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WORKSHOP ON
"NON-LINEAR ELECTROMAGNETIC INTERACTIONS IN SEMICONDUCTORS"

1 - 10 AUGUST 1994

"Two-photon transitions to exciton states in quantum wells"

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TWO-PHOTON TRANSITIONS TO EXCITON STATES IN QUANTUM WELLS

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EXPERIMENTS

Q-W STATES IN THE EFFECTIVE MASS APPROXIMATION

INTERACTION WITH E-M FIELD AND GAUGE INVARIANCE

INTERBAND & INTRABAND TRANSITIONS

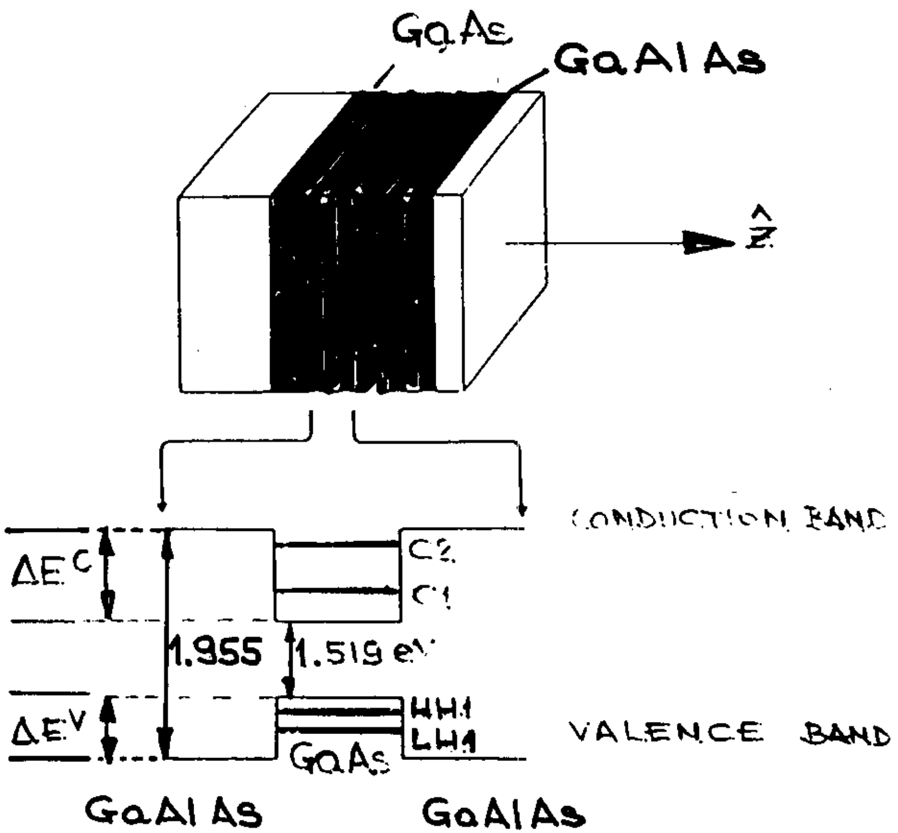
EXCITON STATES

TWO-PHOTON TRANSITIONS TO EXCITON STATES

SUMMATION OVER INTERMEDIATE STATES VARIATIONAL METHOD

$GaAs - Ga_{1-x}Al_xAs$

QUANTUM WELLS



Two-Photon Absorption Spectroscopy in GaAs Quantum Wells

K. Tai, A. Mysyrowicz, R. J. Fischer, R. E. Slusher, and A. Y. Cho

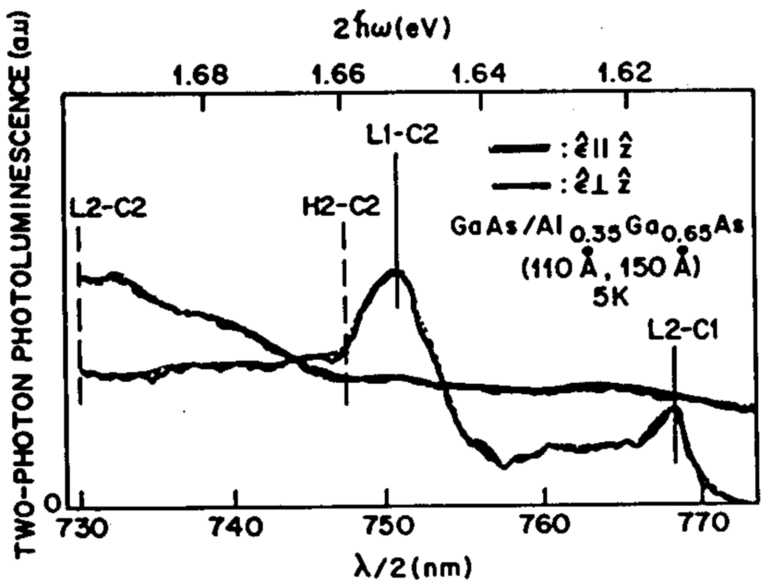


FIG. 3. Two-photon absorption spectra of GaAs/Al$_{0.35}$Ga$_{0.65}$As QW's with $l_z = 110$ Å for the $\hat{\epsilon} \parallel \hat{z}$ (solid line) and $\hat{\epsilon} \perp \hat{z}$ (dashed line) configurations. Vertical lines position the exciton features observed in two-photon (solid line) and in one-photon (dashed line) absorption. Other exciton features seen in linear absorption, $L1$-$C1$, $H1$-$C1$, and $H3$-$C3$, are at 1.552, 1.565, and 1.815 eV, respectively.

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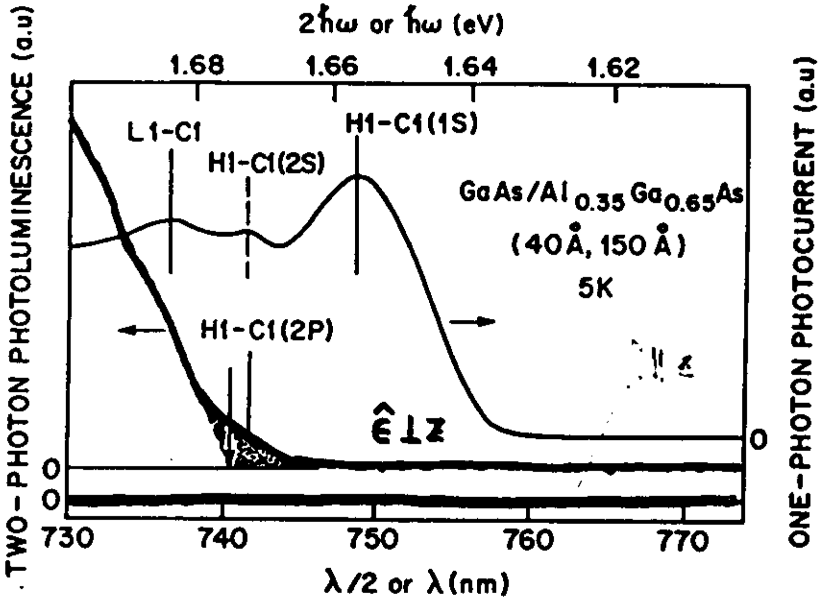


FIG. 2. One-photon (solid line) and two-photon (solid and dashed line) absorption spectra of GaAs/$Al_{0.35}Ga_{0.65}As$ QW's with $l_z = 40$ Å. One-photon spectrum was measured with light incident normal to the sample. Two-photon absorption for $\hat{\epsilon} \parallel \hat{z}$ (dashed line) was found to be zero in the spectral range shown. Nonvanishing two-photon absorption was obtained only for $\hat{\epsilon} \perp \hat{z}$ (solid line). Vertical lines position the observed exciton features. Vertical arrows indicate the $H1$-$C1$ band edge from the linear extrapolation indicated by the dashed line. The shaded area is the broadened $2P$ state. The $1S$ state splitting between $H1$-$C1$ and $L1$-$C1$ agrees with numerical calculations.

K. PLOOG SOLID STATE COMMUN. 71, 217 (1989)

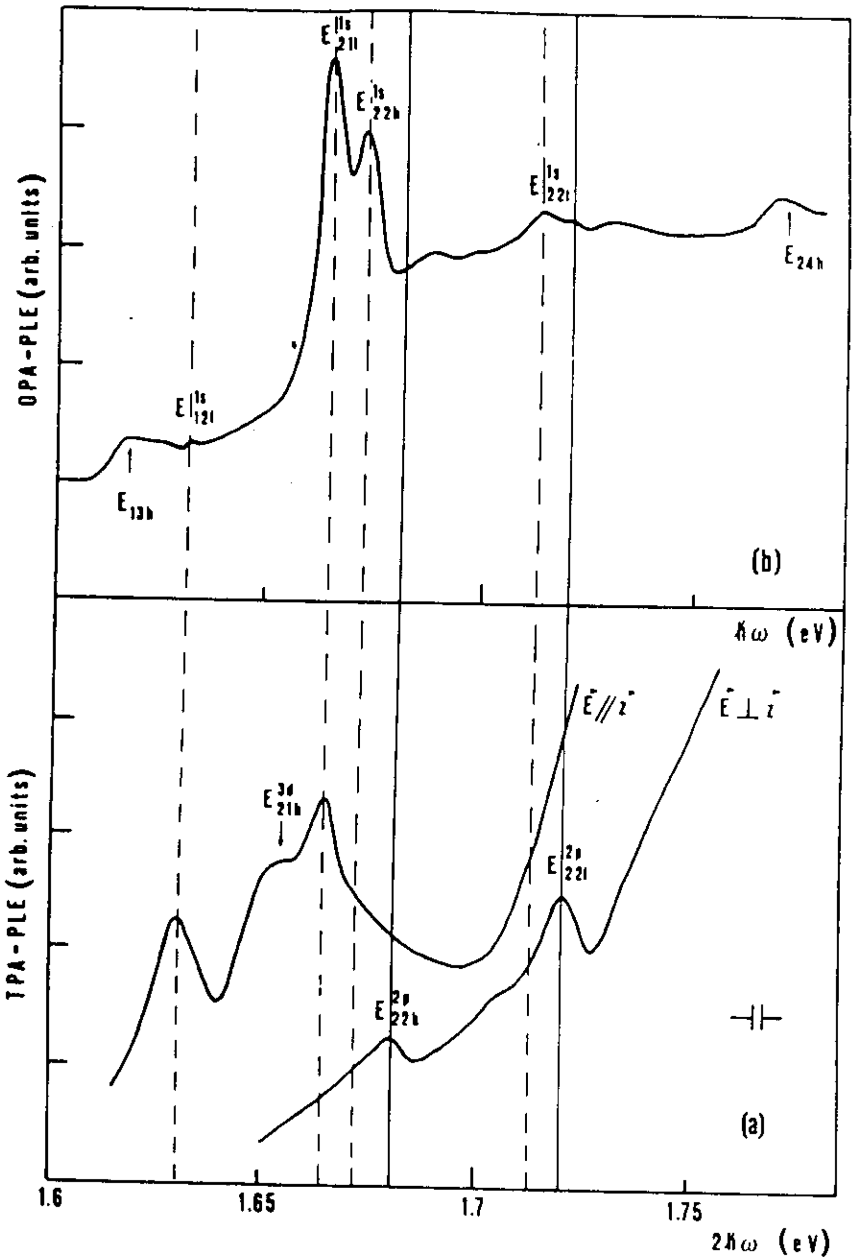


Fig. 1. (a) Two-photon absorption photoluminescence excitation spectra (TPA–PLE) in $E \parallel z$ and $E \perp z$ polarization configurations for the sample A. (b) One-photon absorption ...

EFFECTIVE MASS EQUATIONS

GaAlAs | GaAs | GaAlAs $\rightarrow \hat{z}$

$-L/2$ $\quad$ $L/2$

ONE ELECTRON SCHRÖDINGER EQU.

$$\left[p^2/2m_0 + V_{per}(\vec{r}) + U(\vec{r})\right]\psi(\vec{r}) = E\psi(\vec{r})$$

BOUNDARY CONDITIONS

$$\psi^A\Big|_{L^-/2} = \psi^B\Big|_{L^+/2} \; ; \; \frac{\partial \psi^A}{\partial z}\Big|_{L^-/2} = \frac{\partial \psi^B}{\partial z}\Big|_{L^+/2}$$

ENVELOPE FUNCTION

$$\psi(\vec{r}) = \sum_k A(k)\, e^{i\vec{k}\vec{r}}\, u_n(\vec{k},\vec{r})$$

$$FT(A(k)) = F(\vec{r})$$ ENVELOPE FUNCTION.

EQUATIONS FOR THE ENVELOPE FUNTIONS.

GaAlAs | GaAs | GaAlAs

B $\quad$ A $\quad$ B

$$u_n^A(\vec{k},\vec{r}) \underset{!}{\approx} u_n^B(\vec{k},\vec{r})$$

CONDUCTION BAND

$$\left[-\frac{\hbar^2}{2m(z)}\nabla^2 + V^E(z)\right]F(\vec{r}) = E\,F(\vec{r})$$

(1) $V^E(z) = 0 \qquad |z| < L/2$

$V^E(z) = V_0^E \qquad |z| > L/2$ [BAND OFFSET

(2) $m(z) = m(GaAs) \qquad |z| < L/2$

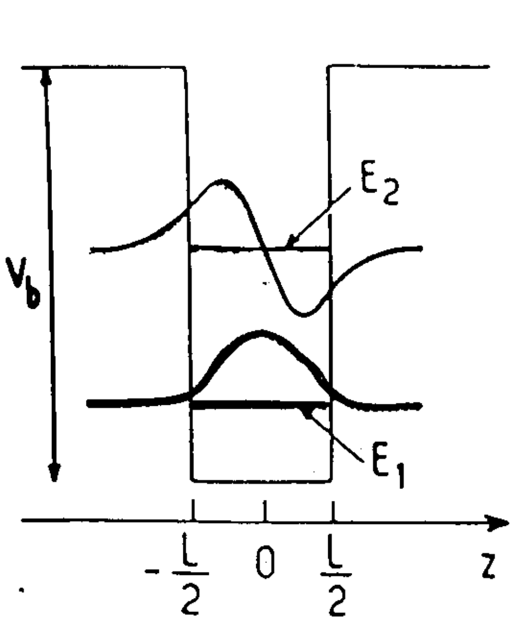
$= m(GaAlAs) \qquad |z| > L/2$

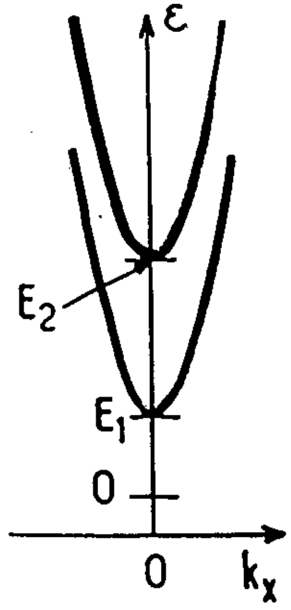
(3) $F^A\Big|_{L^-} = F^B\Big|_{L^+}$

$$\frac{1}{m^A}\frac{\partial F^A}{\partial z}\Big|_{L^-} = \frac{1}{m^B}\frac{\partial F^B}{\partial z}\Big|_{L^+}$$

VALENCE BAND

QUANTUM WELL SUBBANDS





WAVE FUNCTION [EMA]

$$\Psi_{k_{\parallel},n}(\vec{r}) = \underbrace{f_n(z)}_{\text{ENVELOPE F.}}\ \underbrace{\phi_{k_{\parallel}}(\vec{r})}_{\text{BLOCH F.}}$$

VALENCE BAND

FOUR COMPONENT ENVELOPE FUNCTION

$$F = (F_1; F_2; F_3; F_4)$$

$$T(k) \longrightarrow 4\times4 \text{ MATRIX}$$

$$\downarrow$$

$$\left[T(-i\nabla) - V^H(z)\right] F = E F$$

+ boundary conditions

APPROXIMATIONS

BAND MIXING IS NEGLECTED

SEPARATE EQUATIONS FOR

LIGHT & HEAVY HOLE

INFINITE WELL

INTRABAND

$v \rightarrow v$

$c \rightarrow c$

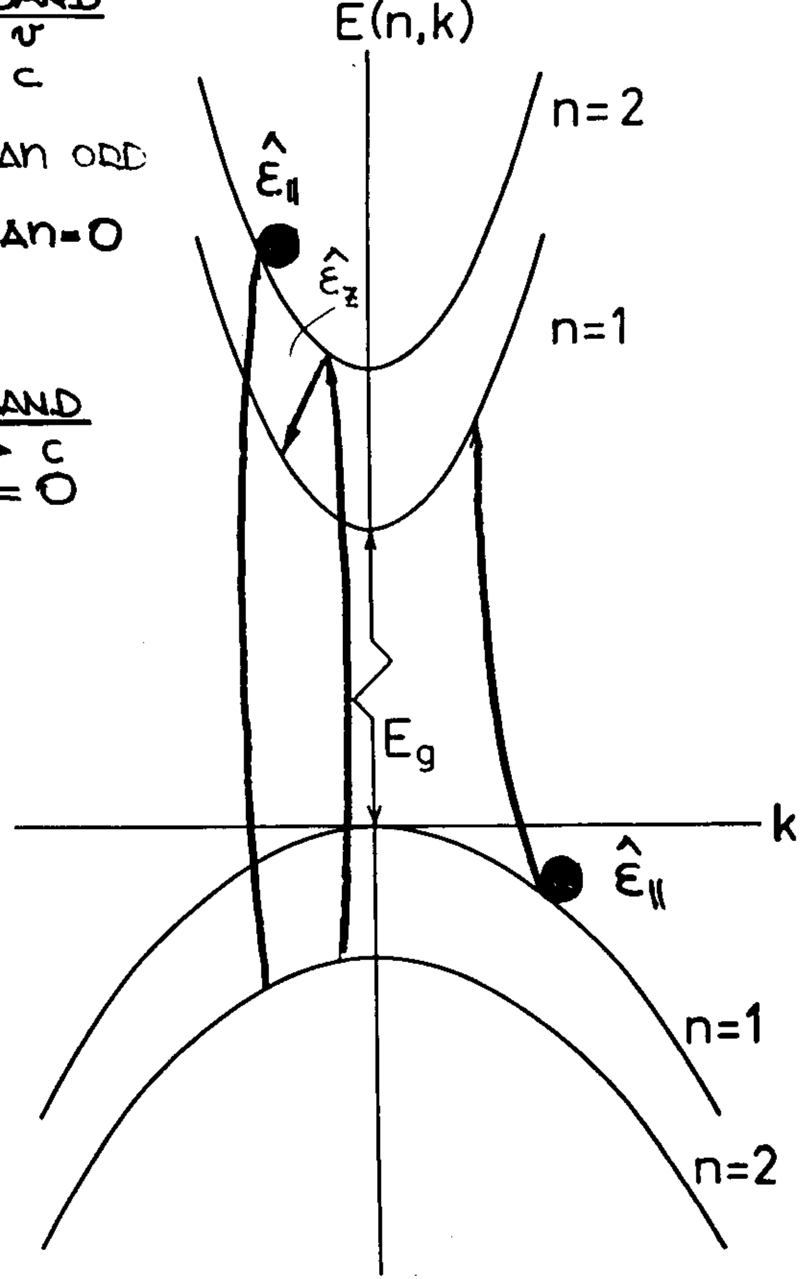
$\hat{\varepsilon}_z$: Δn ODD

$\hat{\varepsilon}_{\parallel}$: $\Delta n = 0$

INTERBAND

$v \rightarrow c$

$\Delta n = 0$



INTERBAND MATRIX ELEMENTS

$$\langle \psi_n^{ck} | \epsilon \cdot \mathbf{p} | \psi_{n'}^{vk} \rangle = \delta_{nn'} \, \epsilon \cdot \mathbf{p}_{cv}$$

$\Delta n = 0$

$$\langle \psi_n^{ck} | \epsilon \cdot \mathbf{v} | \psi_{n'}^{vk} \rangle = \frac{E_n^{ck} - E_{n'}^{vk}}{E_g + \hbar^2 k^2/(2\mu)} \delta_{nn'} \, \epsilon \cdot \mathbf{p}_{cv}/m_0$$

INTRABAND MATRIX ELEMENTS

- <u>*x*-polarization:</u> $\Delta n = 0$

$$\langle \psi_n^{ck} | \mathbf{x} \cdot \mathbf{v} | \psi_n^{ck} \rangle = \langle \psi_n^{ck} | \mathbf{x} \cdot \mathbf{p}/m_0 | \psi_n^{ck} \rangle = \mathbf{x} \cdot \hbar \mathbf{k}/m_c$$

- <u>*z*-polarization:</u> Δn : ODD

$$\langle \psi_n^{ck} | \mathbf{z} \cdot \mathbf{v} | \psi_{n'}^{ck} \rangle = \langle \psi_n^{ck} | \mathbf{z} \cdot \mathbf{p}/m_0 | \psi_{n'}^{ck} \rangle = \langle f_n^c | p_z/m_c | f_{n'}^c \rangle$$

INFINITE QUANTUM WELL

$$\psi^c(\vec{r}, \vec{k}\, n) = F(\vec{r}, \vec{k}\, n)\, u_n(k=0, r)\, \alpha$$

$$F(\vec{r}, k, n) = \exp i\, k_\parallel r_\parallel \quad f_n(z)$$

$$\left\{ \frac{-\hbar^2}{2m^*} \frac{\partial^2}{\partial z^2} + \frac{\hbar^2 k_\parallel^2}{2m^*} \right\} f(z) = E\, f(z)$$

BOUNDARY CONDITIONS

$$f(z) \ : \quad f(L/2) = f(-L/2) = 0$$

$$f_n^+(z) = (2/L)^{1/2} \cos\left(\frac{\pi n}{L} z\right) \qquad n \text{ ODD}$$

$$f_n^-(z) = (2/L)^{1/2} \sin\left(\frac{\pi n}{L} z\right) \qquad n \text{ EVEN}$$

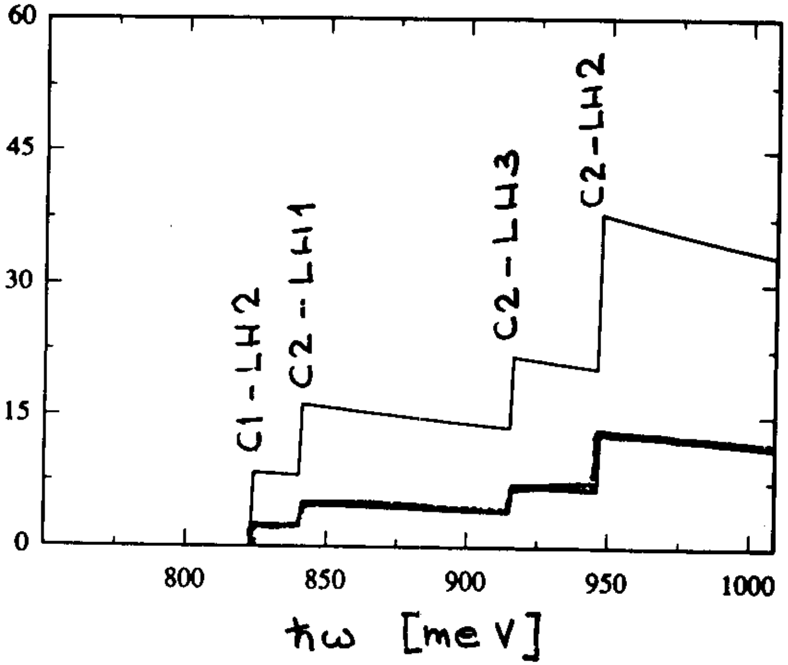
$$E^\pm(n, k_\parallel) = \frac{\hbar^2 k_\parallel^2}{2m^*} + \frac{\hbar^2}{2m^*}\left(\frac{\pi n}{L}\right)^2$$

TRANSITION RATE   Z-POLARIZATION

$\hat{e}_1 = \hat{e}_2 = \hat{z}$ $\omega_1 = \omega_2 = \omega$

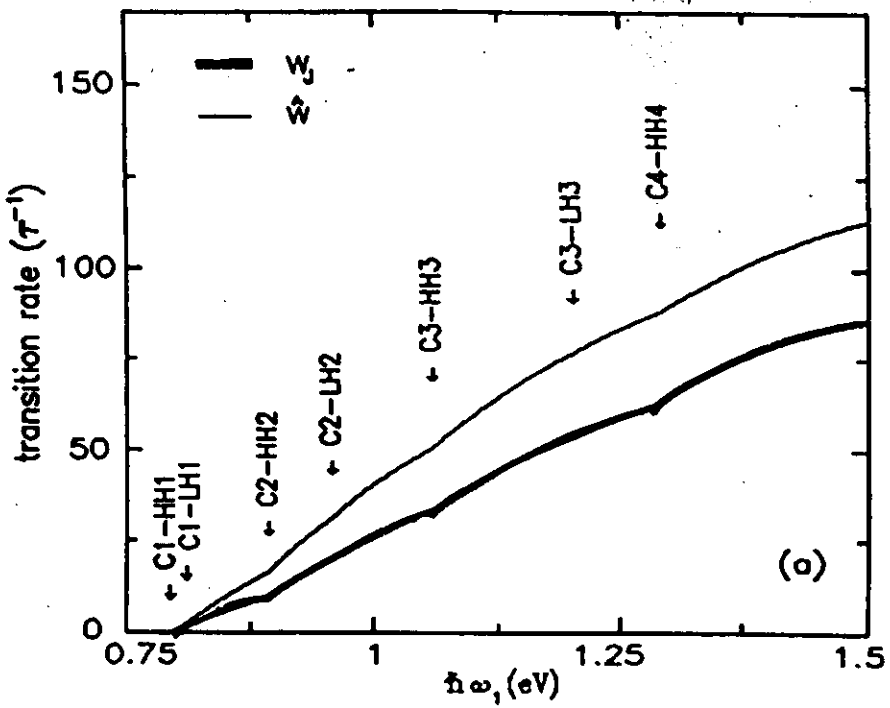
(1) Δn ODD

(2) Z - POLARIZATION :
LIGHT HOLES — RADIATION



(3) STEPS :
$2\hbar\omega = E^c(n, k=0) - E^v(m, k=0)$

(4) INTRABAND MATRIX ELEMENTS
≃ k. INDEPENDENT



TRANSITION RATE $\hat{e}_1 = \hat{e}_2 = e_1$
$\omega_1 = \omega_2$

$\Delta n = 0$

LIGHT & HEAVY HOLES COUPLED
TO THE RADIATION

DISCONTINOUS DERIVATY FON

$2\hbar\omega = E^c(n, k=0) - E^v(n, k=0)$

INTRABAND MATRIX ELENENT $\propto |\vec{k}|$

EXCITON STATES

- The exciton Hamiltonian is given by

KINETIC ENERGIES

$$H = \frac{p_{z_e}^2}{2m_e} + \frac{p_{z_h}^2}{2m_h} - \frac{\hbar^2}{2\mu}\nabla_{\parallel}^2$$

COULOMB INTER. CONFINEMENT

$$-\frac{e^2}{\epsilon[\rho^2 + (z_e - z_h)^2]^{1/2}} + V_{conf}(z_e) + V_{conf}(z_h),$$

where the potentials V_{conf} confine the motion of holes and electrons in the well.

- The exciton wavefunction is given by

$$\psi^{n_e,n_h,m}(\rho,\phi,z_e,z_h) = f_{n_e}^c(z_e) f_{n_h}^v(z_h) R_{|m|}(\rho) \frac{e^{im\phi}}{(2\pi)^{1/2}},$$

where the $f_{n_e}^c(z)$ and $f_{n_h}^v(z)$ are QW-eigenfunctions and where

$$R_{|m|}(\rho) = \rho^{|m|} \sum_{k=1}^{N} A_k \, e^{-\alpha_k \rho}.$$

- Exciton quantum numbers in this model:
  1. m, z-component of angular momentum
  2. n_c, conduction subband label
  3. n_v, valence subband label

TWO-PHOTON TRANSITION RATE

- For an electromagnetic field of vector potential

$$\mathbf{A} = A_1 e^{i\omega_1 t}\epsilon_1 + A_2 e^{i\omega_2 t}\epsilon_2 + \text{c. c.},$$

the two-photon transition rate to an exciton state β is given by

$$W_{0\to\beta} = \frac{2\pi}{\hbar}\left(\frac{e}{c}\right)^4 (A_1 A_2)^2 |(1 + P_{12})$$

LENGTH GAUGE

VELOCITY GAUGE $\hat{\varepsilon}\cdot\vec{v}$

$$\times \sum_{\beta'} \frac{(\beta|\omega_1\epsilon_1\cdot\mathbf{r}|\beta')(\beta'|\omega_2\epsilon_2\cdot\mathbf{r}|0)}{E_{\beta'} - E_0 - \hbar\omega_2}|^2$$

$$\times\delta(E_\beta - E_0 - \hbar\omega_1 - \hbar\omega_2).$$

<u>SELECTION RULES</u> $\beta \equiv (m, n_c, n_v, \nu)$

(1) $\langle\beta'|\hat{\varepsilon}\cdot\vec{r}|0\rangle \neq 0$ FOR : $n_c' = n_v'$, $m' = 0$

$|\beta'\rangle$: s - EXCITON

(2) $\langle\beta|\hat{\varepsilon}_z\cdot\vec{r}|\beta'\rangle \neq 0$ FOR $n_c - n_v$ ODD, $m = 0$

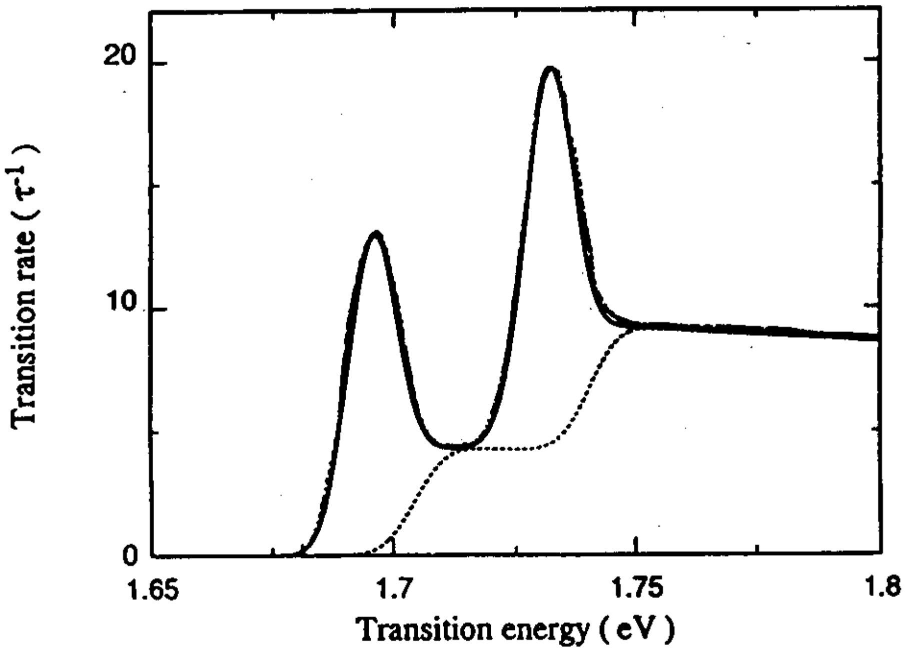
$|\beta\rangle$: s - EXCITON

(3) $\langle\beta|\hat{\varepsilon}_\parallel\cdot\vec{r}|\beta'\rangle \neq 0$ FOR $n_c = n_v$, $m = \pm 1$

$|\beta\rangle$: p - EXCITON

Z POLARIZATION

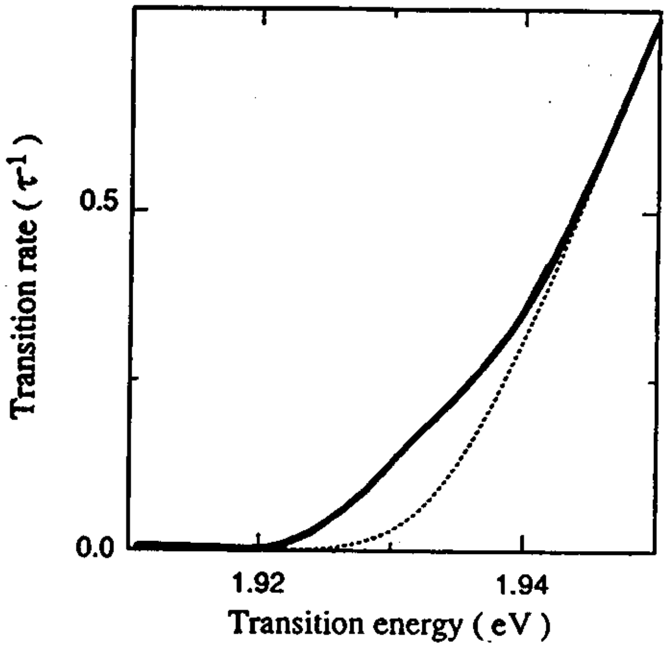
S - EXCITONS



Two-photon transition rate versus transition energy in a 110 Å QW in the case of equal photons of z-polarization. Transitions to s-type C1-LH2 and C2-LH1 excitons as well as between the associated subbands [4] are considered. An overall Gaussian broadening with $\sigma = 5$ meV has been used.

X - POLARIZATION

P- EXCITONS



Two-photon transition rate versus transition energy in a 40 Å QW in the case of equal photons of x-polarization. Transitions to p-type C1-HH1 excitons as well as between the associated subbands [4] are considered. An overall Gaussian broadening with $\sigma = 5$ meV has been used.

NON LOCAL POTENTIALS

$$\boxed{\left(e^{a\frac{\partial}{\partial z}}\right)f(z) = f(z+a)}$$

V : $\langle\vec{x}|V|\vec{x}'\rangle \neq g(\vec{x})\,\delta(\vec{x}-\vec{x}')$

$$\langle\vec{x}|V|\varphi\rangle = \int d^3x' \,\langle\vec{x}|V|\vec{x}'\rangle\,\varphi(\vec{x}')$$
$$= \left[\int d^3x' \,\langle\vec{x}|V|\vec{x}'\rangle\, T_p(\vec{x}'-\vec{x})\right]\varphi(\vec{x})$$

$$T_p(\vec{x}'-\vec{x}) = \prod_{l=1}^{3}\sum_n \frac{1}{n!}\left(\frac{i}{\hbar}\right)^n (x'_l - x_l)^n p_l^n$$

$$\langle\vec{x}|V|\varphi\rangle = \mathcal{V}(\vec{x},\vec{p})\,\varphi(\vec{x})$$

$$H_0 = p^2/2m + \mathcal{V}(\vec{x},\vec{p})$$

$\Downarrow$ e.m. field

$$H = \frac{1}{2m}\left(\vec{p}-\frac{e}{c}\vec{A}\right)^2 + \mathcal{V}\left(\vec{x},\vec{p}-\frac{e}{c}\vec{A}\right)$$

$$H_J^1 = -\frac{e}{mc}\vec{A}_J\cdot\vec{v} + \frac{1}{2i\hbar}\left(\frac{e}{c}\right)^2\vec{A}_J\cdot\left[\vec{x},\vec{A}_J\cdot\vec{v}\right]$$

$$\vec{v} = \left[\vec{x},H_0\right]/i\hbar$$

DIPOLE APPROXIMATION : LENGTH GAUGE

$$H_{J_0}^1 = \frac{e}{c}\,\vec{x}\cdot\frac{\partial}{\partial t}\vec{A} = -e\,\vec{x}\,\vec{\mathcal{E}}$$

NON-LOCAL POTENTIALS [MOMENTUM DEPENDENT POTENTIAL]

INTERACTION WITH RADIATION

EXAMPLE : CONSIDER A PARTICLE IN A MAGNETIC FIELD : $A_M = (-y;0;0)B$

$$H_0 = \frac{1}{2m}\left(\vec{p}-\frac{e}{c}\vec{A}_M\right)^2 + V(x)$$
$$= \frac{1}{2m}p^2 + \underbrace{V(\vec{x}) + \frac{1}{2m}\left(\frac{e}{c}\right)^2 A_M^2 - \frac{e}{mc}\vec{A}_M\cdot\vec{p}}_{\mathcal{V}(\vec{x},\vec{p})}$$

RADIATION FIELD – COULOMB GAUGE : $\vec{A}_R(\vec{x},$

$$H = \frac{1}{2m}\left(\vec{p}-\frac{e}{c}\vec{A}_M-\frac{e}{c}\vec{A}_R\right)^2 + V(x)$$
$$= \frac{1}{2m}\left(\vec{p}-\frac{e}{c}\vec{A}_R\right)^2 + \underbrace{V(\vec{x}) + \frac{1}{2m}\left(\frac{e}{c}\right)^2 A_M^2 - \frac{e}{mc}\vec{A}_M\cdot\left(\vec{p}-\frac{e}{c}\vec{A}_R\right)}_{\mathcal{V}\left(\vec{x},\,\vec{p}-\frac{e}{c}\vec{A}_R\right)}$$

IN GENERAL $\mathcal{V}(x,p)$ IS NOT A LINEAR FUNCTION IN $\vec{p}$.

$$H^{INT} = -\frac{e}{c}\vec{A}\cdot\vec{V} + \frac{1}{2i\hbar}\left(\frac{e}{c}\right)^2 \vec{A}\cdot\left[\vec{x}, \vec{A}\cdot\vec{V}\right]$$

$$\Omega_J(m,n) = (1 + P_{12}) \sum_{\mu} \frac{\langle m|\hat{e}_1\cdot\vec{V}/m|\mu\rangle \langle \mu|\hat{e}_2\cdot\vec{V}/m|n\rangle}{\omega(\mu) - \omega(n) - \omega_2},$$

SECOND ORDER PERTURBATION THEORY

$$\frac{1}{2i}(1 + P_{12})\langle m|[\hat{e}_1\cdot x,\ \hat{e}_2\cdot V]|n\rangle$$

FIRST ORDER PERTURBATION THEORY

TRANSITION RATE

$$W_J^{(2)}(m,n) = \frac{2\pi e^4|A_{01}A_{02}|^2}{\hbar^4 c^4}\cdot \left|\underline{\Omega_J(m,n)} - \underline{\frac{1}{2i}(1 + P_{12})\langle m|[\hat{e}_1\cdot x,\ \hat{e}_2\cdot V]|n\rangle}\right|^2 \delta(\Delta\omega).$$

$$W_J(\varepsilon_1,\varepsilon_2;\omega_1,\omega_2) = \frac{2\pi}{\hbar}\left(\frac{e}{c}\right)^4 (A_1A_2)^2$$

$$\times \frac{S}{(2\pi)^2}\int d^2k \sum_{n_c}\sum_{n_v}\Big|(1+P_{12})\Big\{ \frac{\langle\psi^c(n_c,k)|\varepsilon_1\cdot v|\psi^c(n_v,k)\rangle\langle\psi^c(n_v,k)|\varepsilon_2\cdot v|\psi^v(n_v,k)\rangle}{E^c(n_v,k) - E^c(n_c,k) + \hbar\omega_2}$$

SECOND ORDER

$$+ \frac{\langle\psi^c(n_c,k)|\varepsilon_1\cdot v|\psi^v(n_c,k)\rangle\langle\psi^v(n_c,k)|\varepsilon_2\cdot v|\psi^v(n_v,k)\rangle}{E^v(n_c,k) - E^v(n_v,k) - \hbar\omega_1}$$

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FIRST ORDER

$$- \frac{1}{2i\hbar}\Big[\ \langle\psi^c(n_c,k)|\varepsilon_1\cdot x|\psi^c(n_v,k)\rangle\langle\psi^c(n_v,k)|\varepsilon_2\cdot v|\psi^v(n_v,k)\rangle$$

$$- \langle\psi^c(n_c,k)|\varepsilon_2\cdot v|\psi^c(n_v,k)\rangle\langle\psi^c(n_v,k)|\varepsilon_1\cdot x|\psi^v(n_v,k)\rangle$$

$$+ \langle\psi^c(n_c,k)|\varepsilon_1\cdot x|\psi^v(n_c,k)\rangle\langle\psi^v(n_c,k)|\varepsilon_2\cdot v|\psi^v(n_v,k)\rangle$$

$$- \langle\psi^c(n_c,k)|\varepsilon_2\cdot v|\psi^v(n_c,k)\rangle\langle\psi^v(n_c,k)|\varepsilon_1\cdot x|\psi^v(n_v,k)\rangle\ \Big]\ \Big\}\Big|^2$$

$$\times\,\delta\big(E^c(n_c,k) - E^v(n_v,k) - \hbar\omega_1 - \hbar\omega_2\big). \qquad (8)$$

$\{\varepsilon_{\parallel} \ : \ \Delta n = n_c - n_v = 0\}$

THE TWO PHOTON TRANSITION RATE CONTAINS EXPRESSIONS OF THE FORM :

$$T_{if}(\omega) = \sum_n \frac{\langle\psi_f|p|\psi_n\rangle\langle\psi_n|p|\psi_i\rangle}{E_n - E_i - \hbar\omega}$$

DEFINING $|\phi\rangle$ AS SOLUTION OF

$$(H - E_i - \hbar\omega)\,|\phi\rangle = p\,|\psi_i\rangle$$

IT FOLLOWS

$$T_{if}(\omega) = \frac{\langle\psi_f|p|\phi_N\rangle\langle\phi_N|p|\psi_i\rangle}{\langle\phi_N|H|\phi_N\rangle - E_i - \hbar\omega}$$

WITH

$$\phi_N = \phi/\langle\phi|\phi\rangle^{1/2}$$

CONSIDER A FINITE SET OF STATES WHICH INCLUDES $|\phi\rangle$.
DIAGONALIZATION OF H ON THIS BASIS

$$\langle\varphi_n|H|\varphi_{n'}\rangle = \lambda_n\,\delta_{nn'}\,, \qquad \langle\varphi_n|\varphi_{n'}\rangle = \delta_{nn'}$$

EXPANSION

$$\phi = \sum_n a_n\,\varphi_n$$

THE COEFFICIENTS a_n VERIFY THE EQUATION

$$(\lambda_n - E_i - \hbar\omega)a_n = \langle\varphi_n|p|\psi_i\rangle.$$

DEFINING

$$T'_{if}(\omega) = \sum_n \frac{\langle\psi_f|p|\varphi_n\rangle\langle\varphi_n|p|\psi_i\rangle}{\lambda_n - E_i - \hbar\omega}$$

$$T'_{if}(\omega) = T_{if}(\omega) = \frac{\langle\psi_f|p|\phi_N\rangle\langle\phi_N|p|\psi_i\rangle}{\langle\phi_N|H|\phi_N\rangle - E_i - \hbar\omega}$$

SUMMATION OVER INTERMEDIATE STATES

The summation has been performed using a variational procedure The method consists in summing over the states which diagonalize the Hamiltonian on a variational basis

EXAMPLE
1S-2S TWO-PHOTON TRANSITION AMPLITUDE IN ATOMIC HYDROGEN

ν [Rydberg]	VARIATIONAL D(1s-2s) velocity gauge	EXACT D(1s-2s)
0.3750	11.780483	11.780483
0.5250	14.731873	14.731873
0.6750	41.148411	41.148411
0.6875	49.687778	49.687778
0.7000	62.659474	62.659474
0.7125	84.525170	84.525170
0.7250	128.683525	128.683525
0.7375	262.165418	262.165419
0.7475	1334.32609	1334.32614

THE RADIAL BASIS SET FOR p STATES ARE OF THE FORM

r exp[-r/q^(n-5)]

q = 1.2   n = 1,2, ...10

SUMMATION OVER INTERMEDIATE STATES

The summation has been performed using a variational procedure which consists in summing over the states diagonalizing the Hamiltonian on the variational basis

EXAMPLE
1S-3S TWO-PHOTON TRANSITION AMPLITUDE IN ATOMIC HYDROGEN

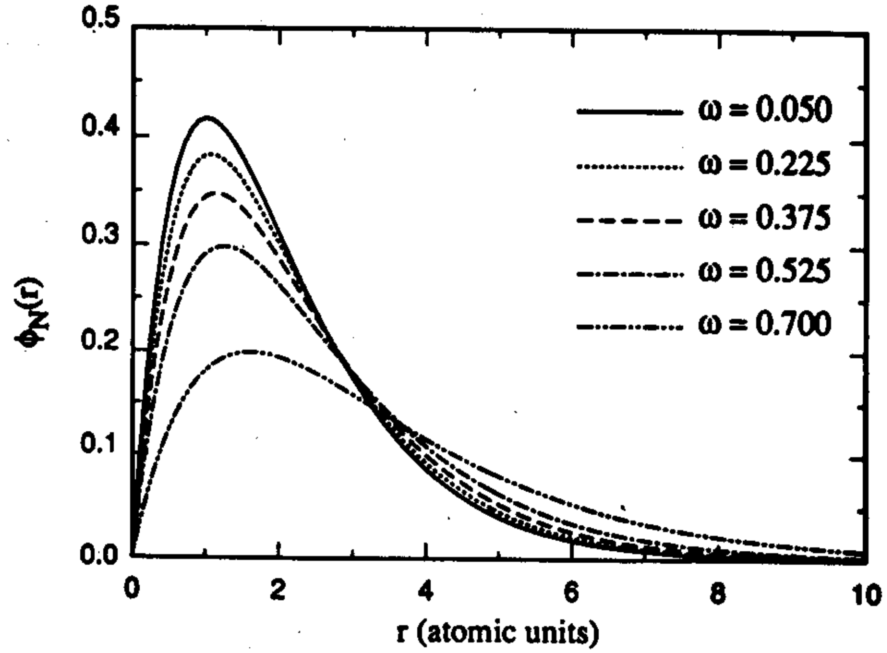
ν [Rydberg]	VARIATIONAL D(1s-3s)	EXACT D(1s-3s)
0.4444	-3.2108	-3.2109
0.6750	-1.6699	-1.6693
0.7000	0.9832	0.9847
0.7250	11.211	11.216
0.7475	226.71	226.81
0.7650	-58.184	-58.200
0.8000	-38.301	-38.310
0.8250	-46.571	-46.580
0.8500	-74.412	-74.420
0.8750	-219.98	-219.98
0.8860	-1116.0	-1117.2

THE RADIAL BASIS SET FOR s & p STATES ARE GAUSSIANS OF THE FORM
exp[-r^2/a] & r*exp[-r^2/a]
24 EXPONENTS IN A GEOMETRICAL SEQUENCE ARE USED
10^-3 < a < 10^5 (a.u.)

s-states   -1.0000 / -0.1111 Rydberg
p-states   -0.25 /-0.1111 /-0.0625 /-0.0397 / -0.0214 /+0.0821 /... Rydberg

1s - 20s TRANSITIONS IN HYDROGEN			
18 Exponentials r (min) = 1, r (max) = 20			
ω	p	x	Exact
0.4950	-0.1087	-0.1092	-0.1088
0.7250	0.3072	0.3058	0.3071
0.7475	5.3453	5.3457	5.3452
0.7525	-5.8903	-5.8941	-5.8906
0.7800	-0.7632	-0.7651	-0.7634
0.8000	-0.5886	-0.5904	-0.5888
0.8500	-0.3801	-0.3817	-0.3803
0.8860	4.5443	4.5362	4.5441
0.9000	-2.3474	-2.3491	-2.3478
0.9200	-1.1739	-1.1783	-1.1743
0.9400	-9.6214	-9.6510	-9.6228
0.9485	-3.5060	-3.5051	-3.5065
0.9660	-6.9256	-6.8370	-6.9226
0.9700	1.4787	1.4803	1.4832
0.9720	101.8448	99.6437	101.7994
0.9740	-21.8227	-21.2615	-21.7944
0.9760	-11.8729	-11.6302	-11.8470
0.9780	0.4384	-0.1501	0.4426
0.9800	-84.5782	-78.7665	84.1434

RADIAL PART OF $\phi_N(\psi_{1s}, \omega)$
FOR DIFFERENT ω [Ry]



$$(H - E_{1s} + \hbar\omega)\phi = p_z \psi_{1s}$$

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