $\vec{k}, \vec{k}'$

Asymptotic WI for Λ^μ

$$\Lambda^\mu(p,q) = -\langle j^\mu a a^\dagger \rangle_{irr} \doteq$$



$$j^\mu_\sigma(\vec{q}) = \sum_{\vec{k}} (1, \vec{v}_{\vec{k}})\, a^\dagger_\sigma(\vec{k}-\vec{q}/2)\, a_\sigma(\vec{k}+\vec{q}/2)$$

Charge/spin conservation $\Rightarrow$ WI

$$q_0 \Lambda^0(p,q) - \vec{q}\cdot\vec{\Lambda}(p,q) = G^{-1}(p+q/2) - G^{-1}(p-q/2)$$

(exact to $O(\vec{q}^2)$)

$$\Rightarrow \quad \boxed{\Lambda^0(p,q) = \frac{G^{-1}(p+q/2) - G^{-1}(p-q/2)}{q_0 - \vec{q}\cdot\vec{v}_p - Y(p,q)}}$$

$$Y(p,q) = \vec{q}\cdot\left[\vec{\Lambda}(p,q) - \vec{v}_p \Lambda^0(p,q)\right] / \Lambda^0(p,q) =$$

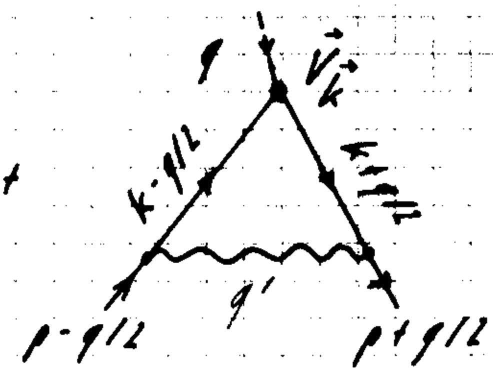
$$- \sum_{\vec{k}} \vec{q}\cdot(\vec{v}_k - \vec{v}_p)\, \langle a^\dagger a a a^\dagger \rangle_{irr} / \Lambda^0$$

$Y \sim 0$ if $|\vec{q}| < \Lambda \ll k_F$!

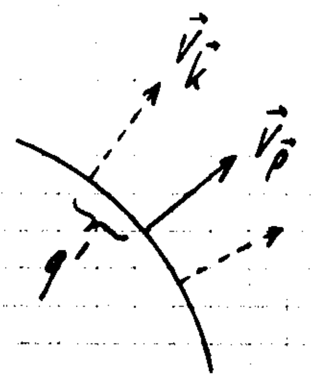
Asymptotic relation

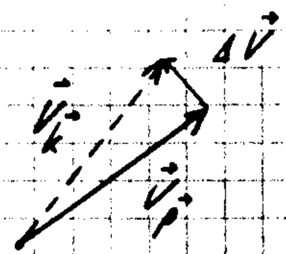
$$\boxed{\vec{\Lambda}(p;q) \sim \vec{v}_{\vec{p}}\, \Lambda^0(p;q)}$$

$$\vec{\Lambda}(p;q) = \vec{v}_{\vec{p}} \quad + $$



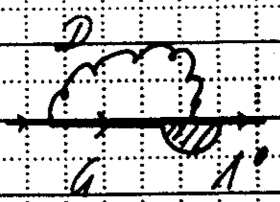
$$+ \dots$$





valid if $\quad \boxed{\dfrac{\langle \Delta v \rangle}{v} \ll 1}$

Tomographic Luttinger model

Dyson equation $\Sigma =$ 

$+$ WI for Λ^0 with $\gamma \sim 0$ $\Rightarrow$

$$(p_0 - \xi_{\vec p})\, G(p) =$$

$$1 + \int_q \frac{i D(q)}{q_0 - \vec v_F \cdot \vec q}\, G(p-q)$$

$$q_r = \vec q \cdot \hat p\,, \quad q_t = (\vec q^{\,2} - q_r^2)^{1/2}$$ radial and tangential

q_t-dependence only in $D(q)$ $\Rightarrow$

$$(p_0 - v_F p_r)\, G(p_r, p_0) =$$

$$1 + \int_{q_r, q_0} \frac{i \bar D_\Lambda(q_r, q_0)}{q_0 - v_F q_r}\, G(p_r - q_r, p_0 - q_0)$$

$$\bar D_\Lambda(q_r, q_0) = A_d \int_0^{\sqrt{\Lambda^2 - q_r^2}} dq_t\, q_t^{d-2}\, D\left(\sqrt{q_r^2 + q_t^2}, q_0\right)$$

"tomographic LM" (cf. Anderson '90)

Results:

- $\bar{D}_{\lambda\Lambda}(\lambda q_r, \lambda q_0) = \lambda^{d-1}\, \bar{D}_{\Lambda}(q_r, q_0) \Rightarrow$

  contributions from forward scattering scale to zero in $d > 1$.

- Momentum distribution

  $n_k - n_{k_F} \propto (k - k_F)^{\eta}$ in $d = 1$

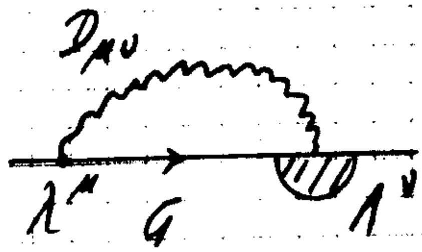
  Δn_{k_F} finite in $d > 1$, vanishes exponentially for $d \to 1$.

- $\gamma \sim 0$ not sufficient for calculating $G(p_r, p_0)$ in $p_0 \sim v_F p_r \to 0$ in $d > 1$.

- FL fixed point stable in $d > d_c = 1$; Quasi particle decay $\Gamma \propto p_r^d$ for $d < 2$.

Fermions + gauge field

Σ =



$$\lambda^\nu_{\vec p} = (1, \vec v_{\vec p}) \,, \quad \Lambda^\nu = (\Lambda^0; \vec\Lambda)$$

Small q processes dominant $\Rightarrow$

$$\vec\Lambda(p;q) \sim \vec v_{\vec p}\, \Lambda^0(p;q)$$

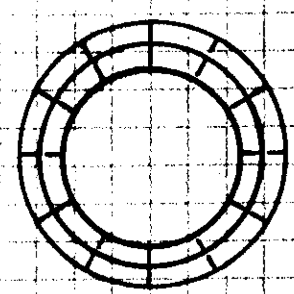
$\Rightarrow$ $G(p)$ given by effective "tomographic Luttinger model", where

$$\boxed{\begin{aligned} D(q_r, q_0) &= A_d \int dq_t\, q_t^{d-2}\, D_{\vec p}(q) \\ D_{\vec p}(q) &= \lambda^\mu_p \lambda^\nu_p D_{\mu\nu}(q) \end{aligned}}$$

Bosonization in $d \geq 1$

Haldane '92
Houghton, Marston '93

Tile FS with squat boxes such that FS is "almost" flat within each box



Box-α density fluctuation operators

$$\rho_\alpha(\vec{q}) = \sum_{\vec{k}}^{\alpha} a^\dagger_{\vec{k}-\vec{q}/2}\, a_{\vec{k}+\vec{q}/2}$$

Approximate action by quadratic form in $\rho_\alpha(\vec{q})$

Reasonable only if $\Delta \vec{v}_p / v_p$ small!

Results essentially equivalent to those from

$$\vec{\Lambda}(p;q) \sim \vec{v}_p \Lambda^0(p;q)$$

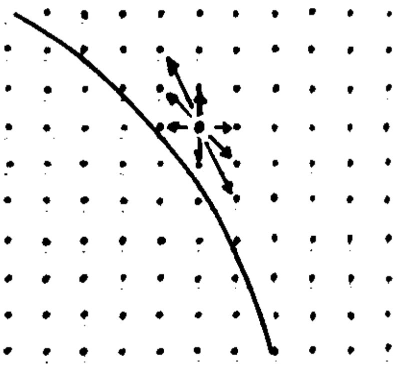
<u>Other alternative:</u> "Eikonal approximation"

Expansion in small scattering angles

(Khveshchenko, Stamp '93)

"Anderson instability"

Castellani, W.M. '94



occupied empty

2 interacting particles on (inert) Fermi surface in finite system ($\Delta k = 2\pi/L$)

$$|\phi\rangle = |\vec{k}_F \uparrow \vec{k}_F \downarrow\rangle \xrightarrow{U>0}$$

$$|\Psi\rangle = \sum_{\vec{q}} C_{\vec{q}} \, |\vec{k}_F + \vec{q} \uparrow \vec{k}_F - \vec{q} \downarrow\rangle \neq |\phi\rangle$$

in $d=2$ even for $L \to \infty$!

<u>Equivalent:</u> Anderson '90

Phase shift $\chi := -\pi \, \delta E / \Delta E \neq 0$

energy shift level spacing

"Phase angle" (phase of T-matrix) $\neq 0$ generically in limit $\vec{k}' \to \vec{k} \to FS$!

Phase angle for $\vec{k} \to \vec{k}' \to FS$

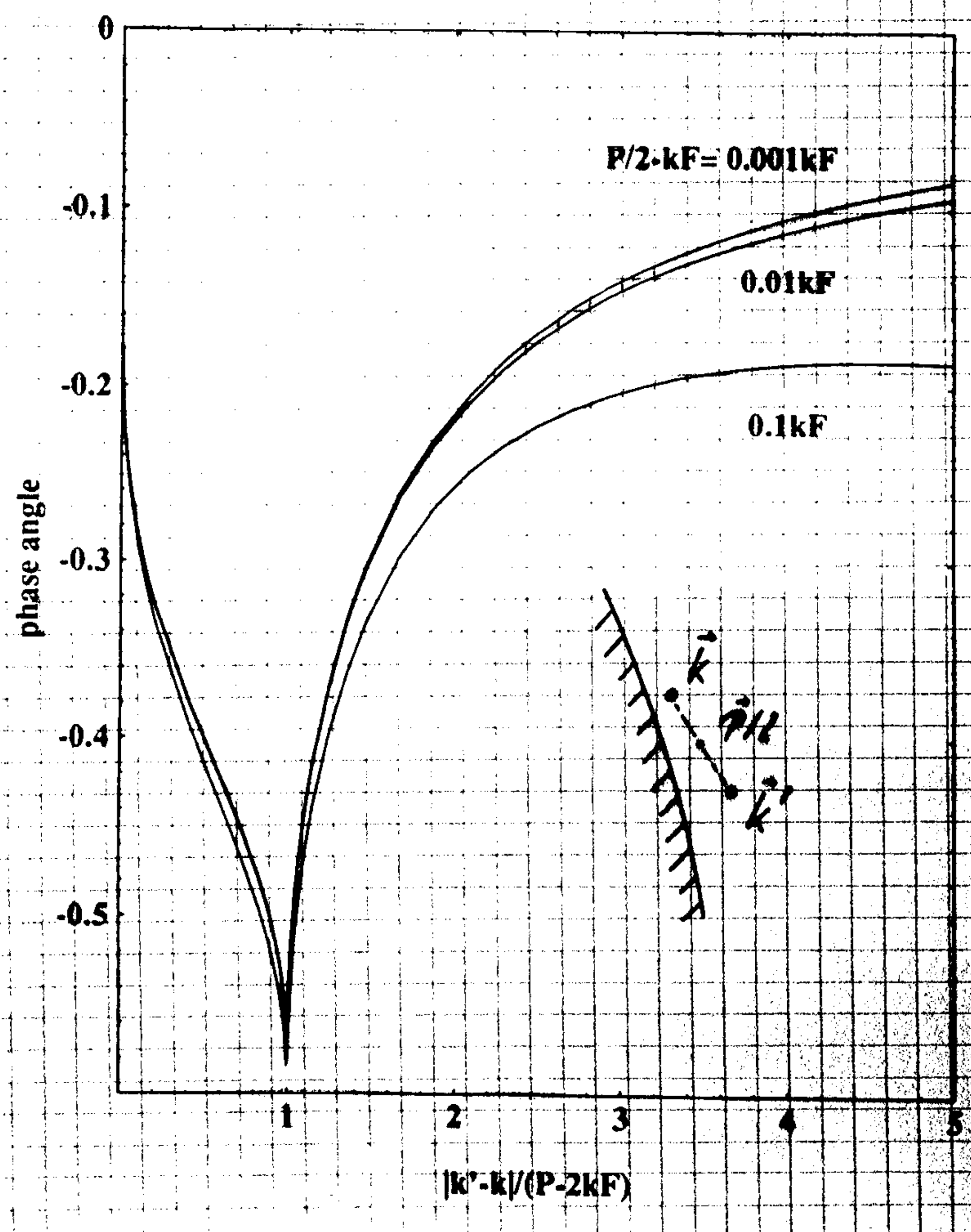


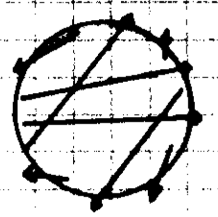
Fig. 7

$\Rightarrow$ Breakdown of FLT ?

- Anderson '67 :

Add local scatterer to Fermi sea $\rightarrow$

Orthogonality catastrophy



$\langle \phi_0 | \Psi_0 \rangle \propto N^{-(\chi_F/\pi)^2}$

χ_F phase shift at Fermi level

- Anderson '90 :

Add (tentative) quasiparticle on FS

Finite phase shift $\rightarrow$

Orthogonality catastrophy $\rightarrow$

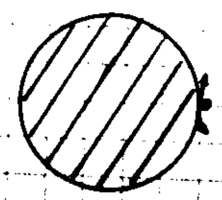
Luttinger liquid

Hovever:

$$|\phi\rangle = |\vec{k}_F\uparrow \vec{k}_F'\downarrow\rangle \xrightarrow{U>0} |\psi\rangle \neq |\phi\rangle$$

only for $\vec{k}_F' = \vec{k}_F$

$\Rightarrow$ Quasi particle affects Fermi sea much less than local scatterer!



No evidence for breakdown of FLT.

Conclusions

- Fermi/Luttinger liquid theory = RG + conservation laws
- Asymptotic Ward identity in theories dominated by small momentum transfers:

$$\vec{\Lambda}(p;q) \sim \vec{v}_{\vec{p}}\, \Lambda^0(p;q)$$

- FL stable in $d > d_c = 1$