



INTERNATIONAL ATOMIC ENERGY AGENCY
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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE CENTRATOM TRIESTE



SMR.769 - 13

**WORKSHOP ON
"NON-LINEAR ELECTROMAGNETIC INTERACTIONS
IN SEMICONDUCTORS"**

1 - 10 AUGUST 1994

"Semiconductor Bulk Media and Surfaces"

E. BURSTEIN
Physics Department
University of Pennsylvania
Philadelphia, PA 19104-6396
U.S.A.

These are preliminary lecture notes, intended only for distribution to participants

MAIN BUILDING STRADA COSTURA, 11 TEL. 22401 TELEFAX 224163 TELEX 460392 ADRIATICO GUEST HOUSE VIA GRIGNANO, 9 TEL. 224241 TELEFAX 224531 TELEX 467
MICROPROCESSOR LAB. VIA BEIRUT, 31 TEL. 224471 TELEFAX 224600 TELEX 460392 GALILEO GUEST HOUSE VIA BEIRUT, 7 TEL. 22401 TELEFAX 2240310 TELEX 46

Inelastic Light Scattering by Electronic Excitations ("Electronic Raman Scattering")

A. Pinczuk

M.Y. Tsang

E. Burstein

Electronic Excitations

Intraband, Interband

Single Particle non-spin flip; collective Charge density

Single Particle spin-flip; collective spin density

Objectives

Elucidate physics underlying ILS as Phenomena

Use ILS as spectroscopic probe of electronic excitations $\omega(q)$ and via excitation profile measurements obtain information about electronic structures

① ILS by "free" carrier excitations in bulk (3-D) Semiconductors; Resonant ILS and mechanisms

② ILS by electronic excitations of 2-D confined electrons (holes) in surface space charge regions; hetero-structure (Quantum Wells and Superlattices).

Focus on Resonant Inelastic Light Scattering (R-ILS)

Outline

Resonant Inelastic Light Scattering

Historical Perspective

Progress during the last 25 years
material synthesis, MBE etc

Instrumentation

Theory (condensed matter)

Major Emphases

Focus on underlying concepts!

Microscopic mechanisms, Selection rules

Incident and Scattered Radiation within
media correspond to phonons.

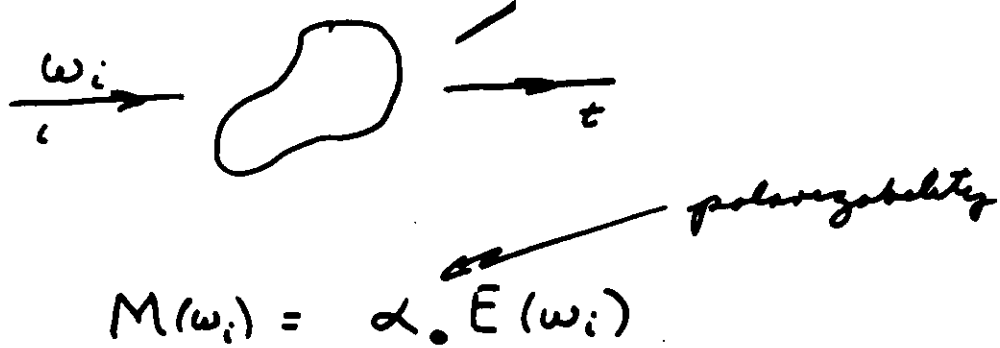
ILS (electronic Raman Scattering) is inelastic

- 2-photon process
 - { 2-step
 - 3-step
- scattering of light by s.p. and collective
electronic excitations and both intraband
and interband electronic excitations -
in Bulk and 2-D heterostructures

Resonant ILS - greatly enhanced scattering
intensities. Enables one to observe
normally forbidden (e-d) scattering
processes.

$$\begin{aligned} \langle \phi_u | & \langle u_\beta | H_{\text{int}} | u_\alpha \rangle \langle \phi_u \rangle \Rightarrow \langle u_\beta | H_{\text{int}} | u_\alpha \rangle \langle \phi_\beta | \phi_\alpha \rangle \\ & + \\ & \langle \phi_\beta | H_{\text{int}} | \phi_\alpha \rangle \langle u_\beta | u_\alpha \rangle \end{aligned}$$

↙ interband
↑ interband



oscillating electric dipole radiates $\rightarrow$ Rayleigh Scattering
 $\boxed{\omega_s = \omega_i}$

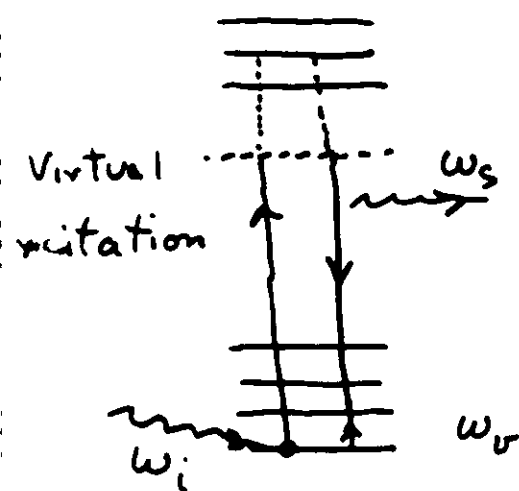
(molecule) $\alpha = \alpha_0 + \frac{\partial \alpha}{\partial u} u(\omega_v) + \dots$

$M(\omega_s) = \frac{\partial \alpha}{\partial u} u(\omega_v) E(\omega_i)$ Raman Scattering

$\boxed{\omega_s = \omega_i \mp \omega_v}$ - Stokes
+ anti-Stokes

$I(\omega_s) \propto \left| \frac{\partial \alpha}{\partial u} u \right|^2 I_0$ $I_0 \propto E_i^2$

In terms of electronic and vibrational levels of molecule



- incident photon induces excitation of e to virtual intermediate state (induced elec. dipole). From virtual state e returns to ground electronic state but different vibrational level

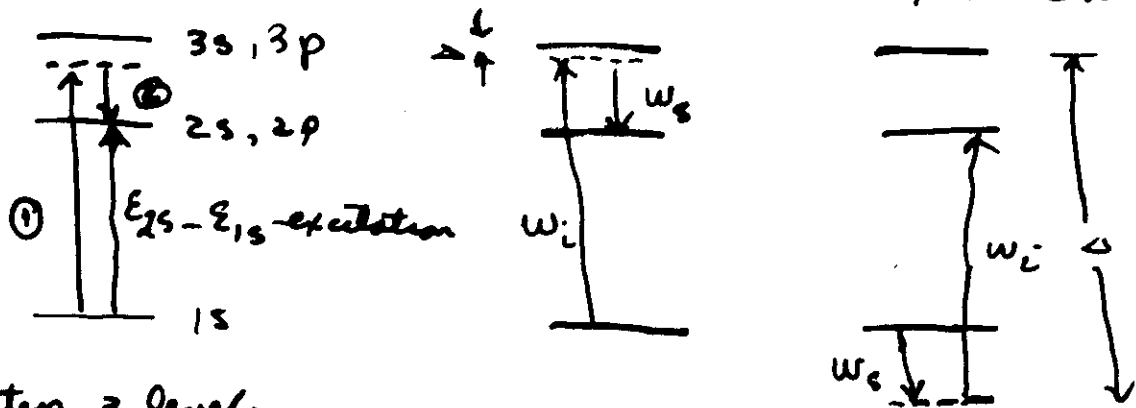
$\omega_i = \omega_s + \omega_v$

$\omega_s = \omega_i - \omega_v$ Stokes RS

Basics of Electronic Raman Scattering

Heitler "Quantum Theory of Radiation"

Raman scattering by atoms (H atom $1s \rightarrow 2s$)
excitation



2 step, 3 levels

2nd order time dependent perturbation theory

$$d_{1s \rightarrow 2s} \propto \frac{\langle 2s | P_z | 3p \rangle \langle 3p | P_z | 1s \rangle}{E_{3p} - (E_{1s} + \hbar\omega_i)} + \frac{\langle 1s | P_z | 3p \rangle \langle 3p | P_z | 2s \rangle}{E_{3p} - (E_{1s} - \hbar\omega_s)}$$

transition polarizability

THE POSSIBLE OBSERVATION OF ELECTRONIC RAMAN TRANSITIONS IN CRYSTALS

R. J. ELLIOTT

Clarendon Laboratory, Oxford, England

and

R. LOUDON

Royal Radar Establishment, Malvern, England

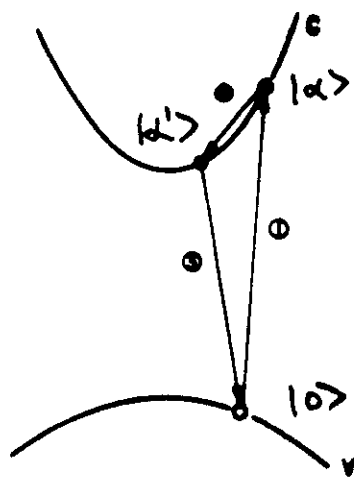
Received 8 December 1962

The Raman transition probability is obtained by second-order perturbation theory ⁴⁾, and is proportional to the square of the matrix element for an isolated atom:

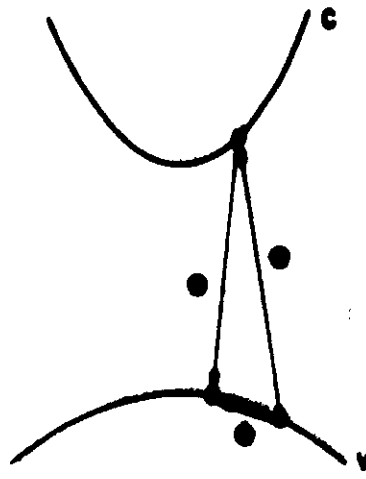
$$\langle i, n_0, n_1 | F | f, n_0-1, n_1+1 \rangle$$

$$= n_0^{\frac{1}{2}} \sum_j \left\{ \frac{\langle i | \epsilon_0 \cdot \mathbf{p} | j \rangle \langle j | \epsilon_1 \cdot \mathbf{p} | f \rangle}{E_j - E_i - \hbar\omega_0} + \frac{\langle i | \epsilon_1 \cdot \mathbf{p} | j \rangle \langle j | \epsilon_0 \cdot \mathbf{p} | f \rangle}{E_j - E_i + \hbar\omega_1} \right\} \quad (1)$$

where n_0 , n_1 are the occupation numbers of the incident and emitted photons with polarisation vectors ϵ_0 , ϵ_1 and wave-vectors q_0 , q_1 . Finally $\mathbf{p}$ is the electronic momentum operator and j denotes the electronic intermediate states (excluding the initial and final states i and f).



(a)

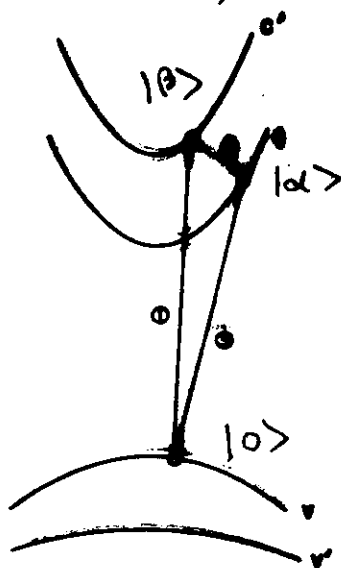


(b)

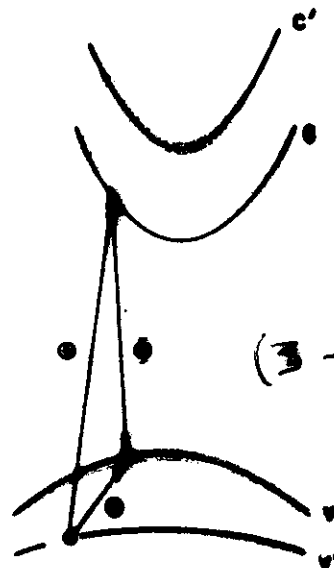
Fig. 2. Schematic diagram of the interband and intraband transitions which play a role in two band scattering processes. The numbers 1, 2, and 3 indicate the time order of the electronic transitions.

(2-BAND RS)

R. LEUDON, *Advances in Physics* (1964)



(a)



(b)

Fig. 3. Schematic diagram of the interband transitions which play a role in three band scattering processes. The numbers 1, 2, and 3 indicate the time order of the electronic transitions.

3-Wave Mixing

vertical transition

(3-BAND RS)

(H_{ep} = electron-phonon interaction)

$$R_i \propto \sum \frac{(\mathbf{p} \cdot \mathbf{A}_s)(H_{ep})(\mathbf{p} \cdot \mathbf{A}_i)}{(\epsilon_{0\beta} - \hbar\omega_s)(\epsilon_{\alpha 0} - \hbar\omega_i)} + 5 \text{ other terms}$$

$$R_j \propto \sum \frac{\langle 0 | P \cdot A_s | \beta \rangle \langle \beta | H_{ep} | \alpha \rangle \langle \alpha | P \cdot A_i | 0 \rangle}{(\epsilon_{0\beta} - \hbar \omega_s)(\epsilon_{\alpha 0} - \hbar \omega_i)} + 5 \text{ other terms}$$

$\alpha, \beta \equiv e-h \text{ pair states}$

$$\langle \beta | H_{ep} | \alpha \rangle = \underbrace{\equiv}_{\beta\alpha}^j + \underbrace{F}_{\beta\alpha}^j + \underbrace{A}_{\beta\alpha}^j$$

$(u_j) \quad (E_j) \quad (A_j)$

Frohlich

$$F_{dd'}^j(k_j) = \pm i e \frac{E_j}{k_j} \delta(k' - k - k_j) ; \hat{e}_i \parallel \hat{e}_s$$

"diagonal" RS

intraband

(2-BAND RS)

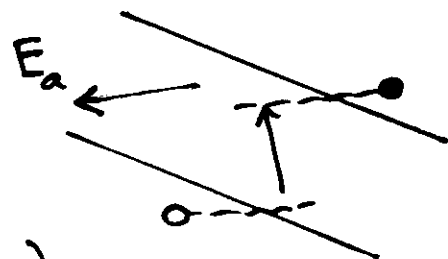
$$(F_{dd'}^j)_e + (F_{dd'}^j)_h = 0 \text{ for } k_j \rightarrow 0$$

(LOUNDON)

Homogeneous $\bar{E}_a$

$$(F_{dd'}^j)_e + (F_{dd'}^j)_h \propto n_{e-h}$$

$\vec{E}_j \parallel \vec{q}_j \parallel \bar{E}_a$ (ie, LO propagating $\parallel \bar{E}_a$)



FRANZ-KELDYSH

Inhomogeneous $\bar{E}_a$ (also δ_{EM} very small)

$$\text{large } k_j \Rightarrow \nabla \bar{E}_j = i k_j E$$

$$(F_{dd'}^j)_e + (F_{dd'}^j)_h \neq 0 \quad \frac{\partial \chi}{\partial \nabla E_j} \nabla E_j = i k_j F E_j$$

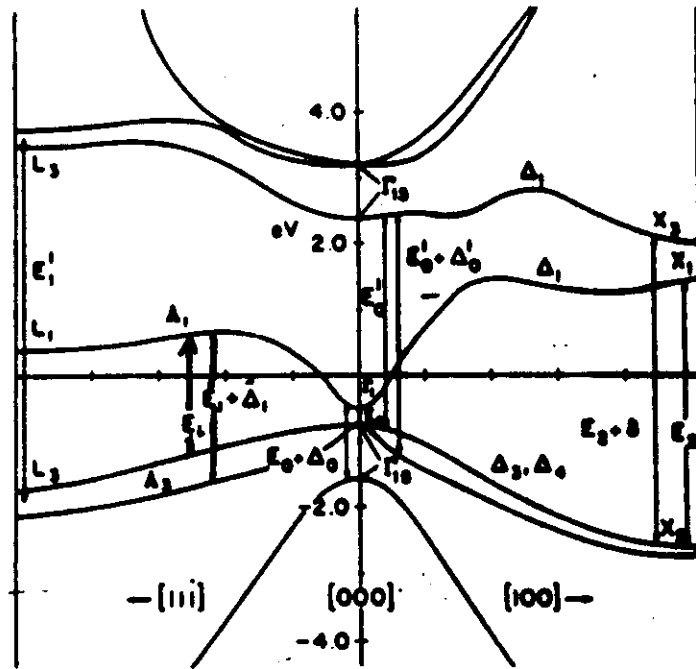


Fig. 7. Band structure of InSb showing the E_0 (Γ point), E_1 (along $\langle 111 \rangle$) and E_2 (X point) critical points. [After F. H. Pollak, C. W. Higginbotham and M. Cardona, J. Phys. Soc. Japan, Suppl. 21, 20(1966).]

Raman Scattering at E , and $E + \Delta$, "gaps"

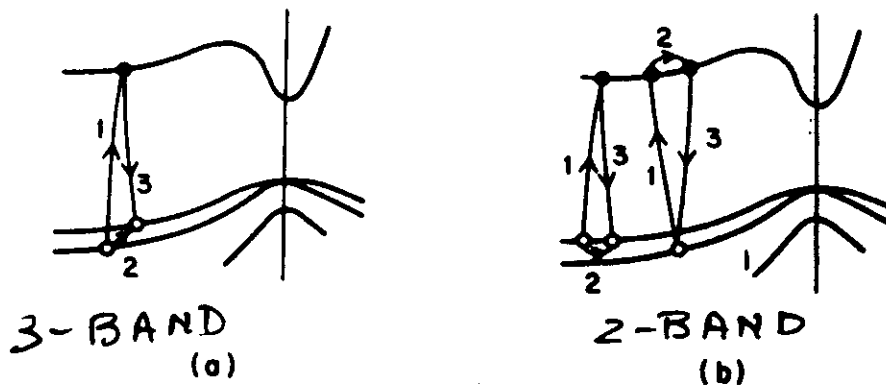
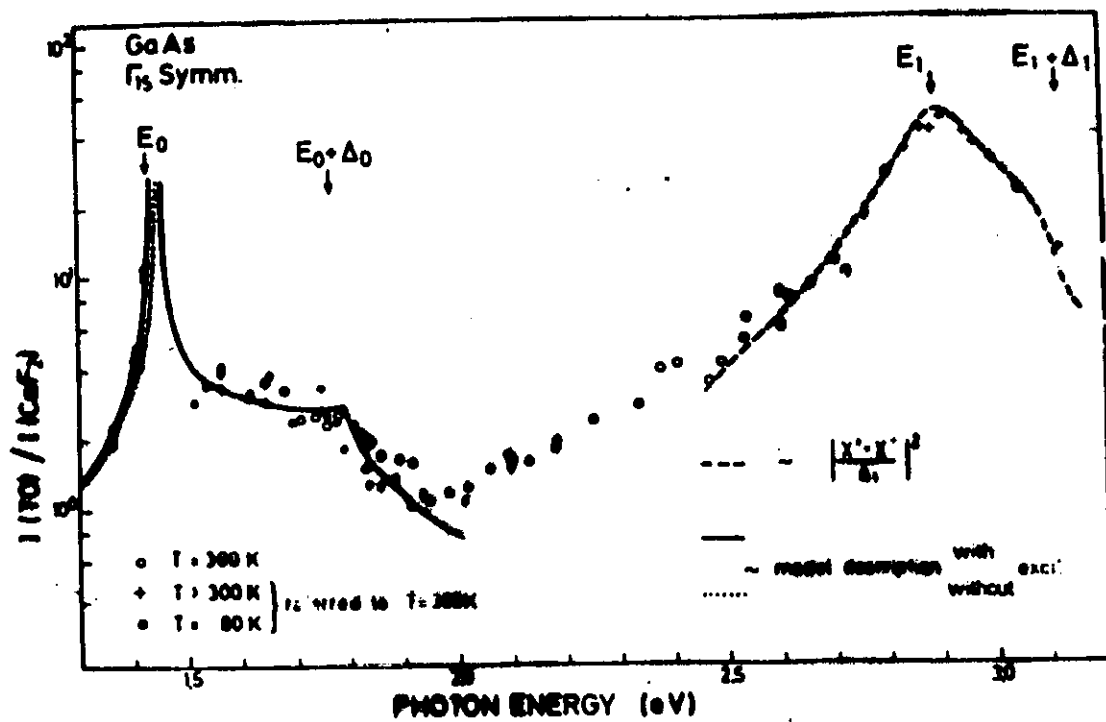


Fig. 6. Schematic diagrams showing (a) the transitions involved in the electro-optic scattering mechanism and in the 3-band deformation potential scattering mechanism, and (b) the transitions involved in the 2-band deformation potential and Fröhlich scattering mechanisms.

(Excitation Profile Measurement)



'RS is a form of Modulation Spectroscopy'
Candau

RAMAN SCATTERING FROM InSb SURFACES AT PHOTON ENERGIES NEAR THE E_g ENERGY GAP*

A. Pinczuk and E. Burstein

Physics Department and Laboratory for Research on the Structure of Matter,
University of Pennsylvania, Philadelphia, Pennsylvania

(Received 8 September 1969)

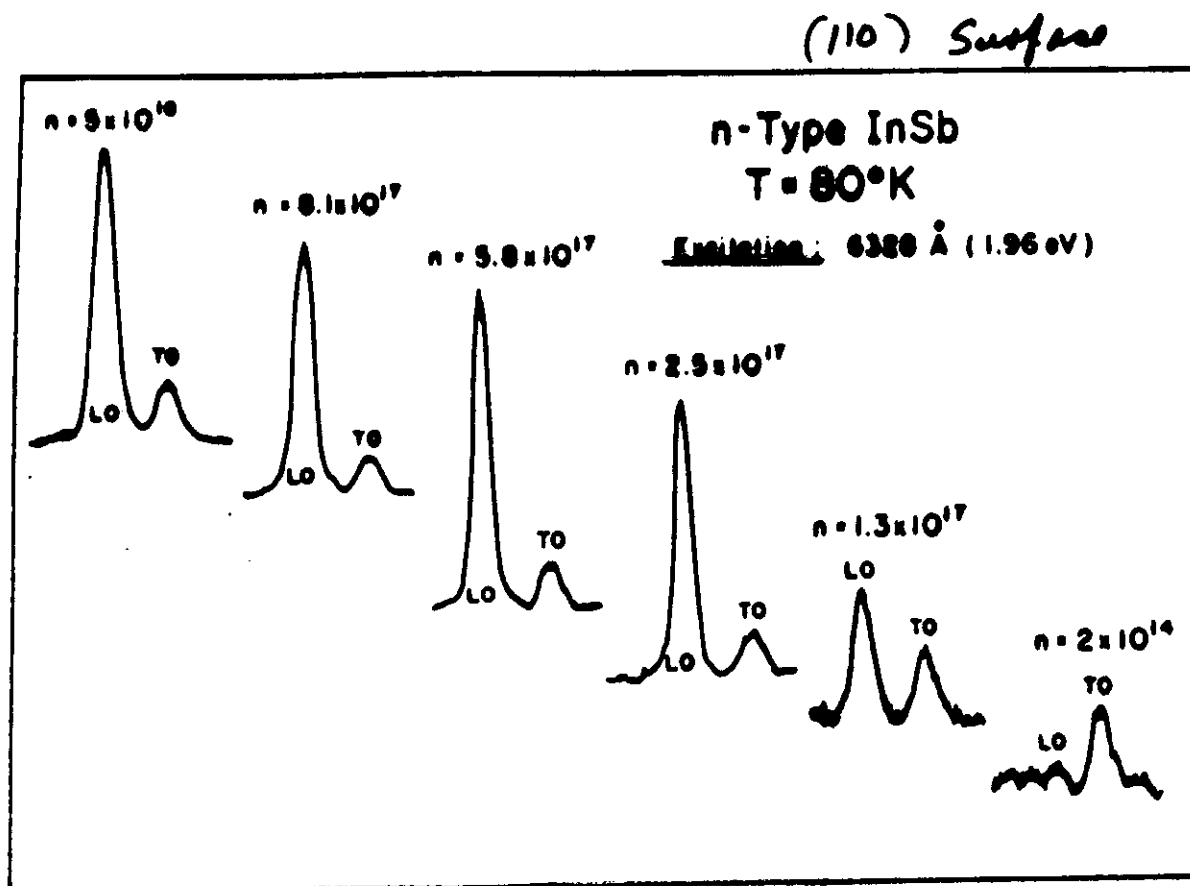


FIG. 2. Recorder traces of Raman spectra of LO and TO phonons of n-InSb at 80 °K.

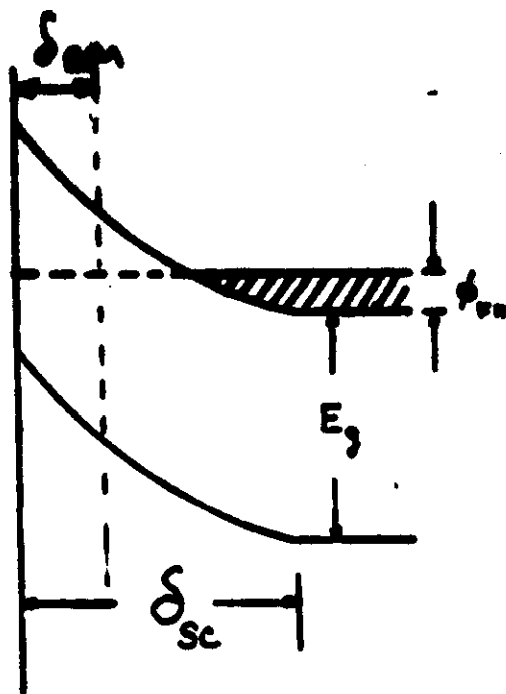
LO forbidden

Surface Electric-Field-Induced Raman Scattering in PbTe and SnTe†

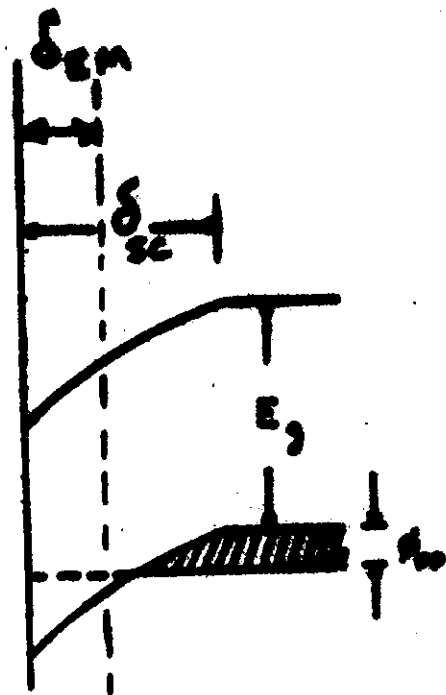
L. Brillson* and E. Burstein

Department of Physics and Laboratory for Research on the Structure of Matter,
University of Pennsylvania, Philadelphia, Pennsylvania 19104

(Received 20 May 1971)



n-type
(a)



p-type
(b)

Raman Scattering Processes

$$\omega_i - \omega_s = \Omega$$

$$\vec{k}_i - \vec{k}_s = \vec{q}$$

"Contact interaction"

Fröhlich (3-step)

e and h scattering contributions
cancel in limit $g \rightarrow 0$

$$E_{\vec{k}\alpha} = \pm i \langle \alpha | e V e^{i\vec{q}\cdot\vec{r}} | \alpha \rangle ; \vec{e}_i \parallel \vec{e}_s$$

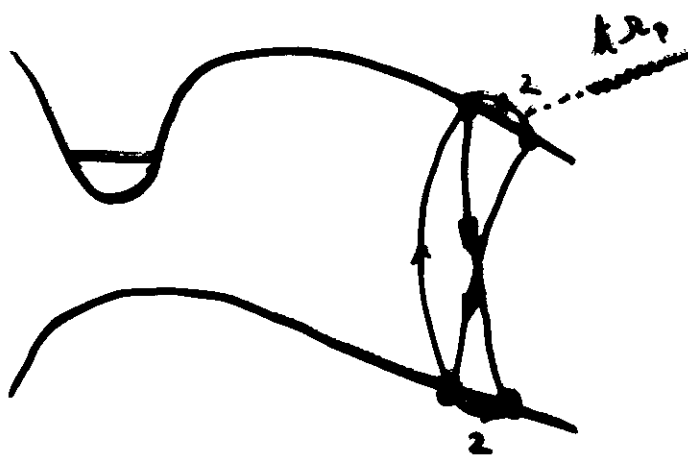
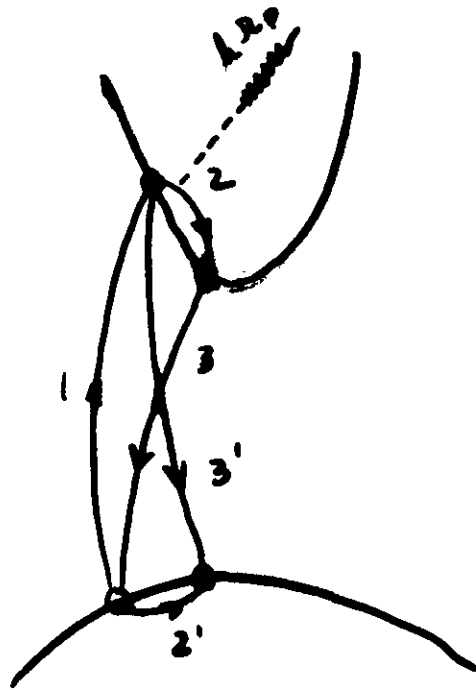
$$I(g) \propto g^2 \text{ providing } m_e \neq m_h$$

(wave vector dependent RS by elem. excitation)

$$I(E_a, g) \propto 4g^2 + 6E_a^2$$

$$I(E_a) \neq 0 \text{ in limit } g \rightarrow 0$$

(Field induced RS by elem. excitation)
(Fröhlich-Altshuler)



Fröhlich-Altshuler

A. RS by plasmons, LO phonons and by coupled
LO phonon-plasmon modes (Ω_+ , Ω_- , ω_L , ω_P)

$$\frac{d^2\sigma}{d\omega_i d\Omega} \propto \left| \sum_{\alpha} \frac{\langle \alpha | \vec{p}_i | 0 \rangle E_{\alpha} \langle 0 | \vec{p}_s | \alpha \rangle}{(E_{\alpha} - \hbar\omega_i)(E_{\alpha} - \hbar\omega_s)} \right|^2$$

2-Band (cont'd)

For excitons as intermediate state

$$(F_{x'x}) = \frac{ie E_{\text{coul}}}{K_{\text{coul}}} \left\{ \langle x | e^{i(m_e/M)K_c \cdot r} e^{-e^{i(m_h/M)K_c \cdot r}} | x \rangle \right\}$$

For hydrogenic exciton $|x\rangle = |1s\rangle$

$$(F_{x'x}) = \frac{ie E_{\text{coul}}}{K_{\text{coul}}} \left(\frac{m_h - m_e}{m} \right) \left[1 - \left(1 + \frac{1}{4} K_c^2 a_{10}^2 \right)^{-2} \right]$$

For $m_e \neq m_h$ (and $K_c \neq 0$) $F_{x'x} \neq 0$

Note: $(A_{a'a})_e = -(A_{a'a})_h$ etc like F

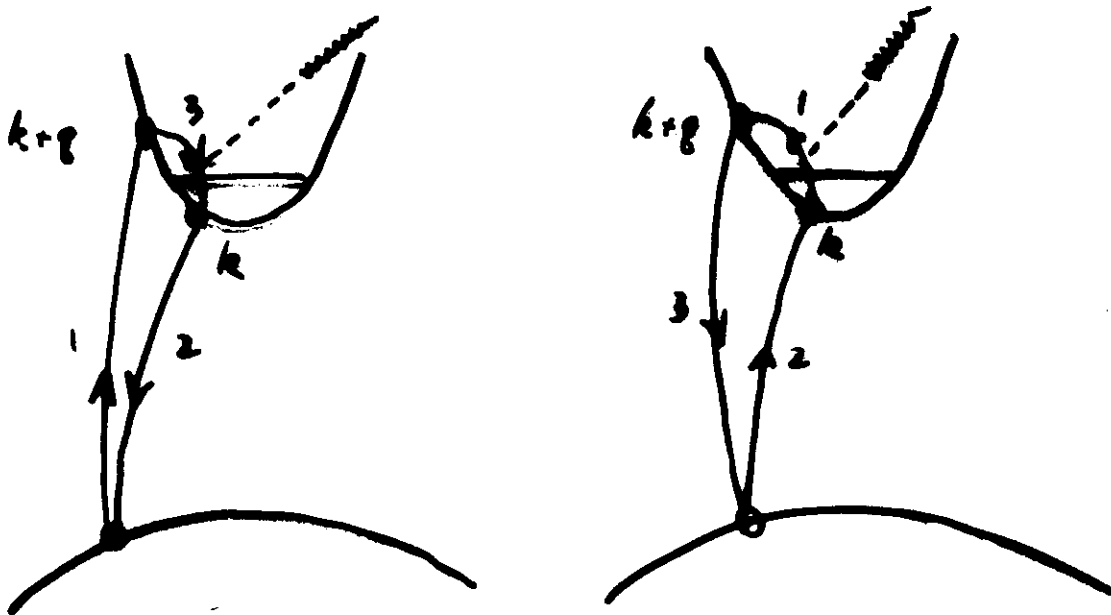
However for $K \neq 0$, the resultant contribution is very small ($A_{ii} = \frac{5}{6} F_{a'a}$)

$(A_{x'x}) = 0$ even for $K \neq 0$ and $m_e \neq m_h$
for hydrogenic exciton

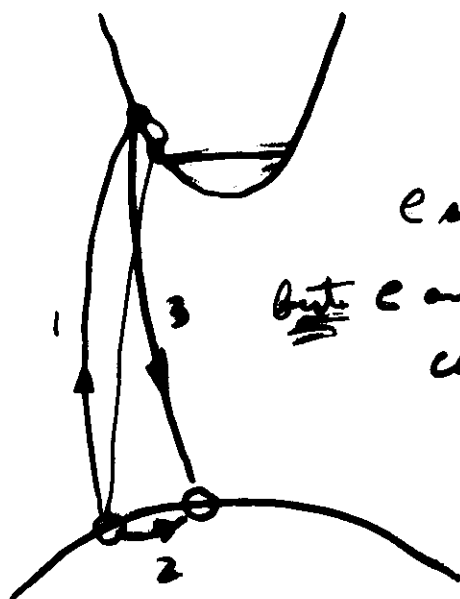
(Ω, q)

A. RS by collective excitations (cont'd)

Hint: $\mathbf{E}$, $\mathbf{F}$ and charge density



$$\frac{d^2\sigma}{d\omega_i d\Omega} \propto \left| \sum_{\mathbf{k}'} \frac{\langle 1 | \mathbf{p} | 0 \rangle F_{\mathbf{k}\mathbf{k}'} \langle 1 | \mathbf{p} | 0 \rangle (f(\mathbf{k}+\mathbf{q}) - f(\mathbf{k}))}{(\mathcal{E}(\mathbf{k}') - \hbar\omega_i)(\mathcal{E}(\mathbf{k}+\mathbf{q}) - \mathcal{E}(\mathbf{k}) - \hbar\Omega)} \right|^2$$



e scattering "blocked"

but e and h contributions do ~~not~~ cancel in limit $q \rightarrow 0$

"Extreme Resonance"

$$\hbar\omega_i = \mathcal{E}_g + \frac{\hbar^2 k_F^2}{2m_v} + \frac{\hbar^2 k_g^2}{2m_c} = \mathcal{E}_g'(n)$$

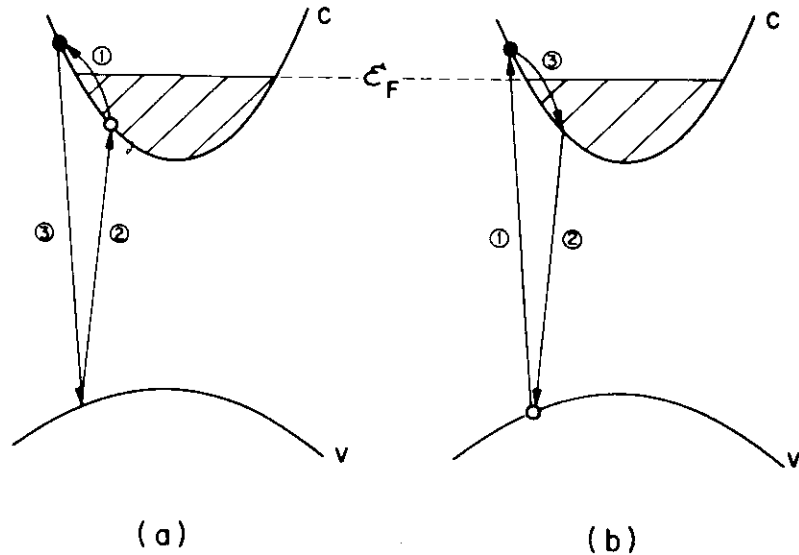


Fig. 6. Schematic diagram of interband and intraband transitions which play a role in two band scattering by plasmons in n-type semiconductors. The numbers 1, 2, and 3 indicate the time order of the electronic transitions.

$$R_j^{(1)}(q) = V^{-1} \sum_k \left\{ (p_{\alpha\alpha}^i)(p_{\beta\beta}^s)(F_{\beta\alpha}^j) / [\omega_P + \epsilon_c(k+q) - \epsilon_c(k)] \right\} \times$$

$$\left\{ 2\omega_a(k) / [\omega_a^2(k) - \omega_l^2] \right\} [f(k) - f(k+q)],$$

$$(F_{\beta\alpha}^j) = e V(q) \delta(k' - k + q) = i e [E_P(q)/q] \delta(k' - k + q)$$

~~~~~

$$\epsilon_T(q, \omega) = \epsilon(\omega) + \frac{4\pi e^2}{q^2} L_0$$

is the total dielectric constant including the electronic contribution. The scattering cross section is then

$$\frac{d^2\sigma}{d\omega d\Omega} = \left( \frac{e^2}{mc^2} \right)^2 \frac{1}{1 - e^{-\omega/T}} \frac{1}{\pi}$$

$$\text{Im} \left\{ \left[ L_2 - \frac{4\pi e^2}{q^2} \frac{L_1^2}{\epsilon_T(q, \omega)} \right] (\hat{\epsilon}_1 \cdot \hat{\epsilon}_2)^2 + K_2 (\hat{\epsilon}_1 \times \hat{\epsilon}_2)^2 \right\}$$

Note that the spin-fluctuation contribution  $K_2 (\hat{\epsilon}_1 \times \hat{\epsilon}_2)^2$  has no screening denominator.

Definition of the terms, in "Light Scattering Spectra of Solids"  
 ed. by Robert F. Meunier (Springer 1969)

$$\frac{d^2\sigma}{d\Omega d\omega} \approx \left[ \frac{E_g^2}{E_g^2 - (\hbar\omega_I)^2} \right]^2 \sigma_T^* (\vec{\epsilon}_I \cdot \vec{\epsilon}_F)^2 \frac{\hbar q^2 V}{4\pi^2 e^2} \epsilon_0^2(\omega) \text{Im} \left[ \frac{1}{\epsilon(\vec{q}, \omega)} \right]$$

$$\sigma_T^* = (e^2/m^*c^2)^2$$

$$\epsilon(0, \omega) \approx \epsilon_0(\omega) - \epsilon_\infty \frac{\omega_p^2}{\omega^2}$$

where

$$\epsilon_0(\omega) = \epsilon_\infty \left[ \frac{\omega_{LO}^2 - \omega^2}{\omega_{TO}^2 - \omega^2} \right]$$

## EXCITON-ENHANCED RAMAN SCATTERING BY OPTICAL PHONONS

E. Burstein\*† and D. L. Mills‡

Physics Department, University of California, Irvine, California

and

A. Pinczuk\* and S. Ushioda\*

Physics Department and Laboratory for Research on the Structure of Matter,  
University of Pennsylvania, Philadelphia, Pennsylvania

(Received 16 September 1968)

The theory of exciton-enhanced Raman scattering is formulated in terms of the scattering of polaritons by optical phonons via the exciton part of the coupled modes. The expression for the exciton contribution to the scattering tensor is given, within a constant factor, in terms of the same parameters that determine the exciton contribution to the frequency-dependent dielectric constant. The theory also provides a new mechanism for the exciton contribution to the electro-optic effect.

Resonant Light Scattering by Single-Particle Electronic Excitations in  $n$ -GaAs†

A. Pinczuk,\* L. Brillson, and E. Burstein

*Department of Physics and Laboratory for Research on the Structure of Matter, University of Pennsylvania, Philadelphia, Pennsylvania 19104*

and

E. Anastassakis

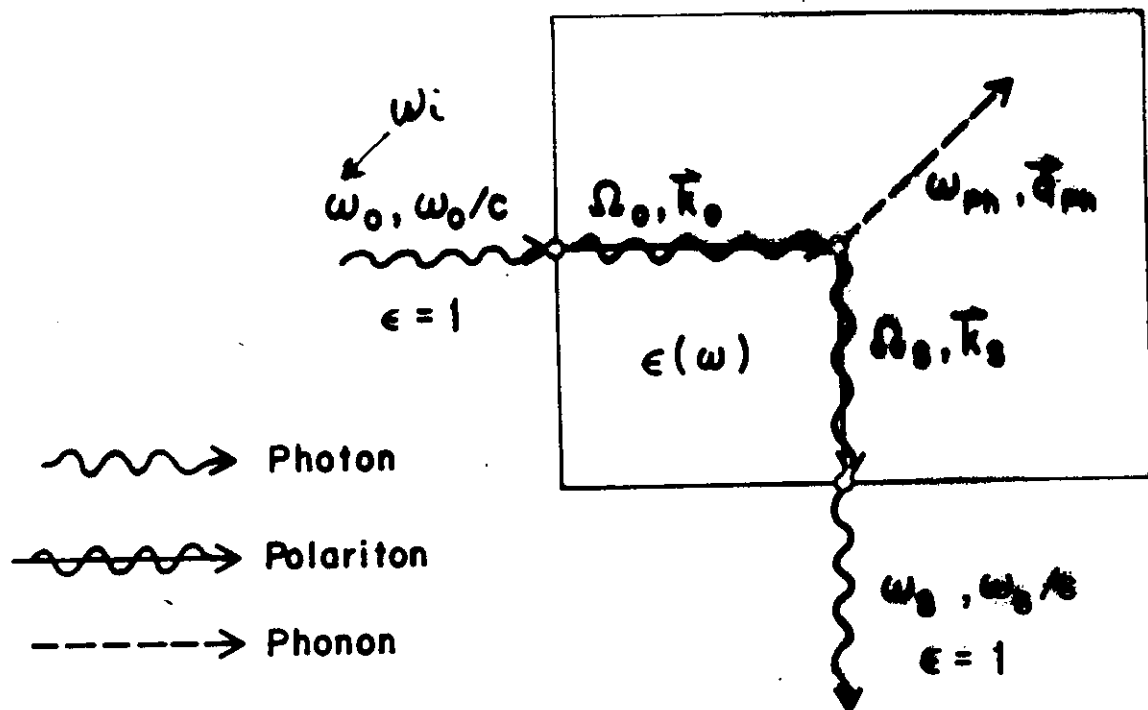
*Department of Physics, Northeastern University, Boston, Massachusetts 02115*

(Received 29 March 1971)

Resonant light scattering by single-particle excitations was observed in  $n$ -GaAs with incident photon energies near the  $E_0 - \Delta_0$  optical energy gap. We find that under extreme resonance conditions the spectra have two components with the scattered light polarization perpendicular and parallel to the incident light. These results are interpreted in terms of a random-phase-approximation theory.

Conceptually very useful!

(Overlander)



$$M_{nm} \sim \sum S_{xi}^{1/2} M_{is} S_{xs}^{1/2} \times F(\omega_{xi}, \omega_{xs}, \omega_s, \omega_c)$$

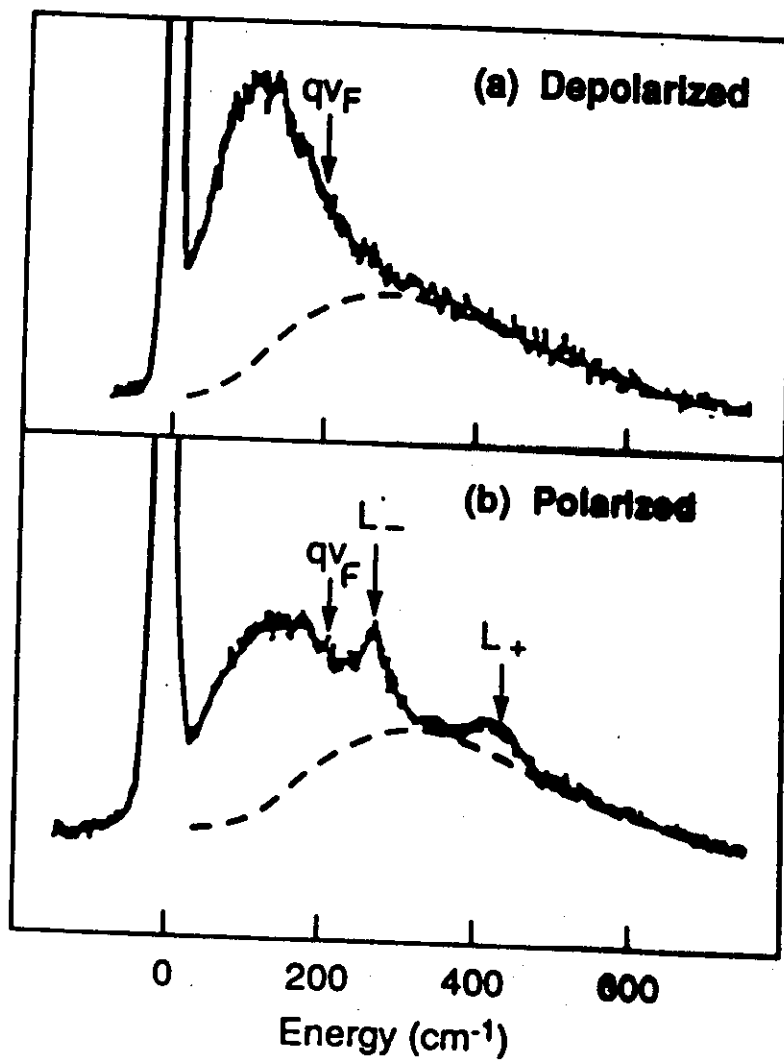
Polariton-phonon  
scattering matrix  
element

exciton-phonon  
matrix element  
(phonon induced transition between  
exciton levels)

$$M_{is}^E > M_{is}^D$$

Overlander  
Mello et al  
Bandaru and Brown  
Hopfield  
London

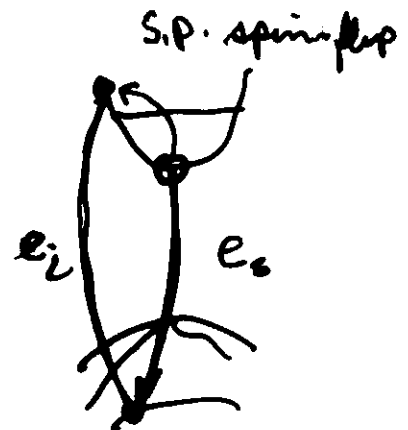
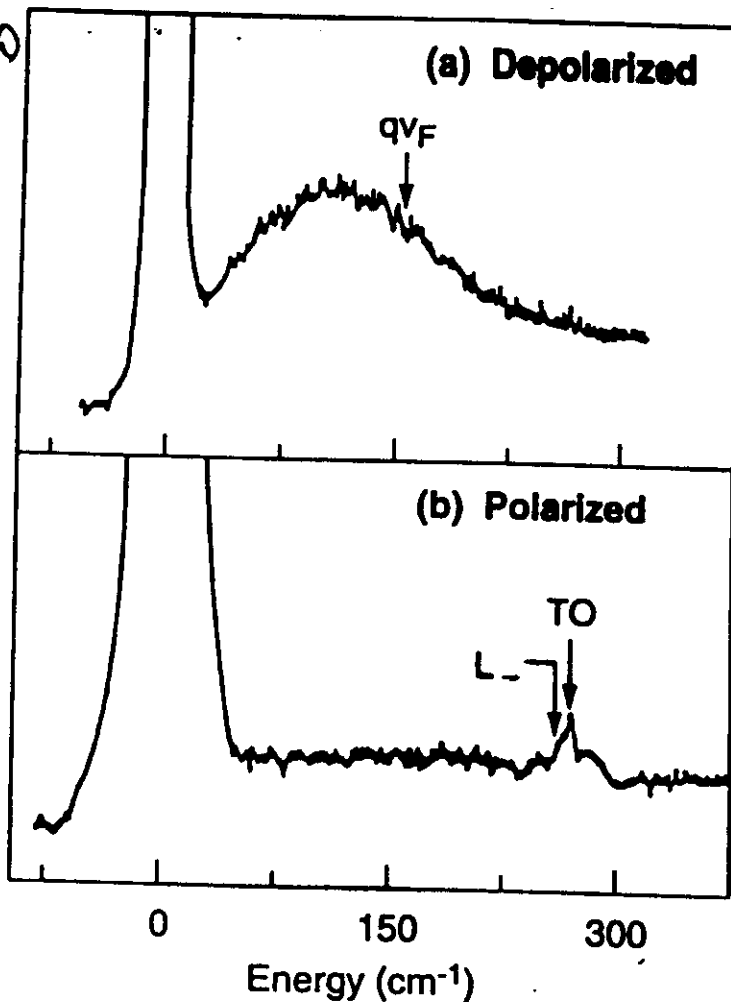
$n\text{-GaAs}$   $n = 1.3 \times 10^{18} \text{ cm}^{-3}$   
 $T = 10\text{K}$   $\hbar\omega_L = 1915.7 \text{ meV}$



$$(\bar{E}_0 + \Delta_0)_{\text{gap}}$$

n-GaAs  $n = 2.0 \times 10^{18} \text{ cm}^{-3}$   
 $T = 10\text{K}$   $\hbar\omega_L = 1915.7 \text{ meV}$

or-spin flip  
 p excitation  
 created by  $\Sigma(k-Q)$



$e_v e_c$   
 spin-flip  
 in  
 semiconductors  
 with large  
 spin-orbit  
 splitting

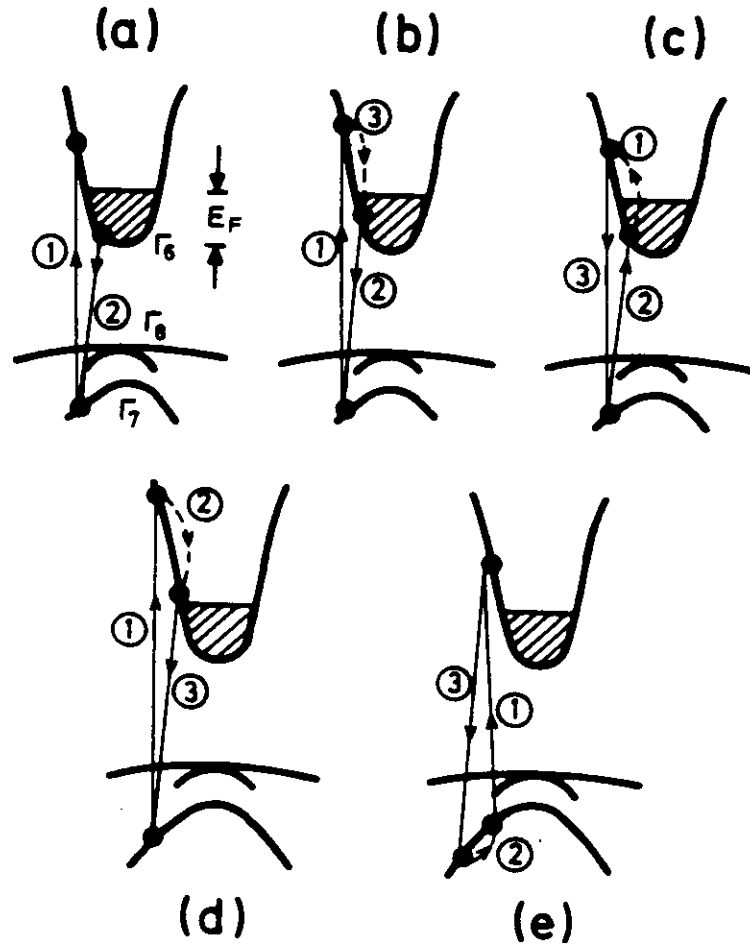


Fig. 1: Diagrams of the various Raman processes discussed in this paper. (a) single particle spin-flip scattering, (b) and (c) electron-phonon collective excitations (charge-density mechanism), (d) and (e) "Fröhlich mechanism". The dashed lines represent the Fröhlich interaction of the longitudinal collective modes. The numbers in circles give the time order of the processes.

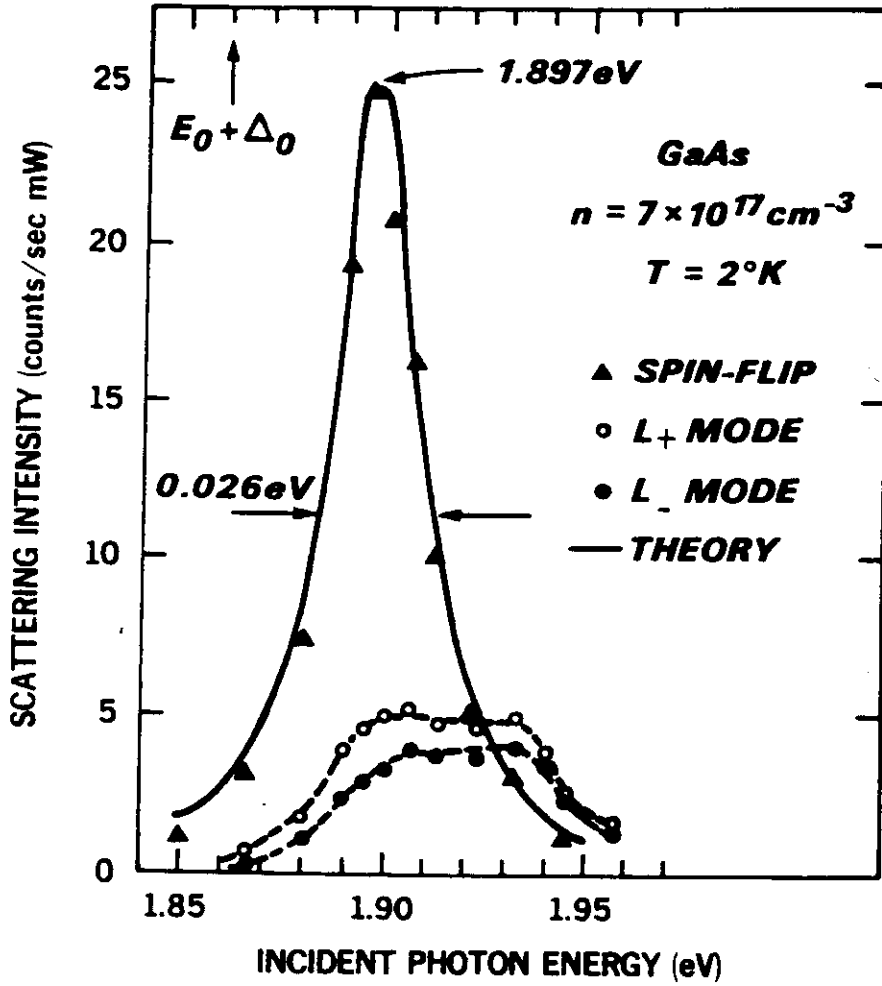


Fig. 2: Scattering intensity as function of incident photon energy of spin-flip single particle excitations at  $90 \text{ cm}^{-1}$  and the coupled modes at  $245 \text{ cm}^{-1}$  and  $350 \text{ cm}^{-1}$ . The full line is the fit with Eq. 3.

