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MINIWORKSHOP ON STRONG CORRELATIONS
AND QUANTUM CRITICAL PHENOMENA
(4 - 22 July 1994)

# THE ELECTRON-PHONON INTERACTION IN THE PRESENCE OF STRONG CORRELATIONS

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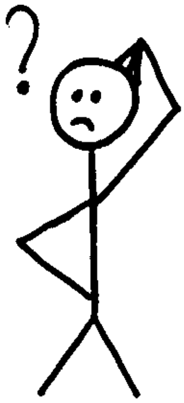
# The electron-phonon interaction in the presence of strong correlation

C. Castellani
and
M. G.

Two questions



1) Take the usual phonon-mediated e-e coupling

$\lambda_0 = \gamma^2 \nu_0$ where $\gamma$ = e-ph coupling

$\nu_0$ = electron d.o.s.

The e-e interaction (e.g. Hubbard-U) can increase the quasiparticle d.o.s. $\nu^* \gg \nu_0$ (for instance close to the insulating phase in a system with Mott-Hubbard MIT)
Could it be that a large $\lambda = \gamma^2 \nu^*$ is obtained between the quasiparticles?

2) Phase separation is a generic feature of systems with strong local e-e repulsion and short-range interactions.
Can phonons be responsible for a phase-separation instability in a system with strong local e-e repulsion?

PS in the presence of strong e-e interaction:

The Kinetic energy (KE) tends to delocalize electrons it favors uniformity extended Bloch states)

short-range interactions (SRI) (magnetic, n.n. Coulombic, ...) tend to favor localization or attraction

KE competes with SRI

Usually KE ($\sim$ eV) $\gg$ SRI and the system is uniform. However

The strong local e-e repulsion reduces the KE ($\mathcal{V}^*(\varepsilon_F) \gg \mathcal{V}^0$) (e.g. close to the insulating phase above a Mott-Hubbard MIT).

Then KE $\sim$ SRI and the localizing-attractive effect of SRI may become important

$\Downarrow$

Phase separation

# Fermi-liquid framework

- General FL analysis

i) The e-ph interaction strongly depends on the $(v_F q/\omega)$ ratio when $\frac{m^*}{m_0} \gg 1$ $\lambda = \begin{cases} \sim \lambda_0 \frac{m^*}{m} & \frac{\omega}{v_F q} \gg 1 \\ \sim \lambda_0 \frac{m}{m^*} & \frac{\omega}{v_F q} \ll 1 \end{cases}$

ii) a phonon-driven phase separation ($q \to 0$ instab.) is possible in principle.

- Specific model: three-band infinite-U Hubbard Model

(i) and (ii) confirmed and quantitatively specified

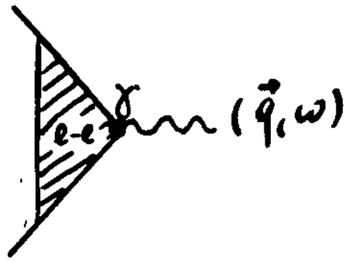
(iii) Superconductivity occurs close to the PS region.

# e-ph interaction: a FL analysis

optical phonon

$$H_{e\text{-}ph} = \gamma \sum_i \underline{\underline{n_i}}\ \underline{\underline{\varphi_i}} \qquad \text{where } \varphi_{\vec q} = \sqrt{\frac{\omega_q}{2}}\,(a_q + a^+_{-q})$$

The e-e interaction dresses the e-ph vertex

$\Lambda(q,\omega)$ 


Particle conservation (continuity eq.) implies the Ward Identities

$$z_\omega \Lambda(q=0,\ \omega\to 0) = 1 \qquad \text{dynamic } \frac{v_F q}{\omega}\to 0 \text{ limit}$$

$$z_\omega\, \underbrace{\Lambda(q\to 0,\ \omega=0)}_{\text{density vertex}} = \frac{1}{1+F_0^{s(e)}} \qquad \text{static } \frac{v_F q}{\omega}\to \infty \text{ limit}$$

wavefunction renormalization

$2\nu^* \Gamma_\omega$ — dynamic scattering ampli.

quasipart. d.o.s.

see e.g. Nozières "Theory of interacting Fermi systems" chapter 6

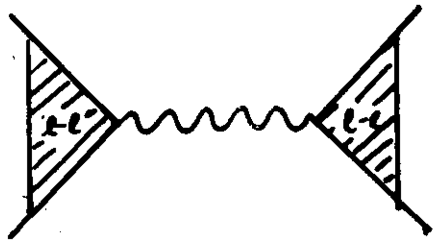
In the *dynamic* limit the e-ph vertex is not dressed (screened)

$$\gamma\Lambda(q=0,\ \omega\to 0) = \gamma$$

In the *static* limit the q.ps screen the e-ph vertex

$$\gamma\Lambda(q=0,\ \omega\to 0) = \frac{\gamma}{1+F_0^{s(e)}} = \gamma\,\frac{\kappa_e}{\nu^*} \sim \gamma\left(\frac{m_0}{m^*}\right)$$ the e-ph vertex is suppressed

Single-phonon exchange



optical phonon

$$\nu^* \Gamma_{eff}^{1ph}(q,\omega) = \nu^* \gamma^2 z_\omega^{e2} \Lambda_e^2(q,\omega) \left[\frac{\omega^2(q)}{\omega^2-\omega^2(q)}\right]$$

Take the small-$q$ and small-$\omega$ limits and use the WI's

$$\nu^* \Gamma_{eff}^{1ph}(q=0,\omega\to 0) = -\nu^*\gamma^2 \qquad \text{dynamic} \quad \frac{v_F q}{\omega}\to 0$$

$$\nu^* \Gamma_{eff}^{1ph}(q\to 0,\omega=0) = -\frac{\nu^*\gamma^2}{[1+F_0^s]^2} = -\gamma^2\frac{\kappa_e^2}{\nu^*} \qquad \text{static} \quad \frac{v_F q}{\omega}\to \infty$$

When $\frac{m^*}{m_0} = \frac{\nu^*}{\nu_0} \gg 1$ (for instance close to insulating phase of a system with Mott-Hubbard MIT) the difference is dramatic

dynamic limit $\frac{v_F q}{\omega}\to 0$

$$\nu^* \Gamma_{eff}^{1ph}(q=0,\omega\to 0) = -\gamma^2\nu^* = -\gamma^2\nu_0\left(\frac{m^*}{m_0}\right)$$

enhanced

static limit $\frac{v_F q}{\omega}\to\infty$

$$\nu^* \Gamma_{eff}^{1ph}(q\to 0,\omega=0) \simeq -\gamma^2\nu_0\left(\frac{m}{m^*}\right)$$

suppressed

Strong ($\omega$ vs. $v_F q$)-dependence of the e-ph vertex

different physical quantities, involving different $\omega / v_F q$ regimes, feel differently the e-ph coupling

e.g. Transport is a "low energy"–"high momentum" process $\Rightarrow$ vertex corrections strongly suppress the e-ph coupling

cf. J. Kim et al. PRB 40, 11378 ('89)

M. Kulić and R. Zeyher, PRB ('94)

How about pairing?

A full Eliashberg treatment is required

Care with momentum-averaged potentials ---

Vertex corrections beyond Migdal approximation

L. Pietronero and S. Strässler, Europhys. Lett. 18, 627 ('92)

again: $v_F q > \omega$ vertex suppression

$v_F q < \omega$ vertex enhancement

This agrees with WI arguments. Moreover

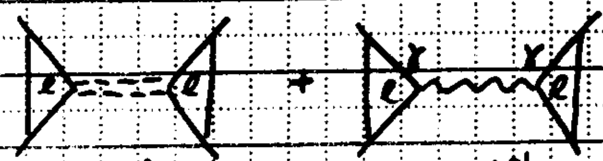
e-e interactions emphasize this behaviour

If phonon vertex corrections give pairing, e-e interaction helps a lot!! $\left(\sim \frac{m^*}{m}\right)$

~~However, something more may occur (see question~~

Consider the $(q \to 0, \omega \to 0)$ limit of the effective scattering amplitude

$$\Gamma_\omega = \Gamma_\omega^e + \Gamma_\omega^{1ph} =$$



$$= \Gamma_\omega^e - \gamma^2$$

$\Gamma_\omega^e$ $\Gamma_\omega^{1ph}$

This could be negative

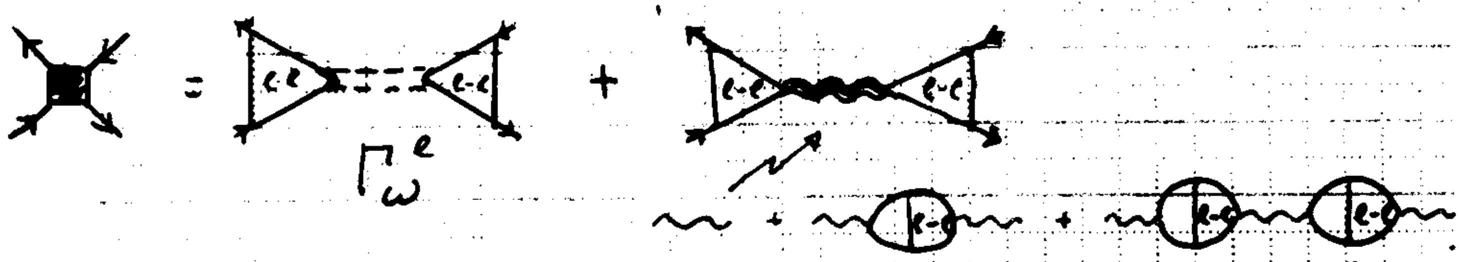
The corresponding compressibility is

$$\kappa = \frac{\nu^*}{1 + 2\nu^* \left[\Gamma_\omega^e - \Gamma_\omega^{1ph}\right]} = \frac{\nu^*}{1 + F_0^{s(1ph)}}$$

Already at this order $F_0^s$ could change sign or even become $< -1$

An instability (infinite or negative compr. $\kappa = \frac{\nu^*}{1+F_0^s}$) can occur if $\Gamma_\omega < 0$ and $\nu^*$ is large enough.

A RPA calculation confirms this result



$\Gamma_\omega^e$

dressed phonon propagator

A divergent compr. is possible

Notice: no phonon softening takes place at the instability

Violation of the stability criterion

$$1 + F_0^s = 1 + 2\nu^* \left[\Gamma_\omega^e - \gamma^2\right] \leq 0$$

## Partial summary

- When the e-e interaction is strong ($\frac{m^*}{m} \gg 1$) the (e-e dressed) e-ph vertex depends strongly on $\omega / v_F q$

- An attractive $\Gamma_\omega$ and a large $\nu^*$ could give rise to an instability.

This is general.

Models are needed to quantify

We now move to present $U = \infty$ models.

Slave boson + $\frac{1}{N}$ expansion

Example: one band Hubbard model ($U=\infty$)

$$\left\{\begin{array}{l} H = -t_0 \sum_{ij\sigma} c^+_{i\sigma} c_{j\sigma} - \mu_0 \sum_{i\sigma} c^+_{i\sigma} c_{i\sigma} \\ n_{i\uparrow} + n_{i\downarrow} \leq 1 \end{array}\right.$$

difficult to treat

old language

$|0\rangle_i$

$|\uparrow\rangle_i = C^+_{i\uparrow}|0\rangle$

new language

$b^+_i |\tilde{0}\rangle_i = |b_i\rangle$

$f^+_{i\uparrow} b_i b^+_i |\tilde{0}\rangle$ to create a fermion one must first destroy the "emptyness boson"

$$\begin{array}{l} c^+ \rightarrow f^+ b \\ c \rightarrow f b^+ \end{array}$$

$\sum_\sigma n_{i\sigma} \leq 1$ ⟶ $\sum_\sigma n_{i\sigma} + b^+_i b_i = 1$

either there is a fermion or nothing

either there is a fermion or a boson

easy to deal with formally:

$\underbrace{\lambda_i \left(\sum_\sigma n_{i\sigma} + b^+_i b_i - 1\right)}$

Lagrange multiplier.

$$\text{if } \langle b\rangle \neq 0 : G_c(p,\varepsilon) = \langle\langle c; c^+\rangle\rangle = \langle\langle b^+ b f f^+\rangle\rangle$$

$$\underset{m.f.}{\approx} \langle b\rangle^2 \langle\langle f f^+\rangle\rangle = \frac{\langle b\rangle^2}{\varepsilon - \varepsilon_p}$$

in mean field $\langle b\rangle^2$ is the residuum $z_K$ of the single particle G-fcn: if $\langle b\rangle \neq 0 \Rightarrow$ FL

$\langle b\rangle$: wave fcn. renormalization.

## $1/N$ expansion

Formal trick to get a small expansion parameter

$\sigma = \uparrow, \downarrow \quad \longrightarrow \quad \sigma = 1, \ldots, N$ with $N$ large

in the constraint $\lambda_i \left[ \sum_\sigma f^\dagger_{i\sigma} f_{i\sigma} + b^\dagger_i b_i - 1 \right]$, $1 \longrightarrow \frac{N}{2}$

only at the end $N \rightarrow 2$ back to the original model

The $U = \infty$ Hubbard model becomes

$$H_{SB} = -\frac{t}{N} \sum_{\langle ij \rangle \sigma} f^\dagger_{i\sigma} f_{j\sigma} b^\dagger_j b_i - \mu_0 \sum_{i\sigma} f^\dagger_{i\sigma} f_{i\sigma} +$$

$$+ i \sum_i \lambda_i \left[ \sum_\sigma f^\dagger_{i\sigma} f_{i\sigma} + b^\dagger_i b_i - \frac{N}{2} \right] = H_0 + \text{Constr.}$$

This comes from the Functional integral formalism

$$Z = \int Df \, Db \, e^{-S} \delta\left( \sum_i \lambda_i \left[ \sum_\sigma f^\dagger_{i\sigma} f_{i\sigma} + b^\dagger_i b_i - \frac{N}{2} \right] \right) =$$

$$= \int Df \, Db \, D\lambda_i \, e^{i \sum_i \lambda_i \left[ \sum_\sigma f^\dagger_{i\sigma} f_{i\sigma} + b^\dagger_i b_i - \frac{N}{2} \right] - S}$$

where $S = \int_0^\beta d\tau \left\{ \sum_{i\sigma} f^\dagger_{i\sigma} \partial_\tau f_{i\sigma} + \sum_i b^\dagger_i \partial_\tau b_i \right\} - H_0$

# The infinite-U Hubbard model: an interesting tutorial example

<u>Holstein</u>

$$H = \underline{H_{sb}} + \underline{\omega_0 \sum_i a_i^+ a_i} + \underline{g \sum_{i\sigma} f^+_{i\sigma} f_{i\sigma} (a_i^+ + a_i)}$$

The $(a^+ + a)$ combination only couples to the fermions. Therefore we define $A_i = \frac{1}{2}(a_i^+ + a_i)$ and $\bar{A}_i = \frac{1}{2}(a_i^+ - a_i)$

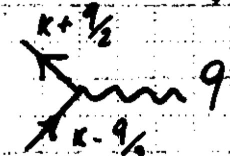
$\bar{A}_i$ is decoupled and its action is quadratic
$\Rightarrow$ it can be integrated away

$$\tilde{H} = H_{sb} + g \sum_{i\sigma} f^+_{i\sigma} f_{i\sigma} A_i + \sum_i \left( \frac{\omega_0^2 + \omega_n^2}{\omega_0} \right) A_i^+ A_i$$

with $H_{sb}$ being the usual slave-boson-large-N "Hamiltonian"

$$H_{sb} = -\frac{t}{N} \sum_{\langle ij \rangle \sigma} f^+_{i\sigma} f_{j\sigma} b_j^+ b_i - \mu_0 \sum_{i\sigma} f^+_{i\sigma} f_{i\sigma} + i \sum_i \lambda_i \Big[ \sum_\sigma f^+_{i\sigma} f_{i\sigma} + b_i^+ b_i - \frac{\Lambda}{2}$$

fermion-boson vertices:




$$\underline{\Lambda^r}\left(k+\frac{q}{2}, k-\frac{q}{2}\right) = -4 t r_0^2 \left[\cos k_x \cos\frac{q_x}{2} + \cos k_y \frac{q_y}{2}\right]$$ (radial gauge)

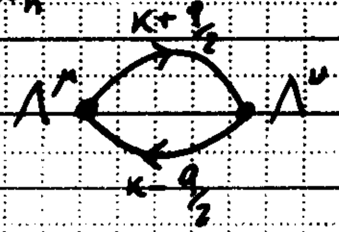
$$\underline{\Lambda^\lambda}\left(k+\frac{q}{2}, k-\frac{q}{2}\right) = i \qquad \underline{\Lambda^g}\left(k+\frac{q}{2}, k-\frac{q}{2}\right) = g$$

Inverse boson propagator (bare)

$$\mathcal{D}^{-1}(q, \omega_n) = N \begin{array}{c} \left[\begin{array}{ccc} \lambda_0 r_0^2 \left(\sin^2\frac{q_x}{2} + \sin^2\frac{q_y}{2}\right) & i r_0^2 & 0 \\ i r_0^2 & 0 & 0 \\ 0 & 0 & \frac{\omega_0^2 + \omega_n^2}{2\omega_0} \end{array}\right] \begin{array}{c} r_q \\ \lambda_q \\ A_q \end{array} \\ \begin{array}{ccc} r_q & \qquad\qquad \lambda_q & \qquad A_q \end{array} \end{array}$$

The $(q=0,\ \omega\to 0)$ dynamical limit of $\Gamma(q,\omega_n)$ is easy: the fermionic bubbles vanish

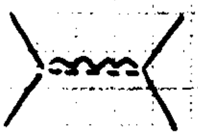
$$\Pi^{\mu\nu}(q,\omega_n) = -\sum_k \frac{f(E_{k+\frac{q}{2}}) - f(E_{k-\frac{q}{2}})}{E_{k+\frac{q}{2}} - E_{k-\frac{q}{2}} + i\omega_n} \Lambda^{\mu}(k+\tfrac{q}{2}, k-\tfrac{q}{2}) \Lambda^{\nu}(k-\tfrac{q}{2}, k+\tfrac{q}{2})$$




$$\xrightarrow[q\to 0]{} 0$$

so that

$$\Gamma^*(q=0,\omega_n\to 0) = -\nu^* \overline{\Lambda^r \Lambda^\lambda \Lambda^g} \left[ \mathcal{D}(0,\omega_n\to 0) \right] \begin{pmatrix} \Lambda^r \\ \Lambda^\lambda \\ \Lambda^g \end{pmatrix} =$$



$$= \underline{4t\frac{\nu^*}{N}\varepsilon_0} - \underline{\frac{2g^2}{N\omega_0}}\nu^* \equiv \Gamma_\omega \nu^*/N$$

where $\varepsilon_0 = \cos(k_{Fx}) + \cos(k_{Fy})$

The compressibility becomes

$$\kappa = \frac{\nu^*}{1+N\nu^*\Gamma_\omega} = \frac{\nu^*}{1+\Big[\underbrace{4t\varepsilon_0}_{\Gamma_\omega^{e}} - \underbrace{\frac{2g^2}{\omega_0}}_{\Gamma_\omega^{ph}}\Big]\nu^*} = \frac{\nu^*}{1 + F_0^{s(e)} + F_0^{s(ph)}}$$

Two interesting peculiarities: (due to $\varepsilon_0 \longrightarrow 0$ for $\delta\to 0$)

i) the purely electronic vertex (static) $\frac{1}{1+F_0^{s(e)}}$ does not vanish in the $\delta\to 0$ limit. Next to nearest neighbor hopping $t'$ makes a big difference!

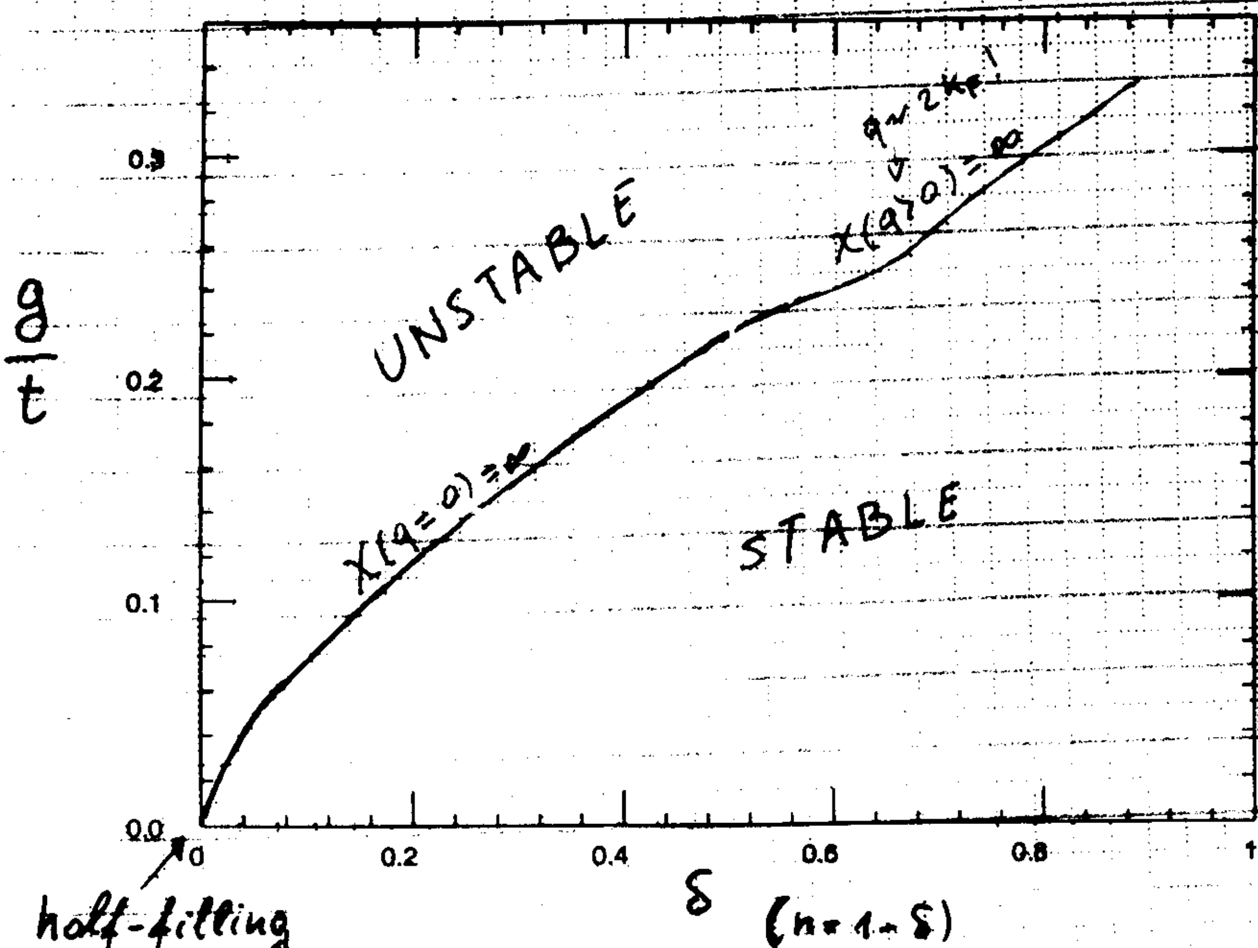
ii) Without $t'$, $F_0^{s(e)} \xrightarrow[\delta\to 0]{} 0$ and $\nu^* \to \infty$ Then

$$\kappa(\delta\to 0) \simeq \frac{\nu^*}{1+F_0^{s(ph)}} = \frac{\nu^*}{1-\frac{2g^2}{\omega_0}\nu^*}$$

this denominator is negative for $\nu^* \sim \frac{1}{\delta} \to \infty$ !!

The model is always unstable at small doping

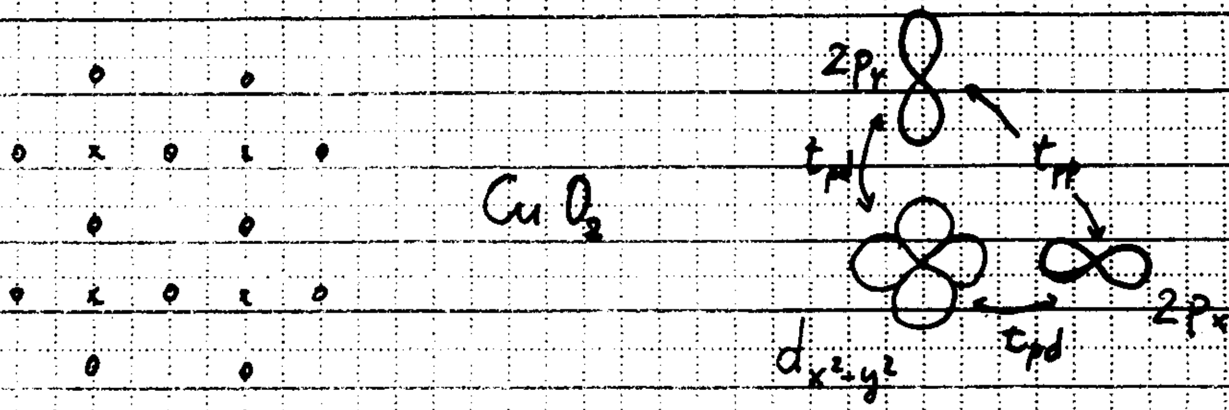
# Infinite-U HM
(n.n. hopping only)




C. Castellani, M. G., M. Tarquini, unpublished

work in progress

A more realistic model: the three band m...




$H = H_0 + H_{ph} =$

$$= \sum_{\langle ij\rangle\sigma}\left(t_{pd}^{ij}\, d^\dagger_{i\sigma} p_{j\sigma} + h.c.\right) + \varepsilon_p \sum_{j\sigma} p^\dagger_{j\sigma} p_{j\sigma} + \varepsilon_d^0 \sum_{i\sigma} d^\dagger_{i\sigma} d_{i\sigma} +$$

$$+ \; U_{dd} \sum_i \left(d^\dagger_{i\uparrow} d_{i\uparrow}\right)\left(d^\dagger_{i\downarrow} d_{i\downarrow}\right) +$$ local e-e Hubbard repulsion on copper

$$+ \; \underbrace{\omega_0 \sum_i a_i^\dagger a_i}_{\text{dispersionless phonon}} + \overbrace{\sum_{i\sigma}\left[g_d\left(n_{di\sigma} - \langle n_d\rangle\right) + g_p\left(n_{pi\sigma} - \langle n_p\rangle\right)\right]\left(a_i^\dagger + a_i\right)}^{\text{e-ph coupling}}$$

local hole density fluctuations

Technical framework:

$U_{dd} \to \infty \;\Rightarrow\;$ use slave boson on $d$ operators

dictionary: $d \to b^\dagger d$; $d^\dagger \to b d^\dagger$

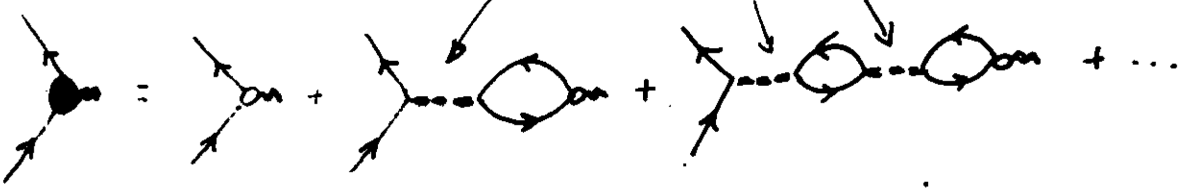
large-N expansion:

constraint $\quad \sum_i i\lambda_i \left[\sum_\sigma d^\dagger_{i\sigma} d_{i\sigma} + b_i^\dagger b_i - \frac{N}{2}\right]$

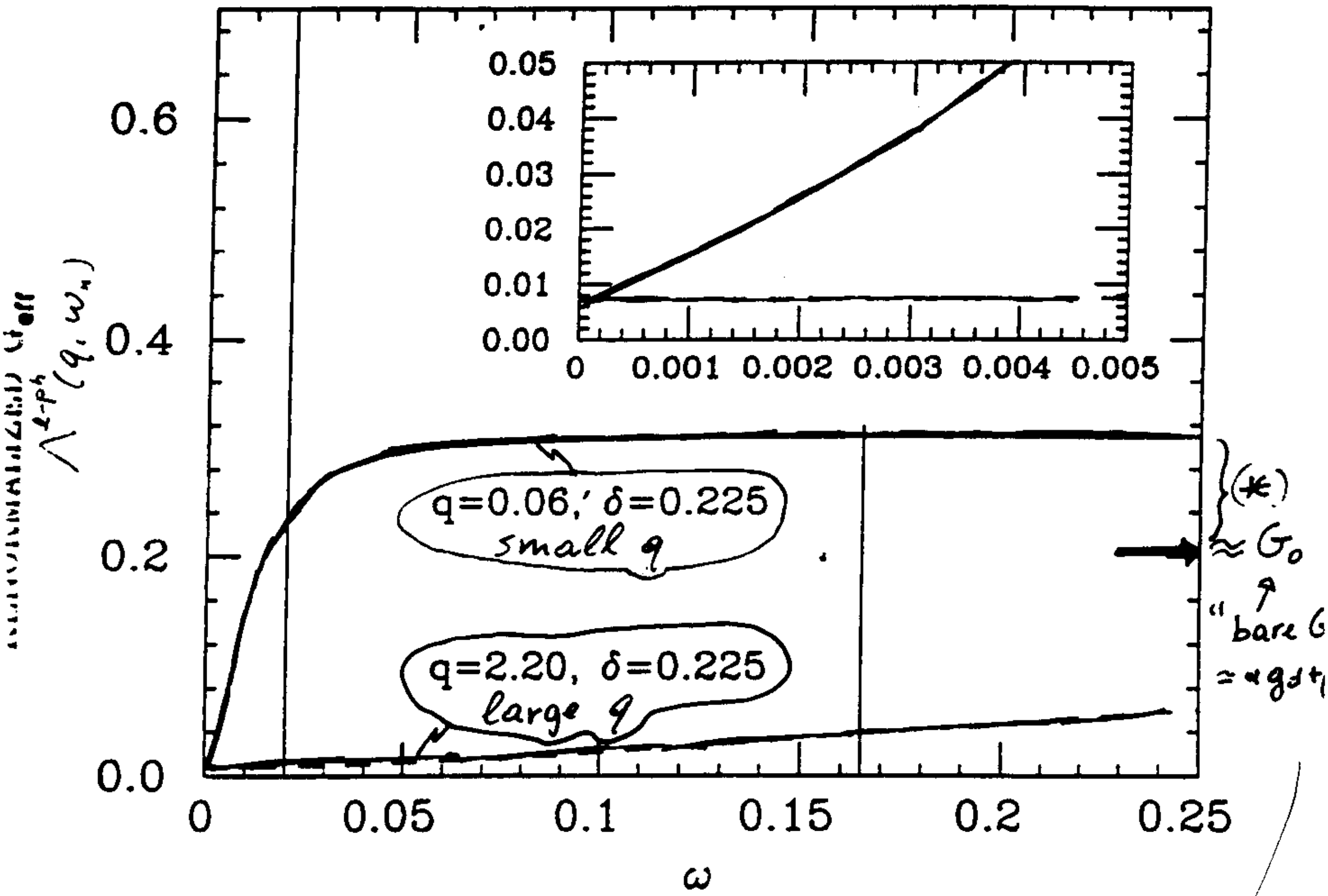
$$t_{pd} \to \frac{t_{pd}}{\sqrt{N}}$$

$$g_{p,d} \to \frac{g_{p,d}}{\sqrt{N}}$$

(e-e) → $\Gamma$ and $\lambda$ propagators



d.o.s. $\nu^* \sim 10\, \nu_0$




strong $\frac{\omega}{v_F q}$ dependence confirmed

**FIG.5**
**M. Grilli and C. Castellani**
**"The electron-phonon interaction in the presence of strong correlations"**

(*) Close inspection shows that interband processes


<u>enhance</u> the vertex at small q's

Quantitative answer to question 2
Possible occurrence of instabilities

General remarks

- Strong e-e local repulsion and (weak) short-range interactions (SRI) generically induce phase-separation (PS)
- The analysis of various models shows the presence of PS irrespective of the nature of the SRI. In particular

Three-band Hubbard model ⊕

magnetic n.n. coupling $J_{Cu-Cu} \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$
C. Castellani, M.G., G. Kotliar, PRB 43, 8000 ('91)

Coulombic n.n. repulsion $V_{pd} \sum_{ij} n_i^d n_j^p$
M.G., R. Raimondi, C. Castellani, C. Di Castro, G. Kotliar
PRL 67, 259 ('91)
Cu O
hole densities

displays PS for large enough $J_{Cu-Cu}$ and/or $V_{pd}$.

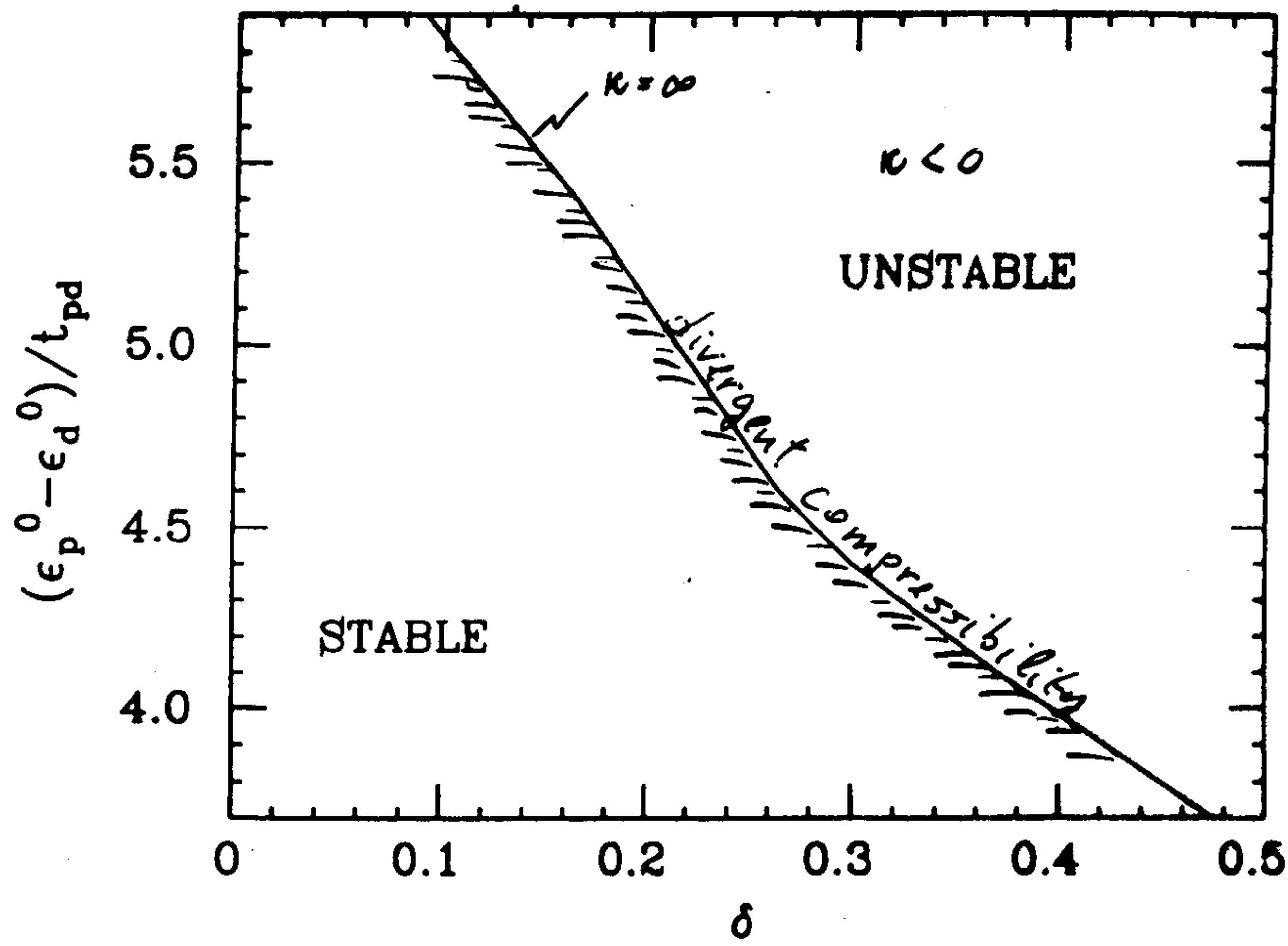
Pairing instabilities (s-wave and d-wave) occur close to the PS regions

What happens for the three-band HM with e-ph coupling?

PS takes place with e-ph coupling too

The required g's are <u>not</u> very large

e.g. $g_d = 0.1\, t_{pd}$ ; $g_p = 0.15\, t_{pd}$. $(\Rightarrow \lambda_0 \lesssim 1)$




SC close to the instability $\begin{cases} \text{d-wave} \\ \text{s-wave} \end{cases}$

**FIG.2**

**M. Grilli and C. Castellani**

**"The electron-phonon interaction in the presence of strong correlations"**

Why are PS and SC related?

A Landau FL analysis

Compressibility (short-range)

$$\kappa_{SR} = \frac{\mathcal{V}^*(\varepsilon_F)}{1+\underbrace{2\,\mathcal{V}^*(\varepsilon_F)\,\Gamma_\omega(q\to 0)}} \equiv \frac{\mathcal{V}^*(\varepsilon_F)}{1+F_0^s}$$

$\Gamma_\omega(q\to 0)$: $q\to 0$ limit of the "unscrened" effective interaction between the quasiparticles (the so-called dynamical limit)

If $\Gamma_\omega(q\to 0) < 0$ is (even weakly) attractive <u>and</u> $\mathcal{V}^*(\varepsilon_F)$ is large enough (here strong local repulsion play a relevant role)

$$F_0^s \equiv 2\,\mathcal{V}^*(\varepsilon_F)\,\Gamma_\omega(q\to 0) \leq -1$$

Landau's criterion for stability violated

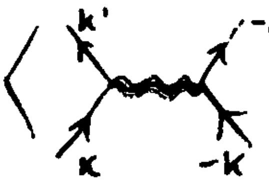
The same occurs for the (total) effective interaction

$$\Gamma(q\to 0) = \frac{\Gamma_\omega(0)}{1+2\,N^*(\varepsilon_F)\,\Gamma_\omega(0)} \longrightarrow -\infty$$

at the instabili[ty]

Large attraction

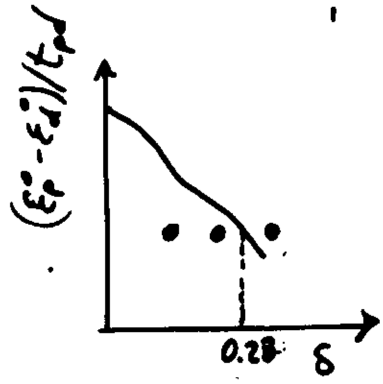
$$\underset{e\text{-}e+ph}{\lambda} \equiv \langle \Gamma_{pp}(q) \rangle_{\substack{\text{Fermi}\\ \text{surface}}} = \langle \quad \rangle_{FS}$$




For small $k-k'=q$ also $\Gamma_{pp}(q) \longrightarrow -\infty$ (in RPA) at the instability and gives an attractive $\lambda$ (e-e+ph) (s-wave and d-wave ...)

Already a static analysis shows that pairing occurs close to PS (frequency helps)

- far from the instability
- closer to the instability
- inside the instability



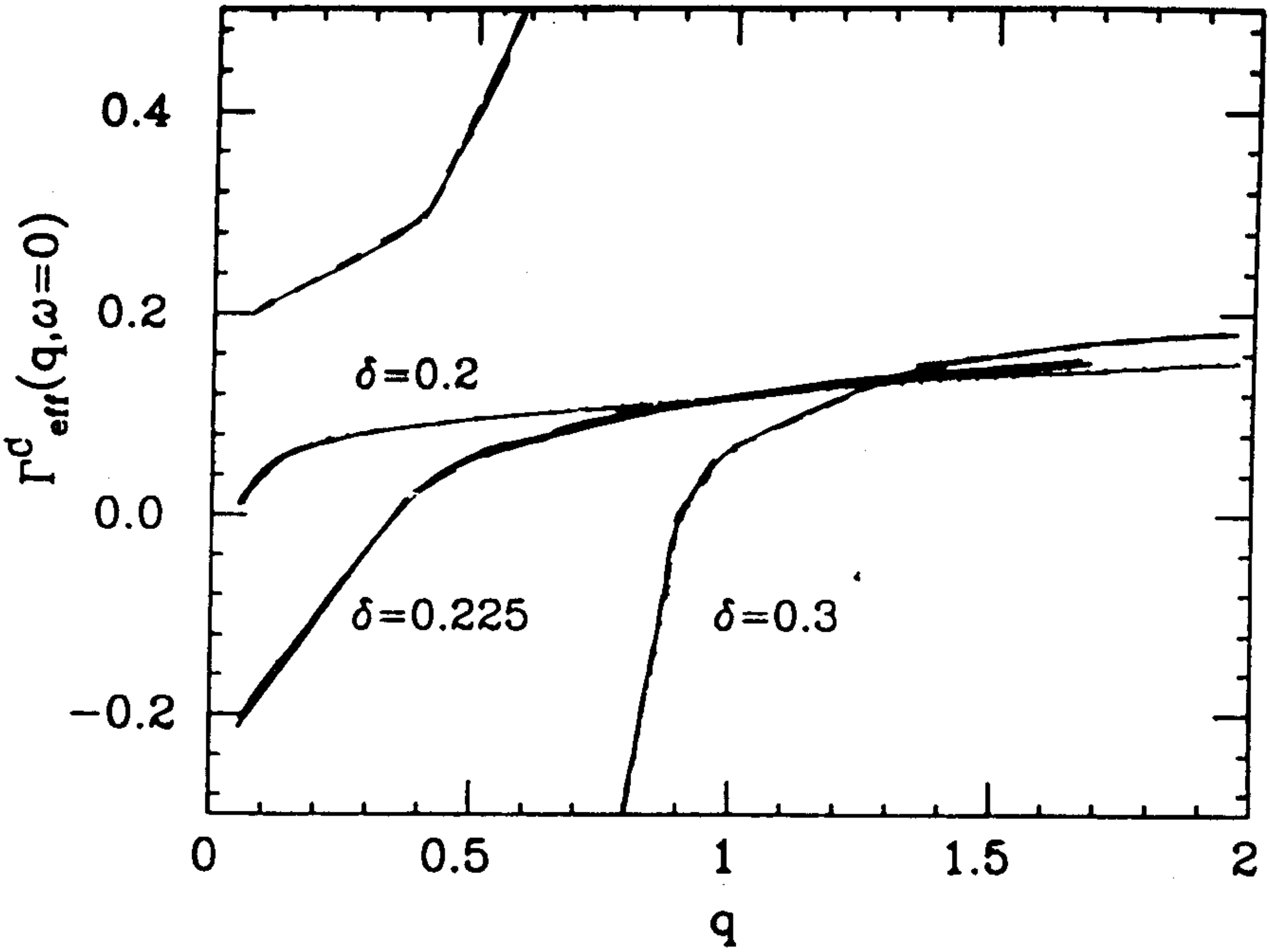




**FIG.3**
**M. Grilli and C. Castellani**
**"The electron-phonon interaction in the presence of strong correlations"**

## TABLES

### TABLE 1

| $\delta$ | 0.15 | 0.20 | 0.208 | 0.22 |
|---|---|---|---|---|
| $\lambda_{s_1}$ | -0.75 | -0.5 | 0.43 | -0.45 |
| $\lambda_{d_1}$ | -0.044 | 0.2 | 0.35 | |

extended s-waves

### TABLE 2

| $\delta$ | 0.15 | 0.20 | 0.229 |
|---|---|---|---|
| $\lambda_{s_1}$ | -0.58 | -0.46 | 1.2 |
| $\lambda_{s_2}$ | -0.55 | -0.65 | 1.5 |
| $\lambda_{d_1}$ | -0.063 | 0.017 | 2.1 |
| $\lambda_{d_2}$ | -0.021 | 0.012 | 1.8 |

d-waves

- repulsive
- attractive

This results from a static analysis

However, the $\omega/v_F q$-dependence of the e-ph vertex favors the attraction (e-ph) at finite frequencies $\Rightarrow$ Eliashberg treatment required (without q-averages!)

Preliminary analysis: $\Gamma^c(q, \omega_n)$

small $q$

$|q| \approx 0.07$

• repulsion (e-e)
• ph. attraction
• total

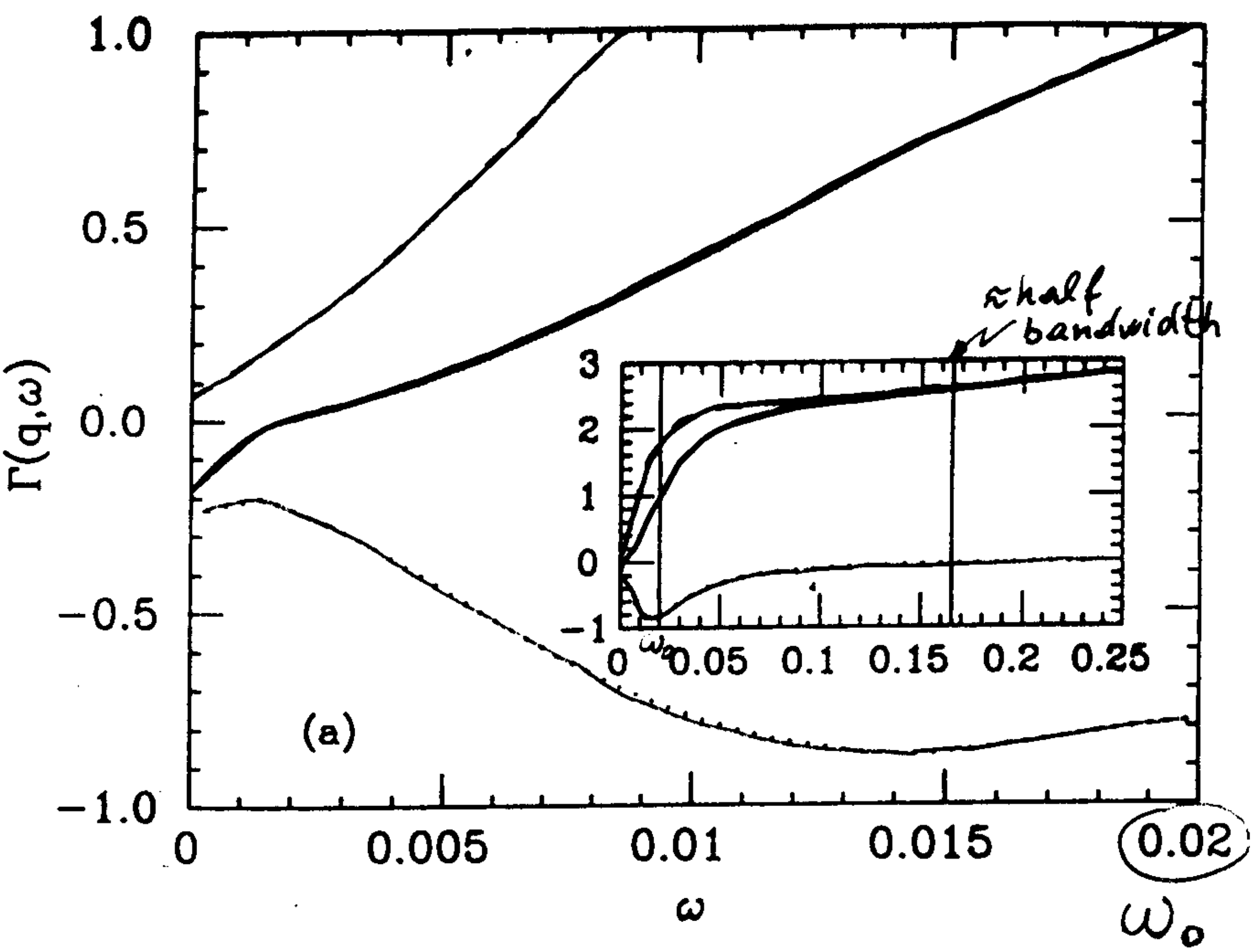



**FIG.7(a)**
**M. Grilli and C. Castellani**
**"The electron-phonon interaction in the presence of strong correlations"**

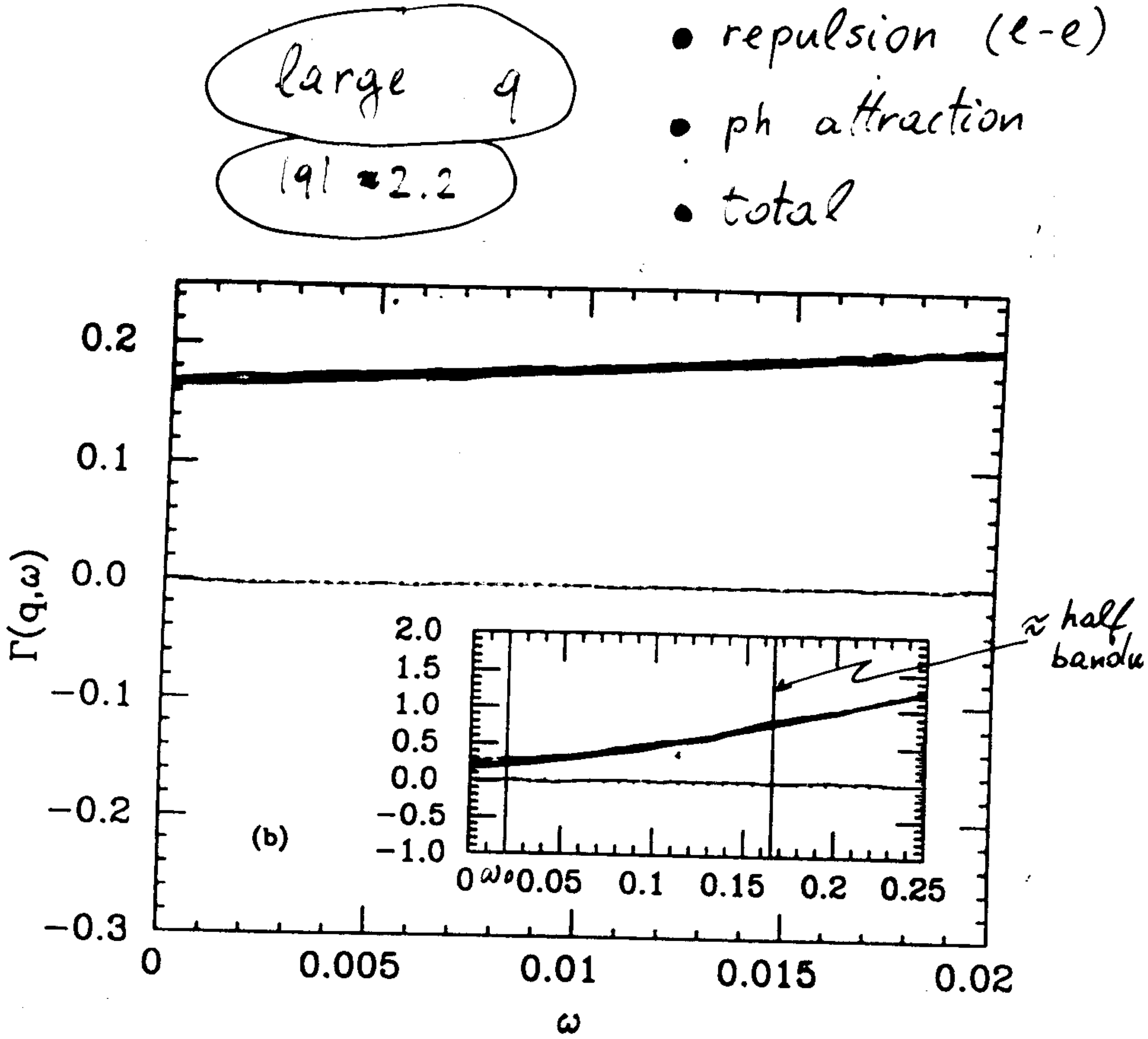



**FIG.7(b)**
**M. Grilli and C. Castellani**
**"The electron-phonon interaction in the presence of strong correlations"**

## A few remarks

- The same forces driving PS also give rise to a static pairing instability: One has to approach the PS instability to get static attractive pairing couplings (particularly for s-wave), but
- The dynamic attraction persists far from the PS instability $\Rightarrow$ (static) pairing only close to PS does <u>not</u> mean SC only close to PS
  Eliashberg needed
- PS $\Rightarrow$ overdamping of zero-sound mode
  $\Rightarrow$ no phonon softening

i) Role of Long-Range Coulomb forces

ii) Polaron formation.

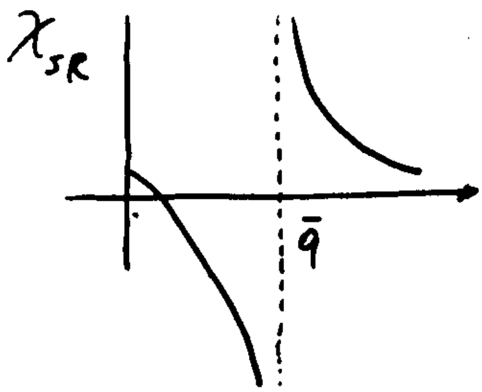
# Effect of Long-Range Coulomb forces

c) • Inside the unstable regions the poles of $\chi_{SR}(q,0)$ occur at finite (sizable) $q$'s

$\Rightarrow$ LR Coulomb forces can stabilize the $q=0$ (PS) instability, but finite-$q$ instabiliti[es] survive (incommensurate CDW)

e.g. RPA approach

$$\chi_{LR}(q) = \frac{\chi_{SR}(q)}{1 + \underset{\substack{\vee \\ 0}}{V_{Coul.}(q)}\,\underset{\substack{\wedge \\ 0}}{\chi_{SR}(q)}}$$

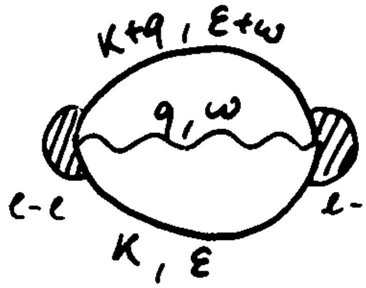



Static analysis:
LR Coulomb forces $\Rightarrow$ CDW (incommensurate)
M. G., R. Raimondi, C. Castellani, C. Di Castro, G. Kotliar PRL ('91
R. Raimondi, C. Castellani, M. G., Y. Bang, G. Kotliar, PRB ('93) $U_{pd}$ driven
(3-band HM + $U_{pd}$)

Dynamical analysis:
slow density fluctuation modes
(V. J. Emery and S. Kivelson, Physica C ('93)) J-driven P.S.

Are polarons favored by the e-e interaction?

Let's look at $\langle n_i \varphi_i \rangle$ : it signals polaron (large lattice displacement if a particle is present.)

$\langle n_i \varphi_i \rangle \sim$ 


phonon prop $\Rightarrow$ relevant freqs in the integral $\approx \omega_0$

integrated momenta $\Rightarrow q \sim k_F$

What matters is $\omega_0 \gtrless v_F k_F$ $\approx$ q.p. bandw

q.p. Fermi velocity

If $\underline{v_F k_F > \omega_0}$ $\Rightarrow$ e-ph vertex suppressed by the e-e screening

$\Downarrow$

polarons more difficult

If $\underline{v_F k_F < \omega_0}$ $\Rightarrow$ e-ph vertex is not screened. + large $\frac{m^*}{m}$

possible close to insulating (Mott) phase

polarons easier $\Leftarrow$ $\begin{cases} \text{effective e-ph coupling} \\ \text{is enhanced} \end{cases}$

# Conclusions

- Strong $\frac{\omega}{v_F q}$ -dependence of $\Lambda_{e\text{-}ph}$
  - transport $\neq$ pairing $\neq$ ....
  - polarons $\to$ easy close to Mott insulating phase; $\to$ difficult well inside metallic phase

- PS (tendency) $\Rightarrow$ SC

  LR Coulomb
  - incommensurate CDW
  - slow collective modes

$$PS \Longrightarrow \Gamma(q \to 0) \longrightarrow -\infty \Longrightarrow \lambda = \langle \Gamma_{pp}(q) \rangle_{FS} < 0$$

The same forces that induce PS may lead to SC close to it.

Moreover

SC may occur instead of PS:

an example is Kondo-lattice model

Cancrini Caprara Castellani Di Castro, M.G. and Raimondi, Europh. Lett '9