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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE CENTRATOM TRIESTE

SMR.769 - 11

WORKSHOP ON "NON-LINEAR ELECTROMAGNETIC INTERACTIONS IN SEMICONDUCTORS"

1 - 10 AUGUST 1994

"Nonlinear Transport in Semiconductors" Parts I & II

J.C. MAAN
Department of Physics
University of NijmegenToernooiveld 1
NL-6500 GL Nijmegen
THE NETHERLANDS

NON LINEAR TRANSPORT IN SEMICONDUCTORS I

Pattern formation with non-linear electrical transport.

- Introduction on spatiotemporal pattern formation.
- N- and S shaped j-E characteristics.
- Breakdown in SI-GaAs.
  - Visualisation technique
  - Topology of domains
  - Bulk j-E characteristic.
- Breakdown in Air
  - Phases in breakdown
  - Topology of sparks
  - Sparks in magnetic field
  - fractal structure

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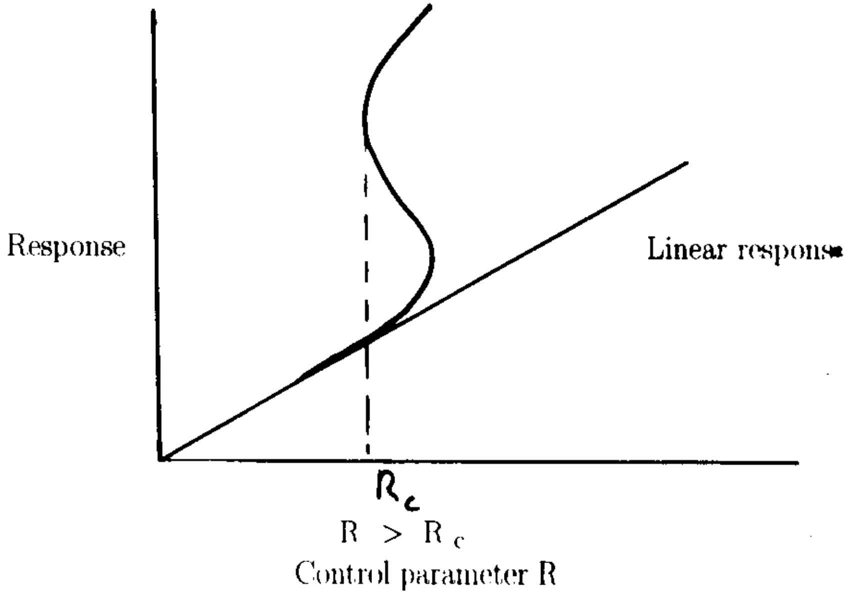
ORIGIN OF PATTERN FORMATION

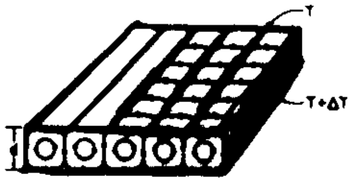


$R > R_c$ more than one response $\Rightarrow$ pattern formation
spontaneous symmetry breaking

- Patterns may be stable in time,
  oscillatory
  chaotic
- Occur in a wide variety of systems

Ref. M.C. Cross and P.C. Hohenberg, Rev. Mod. Phys. 65, 853, 1993.
G. Ahlers, Complex Systems, Vol 7, 1989, Physica D51, 421, 1991

RAYLEIGH - BÉNARD CONVECTION



Instability due to competion

$$\text{buoyancy} \Longleftrightarrow \text{gravity}$$
$$\text{(up)} \qquad\qquad \text{(down)}$$

Navier - Stokes (force balance)

$$\underbrace{\frac{\partial v}{\partial t} + (v \cdot \underline{\text{grad}})\underline{v}}_{\text{change of momentum}} = -\frac{1}{\rho}\,\underline{\text{grad}}\ \text{p} + \frac{\eta}{\rho}\,\Delta v + \text{g}\,\alpha T \underline{Z}$$

$$\underset{\text{pressure}}{\Uparrow} \qquad \underset{\text{friction}}{\Uparrow} \qquad \underset{\text{buoyancy}}{\Uparrow}$$

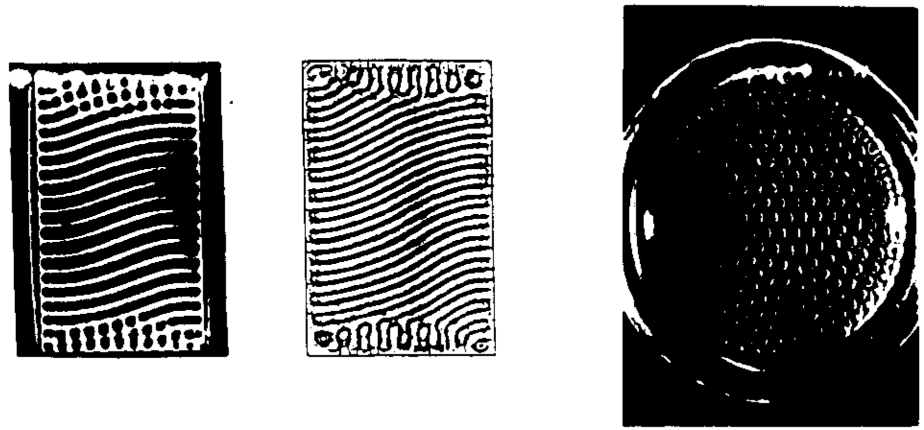
Conservation of heat flow

$$\left(\frac{\partial}{\partial t} + \underline{v} \cdot \nabla\right) C_p T = K \nabla^2 T$$

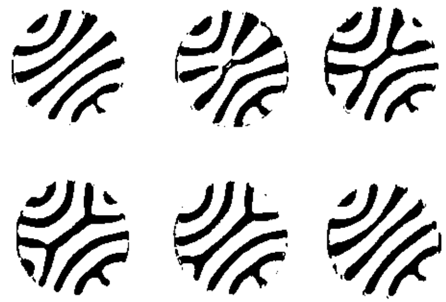
Incompressibility

$$\nabla \cdot \underline{v} = 0$$

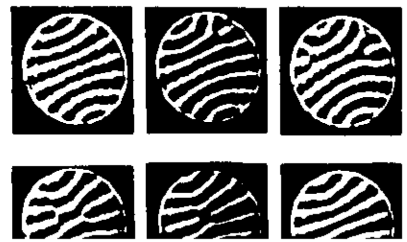
STATIC PATTERNS



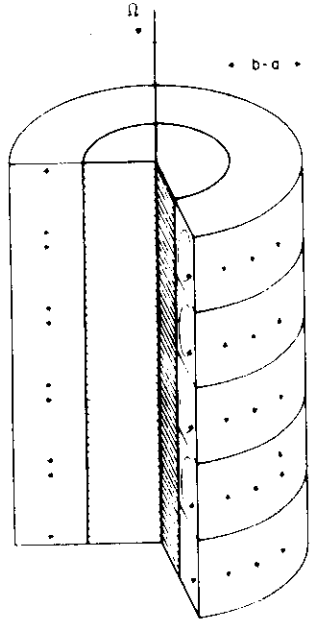
DYNAMIC PATTERNS



(b)



TAYLOR - COUETTE FLOW



Rotate inner cilinder

Instability due to competition

Different centrifugal force $\Leftrightarrow$ incompressibility

PATTERNS IN CHEMICAL REACTIONS

Belousow - Zhabotinsky reaction

$Fe^{3+,\,2+}$ or $Ce^{3+,\,2+}$ in $HBrO_2$

$\Uparrow$ $\Uparrow$

Blue Orange

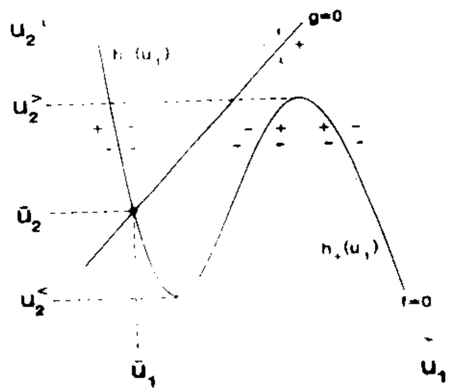
Reaction diffusion equation

$$\frac{\partial}{\partial t}\,u_1 \;=\; D_1\,\frac{\partial^2}{\partial x^2}\,u_1 \;+\; a_1 u_1 \;-\; b_1 u_2$$

$$\frac{\partial}{\partial t}\,u_2 \;=\; D_2\,\frac{\partial^2}{\partial x^2}\,u_2 \;+\; a_2 u_2 \;-\; b_2 u_1$$

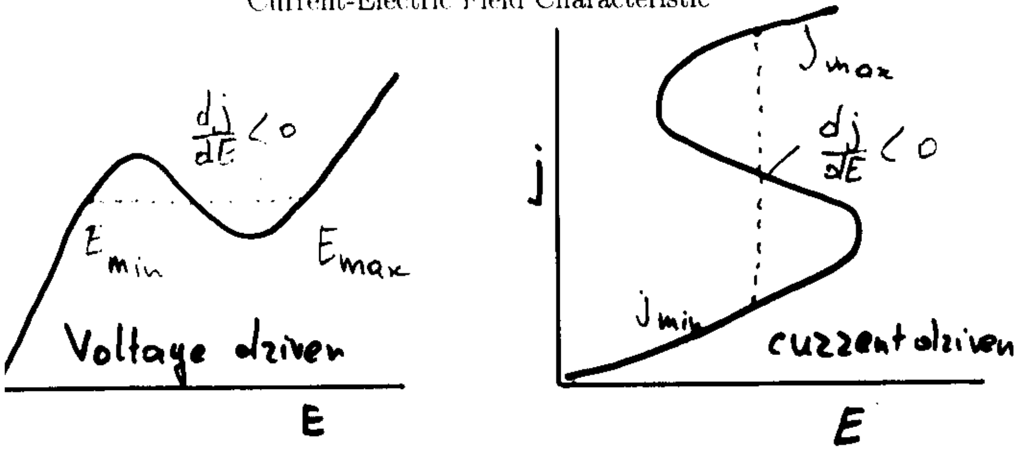
u_1, u_2 concentration 1 or 2.

Time and space dependent patterns, change color in minutes, oscillatory



NON LINEAR TRANSPORT IN SEMICONDUCTORS

Current-Electric Field Characteristic



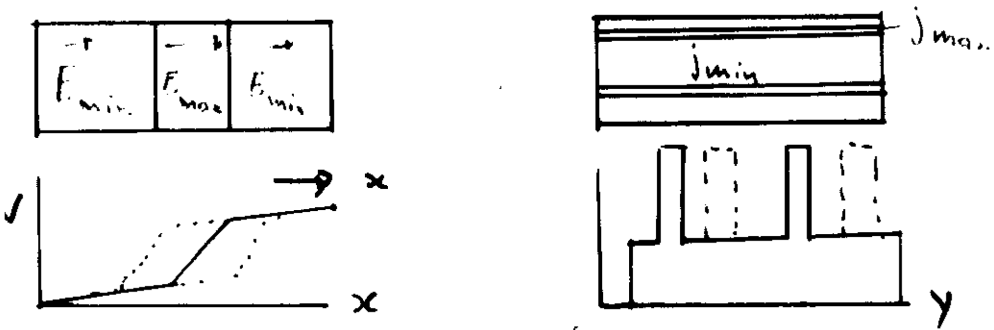
Unstable region $\frac{dj}{dE} < 0$, negative differential conductivity

Domains

(Gunn domains, "slow" domains)

Filaments

(Sparks,)



$\int E(x)dx = V_{applied}$ $\quad\int j(y)dy = I_{total}$

Unstable, many possible realizations of boundary conditions

$j(E) \rightarrow$ topology

BASIC EQUATIONS FOR SLOW DOMAINS

Conservation of charge

$$\frac{\partial j}{\partial X} + \frac{\partial \rho}{\partial t} = 0 \qquad X = x, y, z$$

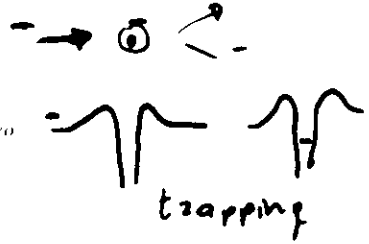
Current continuity Diffusion

$$j = env(E) - e\frac{\partial (Dn)}{\partial X}$$

Rate equations

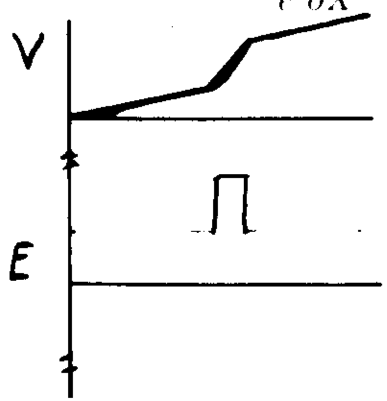
$$\frac{\partial n}{\partial t} = -\frac{1}{e}\frac{\partial j}{\partial X} + \overbrace{g(E)N_t - c(E)n(N_t - N_t)}^{\text{Field dependent capture and generation}}$$

carrier multiplica



Poisson

$$\frac{\epsilon}{e}\frac{\partial E}{\partial X} = N_t + n - n_0$$



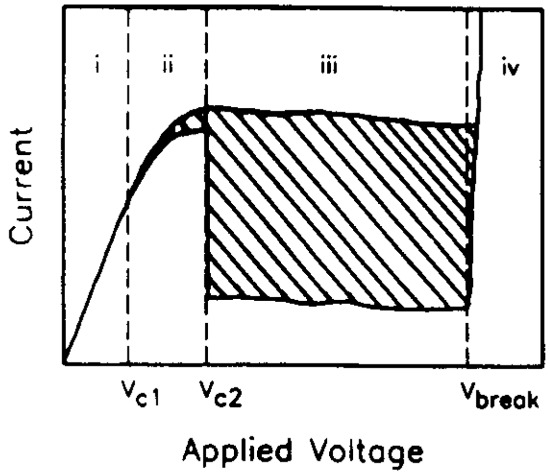
Note:
Since $\frac{dE}{dX} \neq 0$
with domains. $\Rightarrow$
No local charge neutrality

current conductive

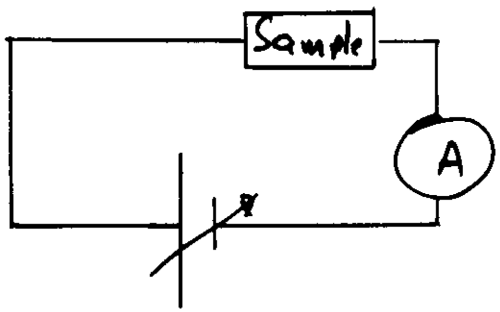
SLOW DOMAINS IN SI - GaAs

(LEC <100> wafers)

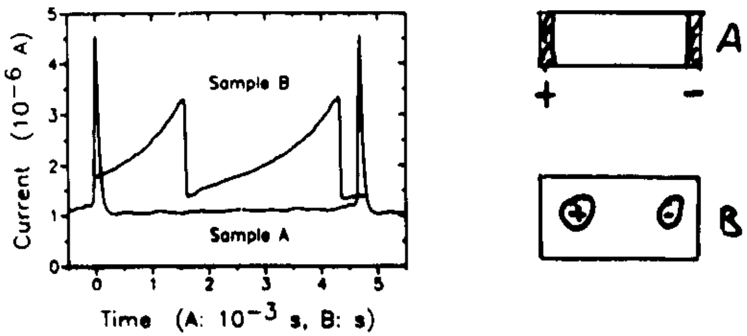
I-V characteristic



- In unstable regions ii and iii spontaneous oscillations
- I-V at contacts cannot measure bulk j - E because of spatial inhomogeneity

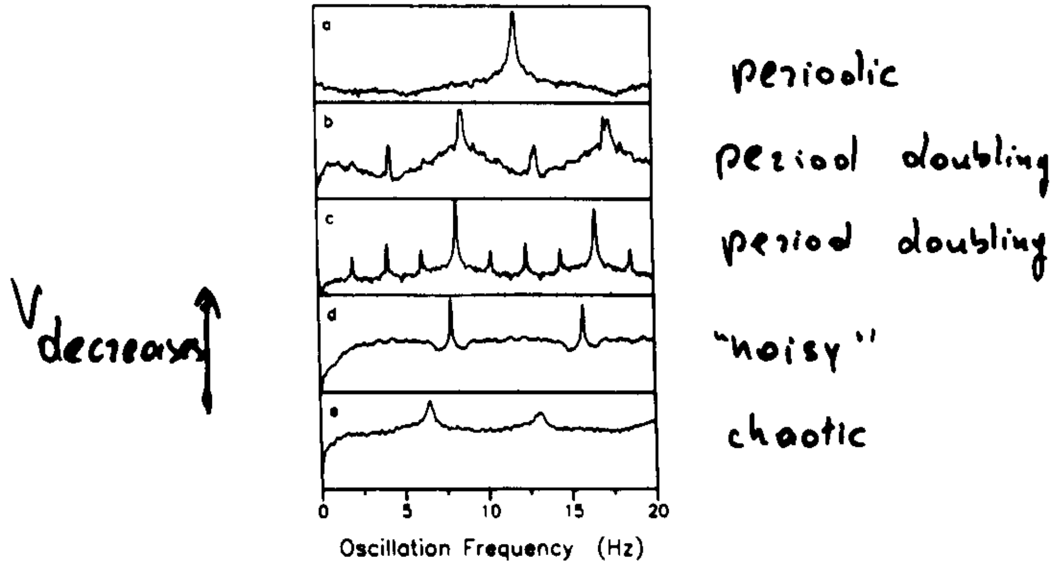


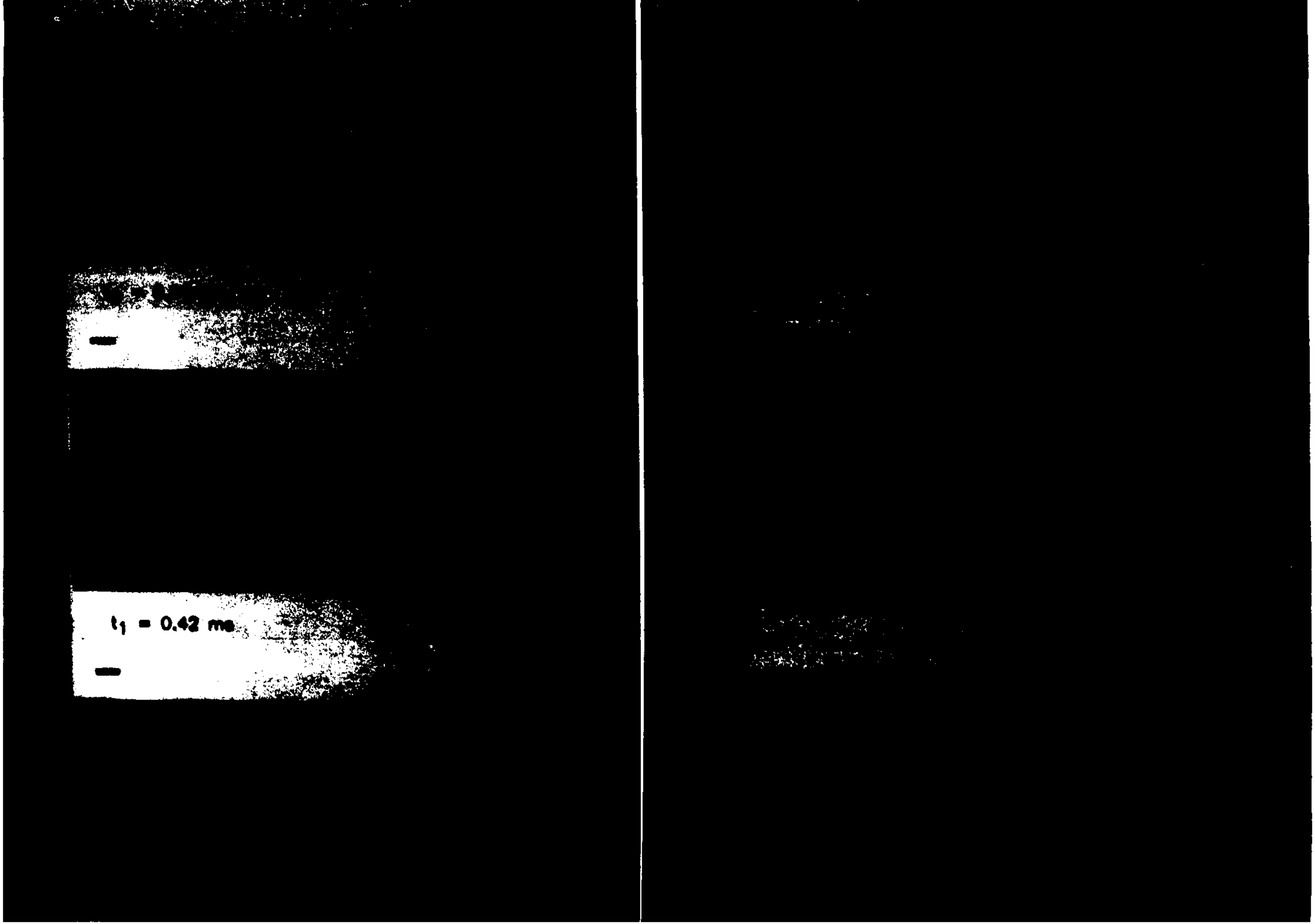
EVOLUTION WITH TIME



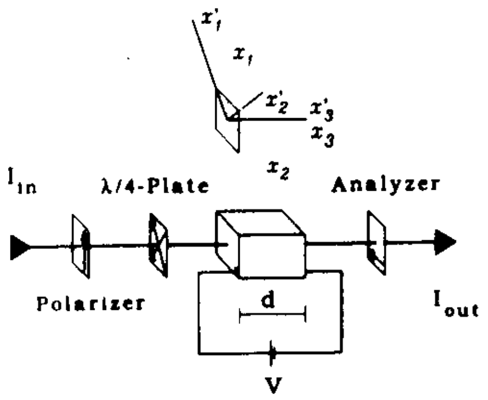
I(t) shape depends on geometry

Fourier spectrum A




t_1 = 0.42 ms

OPTICAL VOLTAGE PROBE



Phase shift $\Gamma = \frac{2\pi d}{\lambda}(\Delta n^{+} - \Delta n^{-})$

For electrooptic crystal:

$$\Gamma = \frac{2\pi}{\lambda} n_0^3 \, r_{41} \, V(x,y) \equiv \text{local voltage.}$$

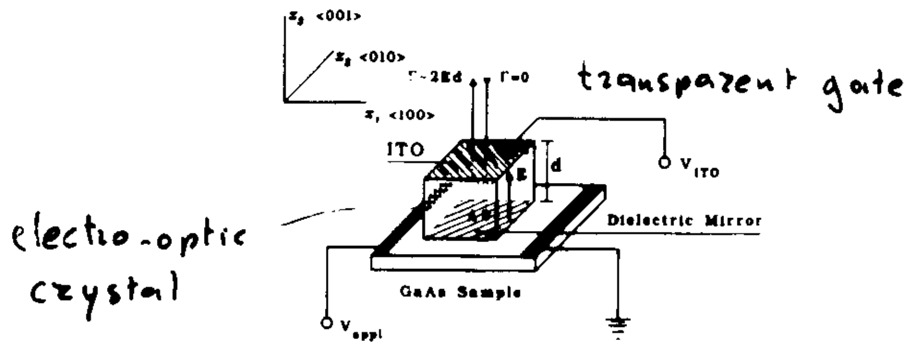
⇑ electro-optic tensor element

$$\Rightarrow \quad \frac{I_{out}}{I_{in}} = \sin^2\left(\frac{\pi}{\lambda} n_0^3 \, r_{41} \, V\right) \quad \text{Without } \frac{\lambda}{4} \text{ plate}$$

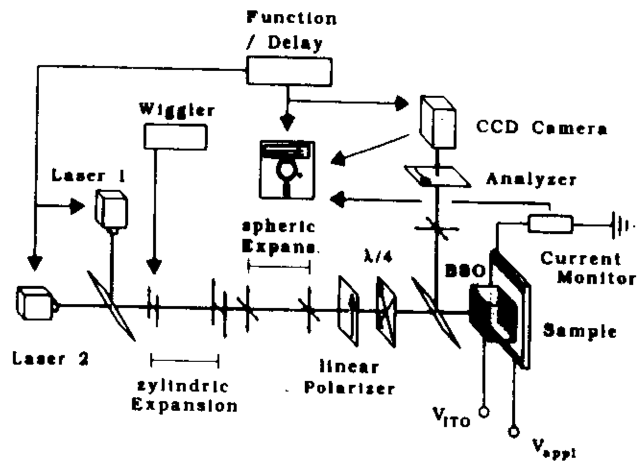
$$I \Rightarrow \text{computer} \Rightarrow \Gamma \Rightarrow V(x,y)$$

LOCAL PROBE

SAMPLE MOUNT



SET UP

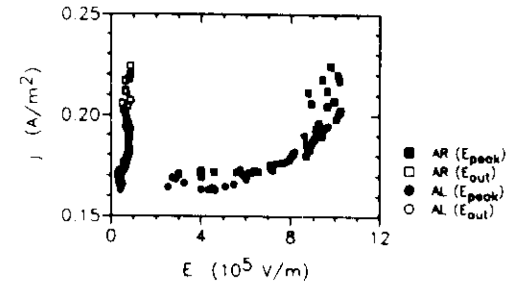


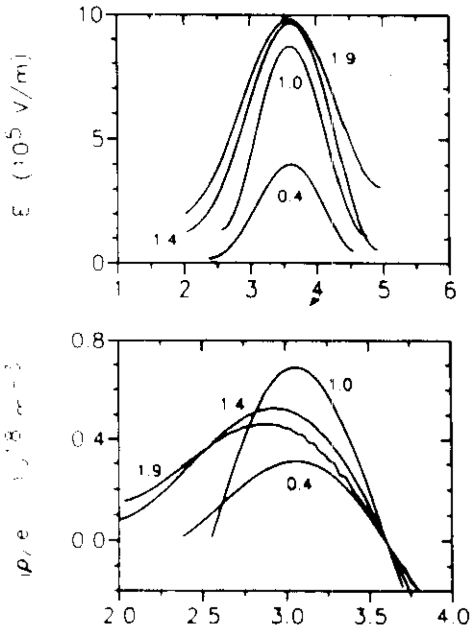
QUANTITATIVE ANALYSIS

From picture $V(x, y)$

$$\Rightarrow \quad E(x,y) = \frac{dV(x,y)}{dx} \quad \Rightarrow \quad j - E \text{ bulk}$$

not accessible before





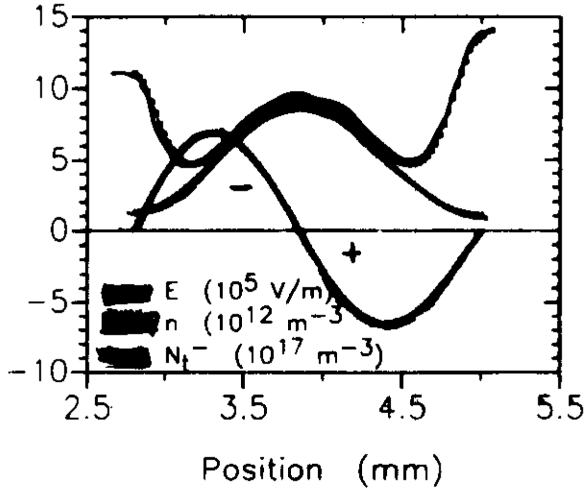
DENSITIES

$$\frac{d^2V(x,y)}{dx^2} = \frac{\rho(x,y)}{\epsilon} = N_t^- + (n - n_o)$$

$$j = \sigma(E)E = nev(E) \cdot E$$

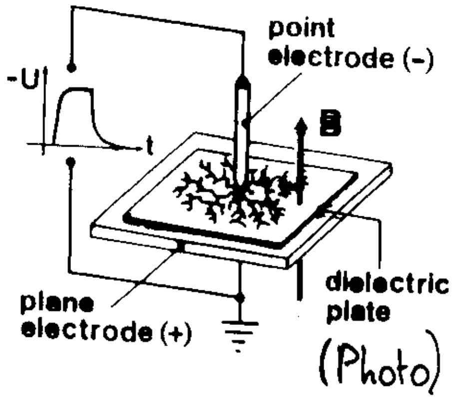
$$\Uparrow$$

From $j - E$ $\Leftarrow$ From literature



SPARKS (DIELECTRIC BREAKDOWN) IN MAGNETIC FIELDS

Set-up



Parameters

V

t_p

medium

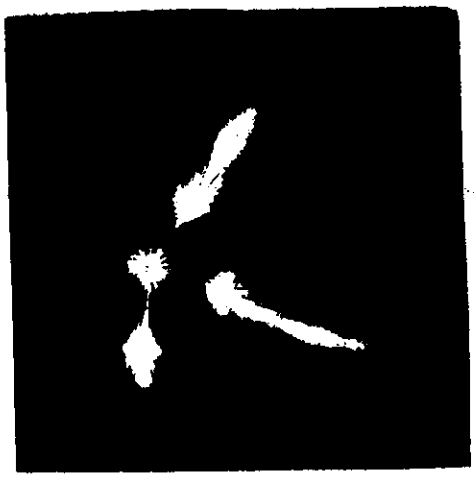
configuration

B

.

.

.



Air

V=-16.5 kV

500 ns

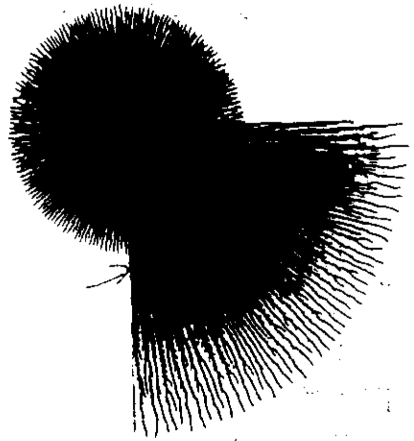
1 cm

? Uhlig 3cm P.Wuler

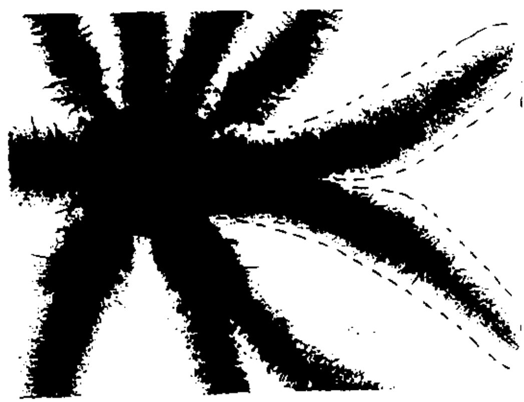
TWO REGIMES; STREAMER AND LEADER

STREAMER	*LEADER*
$V < V_s$	$V > V_s$
Quasi stationary	Avalanche advances with time
Roughly homogeneous	Random
$T_e \simeq 5eV$; $T_{ion} \simeq RT$	$T_e \simeq T_{ion} \simeq 5eV$
$n_e \simeq 10^{17}\ cm^{-3}$	$n_e \simeq 10^{21}\ cm^{-3}$
$E \simeq 15kV/cm$	$E \simeq 0$
(bad conductor)	(good conductor)

V= - 13.6 kV



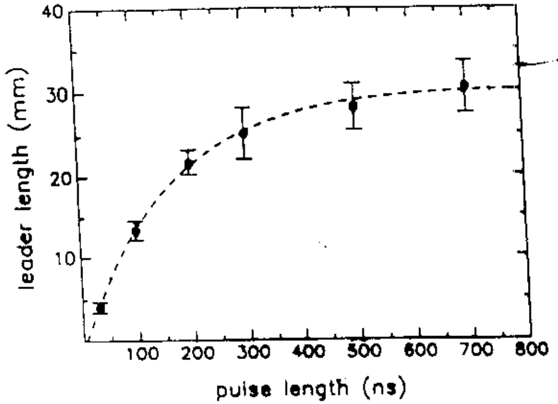
V=-25kV



V=-16 kV

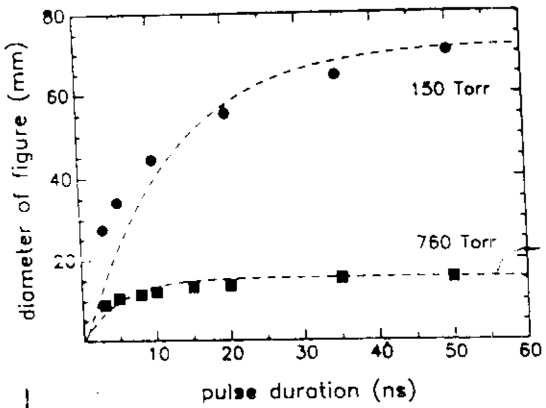
500 ns

STREAMERS AND LEADERS



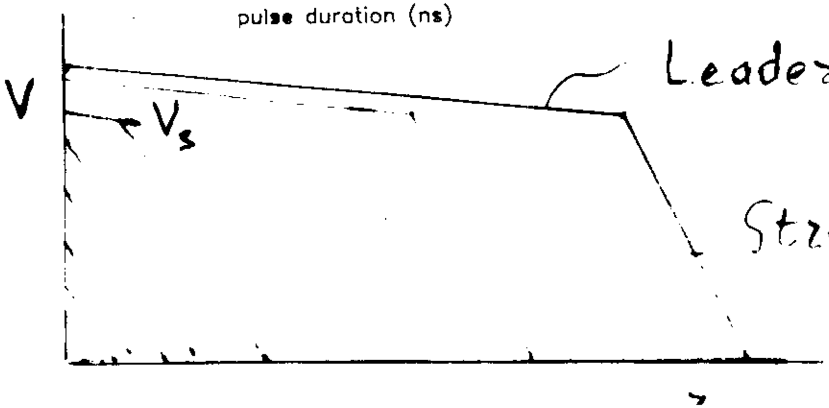
$\frac{dl}{dt} \simeq 10^5$ m/s

Leader



Streamer

$v = \frac{dl}{dt} \simeq c$



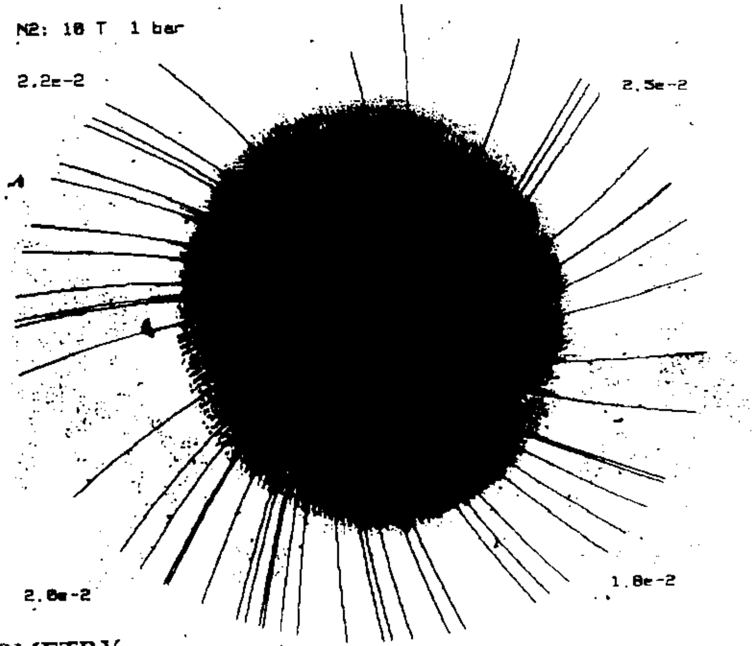
Leader

Streamer

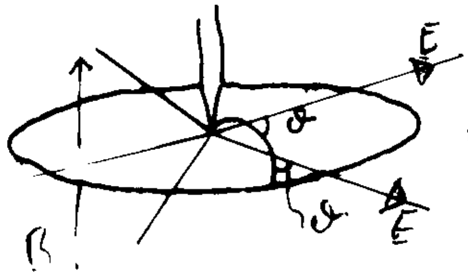
STREAMERS IN A MAGNETIC FIELD

B = 10T

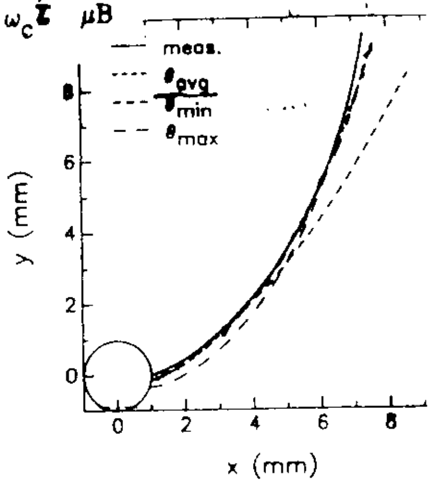
1cm



CORBINO GEOMETRY



$$\mathrm{tg}\,\theta = \frac{1}{\omega_c \tau} = \frac{1}{\mu B}$$

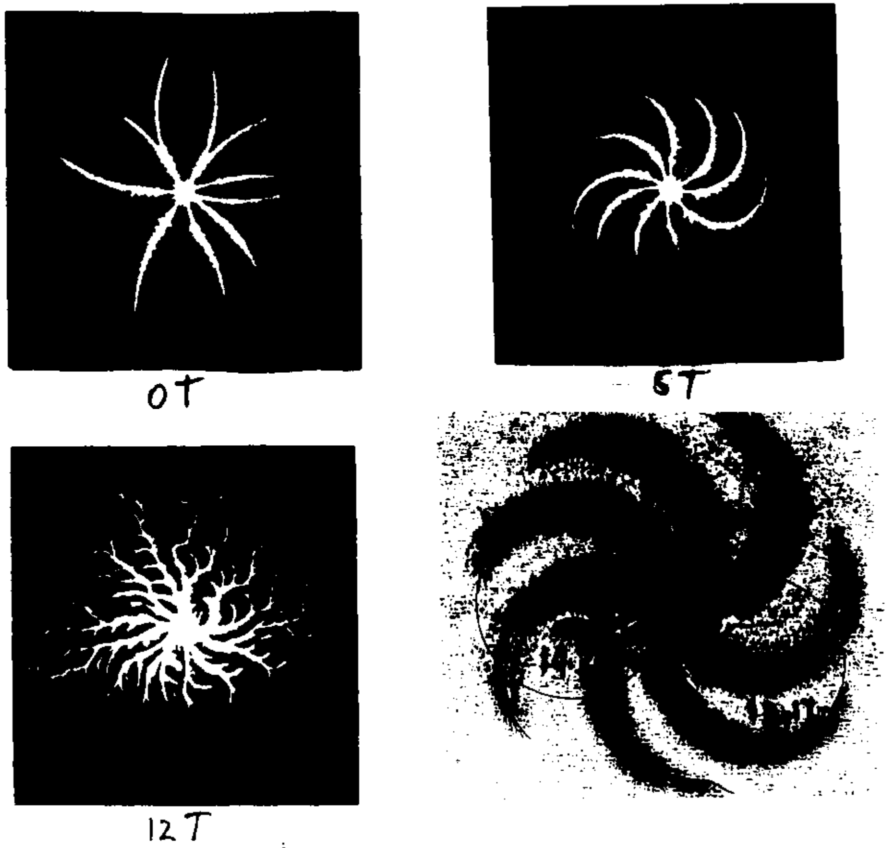


Electron trajectory:

$$r(\phi) = r_0 e^{\left(\frac{\phi - \phi_0}{\mu B}\right)}$$

-> Measure μ in discharge $\simeq$ 500 $\frac{cm^2}{Vs}$

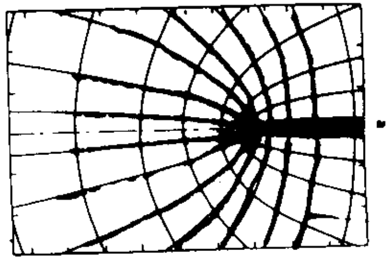
LEADERS IN A MAGNETIC FIELD



Leaders propagate on **circular** trajectories -> Not in a central field.

-> Interference between branches -> increasing complexity.

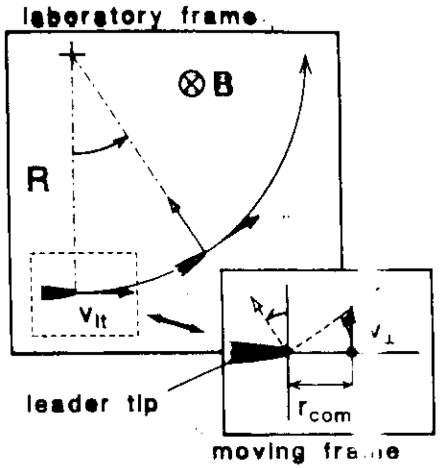
LEADER PROPAGATION



Idealized electric field distribution at leader tip

- Electric field of leader tip is pointed forward
- Predischarge takes places in front of tip due to this field
- Leader propagates in the ionised region which its own field has created.
- Due to Lorentz force the **electrons** in the ionised region will drift also ⊥ to E-field.

-> Circular paths.

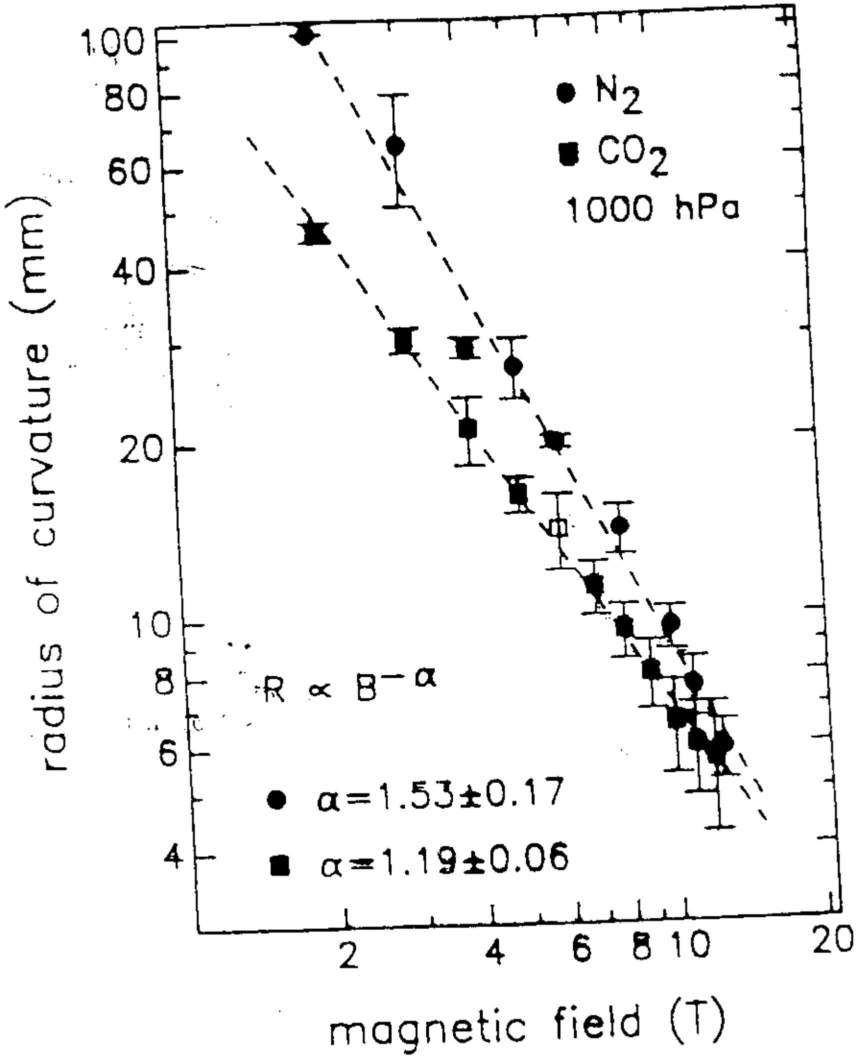


$$R = \frac{v_l}{v_t} r_{com} = \frac{r_{com}}{\mu B}$$

For realistic estimates of r_{com} -> R≃2.5 mm but ≡ B^{-1} measured $B^{-1.5}$

->Polarisation of tip field due to **Hall effect.**

FIELD DEPENDENCE OF RADIUS

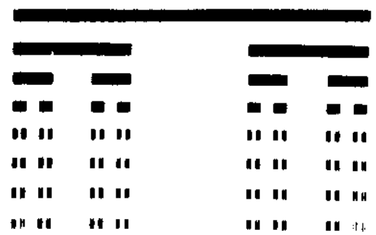


FIELD INDUCED "FRACTAL" BEHAVIOUR

Self - similarity: Patterns look the "same" when viewed at different scales.

or: invariance with respect to scaling.

or: periodicity on logaritmic scale



1D			3D		
x			V		
x'	$\longrightarrow ax$	$\longrightarrow \log ax$	V'	$\longrightarrow a^3 r$	$\longrightarrow 3 \log ar$
x''	$\longrightarrow a^2 x$	$\longrightarrow 2 \log ax$	V''	$\longrightarrow a^6 r'$	$\longrightarrow 6 \log ar$
x'''	$\longrightarrow a^3 x$	$\longrightarrow 3 \log ax$	V'''	$\longrightarrow a^9 r'''$	$\longrightarrow 9 \log ar$

Hausdorf dimension $d_f = \dfrac{\log f}{\log a}$ f is fraction filled with object.

in 1D $d_f = \dfrac{\log \frac{ar}{r}}{\log a} = 1$ 3D $\dfrac{\log \frac{a^3r^3}{r^3}}{\log a}$ 3

For Cantor set: $d_f = \dfrac{\log 0.5}{\log 0.33} = 0,63... < 1$.

Total length of line segments increases slower than Euclidean dimension. "Empty figure"

M. R. Schroeder, "Numbertheory in Science and Communication", Springer, 1986

APPLICATION TO SPARKS

At B = 0 discharges described by DLA model

- Each point has certain chance < 1 to grow
- Probabilities distributed randomly

For 2D

$d_f \simeq 1.7$

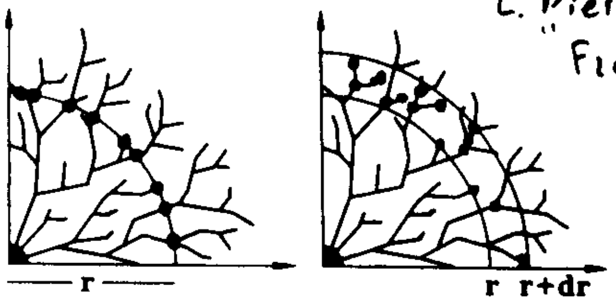
L. Niemeyer, L. Pietronero, H.J. Wiesman PRL 52, 1033, 1984

Statistical analysis d_f obtained from

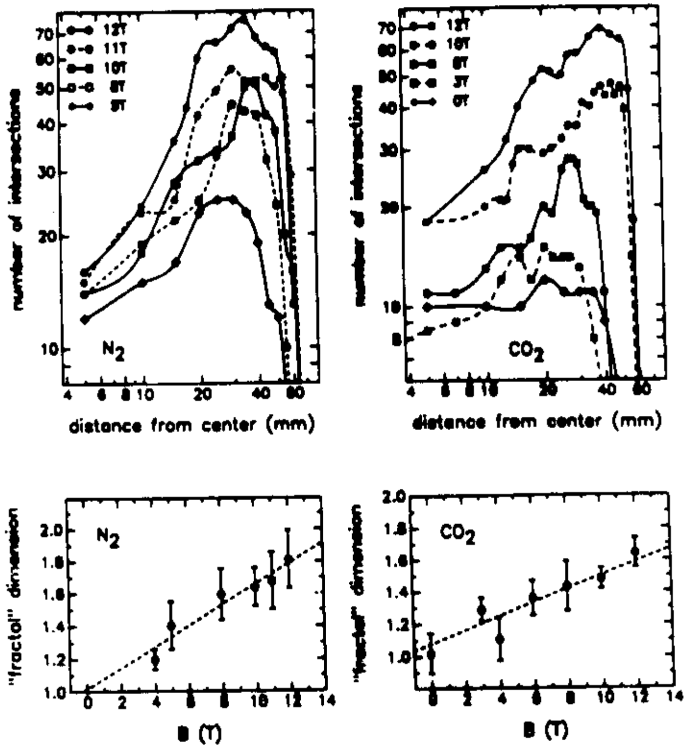
$n(r) \quad \alpha \quad r^{d_f - 1}$ number of intersections at r

$\rho_{BE}(r) \quad \alpha \quad r^{d_f - 2}$ density branching and endpoints

H.J. Wiesman, L. Pietronero, "Fractals in Physics" 1986



"FRACTAL" DIMENSION



$\lim_{B \to 0} = 1$

$\lim_{B \to \infty} \simeq 1.7 \dots 2$

J.C.M. P. Uhlig. B. Willing. Physica

NON LINEAR TRANSPORT
IN
SEMICONDUCTORS II

THz response in Quantum Structures.

- Photon assisted tunneling
- Classical versus Quantum detection
- THz Response doublebarriers resonant tunneling device
- THz response Quantum Point contacts

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THz RESPONSE IN QUANTUM STRUCTURES

Interaction EM-radiation with light.

Power detection · Signal $\equiv$ I $\equiv$ (Amplitude)^2

- Photoconductive $\sigma = ne\ \mu$

$$\Uparrow \quad \Uparrow$$

and/or

Interband (h $\omega > E_g$)

Intersubband (h $\omega > E_1 - E_0$)

Carrier heating $\mu = \mu(E)$

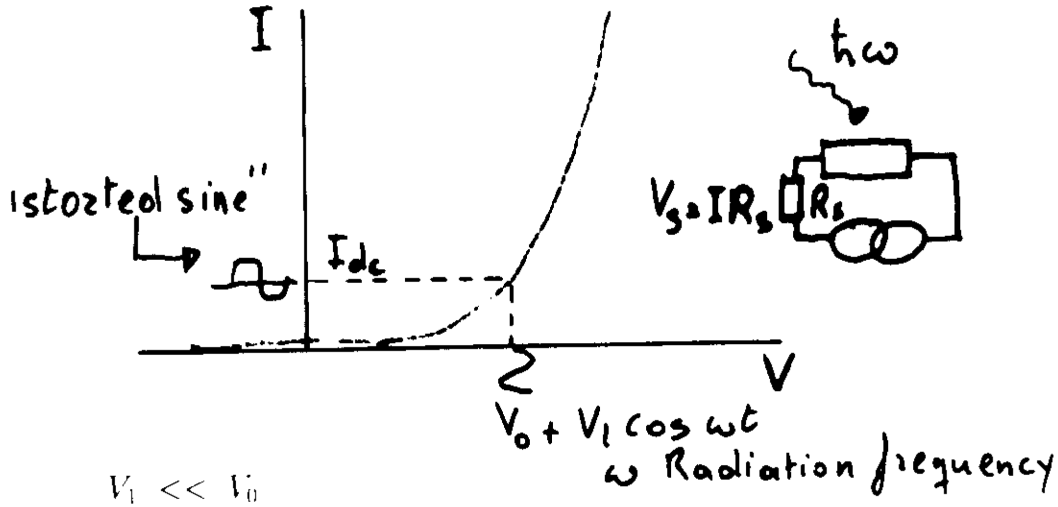
- Bolometric $R = R(T)$
- Phononic (Phonon - drag)
- Classical rectification

Quantum detection signal (Amplitude^2 $h\omega$)

- Photon assisted tunneling

CLASSICAL RECTIFICATION

Non - linear IV $\begin{pmatrix} \text{Classical} & : & \text{diode, heating} \\ \text{Q.M.} & : & \text{tunnel junction} \end{pmatrix}$



$V_1 \ll V_0$

Taylor expansion

$$I(V) = I_{dc}(V_0) + V_1 \cos\omega t \, \frac{dI_{dc}}{dV} + \frac{V_1^2}{2} \cos^2 \omega t \, \frac{d^2 I_{dc}}{dV^2} + \ldots\ldots$$

$V_S = IR_S$ for "slow" Voltage detector

$$V_S = V_S(V_1 = 0) - V_S(V_1 \neq 0) \equiv \frac{V_1^2}{2} \cdot \frac{\int_0^{2\pi/\omega} \cos^2 \omega t \, dt}{\int_0^{2\pi/\omega} dt}$$

- Time averaged Signal $\equiv$ (amplitude)^2

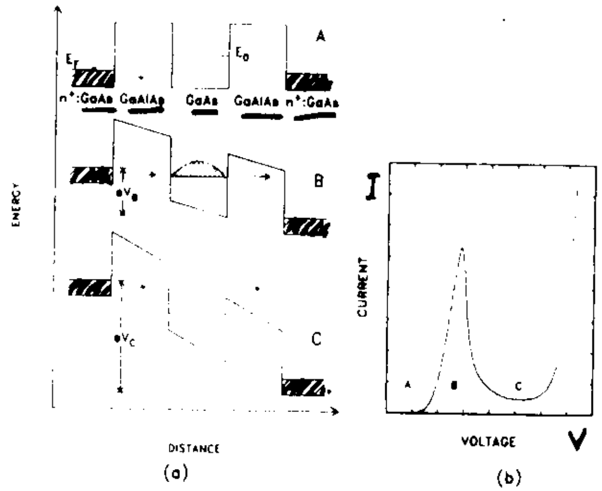
  But also mixing frequency doubling etc.

- Independent of origin non-linearity.

NON-LINEAR I - V BECAUSE OF TUNNELING

Example: double barrier resonant tunneling device. (DBRTD)

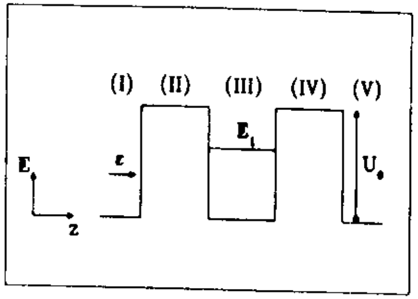
DC response



Non-ohmic response ⇒
Use as microwave mixer.
frequency doubler. Power detector

DC RESONANT TUNNELING

Schematic structure



Hamiltonian $\perp$

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dz^2} + U(z)\right]\psi(z) = \epsilon\psi(z)$$

General solution $\quad l \to z \quad z \to l$

$$\psi_i(z) = A_i \exp(ik_i z) + B_i \exp(-ik_i z)$$

i = I,II,III etc. $\perp$ $\qquad \epsilon = \epsilon_\perp + \frac{\hbar^2 k_\parallel^2}{2m}$

$$k_i = \left(\frac{2m(\epsilon - U_i(z))}{\hbar^2}\right)$$

Match wavefunctions and current continuity at interfaces:

$$j_z(z_{ij}) = \frac{i\hbar}{2m}\left(\psi\frac{d\psi^*}{dz} - \frac{\psi^* d\psi}{dz}\right)_{ij}$$

For each $k_\parallel$ (// momentum conservation)

TRANSMISSION COEFFICIENT AND CURRENT

I ⇒ V $\quad$ Transfer matrix method

$$\begin{pmatrix} A_I \\ B_I \end{pmatrix} = M_T \begin{pmatrix} A_V \\ B_V \end{pmatrix} \qquad \text{with}$$

$$M_T = M_I M_{II} M_{III} M_{IV} \qquad \text{for each interface}$$

Where

$$M_i = \frac{1}{2k_i}\begin{pmatrix} (k_i + k_{i+1})\exp[i(k_{i+1} - k_i)z_i] & (k_i - k_{i+1})\exp[-i(k_i + k_{i+1})z_i] \\ (k_i - k_{i+1})\exp[i(k_i + k_{i+1})z_i] & (k_i + k_{i+1})\exp[-i(k_i - k_{i+1})z_i] \end{pmatrix}$$

Defining the transmissioncoefficient

$$T(\epsilon) = \frac{j_t}{j_i} = \frac{k_V}{k_I}\frac{|A_V|^2}{|A_I|^2} = \frac{k_V}{k_I}|M_T|^2 \quad \text{since } B_V = 0$$

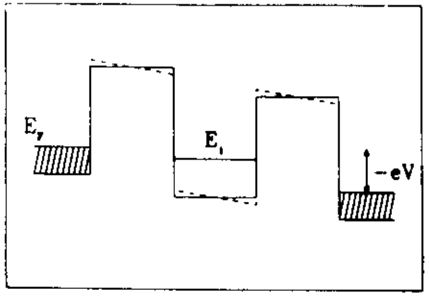
The current is obtained from

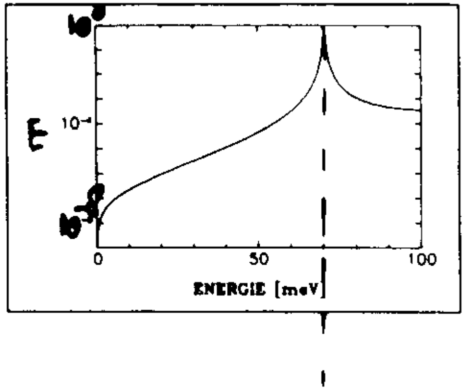
$$J = \frac{e}{4\pi^3\hbar}\int d^2k_\parallel d\epsilon [f(E) - f(E + eV)] T(\epsilon) \quad \text{Transmission coefficient}$$

⇑ Conservation in plane momentum $\quad$ ⇑ Fermifunction $\quad$ ⇑ Energy difference left - right

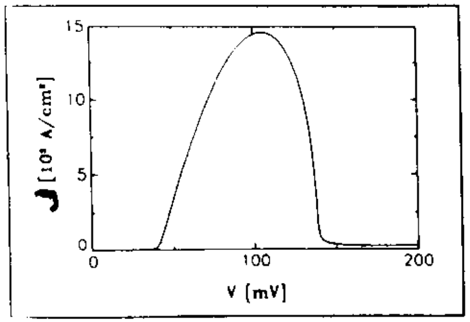
Sum over all $k_\parallel$ values

TRANSMISSION COEFFICIENT



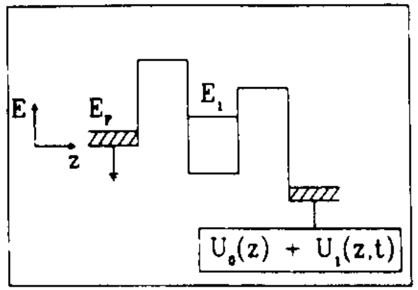


Sharp peak at Resonance



Broadened because of combination

AC RESONANT TUNNELING



HF field modulates U with $H_1 = U_1 \cos \omega t$

$\Uparrow$

Radiation frequency

Hamiltoniaan

$$i\hbar \frac{\partial \psi}{\partial t} = (H_0 + H_1)\psi \qquad H_0 = \frac{-\hbar^2}{2m^*} \frac{\partial^2 \psi}{\partial x^2} + U_0(z)$$

General solution:

$$\psi = \left[A(\epsilon)e^{ikz} + B(\epsilon)e^{-ikz}\right] e^{\left(-\frac{i\epsilon t}{\hbar} - \frac{iu_1}{\hbar\omega}\sin\omega t\right)}$$

$$= - - - - - - - - - - - - - - - - e^{-\frac{i\epsilon t}{\hbar}} \sum_{n=-\infty}^{n=\infty} J_n(\alpha) e^{-in\omega t}$$

$$= \sum_{n=-\infty}^{n=\infty} \left[A(\epsilon \pm n\hbar\omega)e^{ikz} + B(\epsilon \pm n\hbar\omega)e^{-ikz}\right] e^{-\frac{i(\epsilon \pm n\hbar\omega)t}{\hbar}} J_n(\alpha)$$

$$\alpha = \frac{u_1}{\hbar\omega} = \frac{\text{Amplitude}}{\text{Quantumenergy}} \quad j \quad J_n \quad \text{Bessel function n}$$

ψ = linear combination of plane waves with components $\epsilon, \epsilon \pm \hbar\omega, \epsilon \pm 2\hbar\omega$ with weight given by J_n

D.D. Coon, H.C. Liu J. Appl. Phys. 58, 2230, 1985

AC TRANSMISSION COEFFICIENT

Like in DC - case but:

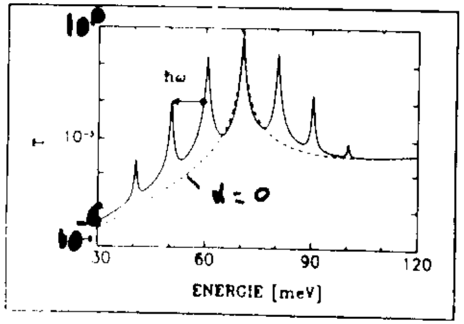
$$A_i(\epsilon) \quad \Rightarrow \quad \begin{pmatrix} \\ A_i(\epsilon + \hbar\omega) \\ A_i(\epsilon) \\ A_i(\epsilon - \hbar\omega) \\ A_i(\epsilon - 2\hbar\omega) \\ \\ \end{pmatrix}$$

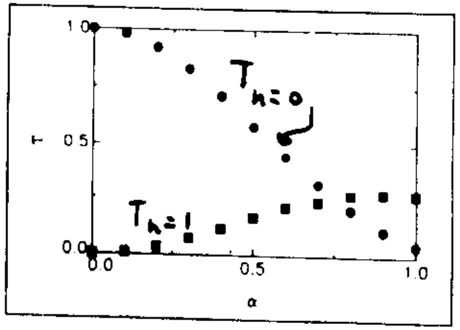
and match wavefunctions at interfaces for the same n

Weight of n-th component given by $J_n(\alpha) \;=\; J_n\left(\frac{u_1}{\hbar\omega}\right)$

$\alpha \;\Rightarrow\; 0 \;\Rightarrow\;$ only $A_i(\epsilon)$ relevant.

$\alpha \;\neq\; 0 \;\Rightarrow\;$ $n = 1, 2, ...$ components increase weight

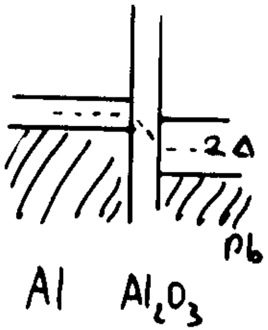


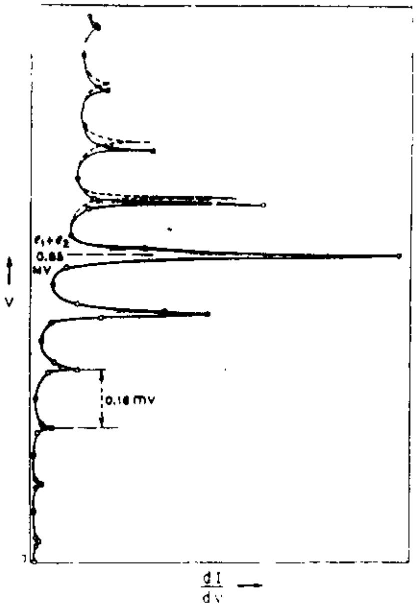


Additional transmission peaks at $\pm\hbar\omega$ appear.

"HISTORICAL EXPERIMENT"

Superconductor - isolator - superconductor





V

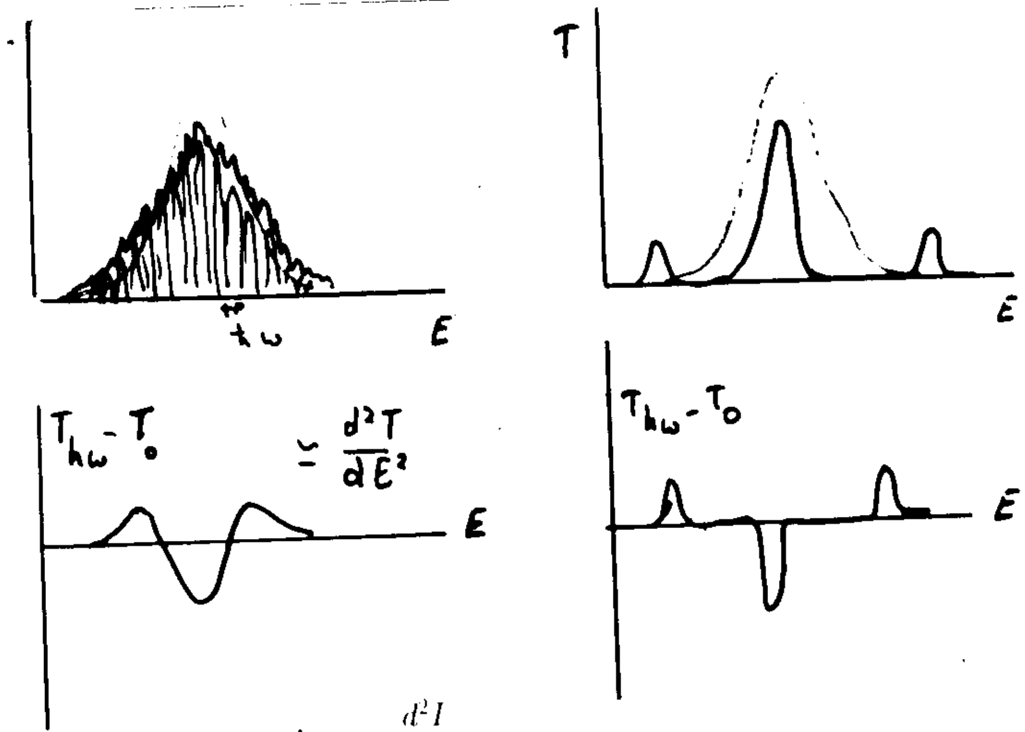
– without

– with

I

$\omega = 24$ GHz $\qquad \omega = 63.$ GHz

CLASSICAL $\Leftrightarrow$ QUANTUM DETECTION



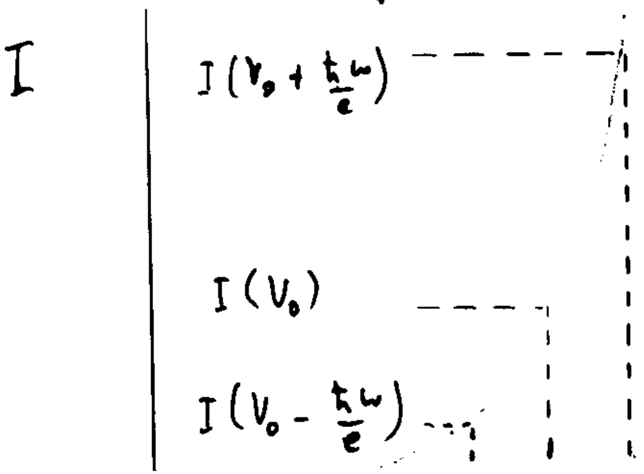
$\simeq I_{h\omega} - I_0 \simeq \frac{d^2 I}{dV^2}$

$h\omega << \Gamma$

$h\omega >> \Gamma$

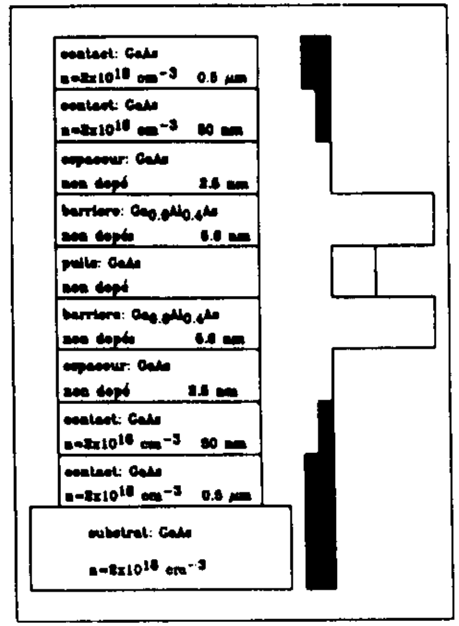
Tucker criterium

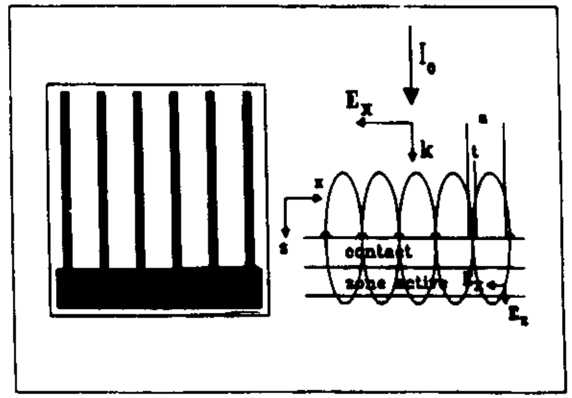
$$1 \simeq \left| \frac{I_{dc}\left(V_0 + \frac{\hbar\omega}{e}\right) - I_{dc}(V_0)}{I_{dc}(V_0) - I_{dc}\left(V_0 - \frac{\hbar\omega}{e}\right)} \right| >> 1$$



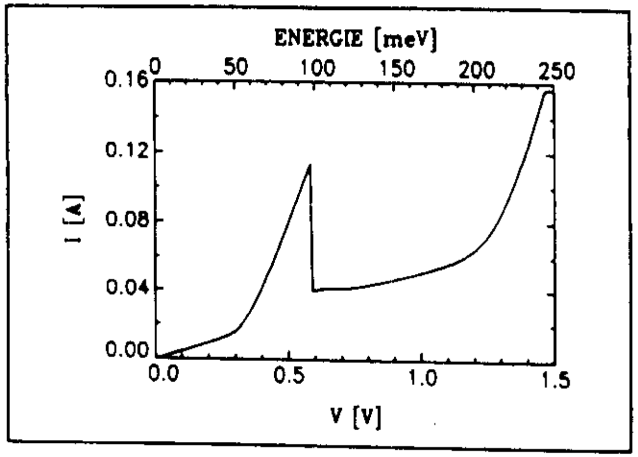
J. Tucker
IEEE-QE 15, 1235, 1979

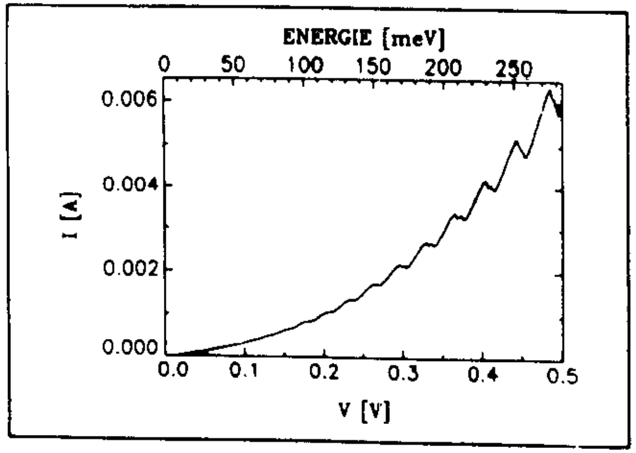
DEVICES STUDIED



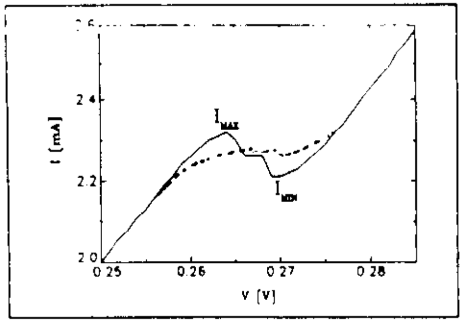


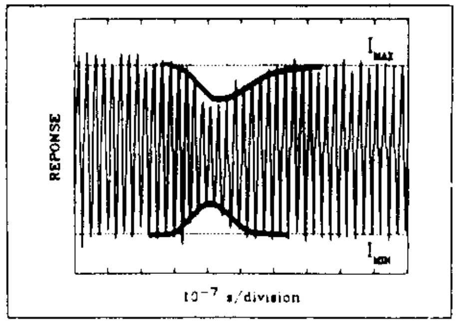
I - V - CHARACTERISTIC



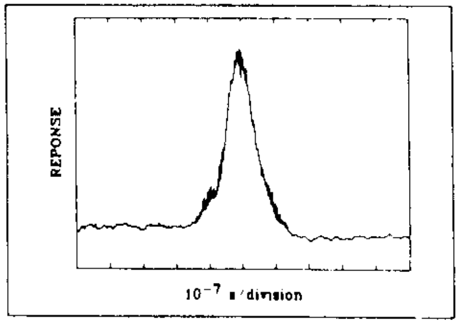


SUPPRESSION HF OSCILLATION



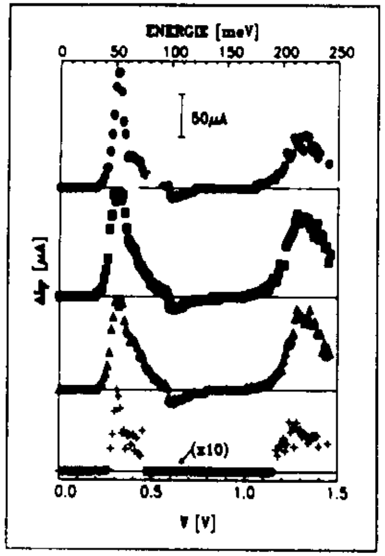


FIR - pulse



V. Chitta, C. Kutter, R.E.M de Bekker, JCM, S.J. Hawkswor

"CLASSICAL" RESULTS



$\hbar\omega$

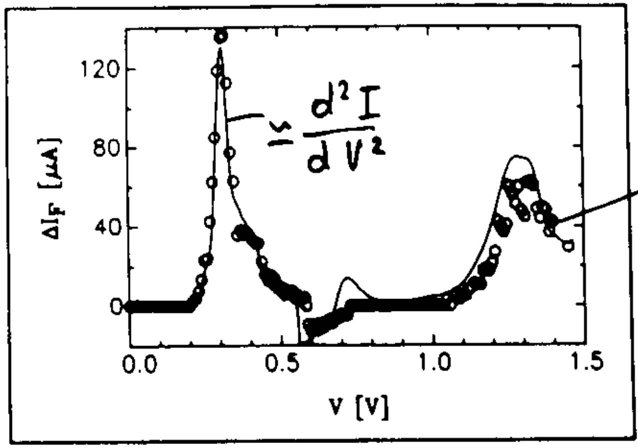
13.6 meV

8.2 meV

4.2 meV

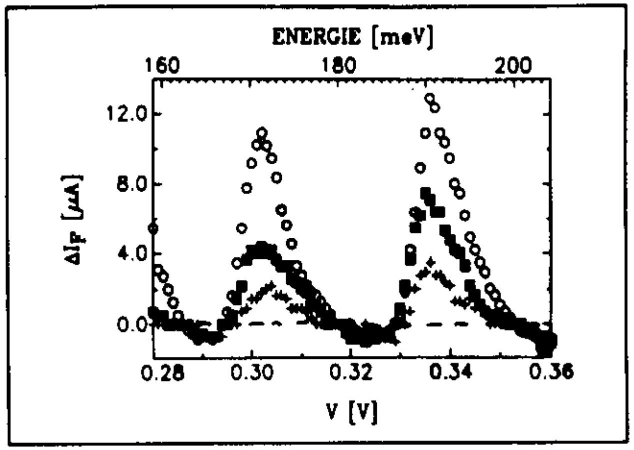
2.5 meV

No frequency dependence



measured signal

NOT SO CLASSICAL RESULTS



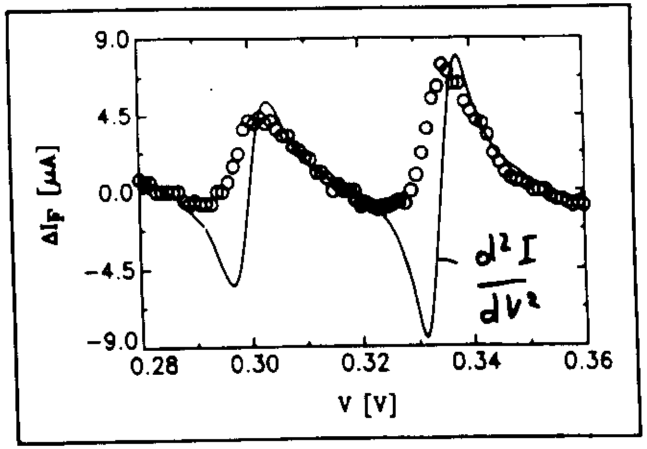
$\hbar\omega$

+ 2.5 meV

■ 13.6 meV

○ 13.6 meV 2 × power

No ω dependence Asymmetric signal



$$\text{Not} \equiv \frac{d^2 I}{dV^2}$$

Rectification at THz frequencies in quantum point contacts

T.J.B.M. Janssen

J.C. Maan
High Field Magnet Laboratory
University of Nijmegen
The Netherlands

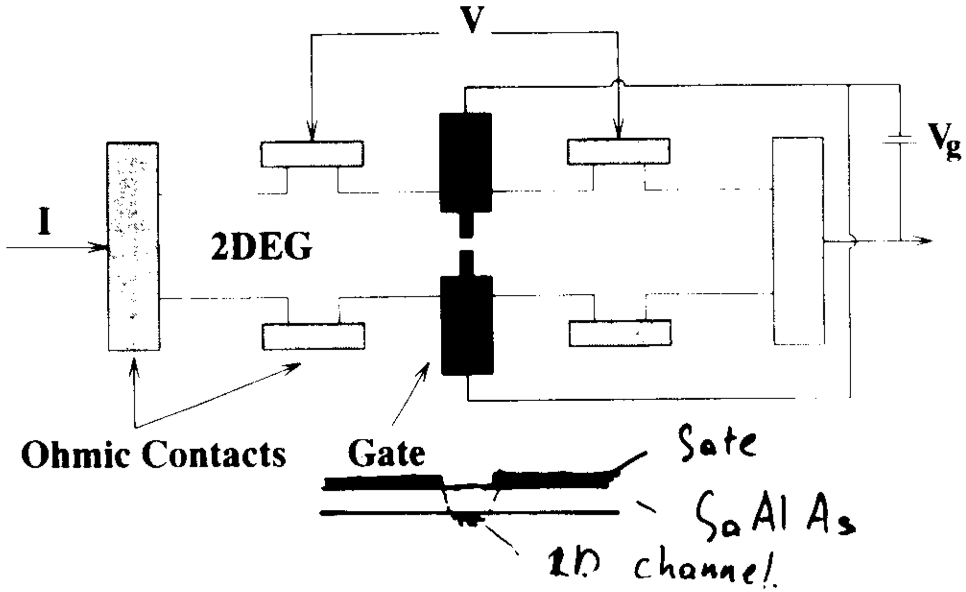
J. Singleton
Clarendon Laboratory
University of Oxford

N.K. Patel, M. Pepper
Cavendish Laboratory
Unversity of Cambridge

MAGNET LABORATORY

Experimental Details

Device

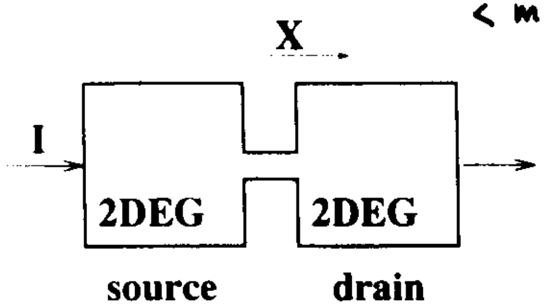


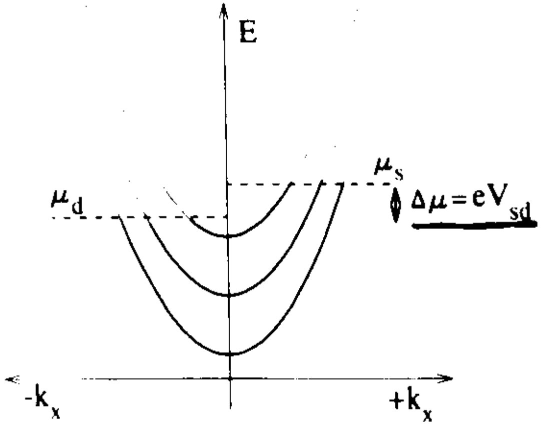
GaAs-Ga_xAl_{1-x}As Heterojunction;

2DEG with $N_s = 2.0 \times 10^{11}$ cm^{-2}
$\mu = 1 \times 10^6$ cm^{2}V^{-1}s^{-1} at 4 K
Split gate: 0.5 μm $\times$ 0.5 μm

Ballistic Transport in a QPC

Width $\approx \lambda_F \Longrightarrow$ size quantization
< mf path





$I_n = e v_n g_n \Delta\mu$; $\quad v_n = (dE_n/\hbar dk)$; $\quad g_n = (\pi dE_n(k)/dk)^{-1}$

$\Longrightarrow I_n = (2e/h)\Delta\mu$; $\quad \boxed{G = N_c 2e^2/h}$

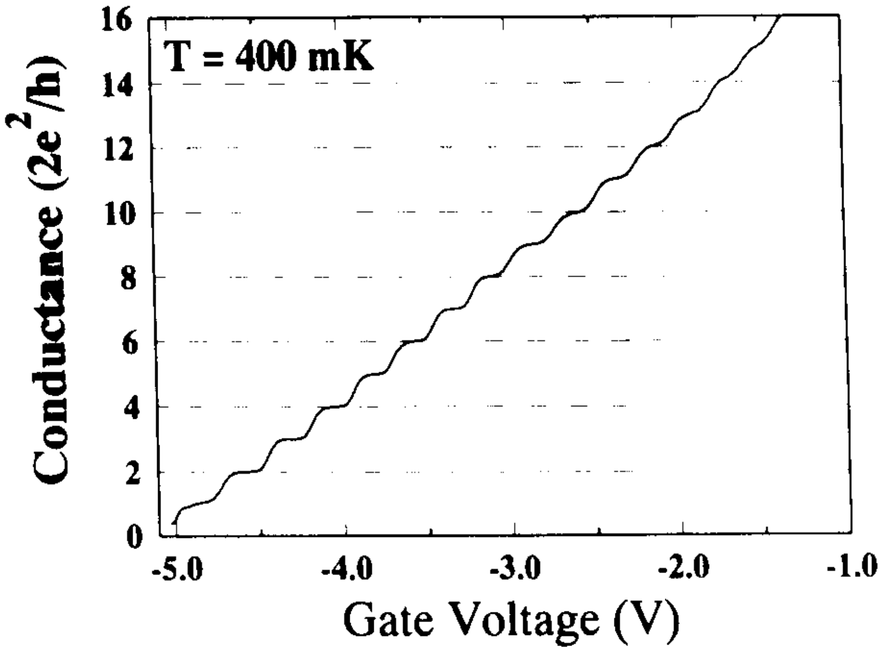
per subband.

Number of channels

No radiation

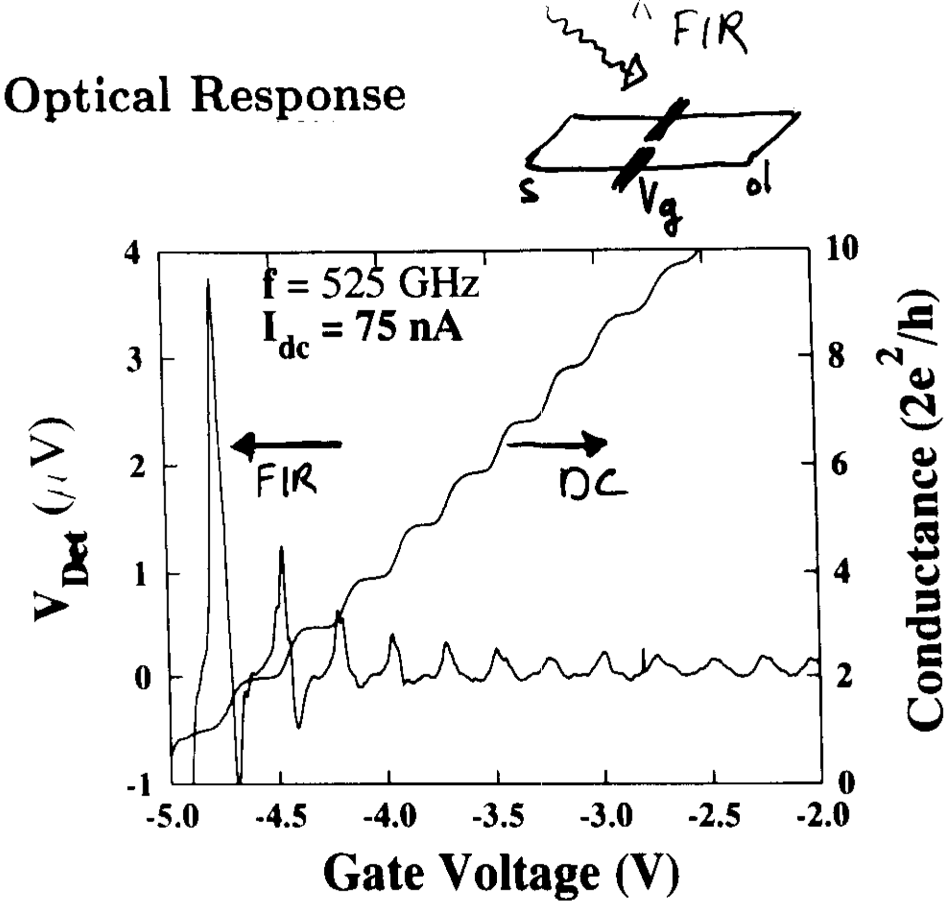
Linear regime: $I_{ac} = 5$ nA

$eV_{sd} \ll \Delta E_n$



16 plateaus $\Longrightarrow$ good quality

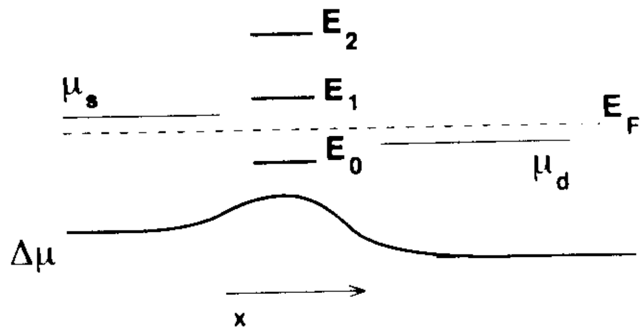
Optical Response



- Oscillations line-up with conductance steps
- No frequency dependence, FIR
- Response disappears above 2.5 THz (~ 10 meV)

Nonlinear Transport in a QPC

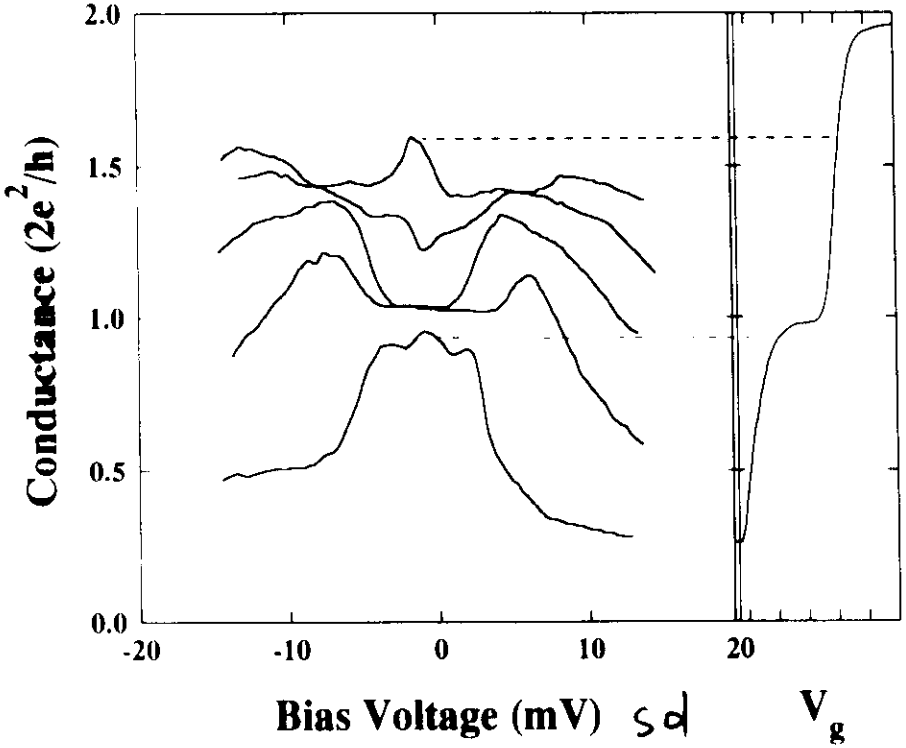
- Occurs when eV comparable to subband separation
- Assume ballistic transport
- Source-drain bias dropped symmetrically

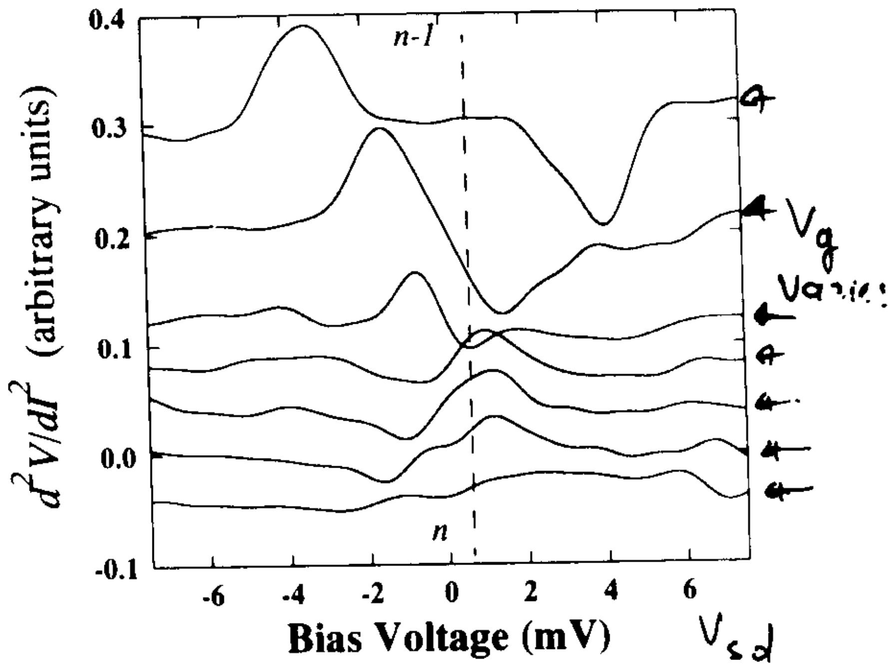


$I_n = (2e/h)[\mu_s - \max(\mu_d, E_n)]$; $g = \frac{dI_n}{dV}$

and $\mu_s = E_F + \frac{eV}{2}$; $\mu_d = E_F - \frac{eV}{2}$

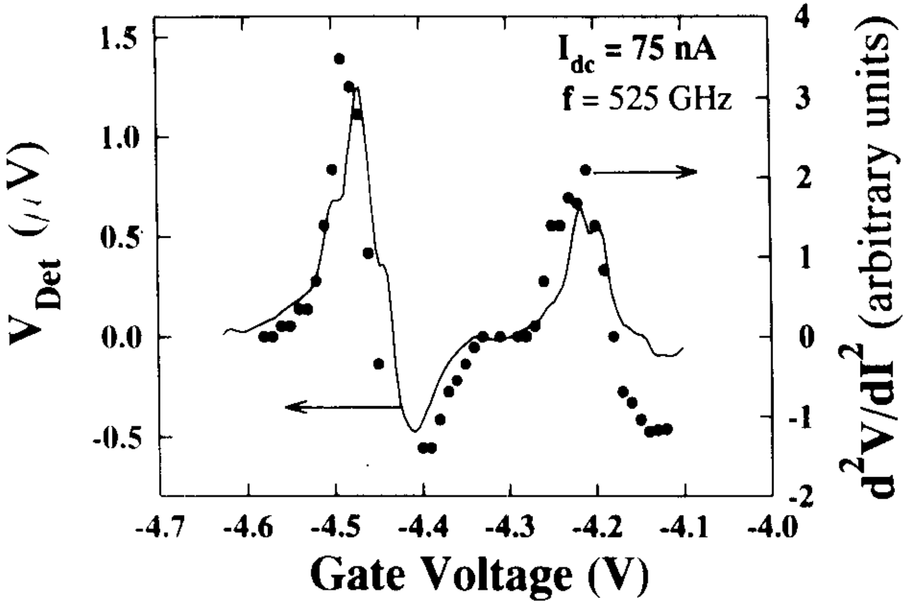
Nonlinear transport results





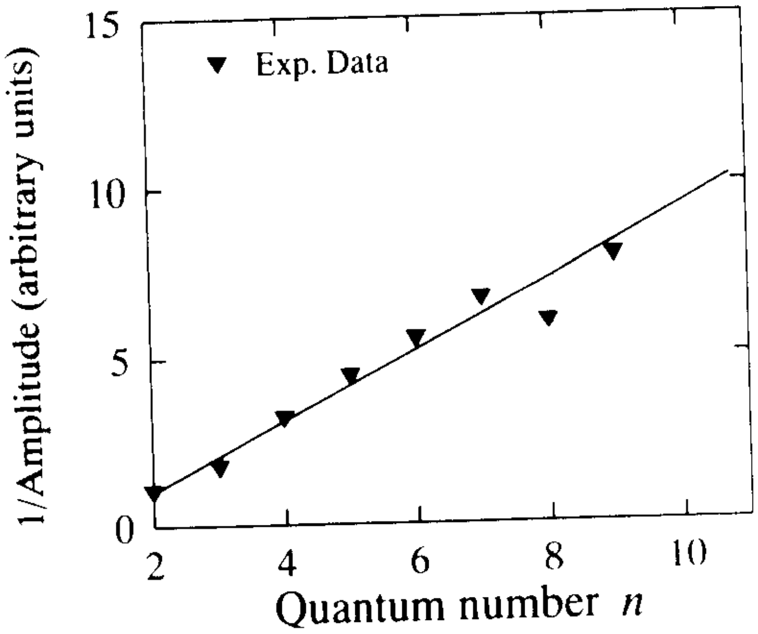
$(I\text{-}V)_{sd}$ non ohmic and depends on V_g

$\left(\frac{d^2V}{dI^2}\right)_{sd} \neq 0$, depends on V_g



Good agreement between second-derivative and optical response

$\Longrightarrow$ **Classical Rectification**



$d^2V/dI^2 = RdR/dV$: R is quantised

dR/dV = non-zero at zero bias

$\Longrightarrow$ quantised response, linear in N as observed.