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WORKSHOP ON "NON-LINEAR ELECTROMAGNETIC INTERACTIONS IN SEMICONDUCTORS"

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"Strongly correlated electrons in a quantum zing"

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Strongly correlated electrons in a quantum ring.

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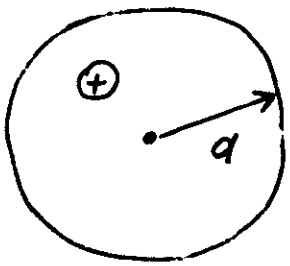
1. Free electrons
2. Coulomb int. and adiabatic approximations
3. Selection rules for 1D Wigner molecule
4. Persistent current
5. Importance of the finite width of QR
6. Optical properties of QR (FIR absorption)
7. Scattering by impurities

Minsky, Chupeliev, Benoit
(1993)

Model of free electrons

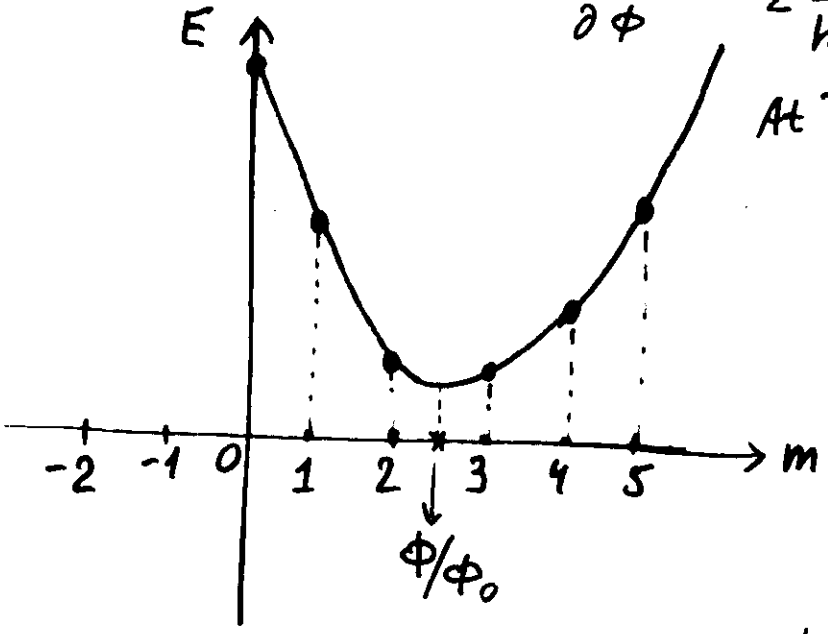


$$E_m = B_0 (m - \Phi/\Phi_0)^2 \qquad B_0 = \hbar^2/2m^*a^2$$

$$\psi_m = \frac{1}{\sqrt{2\pi}} e^{im\varphi} \qquad \Phi_0 = hc/e$$

$$m = 0, \pm 1, \pm 2$$

$$j_m = -c \frac{\partial E_m}{\partial \Phi} = 2\frac{e}{h} B_0 (m - \Phi/\Phi_0)$$



At T=0 the net current is determined by the „last" electron.

In a ballistic system oscillations of the total current $J = \sum_m j_m$ are due to a redistribution of electrons between energy levels which are shifted by the varying magnetic field.

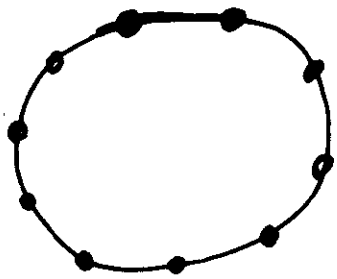
$$T=0, \quad J = (-1)^N \frac{2eB_0 N}{h} \left[\frac{\Phi}{\Phi_0} - Int\left(\frac{\Phi}{\Phi_0} + \frac{1}{2}\right) \right]$$

Sawtooth like curve, the period = Φ_0

The importance of the Coulomb int. depends on the parameter

$$\frac{\text{kin. energy}}{\text{Coulomb en.}} \sim \frac{B_0 m^2}{N\tilde{e}^2/a} = \frac{\hbar^2 m^2}{2m^* \tilde{e}^2 a} \sim \frac{a_B^*}{Na}\left(\frac{N}{4}\right)^2$$

For a few electron ring e-e int. predominates, if $T \lesssim \tilde{e}^2/a \sim 1\,K$ for $a \sim 1\mu$ in GaAs.



1D ring-shaped molecule, strong corr.

$$E_{Fermi} \lesssim \frac{\tilde{e}^2}{a}$$

(Krave et al (1993) - 1D Wigner crystal, continual model, described by the Hamiltonian of a string).

In the molecule the vibr. frequency

$$\omega_v^2 \sim \tilde{e}^2/m a^3$$

$$\omega_{rot}/\omega_{vibr} \sim B_0/\hbar\omega_v \sim \left(a_B^*/a\right)^{1/2} \ll 1$$

For $N \gg 1$ $\omega_{rot} \sim 1/N$, $\omega_v \sim \sqrt{N \ln N}$

and adiabatic criterion $\omega_{rot} \ll \omega_{vibr}$ is satisfied still better.

Separation of rotation and vibrations

$$\varphi_0 = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} \varphi_k , \quad \theta_1 = \frac{\varphi_1 - \varphi_2}{\sqrt{1\cdot 2}} , \quad \theta_2 = \frac{\varphi_1 + \varphi_2 - 2\varphi_3}{\sqrt{2\cdot 3}} \ldots$$

$$\Psi = \exp i\left(\frac{\Phi}{\Phi_0}\sqrt{N}\,\varphi_0\right) \cdot \chi(\varphi_0; \theta_k)$$

$$\boxed{A\varphi = \Phi/2\pi q}$$

$$-B_0\left(\frac{\partial^2}{\partial \varphi_0^2} + \sum_{k=1}^{N-1} \frac{\partial^2}{\partial \theta_k^2}\right)\chi + W_{int}(\theta_k)\chi = E\chi$$

$$E = B_0 x^2 + E_{int} \qquad (B_0 x^2 = E_{rot})$$

The periodicity cond. for rigid rotation $\varphi_k \to \varphi_k + 2\pi$

$$\frac{2\pi N}{\sqrt{N}}\left(\frac{\Phi}{\Phi_0}\sqrt{N} + x\right) = 2\pi J , \quad J = 0, \pm 1, \pm 2$$

$$E_{rot} = B_0 x^2 = \frac{B_0}{N}\left(J - N\Phi/\Phi_0\right)^2$$

AND E_{int} DOES NOT DEPEND ON Φ,
HENCE, HAS NO EFFECT ON THE PERS. CURRENT.

Formally E_{rot} corresponds to the rotator with the mass Nm^* and the charge Ne. This would give the pers. current oscillating with the period Φ_0/N had we not a deal with fermions.

But the Pauli exclusion principle selects allowed quantum numbers J.

For 2 electrons: ortho- and parastates

$S = 0$ J = even ; $S = 1$ J = odd

For 3 electrons:

$S = 3/2$ Term A_2 $J = 0, \pm 3, \pm 6 \ldots$

$S = 1/2$ Term E $J = \pm 1, \pm 2; \pm 4, \pm 5; \ldots$

N electrons in superorthostate $S = N/2$

Rotation by the angle $\frac{2\pi}{N}$ = cyclic perm. of N fermions

$$e^{i \cdot 2\pi J/N} = (-1)^{N-1} = e^{i\pi[(N-1)+2n]}$$

n = integer

For odd N: $J = N \times$ integer

For even N: $J = N \times$ half-integer

Other values of J are possible for $S < \frac{N}{2}$.

Persistent current

Rotational part of the stat. sum

$N = odd \qquad Z_N = \sum_{k=-\infty}^{+\infty} \exp\left[-\beta N B_0 (K \cdot \Phi/\Phi_0)^2\right] \qquad \beta = \frac{1}{T}$

$N = even \qquad \ldots\ldots \qquad \Phi/\Phi_0 \to \Phi/\Phi_0 + 1/2$

Fourier expansion for Z_N:

$$Z_n = \sqrt{\pi} \sum_{z=-}^{+} \begin{Bmatrix} 1 \\ (-1)^z \end{Bmatrix} \frac{\exp(-\pi^2 z^2/N\beta B)}{\sqrt{N\beta B}} \cos 2\pi z \frac{\Phi}{\Phi_0}$$

For $T=0$ only one term „survives" in Z_n

$$J = (-1)^N \frac{2eNB}{h}\left[\frac{\Phi}{\Phi_0} - Int\left(\frac{\Phi}{\Phi_0} + \frac{1}{2}\right)\right]$$

This coincide with cont. model of Wigner crystal and with free electron model.

But both for spinless fermions ($S = N/2$)

For $T \gtrsim NB$

$$J = (-1)^N \frac{4\pi Tc}{\Phi_0} e^{-\pi^2 T/NB} \sin 2\pi \Phi/\Phi_0$$

Coinc. with cont. model of the Wigner crystal but <u>differs</u> from t.e.m. (Kulik 1970)

$$J = -\frac{4\pi c T}{\Phi_0} N e^{-\pi^2 T/B} \sin 2\pi \Phi/\Phi_0$$

QR of a finite width

The number of the vibr. degrees of freedom $= 2N-1$

New effects: nonhardness of the rotator +
interaction of vibrations and rotation

Parabolic radial confinement $U = \frac{m^* \Omega^2}{2}(\rho - a)^2$

1) AB-situation (electrons outside the magn. field)

a) renormalization of B_0 of the order of $B_0/\hbar\Omega \ll 1$

b) contribution to $E_{rot} \sim (J - N\lambda)^4$, what gives an addition to the temp. dependence of pers. cur.

$$\Delta J/J \sim T^3/B_0^2 m \Omega^2 a^2 \ll 1$$

SAME FLUX DEPENDENCE $\sin 2\pi\Phi/\Phi_0$

2) Magn. field everywhere $(A_\varphi = B\rho/2)$

Violation of the periodic dependence $J(\Phi)$

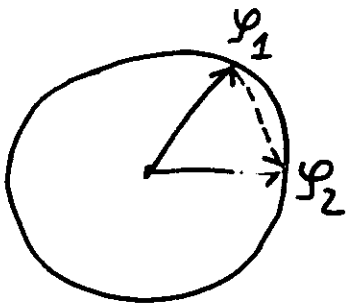
$$J = J_{\text{ideal ring}}(\tilde{\Phi}_0\ \tilde{B}_0) - \text{const}\ \frac{cB_0}{\pi\Omega}\cdot\frac{\Phi}{\Phi_0^2}$$

$\tilde{\Phi}_0$ and $\tilde{B}_0$ are renormalized values of Φ_0 and B_0

The period of oscillations is now (slightly!) different from Φ_0.

Optics of 2-electron QR

(zing shaped „helium)

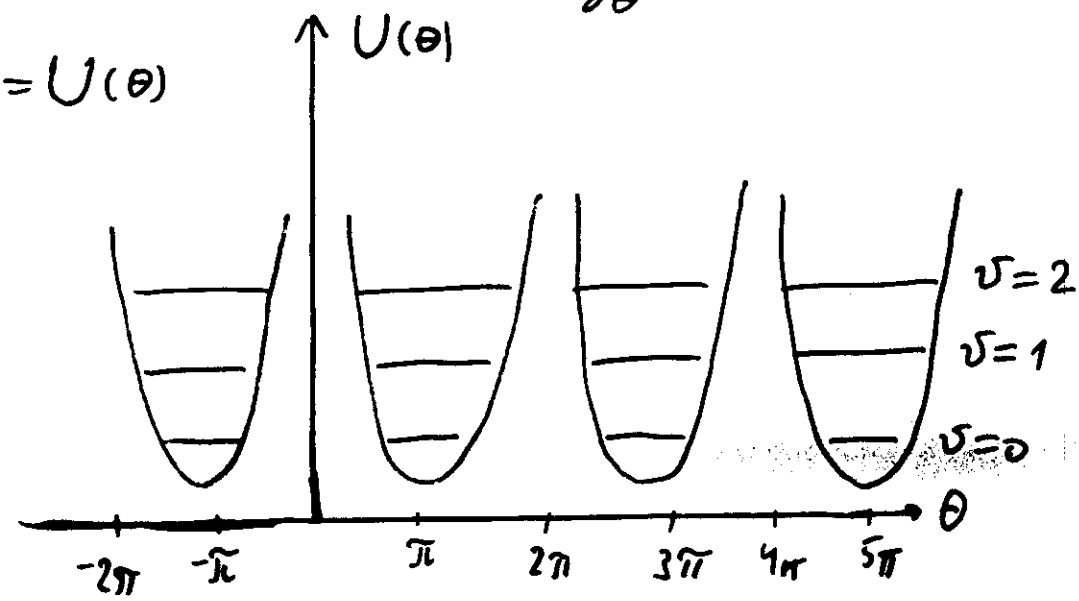


$$U_{int} = \frac{\tilde{e}^2}{2a\left|\sin\frac{\varphi_1-\varphi_2}{2}\right|}$$

$$\varphi = \frac{\varphi_1+\varphi_2}{2}, \quad \theta = \varphi_1-\varphi_2$$

$$\hat{\mathcal{H}} = \frac{B}{2}\left(i\frac{\partial}{\partial\varphi} + 2\frac{\Phi}{\Phi_0}\right)^2 - 2B\frac{\partial^2}{\partial\theta^2} + U_{int}(\theta)$$

$U(\theta+2\pi) = U(\theta)$



The total wave function

$$\Psi(\varphi,\theta) = e^{im\varphi} f(\theta)$$

where $f(\theta)$ has the Bloch form

$$f(\theta) = e^{ip\theta} u(\theta)$$

$$-\frac{1}{2} < p \leq \frac{1}{2} \qquad u(\theta+2\pi) = u(\theta)$$

Shift of φ_1 and φ_2 INDEPENDENTLY by 2π must retain Ψ unchanged.

$$e^{im\pi \pm ip2\pi} = 1$$

a) $m = 2k \rightarrow p = 0$ (center of Br. zone)

b) $m = 2k+1 \rightarrow p = \pm\frac{1}{2}$ (edge of Bz zone)

$$f(\theta) = \sum_{z=-\infty}^{+\infty} \Psi_{osc}[\theta - (2z+1)\pi]\, e^{i\pi p(2z+1)}$$

$f_{osc.}$ is even or odd with resp. to $\delta_z = \theta - \pi(2z+1$

Pauli principle

$$\varphi_1 \rightleftarrows \varphi_2 = \begin{cases} \varphi \rightarrow \varphi \\ \theta \rightarrow -\theta \end{cases}$$

Selection rules for the energy terms

↑↑ $S=1$ $p=0$ m = even v = odd

↑↑ $S=1$ $p=\frac{1}{2}$ m = odd v = even

For ↑↓ $S=0$ — vice versa

Dipole transitions.

$$V_{int} = -eF(\cos\varphi_1 + \cos\varphi_2) =$$

$$= -2eF\cos\varphi \cdot \cos\frac{\theta}{2}$$

Only vicinities around the points $\theta = \pm\pi, \pm 3\pi \ldots$ are essential; $V_{int} \sim (\theta - \pi) \ldots$

$\langle \cos\varphi \rangle_{mm'} \neq 0$ for $\Delta m = \pm 1$

$\left\langle \cos\frac{\theta}{2} \right\rangle_{vv'} = \left\langle \sin\frac{\delta}{2} \right\rangle_{vv'} \neq 0$ for v, v' of different parities.

Only combined vibrational-rotational transitions are possible (non-spin-flip!)

For small oscillations: $\Delta v = \pm 1$

$$\omega_{FIR} = \omega_v \pm mB + B/2$$

If δ is not small the v-selection rule is

$$\Delta v = \text{odd}$$

$$\omega_{FIR} = 3\omega_v, 5\omega_v \ldots \pm mB + B/2$$

Scat. by Impurities

$$V_{sc.} = \sum_{\kappa} V_{im.}(\varphi_\kappa) = \sum_{\kappa} V_{im.}(\varphi_0;\ \text{lin. comb. } \theta_\kappa)$$

For 2 electrons

$$V_{sc.} = V_{im.}(\varphi_0 + \theta/2) + V_{im.}(\varphi_0 - \theta/2)$$

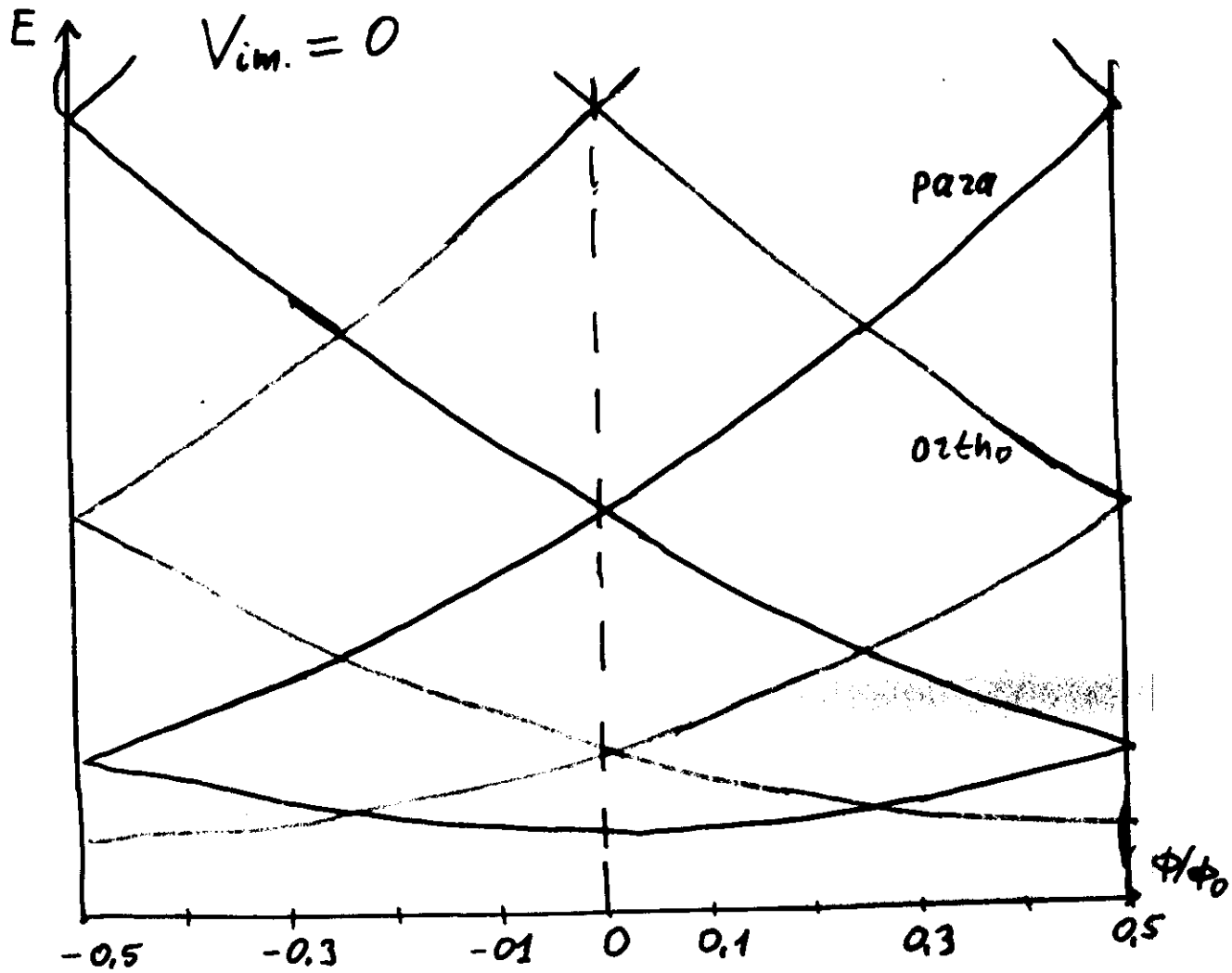
Thus, impurities can be treated as a mechanism of interaction between rotation and vibrations in Wigner molecule.

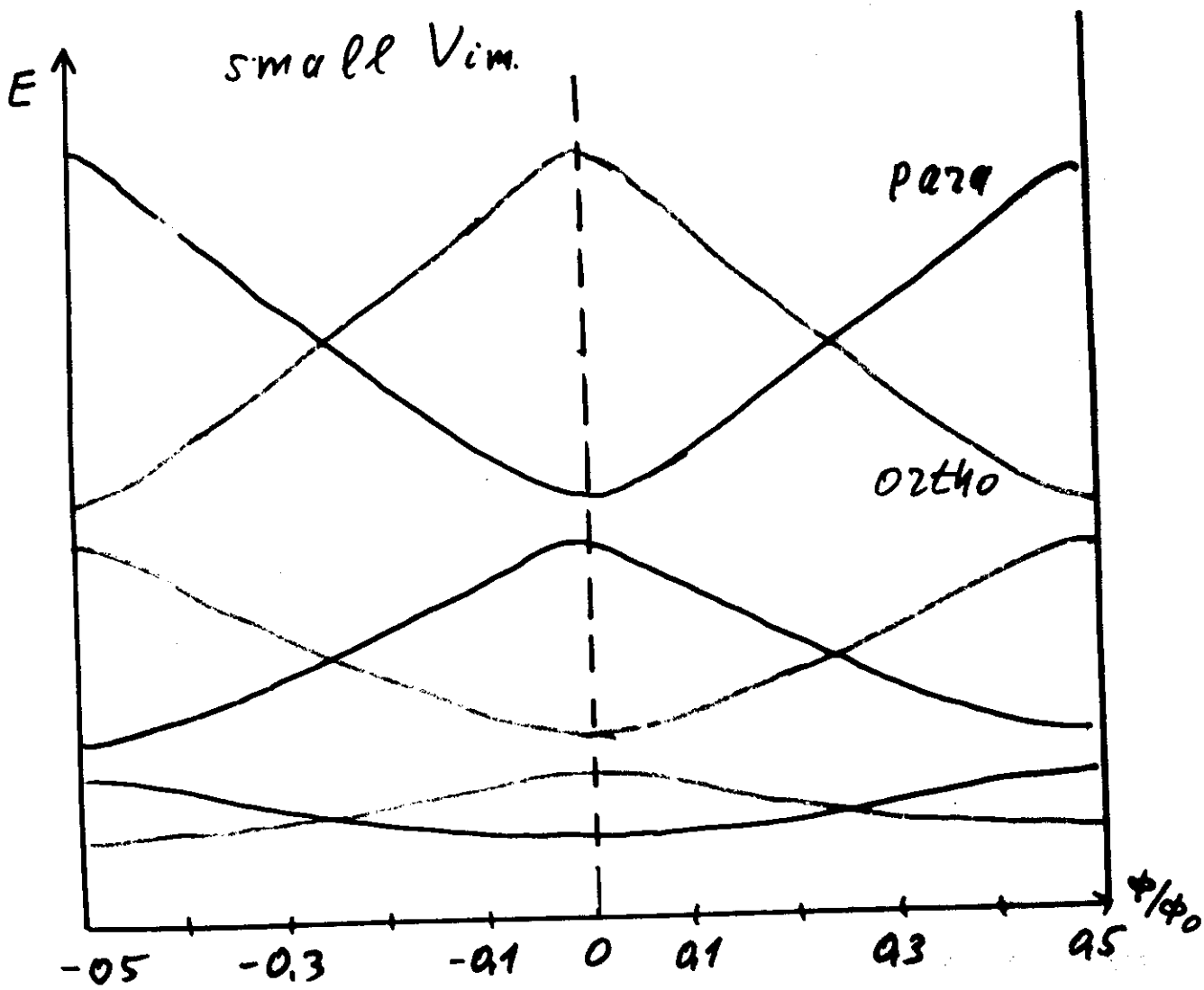
Adiabatic potential is $V_{sc.}$ averaged over "fast" variables θ_κ. For 2 electrons, zero point oscillations and δ-like V_{imp}:

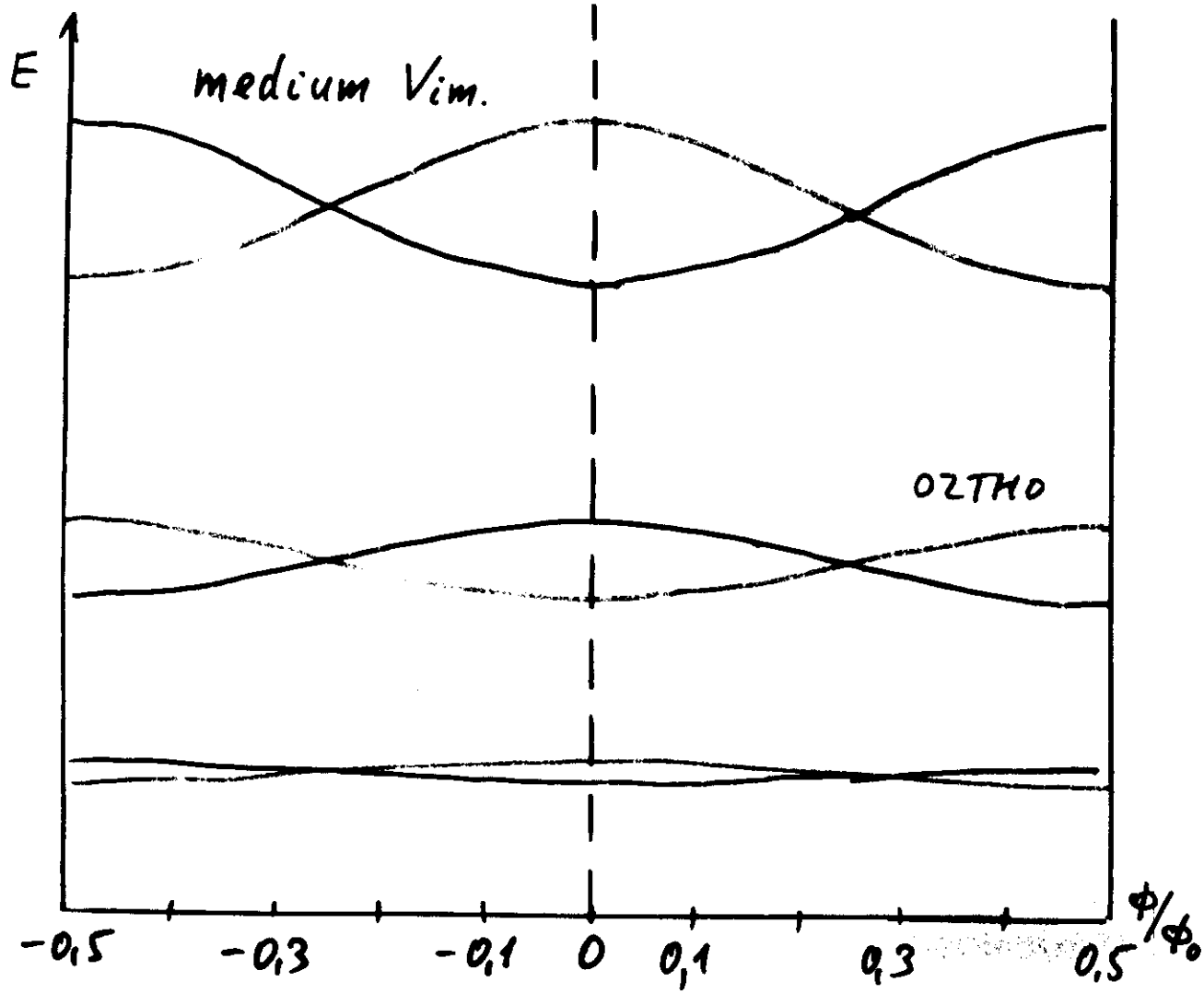
$$V_{ad.}(\varphi_0) = \frac{V_0}{\sqrt{\pi\gamma}} e^{-(\varphi_0 - \pi/2)^2/\gamma} + \frac{V_0}{\sqrt{\pi\gamma}} e^{-(\varphi_0 + \pi/2)^2/\gamma}$$

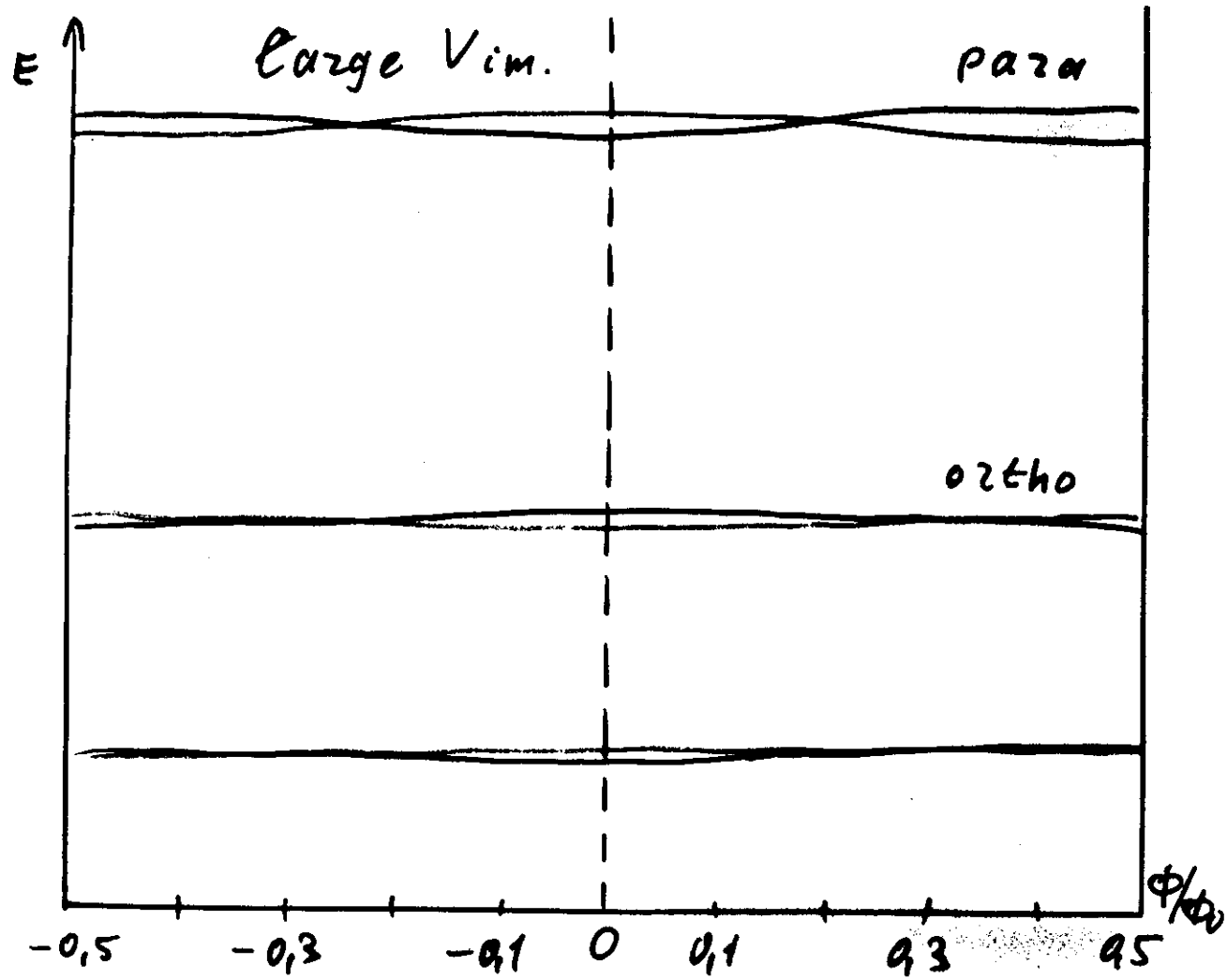
$$\gamma = \langle \delta^2 \rangle$$

This is effective potential energy for a plane rotator; φ_0 = "slow" variable.


E
$V_{im.} = 0$
para
ortho
Φ/Φ_0
-0,5
-0,3
-01
0
0,1
0,3
0,5


E
small Vim.
para
ortho
-05
-0.3
-01
0
01
03
05
Φ/Φ0


E
medium Vim.
ORTHO
-0,5
-0,3
-0,1
0
0,1
0,3
0,5
Φ/Φ0


E
Large Vim.
para
ortho
Φ/Φ0
-0,5
-0,3
-0,1
0
0,1
0,3
0,5