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SMR. 767 - 3

MINIWORKSHOP ON STRONG CORRELATIONS
AND QUANTUM CRITICAL PHENOMENA
(4 - 22 July 1994)

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UNIVERSAL MAGNETIC PROPERTIES OF QUANTUM ANTIFERROMAGNETS

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These are preliminary lecture notes, intended only for distribution to participants.

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PART I

Weakly doped quantum antiferromagnets

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Plan

- ☆ Half-filling (lecture 1)

  Heisenberg AFM, quantum-classical crossover

- ☆ Low doping (lecture 2)

  SDW description

  - pockets of holes
  - uniform susceptibility
  - spin stiffness
  - incommensurability in La(Sr)CuO

- ☆ How long in doping can we extend SDW description

Universal magnetic behavior in $La_{2-x}Sr_xCuO_4$

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Yale University

Plan

★ Introduction :

La_2CuO_4 as 2D antiferromagnet

★ Classical approach (Chakravarty, Halperin & Nelson)

possibility for a new behavior above 600K

★ Quantum-critical approach

- uniform susceptibility
- structure factor
- correlation length
- spin-lattice relaxation rates, T_1; T_2

- magnetism vs doping

★ Conclusions

Large La_2CuO_4 crystals the antiferromagnetic parent compound of a family of cuprate superconductors mounted on a goniometer for inelastic neutron-scattering studies of spin excitations. These plate-like crystals are grown by Sang-Wook Cheong (AT&T Bell Laboratories) and Zachary Fisk (Los Alamos National Laboratory) using CuO as a solvent flux technique. The in-plane size reaches up to 8–10 cm. **Figure 1**

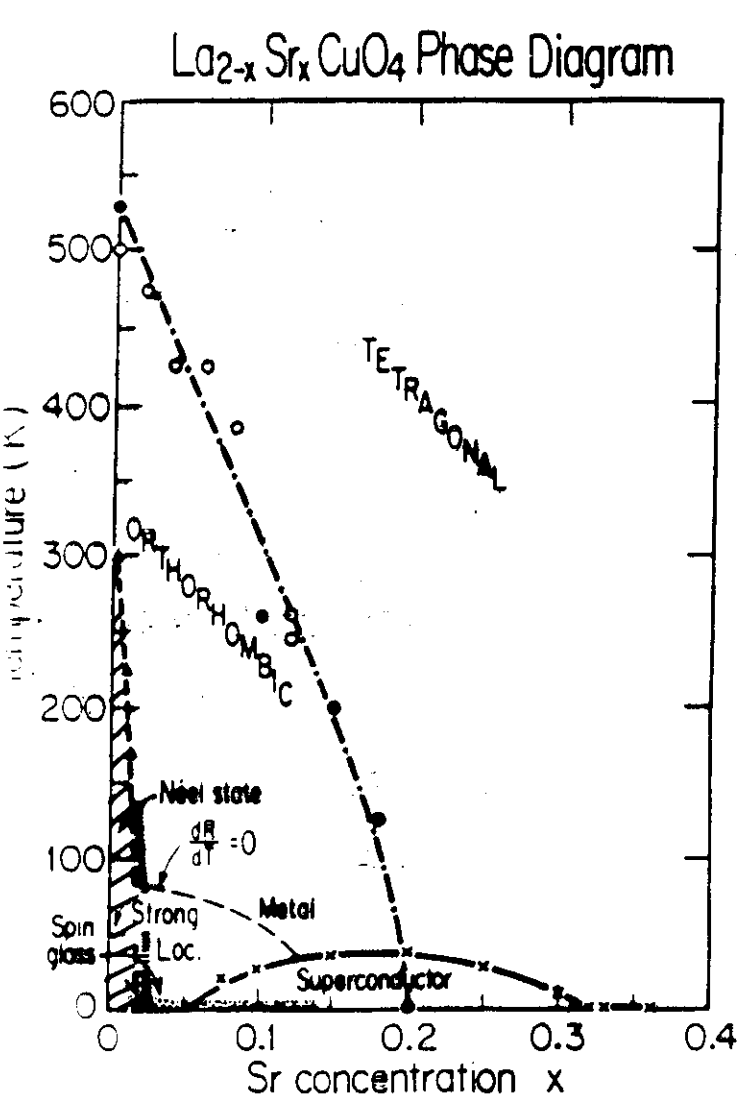
are very similar despite T_c's that differ by as much as a factor of five.

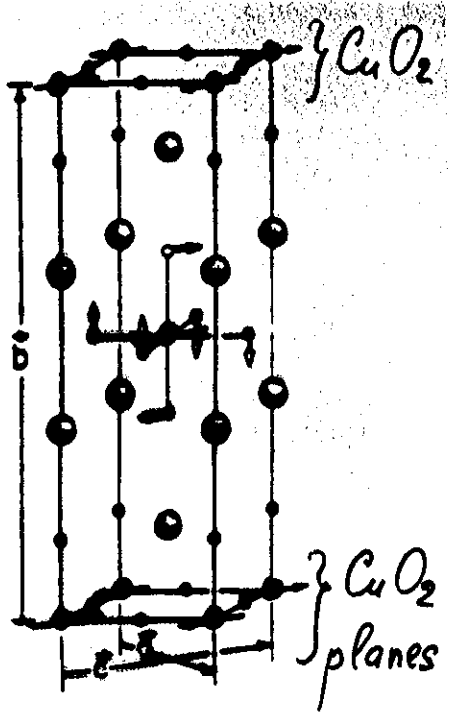
Electronic states and Fermi surface

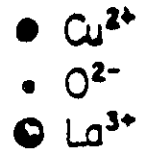
An initial question concerns the nature of the electronic states near the Fermi energy E_F which are responsible for the transport properties. Bonding between nominal Cu^{2+} and O^{2-} has both ionic and covalent character and involves the Cu $3d$ and O $2p$ orbitals. The origin of the covalency is ultimately rooted in the proximity in energy of the $3d$ and $2p$ levels, and thus it is specific to the combination of these two elements. A similar energy degeneracy occurs between the $6s$ levels of Pb and Bi and the $2p$ level of O in the $BaBi_{1-x}Pb_xO_3$ superconductors.[1] Various types of electron spectroscopies confirm this covalent mixing and reveal that the highest occupied electronic states—the ones near E_F—have both $3d$ and $2p$ character with a predominance of the latter. Consequently, the holes introduced in the CuO_2 layers by doping are not located only at the Cu sites or only at the O sites, but have a mixed character.

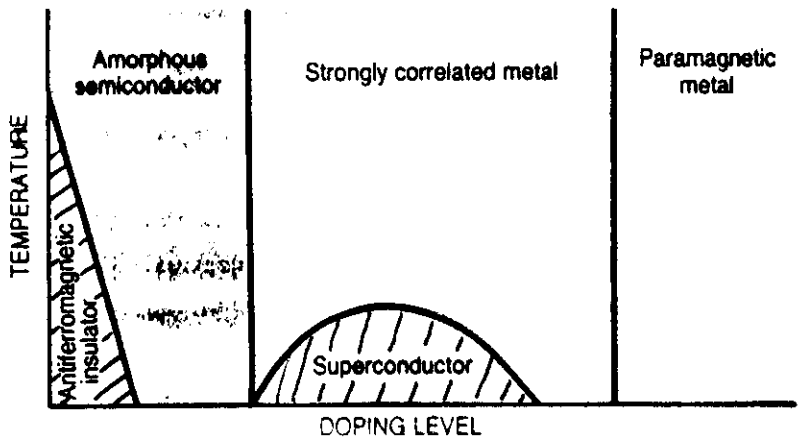
More important is the question of the existence of a Fermi surface and its shape. The Fermi surface is the locus in momentum space where the occupation of the electronic states drops abruptly and where the energy required to create particle–hole excitations vanishes. In a simple metal, with a spherical Fermi surface, at $T = 0$ all states with momentum k less than the Fermi momentum k_F are filled, and states with $k > k_F$ (and hence energy $E > E_F$) are empty. In general, the shape of the Fermi surface is determined by the momentum dependence of electronic energies $E(\mathbf{k})$, also called electronic band structure. $E(\mathbf{k})$ is material specific, and great advances have been made in calculating $E(\mathbf{k})$ even for complicated compounds such as the cuprates. When one considers the strong correlations among the electrons in the cuprates, the existence of a Fermi surface is not obvious.

Photoemission electron spectroscopy is one of the experimental tools used to study $E(\mathbf{k})$. The emission intensity is measured as a function of energy and emission angle along the principal symmetry directions of the crystal. The best results available are for $Bi_2Sr_2CaCu_2O_{8+\delta}$, for which cleaving in a vacuum produces clean and chemically stable surfaces.[2] The main result is that the measured energy bands $E(\mathbf{k})$ cross the Fermi energy at points in $\mathbf{k}$ space that are given by the calculations, confirming the presence of a Fermi surface. In addition, more subtle but important details of the emission spectra,


La2-x Srx CuO4 Phase Diagram
600
500
400
300
200
100
0
Temperature (K)
TETRAGONAL
ORTHORHOMBIC
Néel state
dR/dT = 0
Strong Loc.
Metal
Spin glass
Superconductor
0
0.1
0.2
0.3
0.4
Sr concentration x


CuO2
b
CuO2
planes

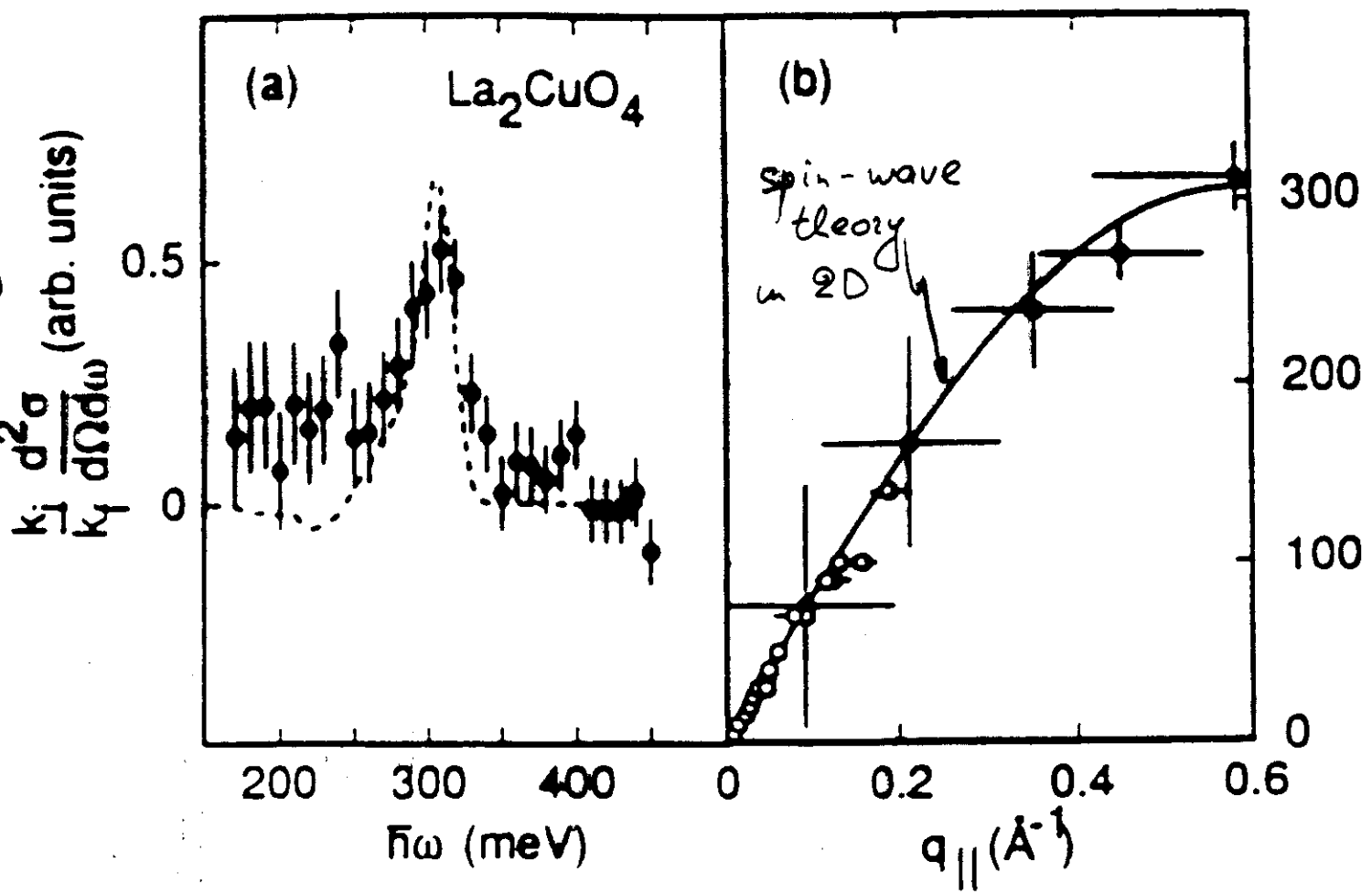

Cu2+
O2-
La3+

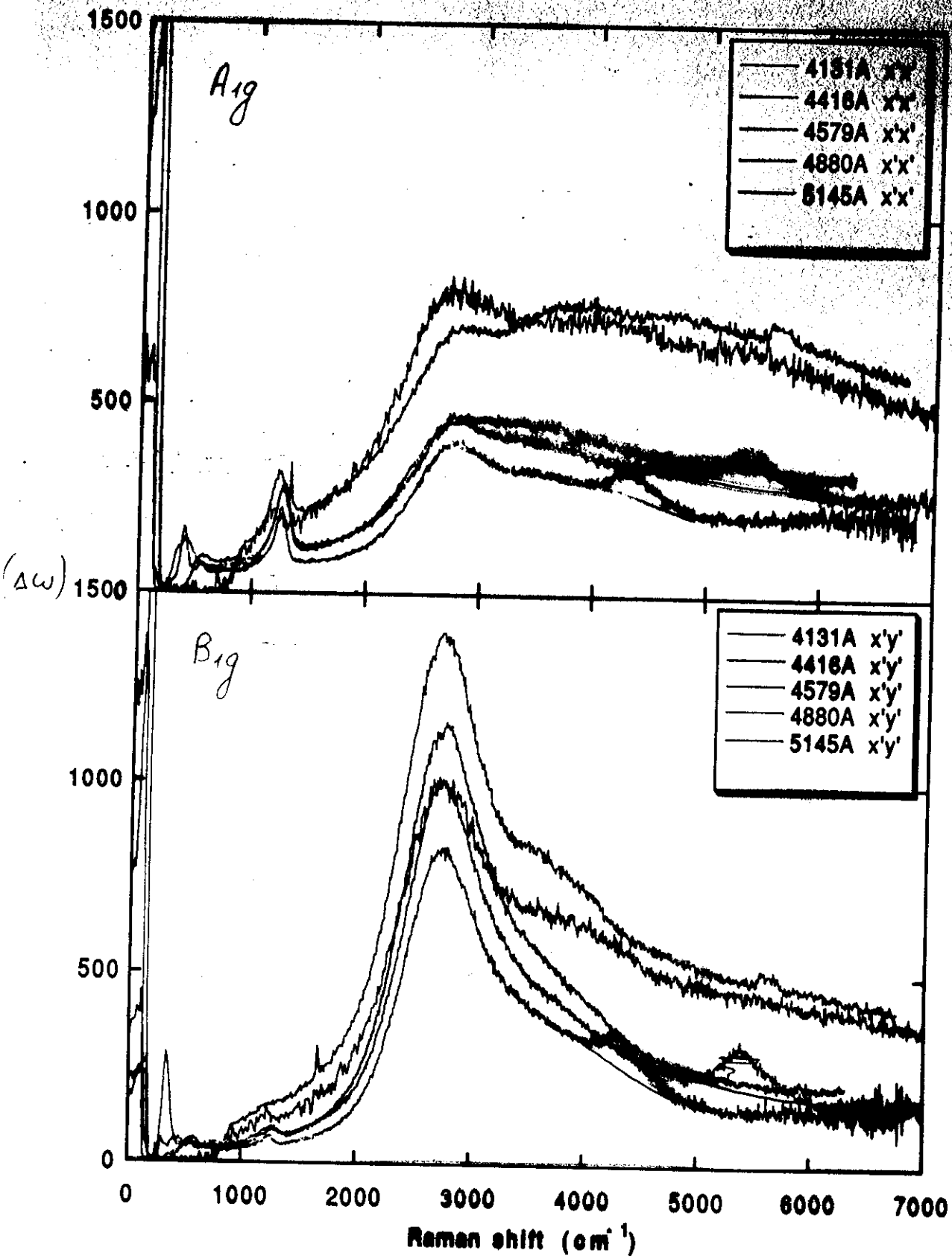

Amorphous semiconductor
Strongly correlated metal
Paramagnetic metal
TEMPERATURE
Antiferromagnetic insulator
Superconductor
DOPING LEVEL

$J = 0.13 \pm 0.01$ eV

Raman scattering gives

$J = 0.128$ eV





$(\Delta\omega)$

$\Delta\omega$

For ordered antiferromagnet

for $\vec{k} \approx (\pi, \pi)$

$$\chi(q;\omega) = \frac{N_0^2}{\rho_s q^2 - \chi_\perp \omega^2} \qquad \vec{q} = (\pi,\pi) - \vec{k}$$

ρ_s - spin stiffness

$\chi_\perp$ - transverse susceptibility

$$c^2 = \rho_s/\chi_\perp$$

N_0 ordered moment (sublattice magnetization

2D magnetism

- long-range order at $T = 0$ (even for $S = \frac{1}{2}$)
  $\boxed{\rho_s > 0}$
- no LRO and no phase transitions at finite T

  (thermal fluctuations produce internal scale)

$$\xi = 0.34 \left(\frac{\hbar c}{2\pi \rho_s}\right) \exp\left(\frac{2\pi \rho_s}{k_B T}\right) \qquad \boxed{2\pi\rho_s \gg T}$$

↳ (Chakravarty et al ; 88, 89)

(Hasenfratz et al ; 90; 91)

2D $S = 1/2$ square-lattice AFM

$c = 1.67\ J$ — spin-wave velocity

$\rho_s = 0.18\ J$ — spin stiffness

$M_0 = 0.305$

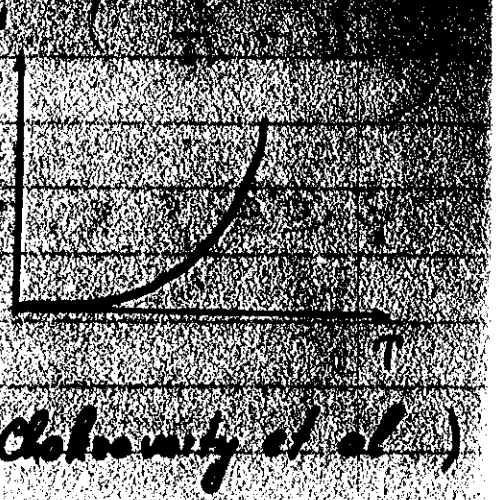
Singh, 89

Igarashi, 92

$(2\pi\rho_s \gg T)$

correlation length

$$\xi^{-1} \sim \exp\left\{-\frac{2\pi\rho_s}{T}\right\}$$

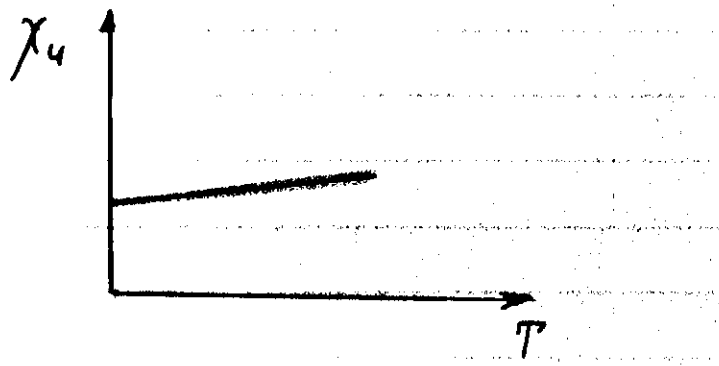


(Chakravarty et al.)

uniform susceptibility

$$\chi_u = \frac{2}{3}\chi_\perp^{T=0} + \frac{1}{3\pi}\frac{k_B T}{c^2}$$

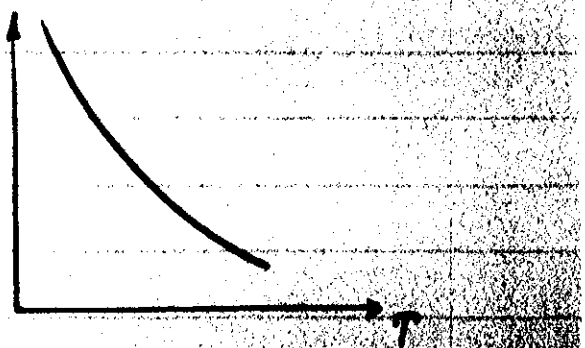
(Chubukov & Sachdev, Hasenfratz & Niedermayer)



spin-lattice relaxation rate

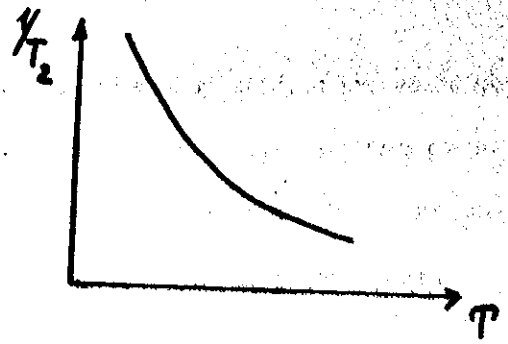
$$\frac{1}{T_1} \sim \frac{T}{\omega}\int d^2q\, \chi''(q,\omega)\Big|_{\omega\to 0} \sim T^{3/2}\xi$$

(Chakravarty & Orbach)



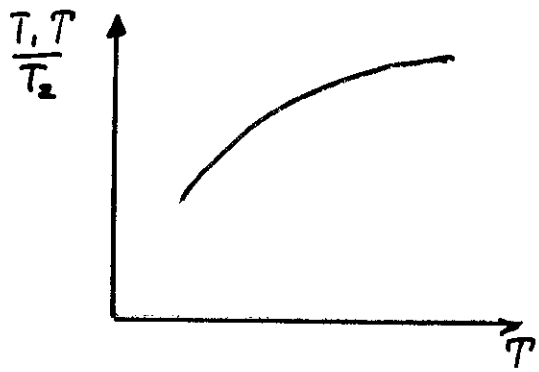
spin - lattice decay rate (Sokol & Pines)

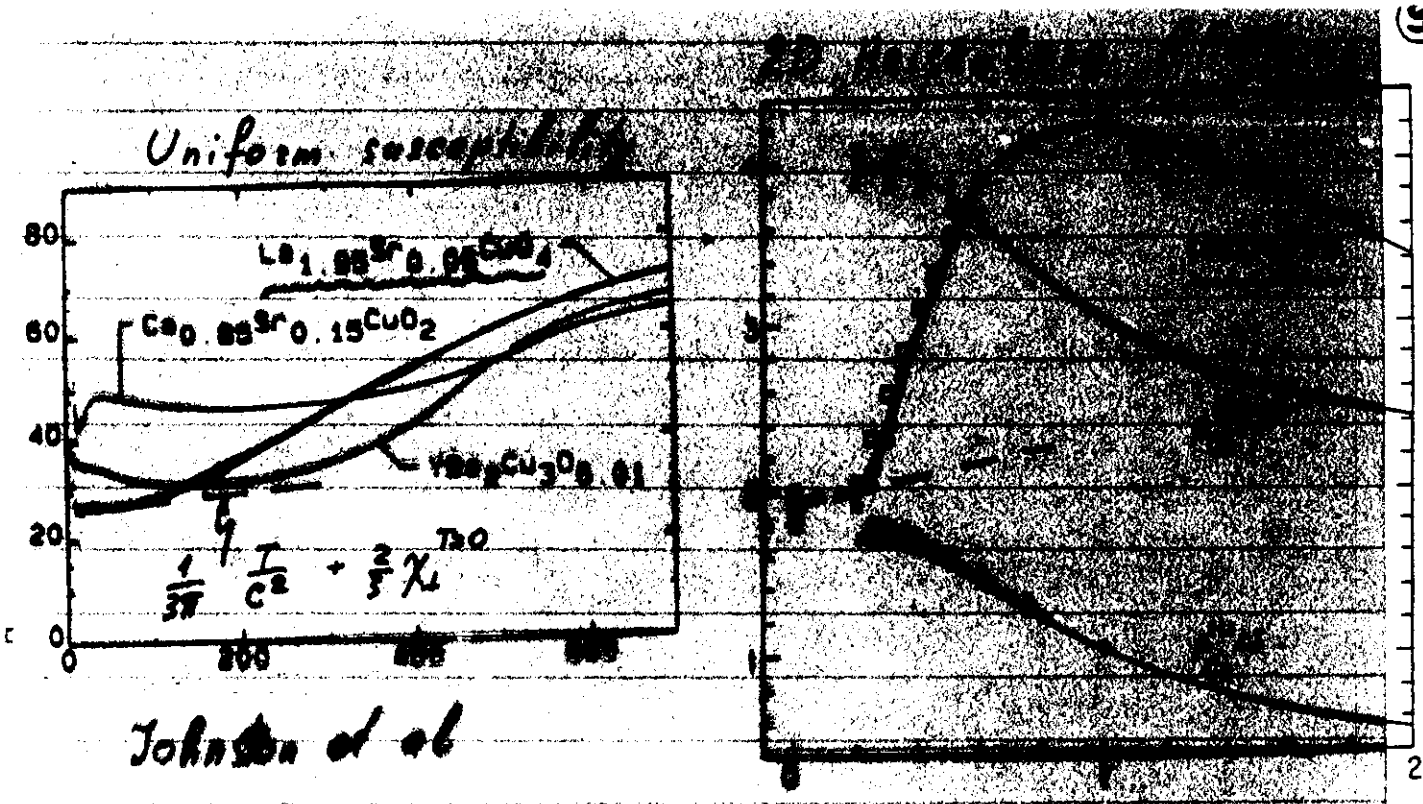
$$\frac{1}{T_2} \sim \left(\int d^3q \; \chi^2(q,0) \right)^{1/2} \sim T^{q}$$



ratio of $\frac{T_1}{T_2}$

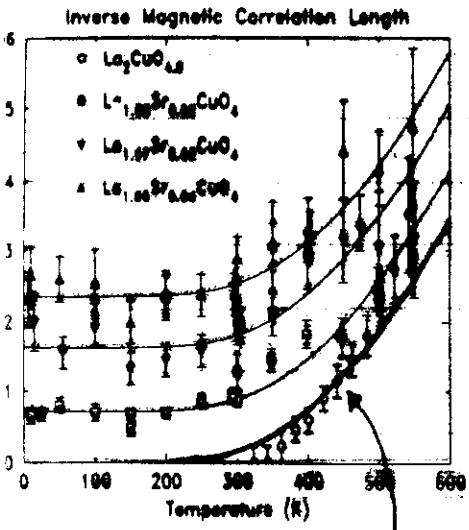
$$\frac{T_1 T}{T_2} \sim T^{1/2}$$



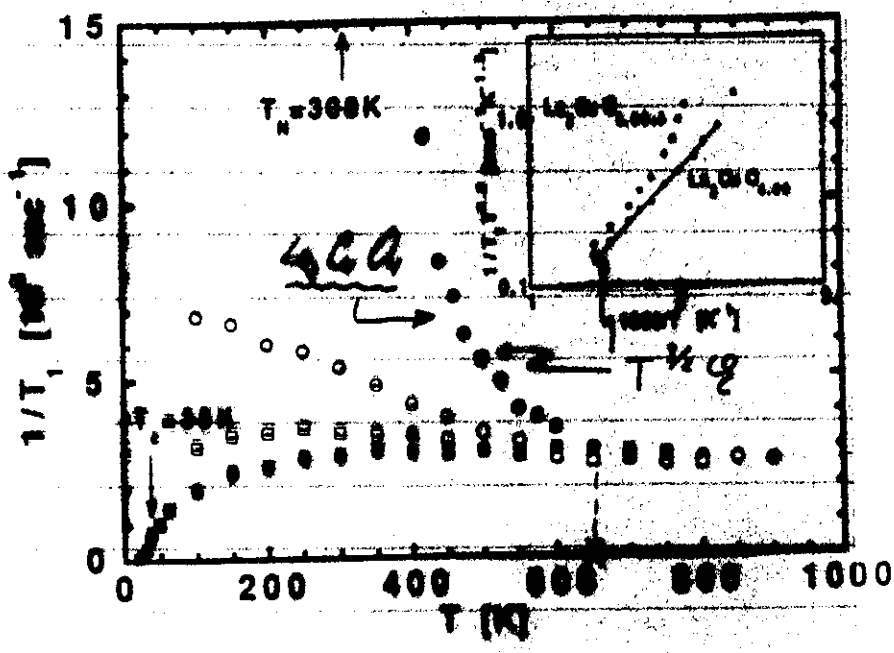


Johnston et al

Singh & Gelfand,
Ding & Makivic

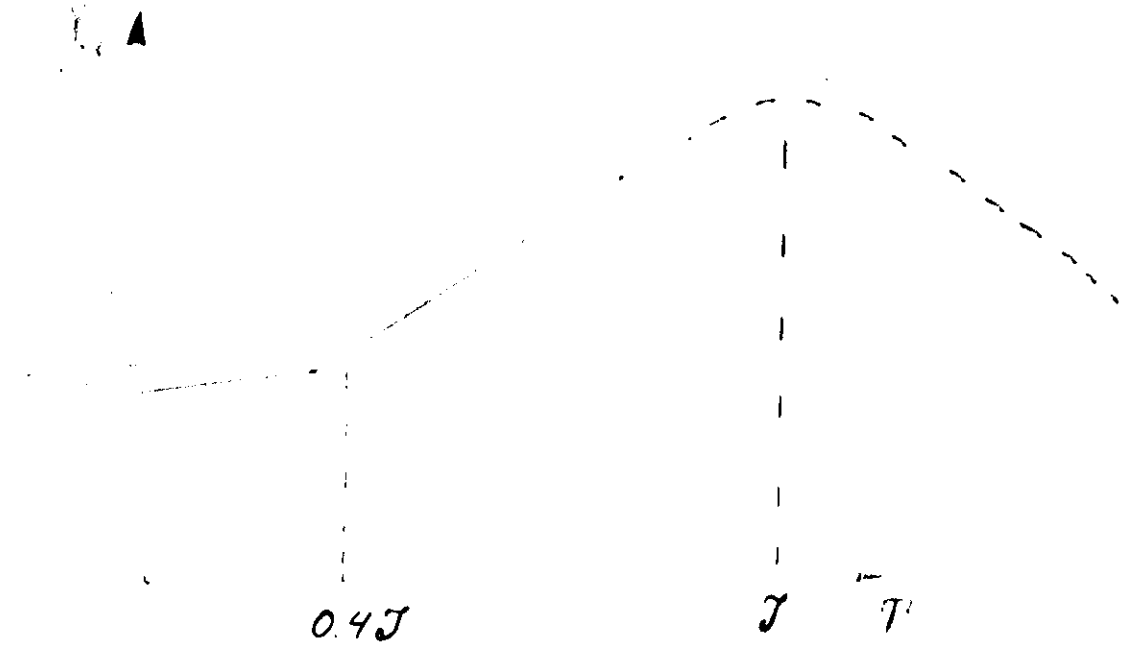


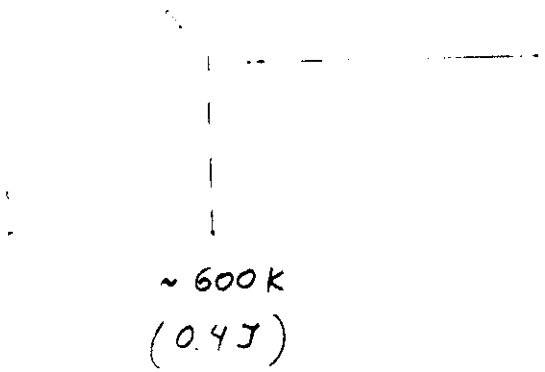
Keimer et al



Imai et al

$$\xi^{-1} \sim \exp\left\{\frac{A}{T}\right\}$$


0.4J
J
T


~ 600 K
(0.4J)

☆ Possibility for another type of behavior at intermediate T, $0.4J < T < 0.6J$

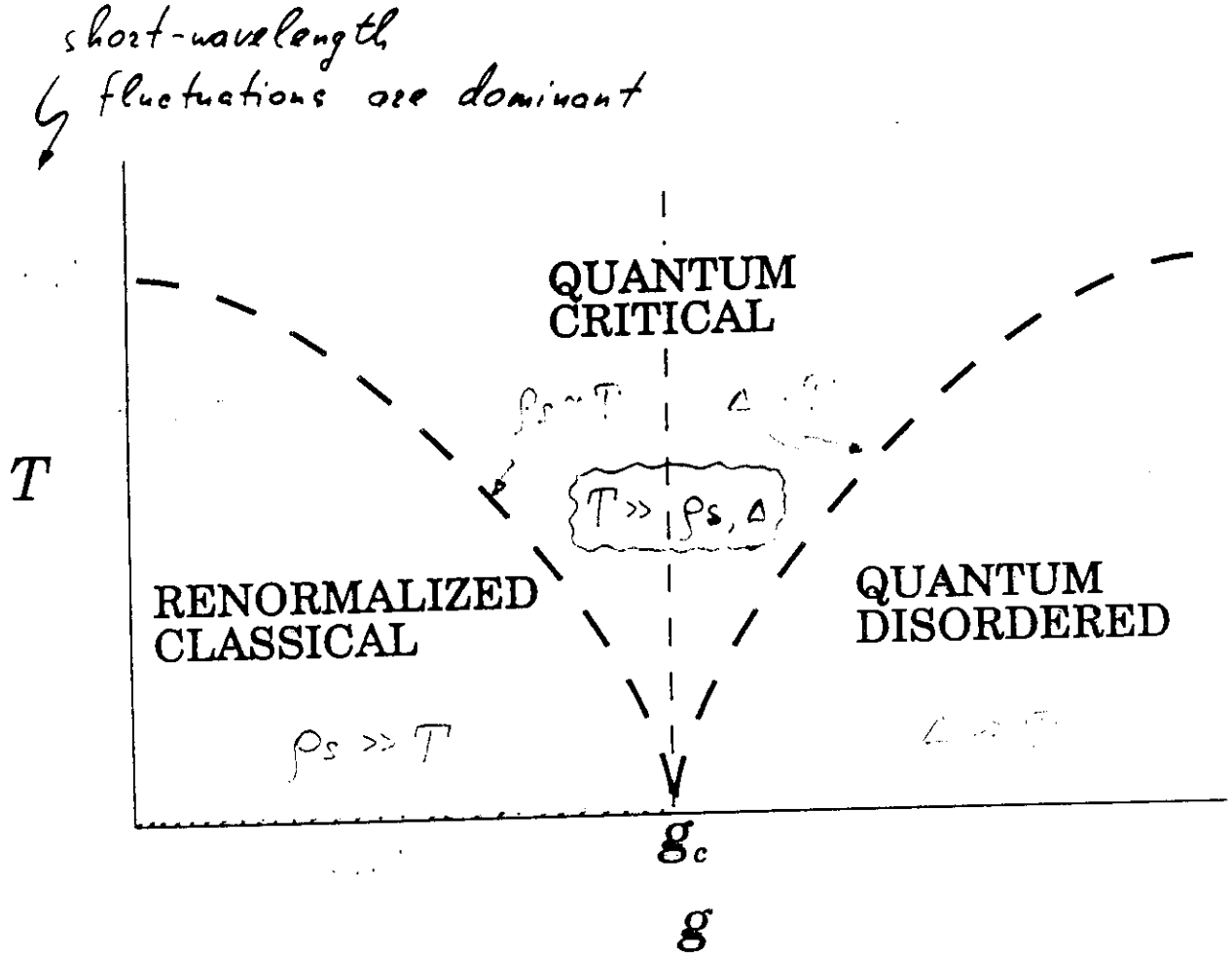
$600\,K < T < 900\,K$ (pure La_2CuO_4)

- no phase transitions at $T \neq 0$, only smooth crossover (in 2D)

☆ We never considered the role of quantum fluctuations (apart from renormalization of ρ_s, c)

- let g measure the strength of quantum fluctuations ($g \sim 1/S$ at $S \gg 1$)

quantum-disorder region



We argue that most of expeeiments on uniform susceptibility, T_1 and T_2 are done in the quantum-critical region

Chubukov & Sachdev, 93

To this end, we consider antiferromagnets which have

$T \ll J$, $\rho_s \ll J$ but T/ρ_s arbitrary

(on the ordered side)

$T/\rho_s \ll 1$ zero-classical region

$T/\rho_s \gg 1$ quantum-critical region

- The system realizes (at $\rho_s \ll J$) that it is not at the critical point only at very large scales $\Rightarrow$ universality!

- we can use scaling

$\rho_s = \frac{\hbar c}{\xi_J} \times (\text{number})$ $\quad \xi_J \sim (g_c - g)^{-\nu}$

at $\rho_s = 0$, $\quad L_t = \frac{\hbar c}{T}$

"finite size" of a critical theory

From general principles of scaling any observable is a function of three dimensionless quantities

$$\bar{k} = k L_t \qquad \bar{\omega} = \frac{\omega}{c} L_t \qquad x \sim \xi / L_t$$

$$\Downarrow \qquad \Downarrow \qquad \Downarrow$$

$$\bar{k} = \frac{\hbar c k}{T} \qquad \bar{\omega} = \frac{\hbar \omega}{T} \qquad x = \frac{N T}{2\pi \rho_s}$$

N - number of components of the order parameter ($N = 3$ for AFM)

Scaling predictions for [illegible]

★ $\chi_u(T) \sim T$

★ $\xi(T) \sim 1/T$

★ $1/T_1 \sim T^{\eta}$ } $\eta = 2 - z$

★ $1/T_2 \sim T^{-1+\eta}$

$$\frac{T_1 T}{T_2} = \text{const}$$

★ $S(k) \sim T^{\eta-1} f(ck/T)$

★ $\chi''(\omega) \sim T^{\eta} \varphi(\omega/T)$

$$\int \frac{d^2q}{4\pi^2} \chi''(q,\omega)$$

$$\chi_S(k,\omega) = \frac{N_0^2}{\rho_S} \left(\frac{\hbar c}{k_B T}\right) \left(\frac{k_B T}{2\pi\rho_S}\right)^{\eta} \Psi(\bar{k}, \bar{\omega}, x)$$

Calculation of universal functions

RG approach - perturbative expansion around ordered state (only transverse fluctuations)

We use $1/N$ expansion for $O(N)$ β-model;

$N = 3$ is a goal.

Disadvantage : no "real" small parameter for the expansion

Advantage : the starting point is a mean-field solution for the Green function

$$G = \frac{1}{k^2 + \omega^2 + m_0^2}$$

this implies that the symmetry is unbroken

We do not have to separate classical and quantum fluctuations.

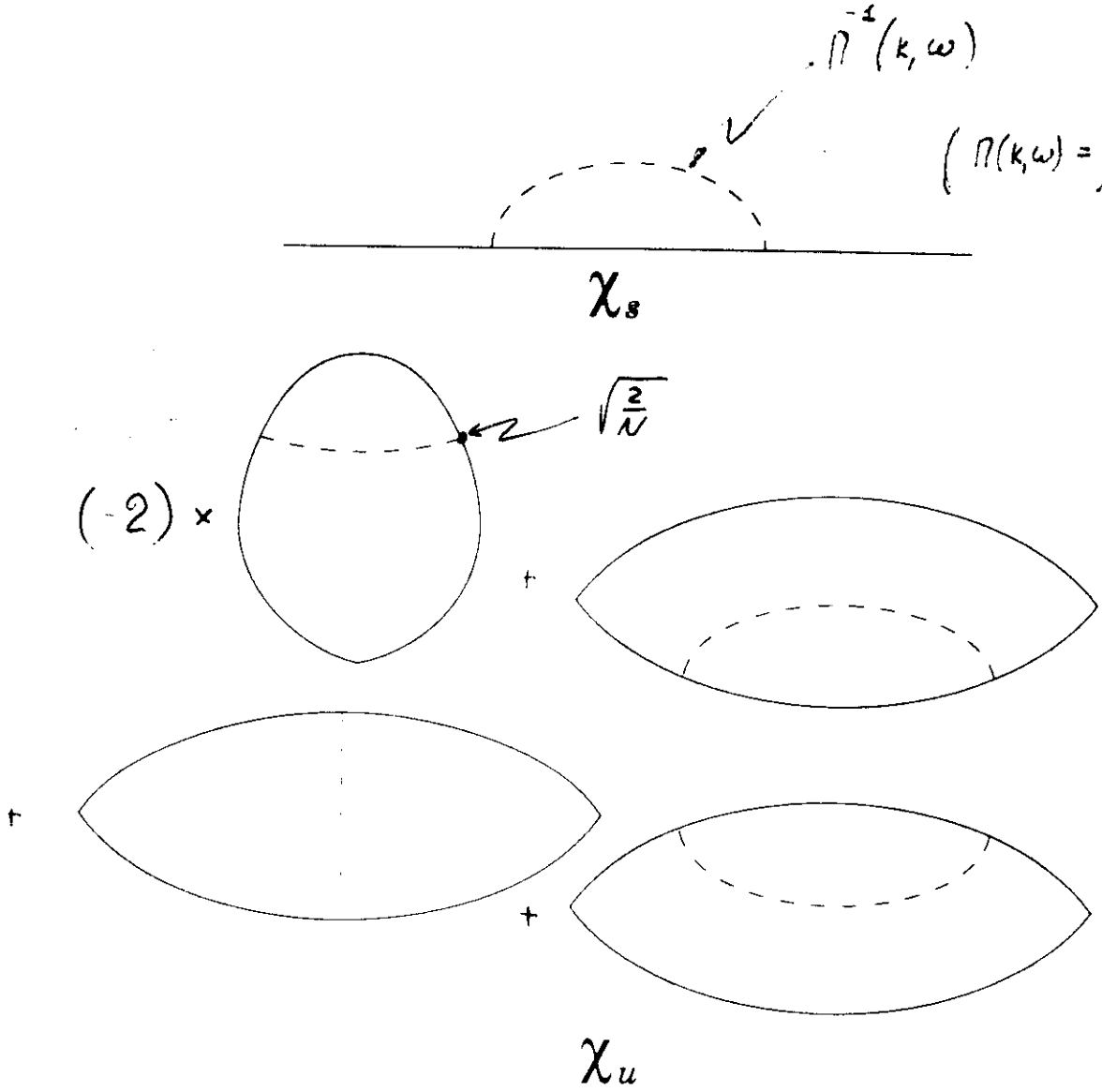

$\Pi^{-1}(k,\omega)$
$\Pi(k,\omega) = \int G_{k+q,\omega} G_{q,\omega}$
χ_s
$\sqrt{\frac{2}{N}}$
$(-2) \times$
+
+
+
χ_u

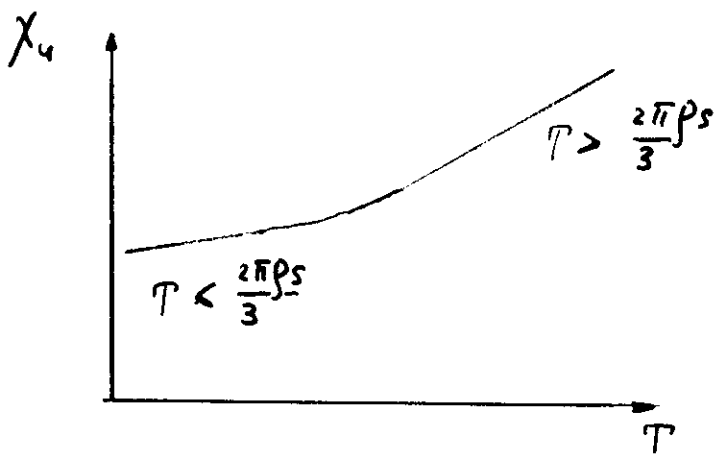
FIG 6.

Uniform susceptibility

$$\chi_u(T) = \left(\frac{g\mu_B}{\hbar}\right)^2 \bar{\chi}$$

$$\bar{\chi} = \begin{cases} 0.66\, \chi_\perp^{T=0} + 0.11\, \dfrac{T}{c^2} & T \ll \dfrac{2\pi\rho_s}{3} \\ 0.4\, \chi_\perp^{T=0} + 0.2\, \dfrac{T}{c^2} & T \gg \dfrac{2\pi\rho_s}{3} \end{cases}$$

quantum-critical result (obtained in $\frac{1}{N}$ expansion, $N=3$ is the goal)



Slopes

- renormalized - classical

$$0.11 = \frac{1}{4\pi}\,\frac{N-2}{N}$$

- quantum - critical

$$0.27 = \underbrace{\frac{\sqrt{5}}{4\pi}\log\frac{\sqrt{5}+1}{2}}_{0.34}\left(1-\frac{0.62}{N}\right)$$

our $N=\infty$ result is two times larger than in mean-field Schwinger-boson approach

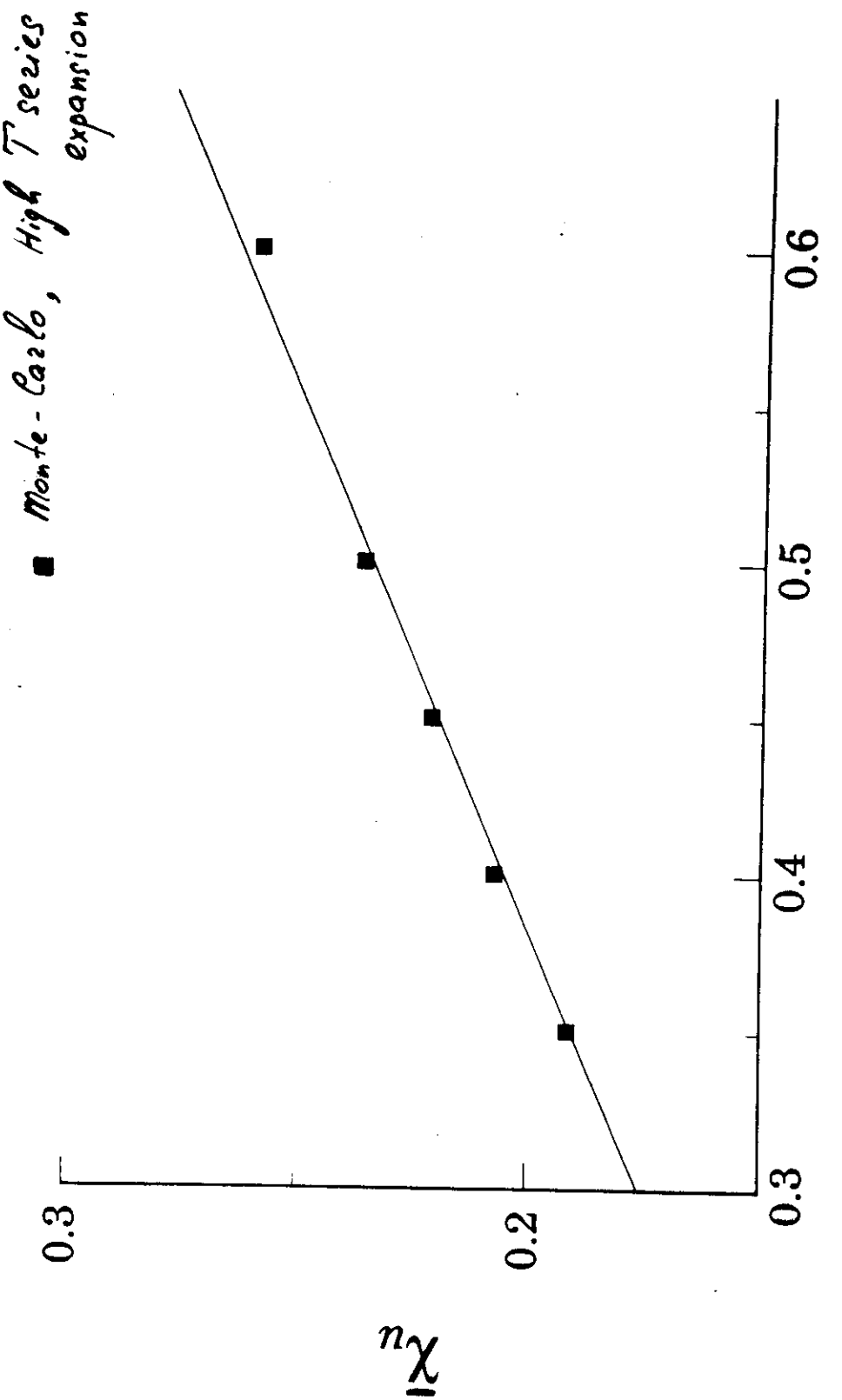


FIG 7

PART II

	Ren.- classical	Quantum - critical
χ_u uniform susceptibility	$0.66\,\chi_\perp^{T=0} + 0.11\,\frac{T}{c^2}$	$0.4\,\chi_\perp^{T=0} + 0.27\,T/c^2$
ξ correlation length	$\sim e^{2\pi\rho_s/T}$	$\sim 1/T$
$1/T_1$ spin-lattice relaxation rate	$\sim T^{3/2}\,\xi$	$\sim T^{\eta} \approx \text{const}$
$1/T_2$ spin-lattice decay rate	$\sim T\,\xi$	$\sim T^{-1+\eta} \approx T^{-1}$
$\frac{T_1 T}{T_2}$	$\sim T^{1/2}$	const

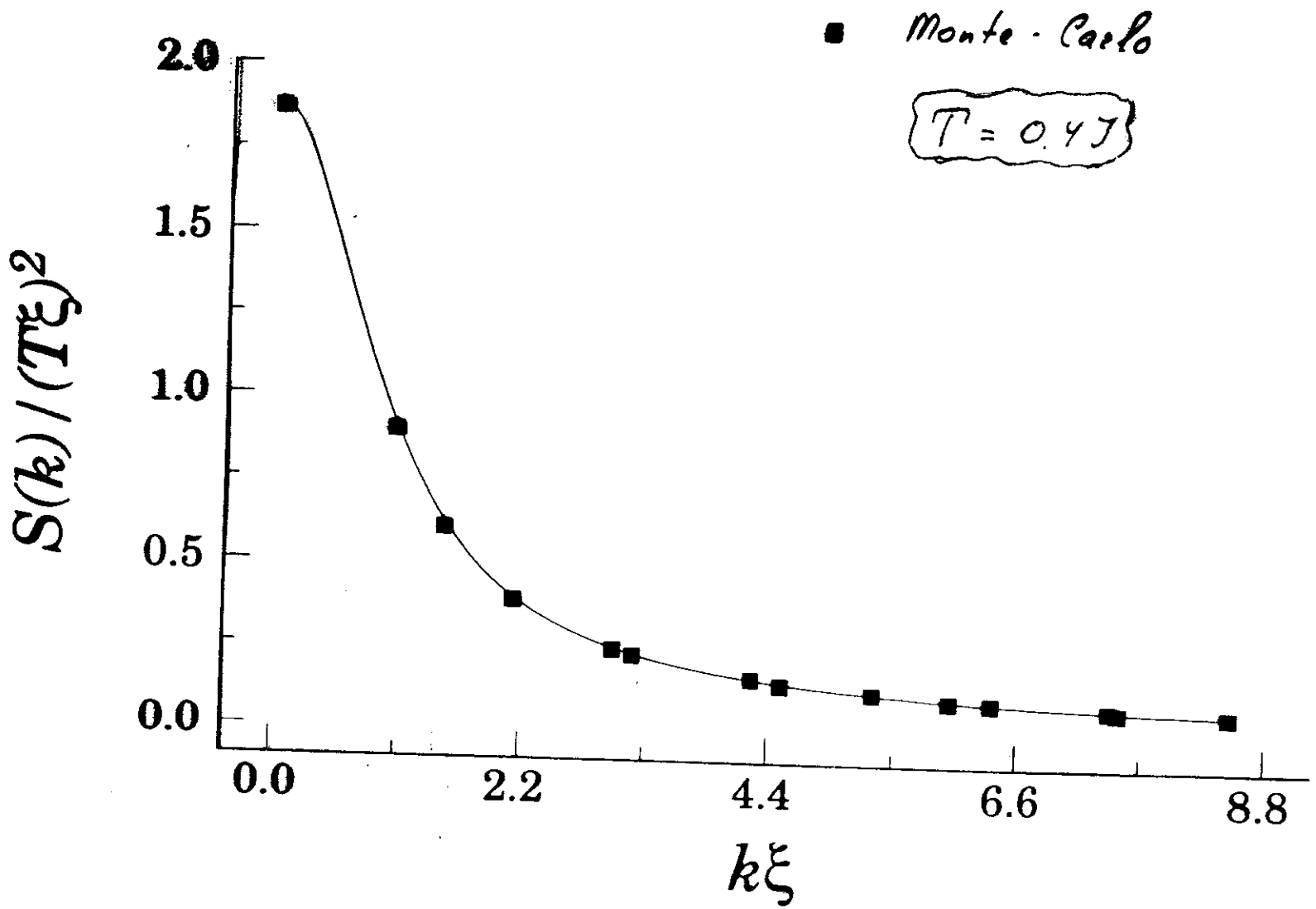


FIG. 9

Crossover temperature

- In the large - N expansion :

$$x = \frac{2\pi \rho_s}{N T}$$

$x > 1$ classical region

$x < 1$ quantum - critical

$\rho_s = 0.18\,J$, $N = 3$ $\Rightarrow$ $x = \frac{T_{c2}}{T}$, $T_{c2} \sim 0.38 J \sim$ $\sim 600\,K$

- Deep in the zen-classical region, one has (at arbitrary N)

$$\xi \sim \exp\left[\frac{2\pi \rho_s}{(N-2)\,T}\right]$$

Formally, the expansion holds in $\frac{2\pi\rho_s}{T} \sim \frac{\overline{T}_{c2}}{T}$,

$\overline{T}_{c2} \sim 1.13\,J \sim 1700\,K$

• We found in $\frac{1}{N}$ expansion, that

$N \to N-2$ substitution results from

the series of $\frac{1}{N}\log(T\varphi)$ terms $\Rightarrow$

$\Rightarrow$ it holds only if $\log(T\varphi) \gg 1 \Rightarrow \rho_s \gg T(?)$

$$T\varphi \sim e^{\frac{2\pi\rho_s}{(N-2)T}} \to e^{\frac{2\pi\rho_s}{NT}} e^{\frac{4\pi\rho_s}{N^2T}}$$

$$\to e^{\frac{2\pi\rho_s}{NT}} \left[1 + \frac{1}{N}\frac{2\pi\rho_s}{NT} + \dots\right]$$

$\log T\varphi_0 = \frac{2\pi\rho_s}{NT}$, therefore

"$N=\infty$" result

$$T\varphi = (T\varphi_0)\left[1 + \frac{1}{N}\log T\varphi_0 + \frac{1}{2N^2}\log^2 T\varphi_0 + \dots\right.$$

$$\left. + \frac{1}{N} \times \text{const} + \dots\right]$$

[ ] this is what we obtain in $\frac{1}{N}$ expansion

- longitudinal relaxation, $1/T_1$

$$\mathcal{H} = \sum I_r S_{r'} A_{r,r'} \equiv \sum_r I_r \times H_{eff}(r)$$

$$H_{eff} = \sum_{r'} S_{r'} A_{rr'}$$

In general, $H_{eff}(r,t)$

Suppose, we apply transverse oscillating field with ω_0

$$1/T_1 \sim \int dt \langle H_{eff}(r,t)\, H_{eff}(r,0)\rangle e^{i\omega_0 t}$$

$$1/T_1 \sim A^2 \int d\vec{k}\; S(k,\omega_0)$$

- transverse relaxation, $1/T_2$

$$\mathcal{H} = \sum I_r S_{r'} A_{rr'} \Rightarrow \sum_r S_{r'} H_{eff}(r')$$

$$H_{eff}(r') = \sum_r I_r A_{rr'} \Rightarrow H_{eff}(q) \equiv\!\Rightarrow \tilde{S}(q) = \chi'(q) H_{eff}(q)$$

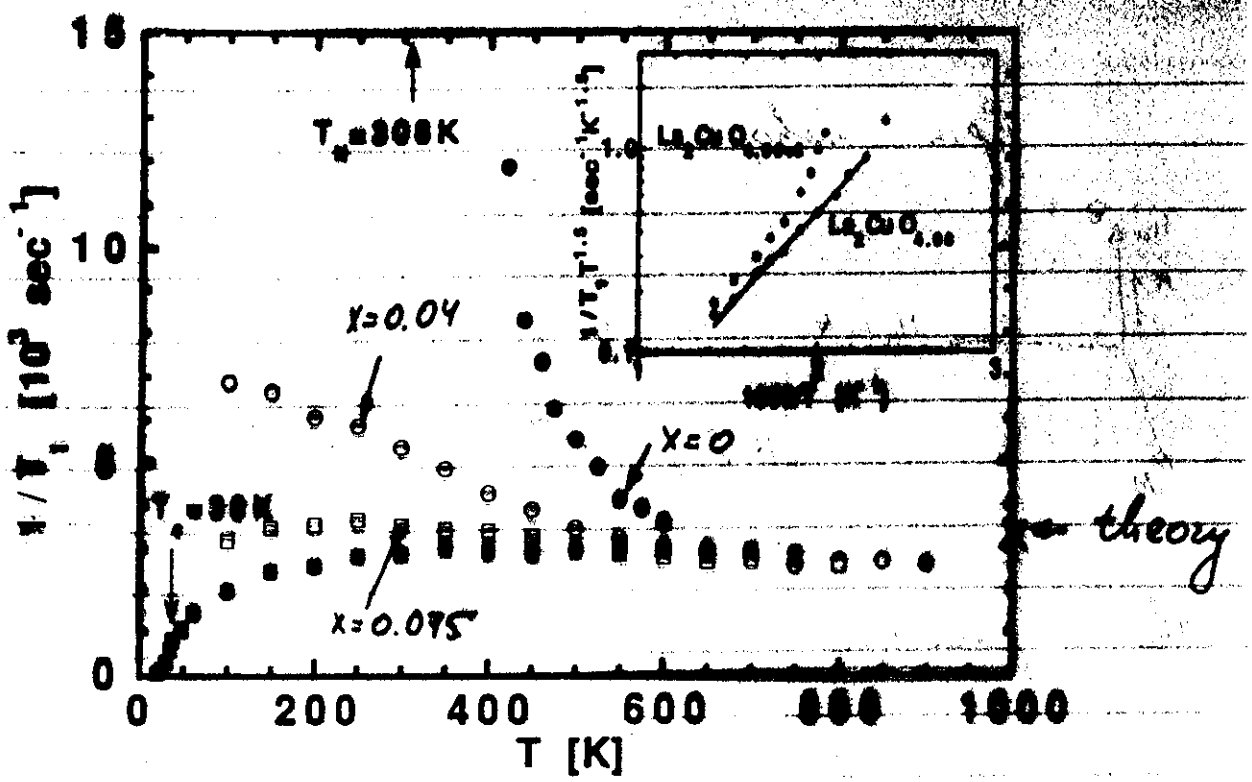
$\tilde{S}$: polarization

$$\equiv\!\Rightarrow \tilde{S}(\tilde{r}) \Rightarrow \mathcal{H} = A_{\tilde{r}\tilde{r}'} \tilde{S}_{\tilde{r}} I_{\tilde{r}'}$$

$$\boxed{\mathcal{H}_{eff} = \sum_{\langle ij\rangle} a_{ij} I_i I_j} \qquad a_{ij} \sim A^2 \chi'$$

spin-echo technique : $\text{signal}(t) \sim e^{-t^2/2t^2}$, $\frac{1}{t^2} \sim \sum_k a_k^2$

$$1/T_1 \sim \int d^2q \; \frac{\chi''(q,\omega)}{\omega}\Big|_{\omega\to 0}$$



Theory (quantum-critical)

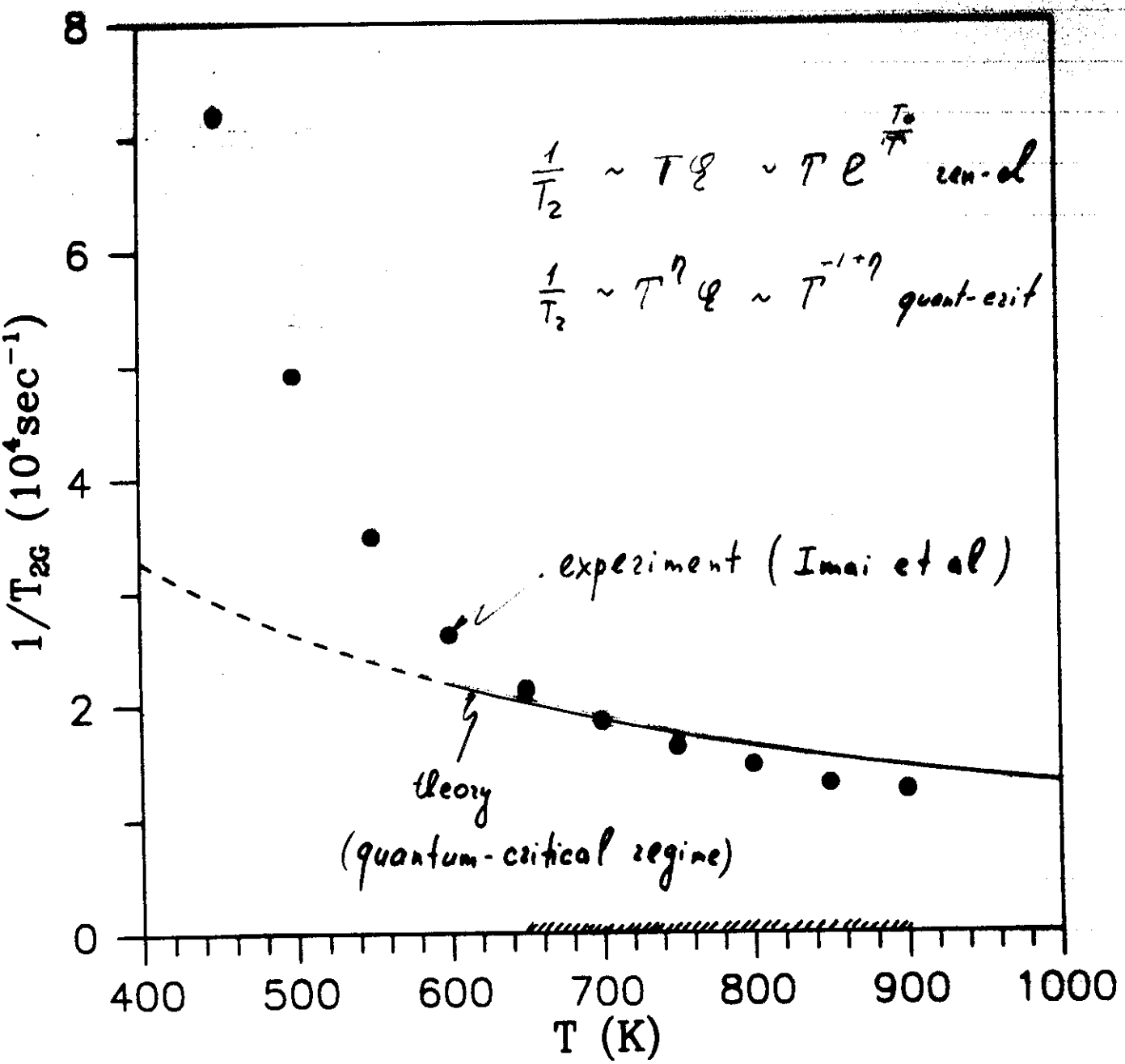
$$1/T_1 \simeq 3.2 \pm 0.5 \times 10^3 \; sec^{-1}$$

Experiment

$$1/T_1 \approx 2.7 \cdot 10^3 \; sec^{-1} \quad at \; T \gtrsim 700k$$

Spin-echo decay rate, $1/T_2$

$$1/T_2 \sim \left[\int d^2q \, \chi^2(q,0) \right]^{1/2}$$



Chubukov, Sachdev & Sokol

Explicit calculations at large N.

$$\frac{1}{T_2} = \frac{\hbar^2}{\sqrt{4\pi}} \left(\frac{N_0^2}{\rho_s}\right) \left(\frac{MT}{2\pi\rho_s}\right)^{1} \varphi_a \times$$

$$\times \left(1 + \frac{0.46}{N}\right)$$

$$\bar{\varphi}^{1}(T) = 0.962 \frac{T}{\hbar c} \left(1 + \frac{0.237}{N}\right)$$

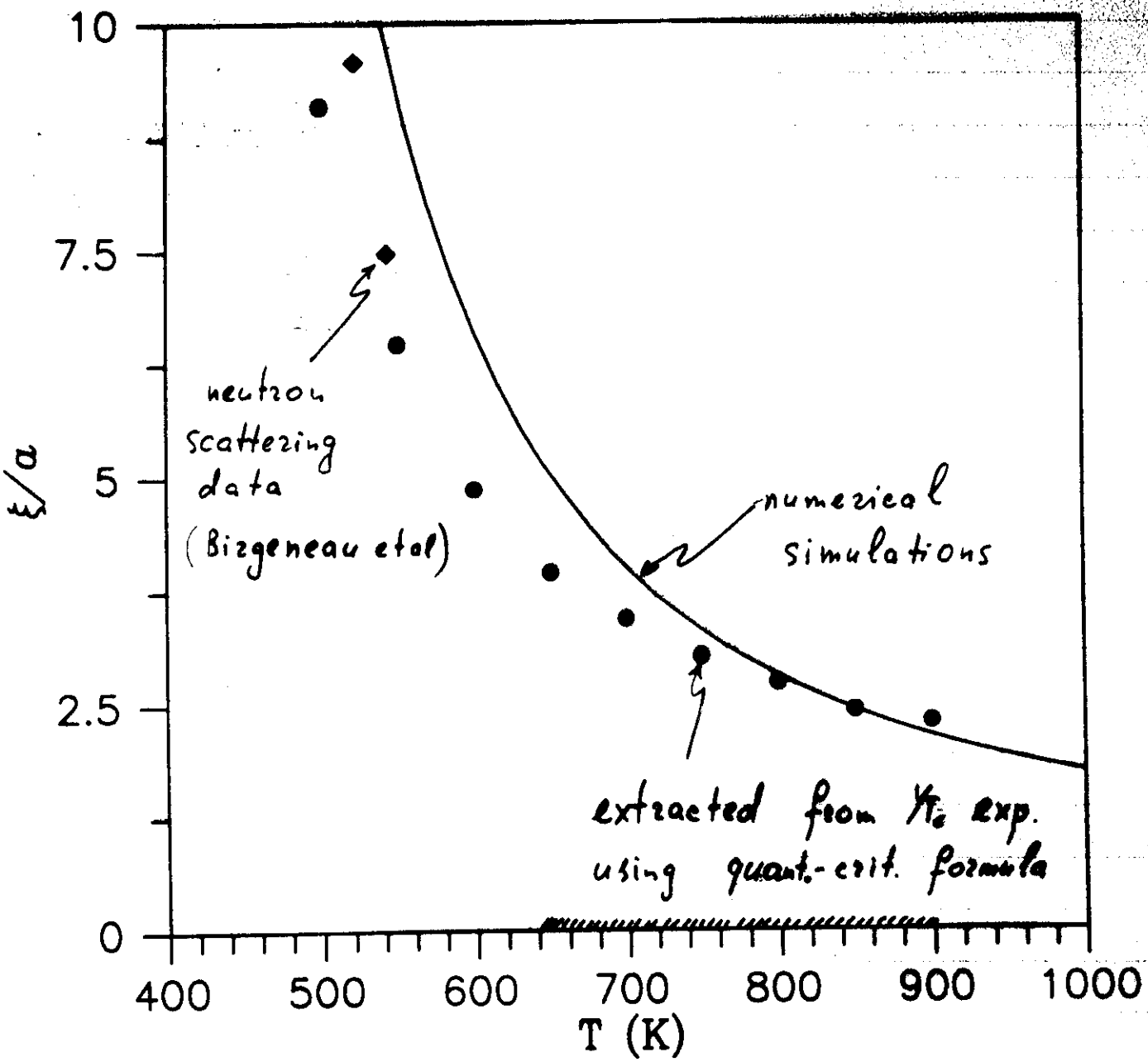
$N_0 = 0.305$ spontaneous magnetization

$N = 3$ is a physical case

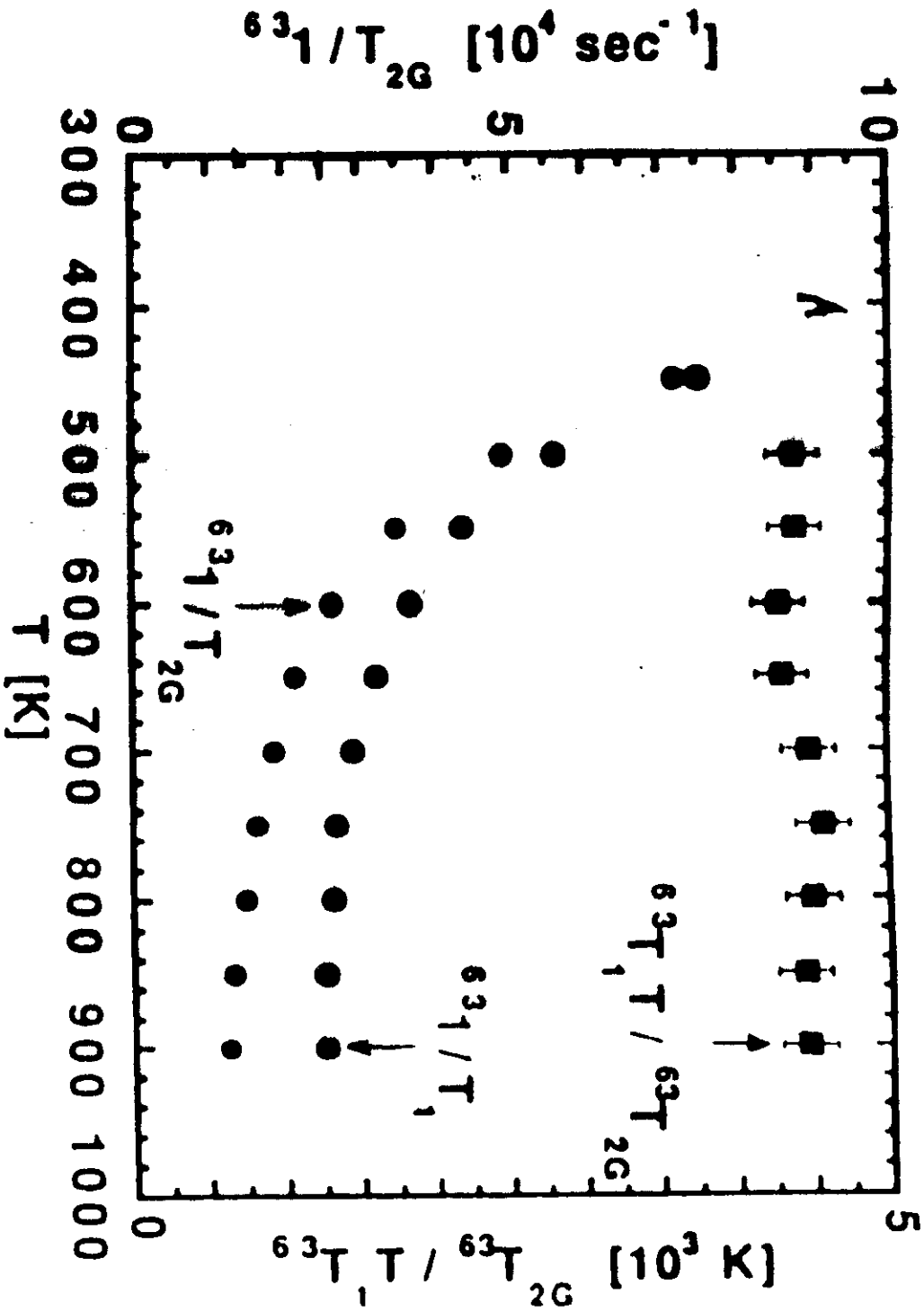
$$\frac{1}{T_2} \approx 0.546 \; \varphi_a \times 10^{4} \; \text{sec}^{-1}$$

Fig 1

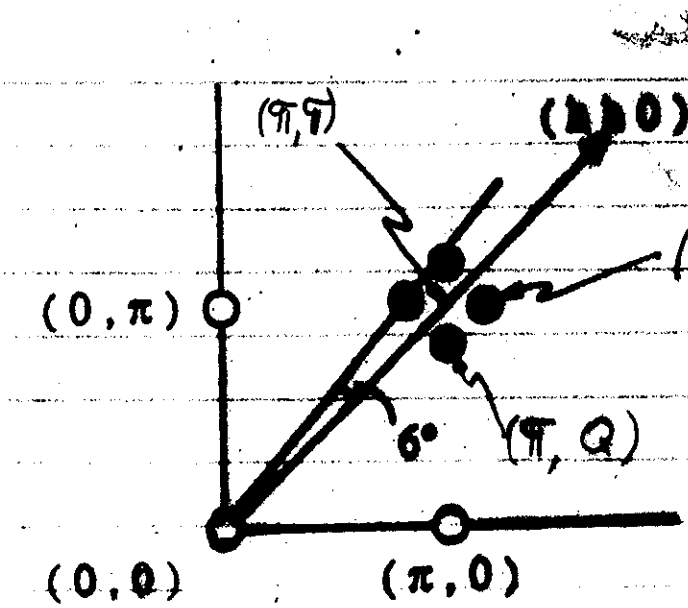
Correlation length



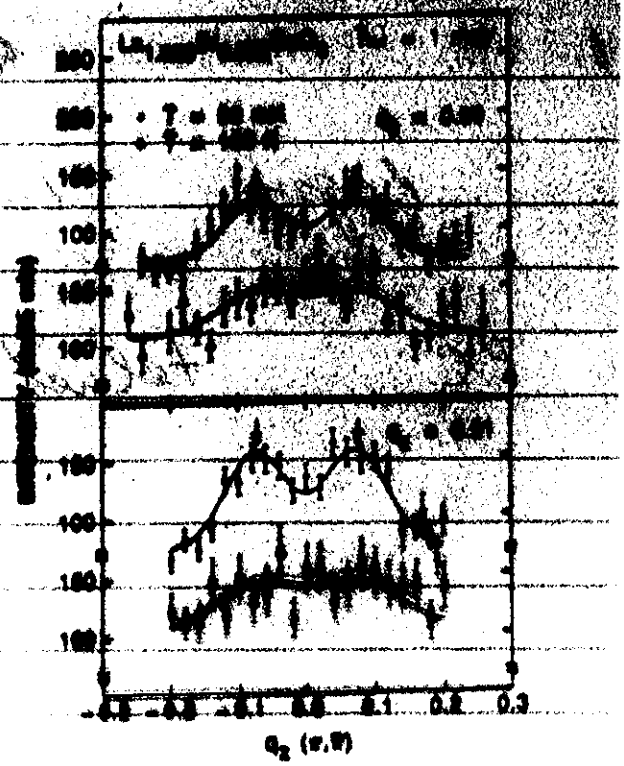
Chubukov, Sachdev & Sokol


$^{63}1/T_{2G}$ [10^4 sec^{-1}]
0
5
10
300
400
500
600
700
800
900
1000
T [K]
$^{63}1/T_{2G}$
$^{63}1/T_1$
$^{63}T_1T/^{63}T_{2G}$
0
5
$^{63}T_1T/^{63}T_{2G}$ [10^3 K]

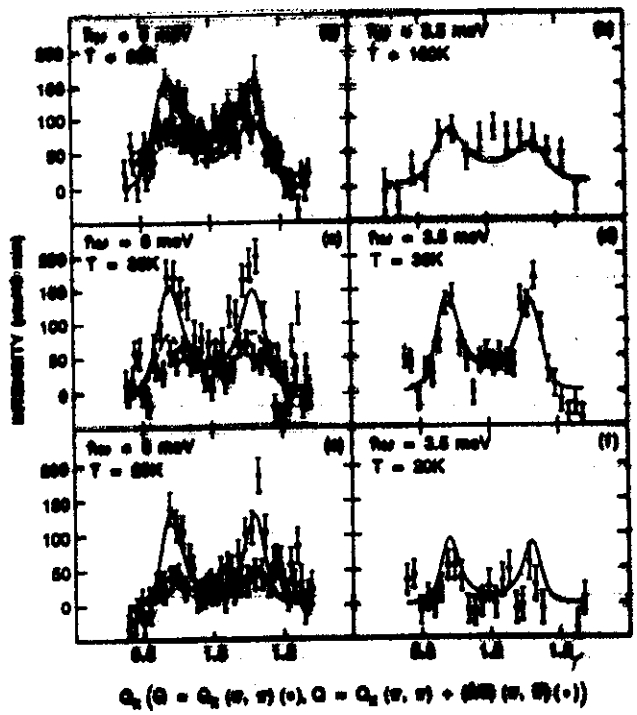
$La_{2-x}Sr_xCuO_4$



$\chi''(q,\omega)$



$La_{1.86}Sr_{0.14}CuO_4$



$\delta \approx 2x$

Max in $\chi''(q,\omega)$ at $q = (\pi,\pi) \pm \delta(\pi,0)$

$(\pm\delta(0,\pi))$

χ'' can also be inferred from NMR

$$\frac{1}{T_1} = T \int d^2q \; F(q) \; \frac{\chi''(q,\omega)}{\omega}\Big|_{\omega \to 0}$$

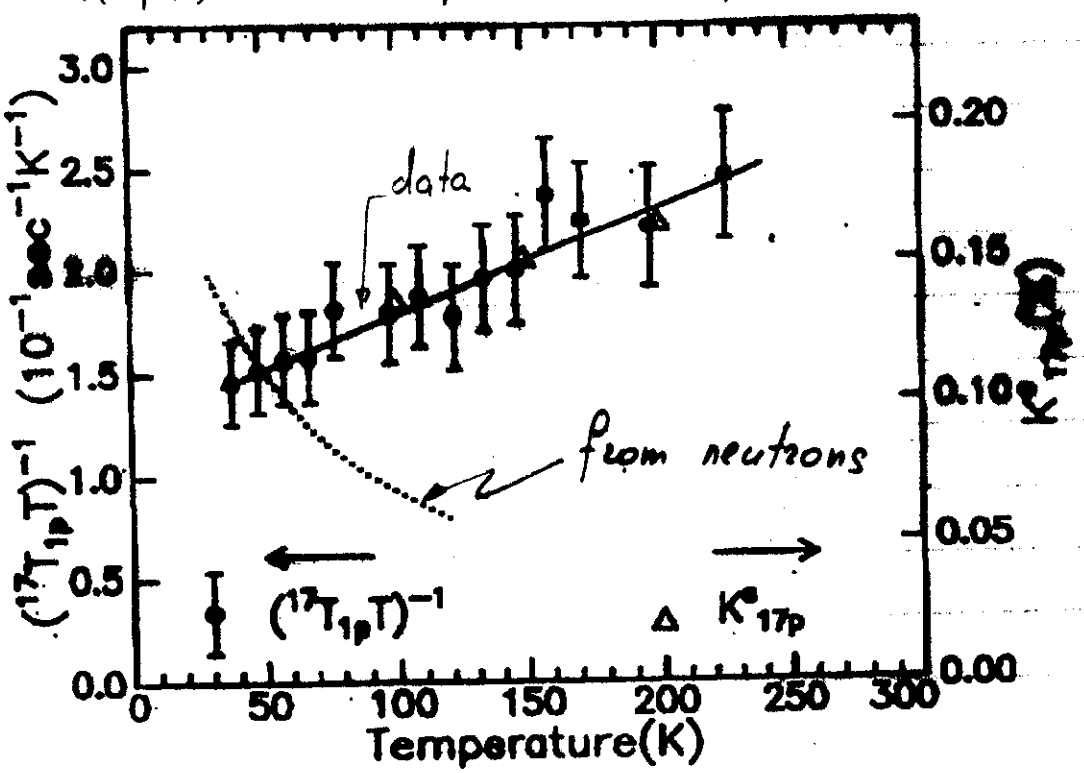
$F(q) = \begin{cases} \text{const}, & {}^{63}Cu \\ 0\,(!), & {}^{17}O \end{cases}$ $\Rightarrow$ $F(q) \sim (\pi - q)^2$ for ^{17}

In quantum-critical region,

$1/T_1 T \sim 1/T$ if max in χ'' is away from (π,π)

$1/(T_1 T) \sim T$ if max in χ'' is at (π,π).

^{17}O



Finite doping

- $g \to g_c$
- this is not the only effect of doping (Millis)

SDW theory of 2D Hubbard model

$$\mathcal{H} = -t \sum_{\langle ij \rangle} c_{i\sigma}^{\dagger} c_{j\sigma} - t' \sum_{\langle\langle ij \rangle\rangle} c_{i\sigma}^{\dagger} c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

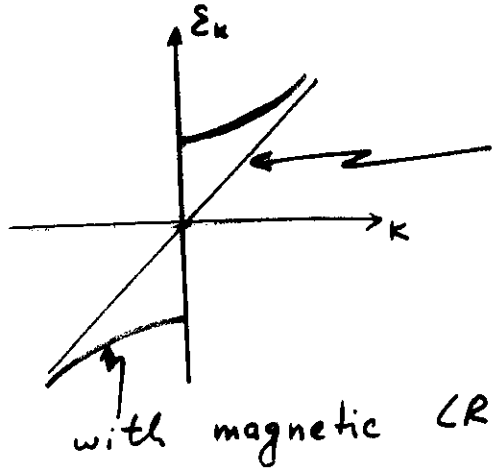
- antiferromagnetism at half-filling

$$\vec{S} = c_\alpha^{\dagger} \vec{\sigma}_{\alpha\beta} c_\beta$$

$$\langle S_z(i) \rangle = (-1)^i \lambda$$

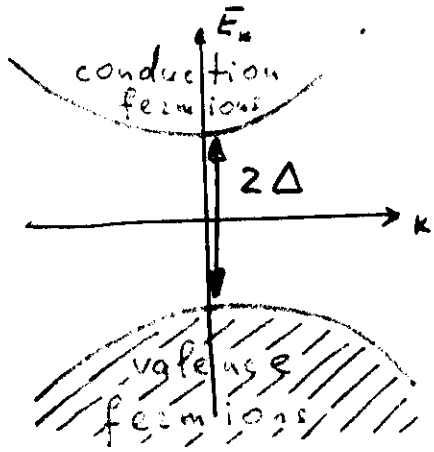
- (π, π) antiferromagnetism if $|t'| < t/\sqrt{2}$

- Electronic spectrum at half-filling



$$E_k = \pm \sqrt{\Delta^2 + 4t^2(\cos k_x + \cos k_y)^2} - 4t' \cos k_x \cos k_y$$

$$\boxed{\Delta = U \cdot \langle S_z \rangle}$$

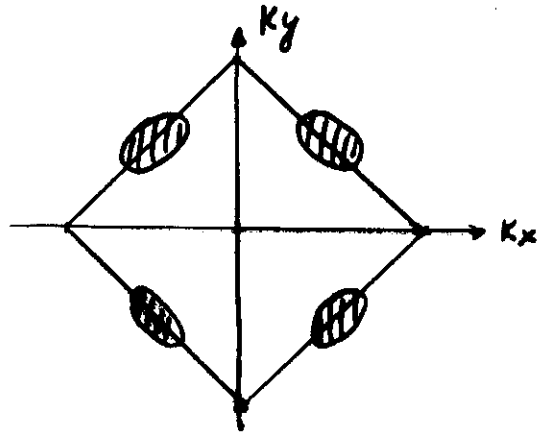


Band minima at $(\pm \pi/2, \pm \pi/2)$ if $t' < 0$ and $|t'| < J$

(In $La_2Cu_4O_4$,

$t' \approx -0.2\,t$

$J \approx 0.4\,t$)



- in the model with $t' = 0$,

  hole pockets are located at $(\pm\pi/2 \pm \pi/2)$

  variational Monte-Carlo — Manousakis

  variational wave functions — Trugman, Sachdev

  perturbative "$1/s$" expansion — Chubukov & Musaelian

- Bosonic excitations ≡ collective modes

$\chi(q)$ =  + ...

$$\chi(q) = \frac{\chi_0(q)}{1 - U\chi_0(q)} \qquad \chi_0 \sim \frac{1}{U}$$

not $\sim 1/t$ as in a metal

- $\chi(0) = \frac{1}{8J}$; $\chi(q \approx \pi) = \frac{1}{J(q-\pi)^2}$

- dynamical susceptibility ≡ 2 × 2 problem

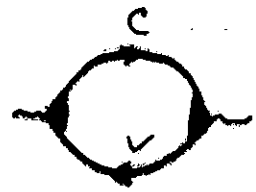
$$\chi(q, q+\pi, \omega) \sim \omega$$

- systematic expansion : large number of "orbitals"

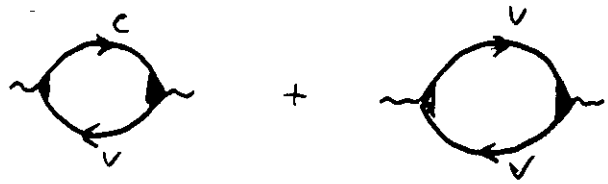
$U \sum_i n_\uparrow n_\uparrow \Rightarrow - U \sum_i S_i^2$ "$1/S$" expansion

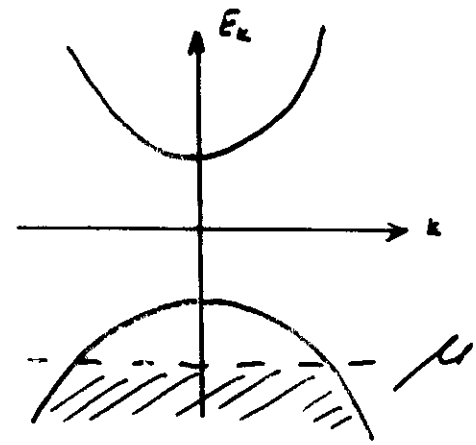
(S = 2 × number of orbitals)

• at half-filling

$\chi_0(q) =$ 

•• away from half-filling

$\chi_0(q) =$ 



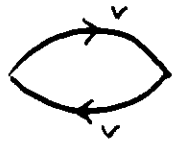
Uniform susceptibility

- **at $T = 0$**, in the ordered phase

★ $\chi \equiv 0$, $\chi_{\parallel} \equiv 0$, $\chi_{\perp} \neq 0$

★ $\chi \neq 0$

$$\chi_{\perp}^{\chi} = \chi_{\perp}^{\chi=0} \left(1 + O(\chi)\right)$$

$\chi_{\parallel}^{\chi}$ =  $\equiv \chi_{Pauli}$

In 2D, $\chi_{Pauli} \equiv \sqrt{m_{\parallel} m_{\perp}}/2\pi$, if

$$E_k = \Delta + \frac{k_{\parallel}^2}{2m_{\parallel}} + \frac{k_{\perp}^2}{2m_{\perp}}$$

χ_{Pauli} does not depend on carrier concentration

at $T \to 0$ $\chi_{tot} = \frac{2}{3}\chi_{\perp} + \frac{1}{3}\chi_{Pauli}$

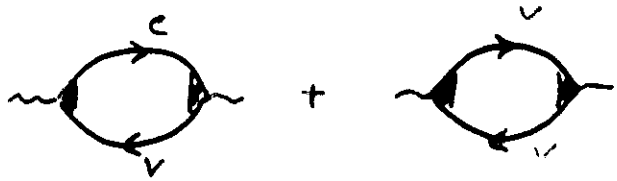
T increases $\left\{\chi_{tot} = \chi_{spin} + \frac{1}{3}\chi_{Pauli}\right\}$

χ_0/μ_B^2 [states/eV-Cu]
0
1
2
3
$\delta=0$
0.08
0.10
0.075
0.10
0.00
0.04
0.10
theoretical slope
Mills, 93

Staggered susceptibility

- $\chi_{tot}(q) = \rho_s \bar{q}^2$, $\bar{q}_i = \pi - q_i$

$$\rho_s = \rho_s^{x=0}(1 - ?)$$

$\therefore$ $\chi_0(q) =$ 

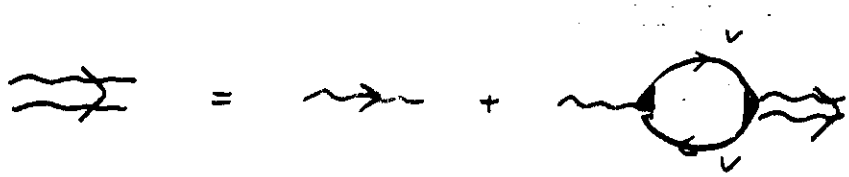
$\chi_0(q)$ has to be substituted into RPA series

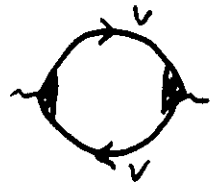
Rule of the game:

valence - valence bubbles are separated by valence - conduction ones

$\chi^{tot} =$ 

$\chi^{tot}_{x=0} =$



$$\rho_s \bar{q}^2 = \rho_s^{x=0} \bar{q}^2 -$$ 

Vertex

$\equiv + (\bar{q}_x \sin k_x + \bar{q}_y \sin k_y)$

- pockets at $(\pm\pi/2, \pm\pi/2) \Rightarrow |\sin k_i| = 1$

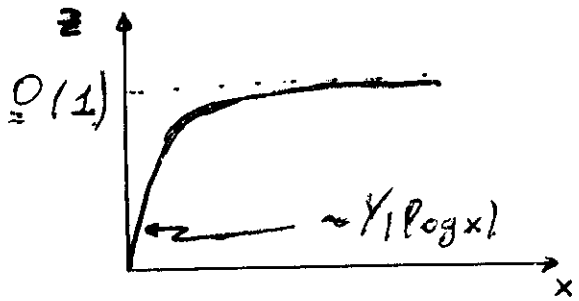
$$\rho_s = \rho_s^{x=0} - \lambda t^2 \chi_{Pauli}$$

does not depend on carrier concentration

$$\rho = \rho^{x=0} \, (1 - z)$$

- mean-field $\quad z = \frac{2t}{\pi} \sqrt{m_\parallel m_\perp} \sim \mathcal{O}(t_\perp) \gg 1$

- with self-energy & vertex corrections + $U \to T$ (scattering amplitude)



In any event, ρ_s rapidly decreases with doping.

- $\rho_s = \rho_s^{x=0} \left(1 - \underline{\underline{Q}}(x)\right)$

$$N_0 = N_0^{x=0} \left(1 - \underline{\underline{Q}}(x)\right)$$

$$\chi_\perp = \chi_\perp^{x=0} \left(1 - \underline{\underline{Q}}(x)\right)$$

- $z \equiv z(\omega)$

$$z(\omega) = z(\omega=0) \left[1 - \left(\frac{\delta^2}{\delta^2+1}\right)^{1/2}\right]$$

$$\delta^2 = \frac{\omega^2 m_\parallel m_\perp}{\bar{q}^2 p_F^2}$$

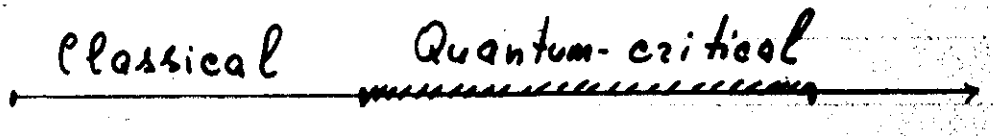
At spin-wave frequencies,

$$\omega = c q \Rightarrow \delta^2 \sim \frac{1}{p_F^2} \sim \frac{1}{x} \gg 1$$

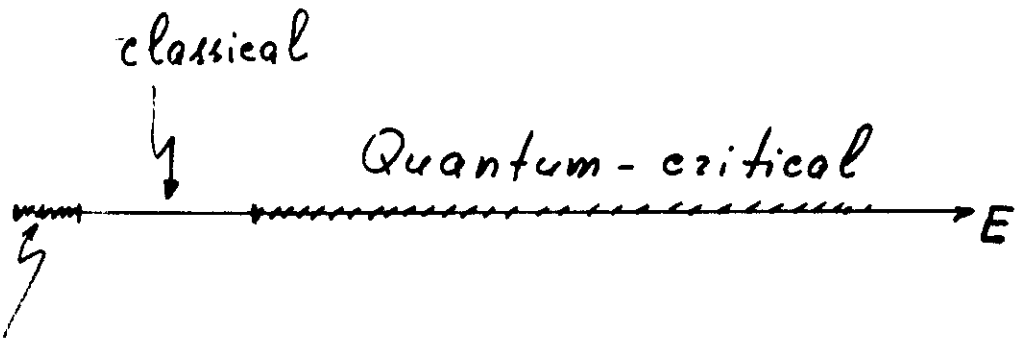
then $z(\omega) \sim \underline{\underline{Q}}(x)$!

We have new scale $\sim \underline{\underline{Q}}(x)$

- at half-filling

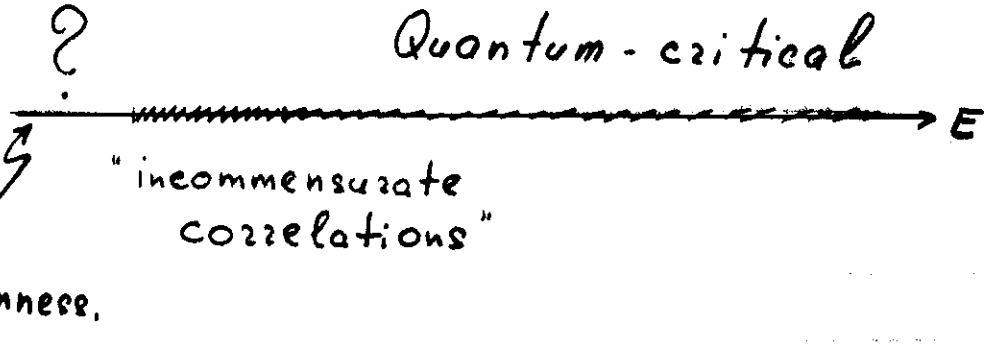


- at finite doping



width ~ x [if ρ_s changes sign before $N_0 \to 0$]

- above 4% doping



$\chi''_z(\omega) \sim I(|\omega|) F(\omega/T)$

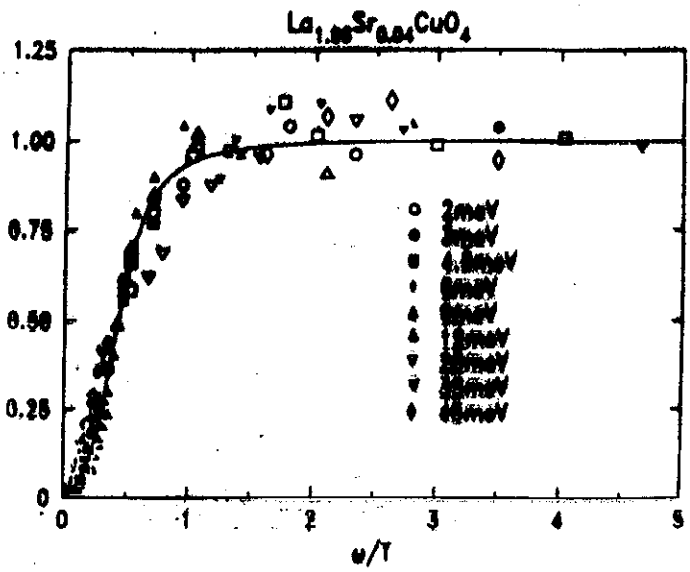


Figure 5: *Neutron scattering results for the scaling function F from Ref. [15]*

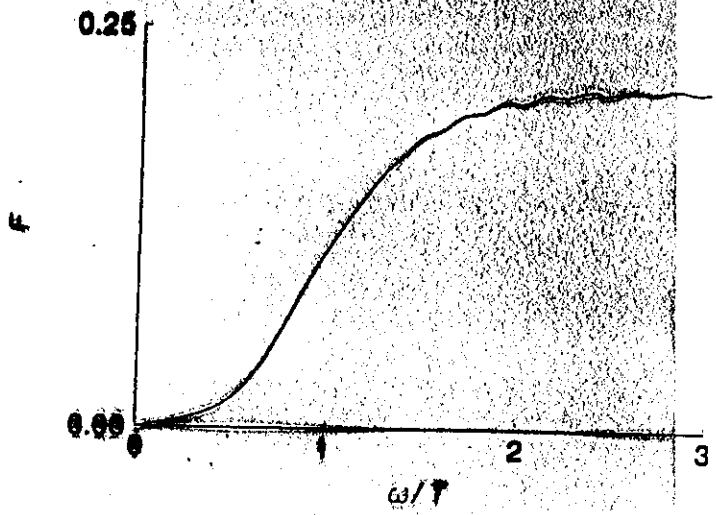


Figure 4: *The imaginary part of the universal local susceptibility, F_z for the same model as in the previous figure. We have $F(y) = y^{-\eta}$ [illegible] $F(x,y)$. The oscillations at large y are due to a finite step-size in the momentum integrations.*

Keimez et al

In a pure system, $I(|\omega|) \sim \omega^{\eta} \approx$ const

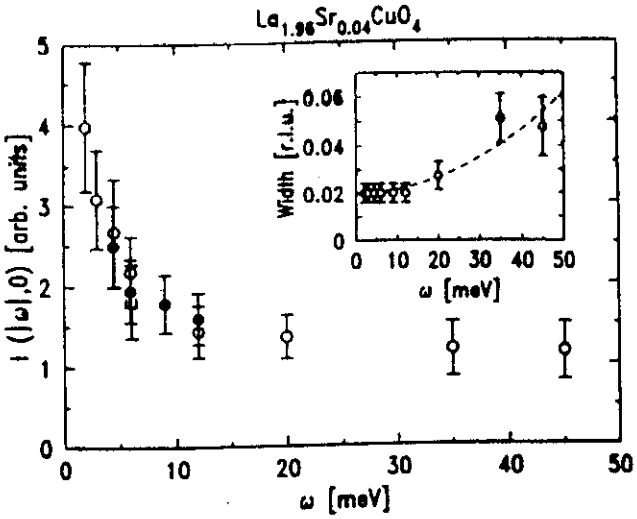


Figure 6: *Neutron scattering results for the normalization factor I from Ref. [15]*

Conclusions

- we found the new low-temperature region in quantum Heisenberg antiferromagnets, which is governed by the quantum-critical fixed point

- theoretical results for uniform susceptibility, static structure factor, local susceptibility and spin-lattice relaxation rates, T_1 and T_2, agree well with the experimental data

- quantum-critical region stretches to lower T with doping.

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