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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
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**College on Medical Physics:
Radiation Protection and Imaging Techniques**

5 - 23 September 1994

Monte Carlo

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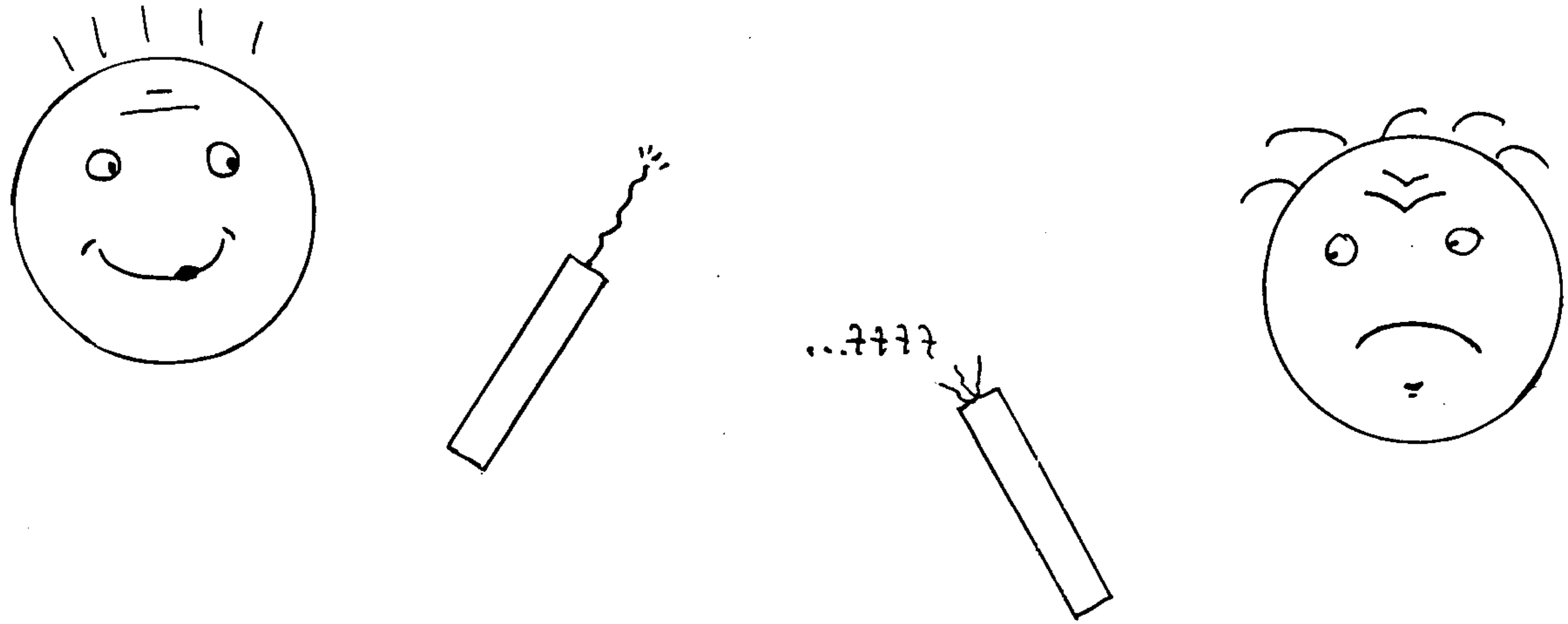
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MONTE CARLO

TRANSFORM

DETERMINISTIC PROBLEMS

INTO

PROBABILISTIC PROBLEMS

COMPUTER PROGRAMMING

$$2\text{ C's} \quad C = 2$$

$$5\text{ B's} \quad B = 3$$

$$8\text{ A's} \quad A = 4$$

Average grade $\langle G \rangle$

$$\langle G \rangle = (2 \times 2 + 5 \times 3 + 8 \times 4)/15$$

$$= \frac{56}{15}$$

$$\langle G \rangle = 3.73$$

$$p(C) = \frac{2}{15} \qquad p(B) = \frac{5}{15}$$

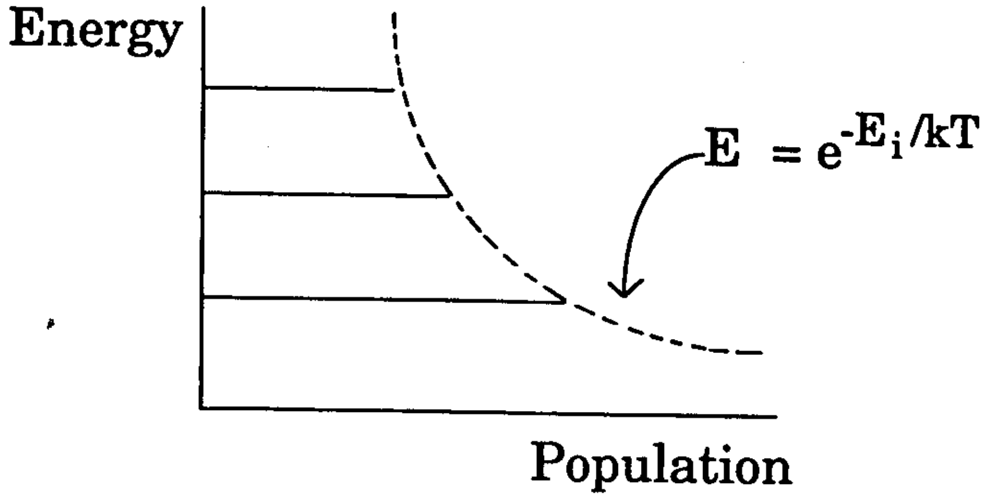
$$p(A) = \frac{8}{15}$$

$$\langle G \rangle = \sum_{i=1}^{3} G_i p_i$$

$$p_i = \frac{n_i}{\sum n_i}$$

$$= \frac{n_i}{N}$$

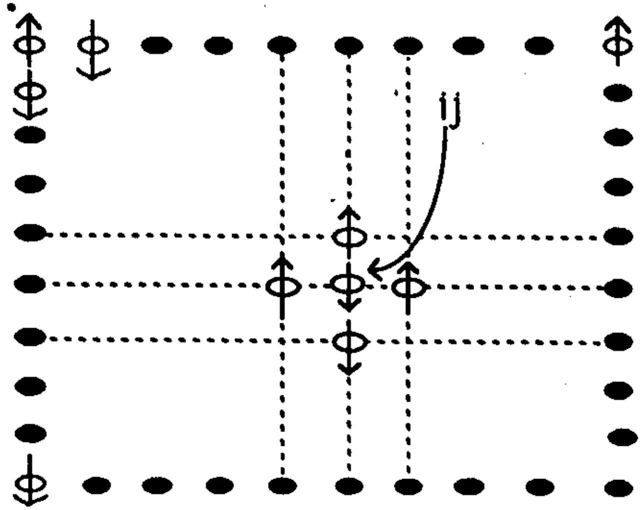
E population $\propto$ Boltzman factor



$$\langle E \rangle = \sum_{i=1}^{n} E_i \cdot P_i$$

$$P_i = \frac{e^{-E_i/kT}}{\sum e^{-E_i/kT}}$$

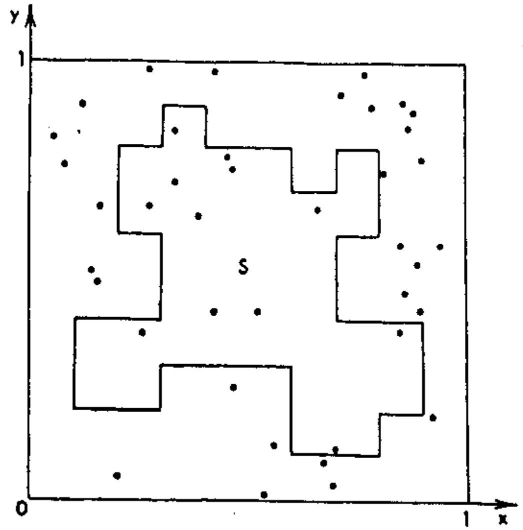
$$= \frac{e^{-E_i/kT}}{Z}$$



$10 \times 10 = 100$

$Z = \sum e^{-E_i/kT}$ partition function

$$E_{ij} = J\left(S_{ij}\left[S_{i,j+1} + S_{i+1,j} + S_{i,j-1} + S_{i-1},S_j\right]\right)$$



Since $S_{ij} = \pm 1$

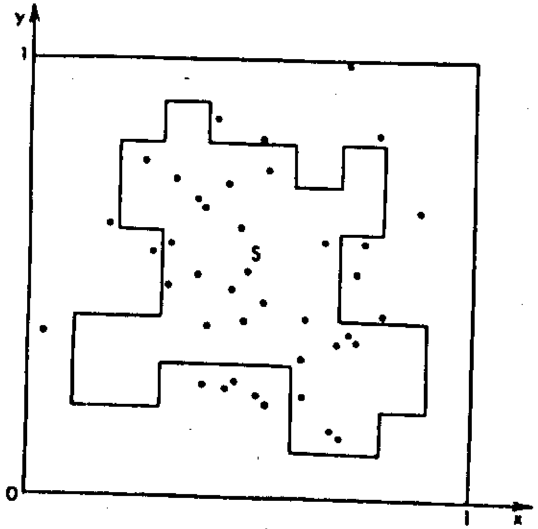
$$E_{ij} = J[-1-1-1+1] = -2J$$

N° of states $2^{100} \sim 10^{30}$

$$\langle E\rangle = \frac{\sum E_i e^{-E_i/kT}}{Z} \rightarrow 1\mu s$$

$t = 10^{30} \times 10^{-6}$ s $= 10^{24}$ s

$= 10^8$ times $\times$ life of Univ!!



Monte Carlo

$N = 40$

$n = 12$

$$\frac{n}{N} = \frac{12}{40} = 0.30$$

$S_c = 0.30$

$S_e = 0.35$

Shotgun

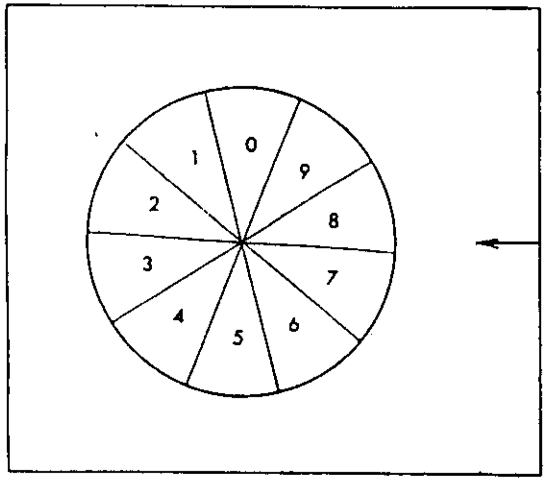
$N = 40$

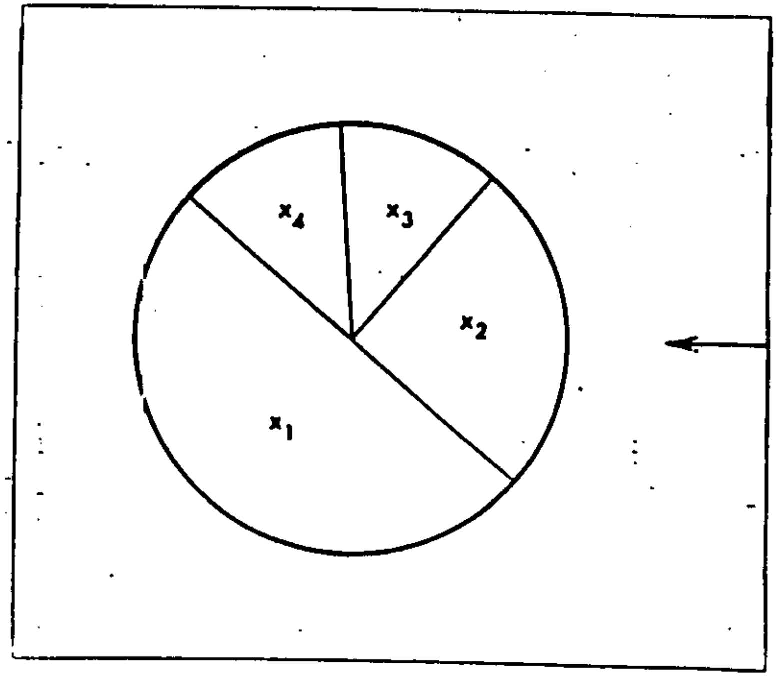
$n = 24$

$$\frac{n}{N} = \frac{24}{40} = 0.60$$

$S_c = 0.60$

$S_e = 0.35$





$$\begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ 0{,}5 & 0{,}25 & 0{,}125 & 0{,}125 \end{pmatrix}$$

$$\xi = \begin{pmatrix} x_1 & x_2 & \dots & x_n \\ p_1 & p_2 & \dots & p_n \end{pmatrix}$$

$$0 < y < 1$$

$$y = p_1,$$

$$y = p_1 + p_2$$

$$y = p_1 + p_2 + p_3,$$

$$y = p_1 + p_2 + \dots + p_{n-1}.$$

$$\mathsf{P}\{0 < \gamma < p_1\} = p_1.$$

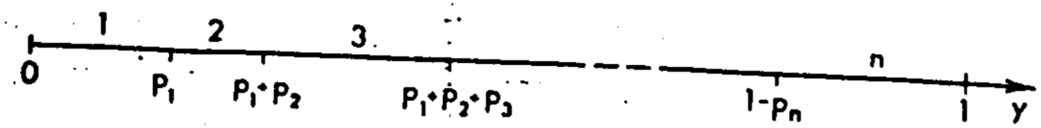
$$\mathsf{P}\{p_1 < \gamma < p_1 + p_2\} = p_2,$$

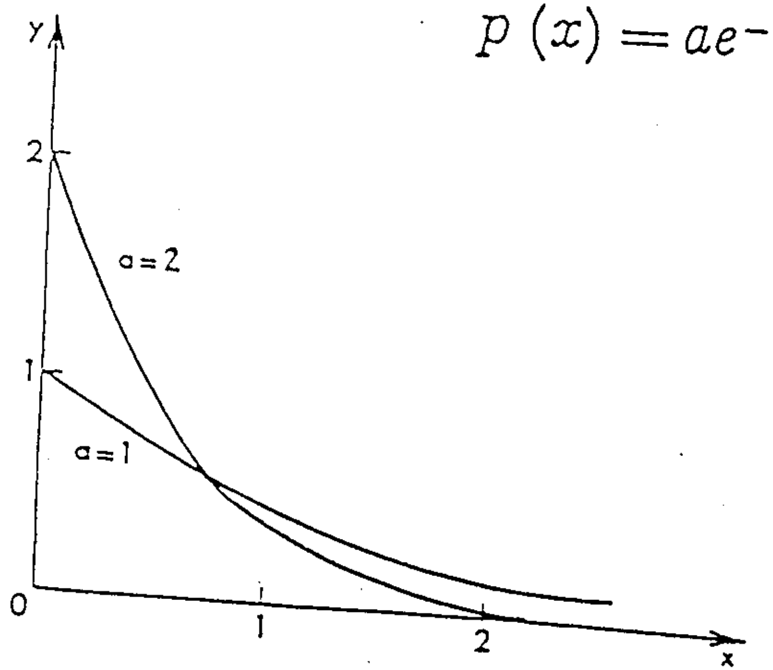
$$\dots\dots\dots\dots\dots\dots\dots\dots$$

$$\mathsf{P}\{p_1 + p_2 + \dots + p_{n-1} < \gamma < 1\} = p_n.$$

$$\xi = x_i$$

$$p_1 + p_2 + \dots + p_{i-1} < \gamma < p_1 + p_2 + \dots + p_i$$





$$p(x) = ae^{-ax}$$

$$\int_a^{\xi} p(x)\,dx = \gamma$$

$$p(x) = \frac{1}{b-a} \qquad a < x < b$$

$$\int_a^{\eta} \frac{dx}{b-a} = \gamma$$

$$\frac{\eta - a}{b-a} = \gamma$$

$$\eta = a + \gamma(b-a)$$

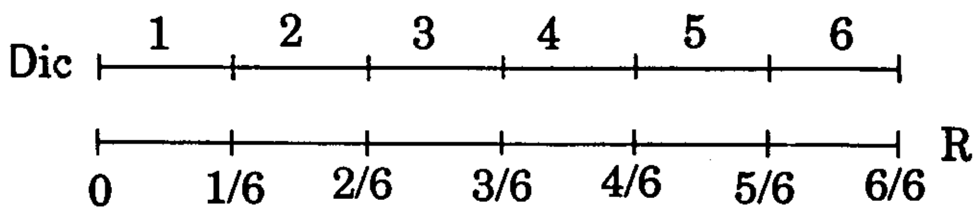
$$R = \int_{a}^{\xi} p(x)\,dx$$

for a dice $p(x) = 1/6$ and

$$\int \rightarrow \sum$$

$$R = \sum_{1}^{6} p(x)$$

For	$x = 1$	$0 < R < 1/6$
For	$x = 2$	$1/6 < R < 2/6$
For	$x = 3$	$2/6 < R < 3/6$
For	$x = 4$	$3/6 < R < 4/6$
For	$x = 5$	$4/6 < R < 5/6$
For	$x = 6$	$5/6 < R < 6/6$



MACHINE THAT PERFORM INTEGRALS

N	R1	R2
1	0.38	0.26
2	0.18	0.73
3	0.73	0.82
4	0.38	0.88
5	0.83	0.06
6	0.89	0.81
7	0.86	0.88
8	0.58	0.55
9	0.07	0.44
10	0.02	0.14
11	0.65	0.67
12	0.12	0.70
13	0.14	0.52
14	0.87	0.60
15	0.72	0.99
16	0.42	0.63
17	0.96	0.09
18	0.88	0.50
19	0.04	0.77
20	0.24	0.56
21	0.19	0.08
22	0.82	0.14
23	0.48	0.64
24	0.53	0.37

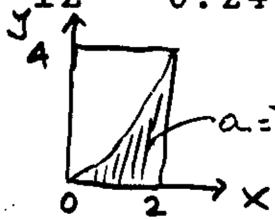
$$I = \int_0^2 x^2 dx$$

25	0.46	0.42
26	0.44	0.80
27	0.90	0.16
28	0.89	0.87
29	0.20	0.41
30	0.67	0.27
31	0.64	0.38
32	0.49	0.57
33	0.96	0.65
34	0.28	0.85
35	0.62	0.01
36	0.50	0.94

MONTE CARLO INTEGRATION

INT(X**2)/0-2=2,333

N	X=2*R1	Y=X**2	R=4*R2	M	S
1	0.75	0.57	1.05	0	0.00
2	0.36	0.13	2.91	0	0.00
3	1.46	2.13	3.30	0	0.00
4	0.77	0.59	3.52	0	0.00
5	1.66	2.75	0.23	1	1.60 *
6	1.77	3.15	3.23	1	1.33
7	1.71	2.94	3.53	1	1.14
8	1.17	1.36	2.21	1	1.00
9	0.14	0.02	1.77	1	0.89
10	0.04	0.00	0.56	1	0.80
11	1.31	1.71	2.67	1	0.73
12	0.24	0.06	2.80	1	0.67



$a = I^*$ $\quad \frac{1}{J} = \frac{I}{2\times4}$ $\quad I = \frac{2\times4}{J} = 1.6$

13	0.27	0.07	2.06	1	0.62
14	1.74	3.04	2.38	2	1.14
15	1.45	2.10	3.96	2	1.07
16	0.85	0.72	2.52	2	1.00
17	1.92	3.69	0.35	3	1.41
18	1.76	3.11	1.99	4	1.78
19	0.08	0.01	3.06	4	1.68
20	0.48	0.23	2.24	4	1.60
21	0.38	0.14	0.34	4	1.52
22	1.63	2.67	0.55	5	1.82
23	0.96	0.93	2.58	5	1.74
24	1.06	1.12	1.47	5	1.67

25	0.91	0.83	1.69	5	1.60
26	0.87	0.76	3.21	5	1.54
27	1.80	3.23	0.65	6	1.78
28	1.78	3.16	3.47	6	1.71
29	0.40	0.16	1.66	6	1.66
30	1.33	1.78	1.10	7	1.87
31	1.28	1.64	1.51	8	2.06
32	0.99	0.97	2.26	8	2.00
33	1.91	3.65	2.59	9	2.18
34	0.56	0.31	3.42	9	2.12
35	1.23	1.51	0.02	10	2.29
36	1.00	1.00	3.75	10	2.22

$\frac{10}{36} = \frac{I}{8} \quad I = \frac{80}{36}$

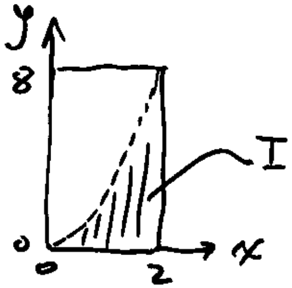
$\boxed{I = 2.22}$

$$\int_0^2 x^3 dx$$

MONTE CARLO INTEGRATION

INT(X**3)/0-2=4.0

N	X=2*R1	Y=X**3	R=8*R2	M	S
1	0.75	0.43	2.10	0	0.00
2	0.36	0.05	5.82	0	0.00
3	1.46	3.10	6.59	0	0.00
4	0.77	0.45	7.03	0	0.00
5	1.66	4.55	0.46	1	3.20
6	1.77	5.58	6.45	1	2.67
7	1.71	5.04	7.06	1	2.29
8	1.17	1.59	4.43	1	2.00
9	0.14	0.00	3.54	1	1.78
10	0.04	0.00	1.12	1	1.60
11	1.31	2.24	5.34	1	1.45
12	0.24	0.01	5.60	1	1.33
13	0.27	0.02	4.12	1	1.23
14	1.74	5.30	4.76	2	2.29
15	1.45	3.04	7.93	2	2.13
16	0.85	0.61	5.05	2	2.00
17	1.92	7.10	0.71	3	2.82
18	1.76	5.49	3.97	4	3.56
19	0.08	0.00	6.12	4	3.37
20	0.48	0.11	4.48	4	3.20
21	0.38	0.05	0.68	4	3.05
22	1.63	4.36	1.11	5	3.64
23	0.96	0.90	5.16	5	3.48
24	1.06	1.18	2.94	5	3.33



$$\frac{n}{N} = \frac{I}{A} \qquad I = \frac{n}{N} A$$
$$= \frac{n}{36} \times 16$$

25	0.91	0.76	3.39	5	3.20
26	0.87	0.66	6.43	5	3.08
27	1.80	5.81	1.30	6	3.56
28	1.78	5.61	6.93	6	3.43
29	0.40	0.07	3.31	6	3.31
30	1.33	2.38	2.19	7	3.73
31	1.28	2.11	3.02	7	3.61
32	0.99	0.96	4.52	7	3.50
33	1.91	6.98	5.17	8	3.88
34	0.56	0.17	6.83	8	3.76
35	1.23	1.86	0.05	9	4.11
36	1.00	0.99	7.50	9	4.00

$$I = \frac{n \times 16}{36}$$

$n = 9$ $$\boxed{I = \frac{9 \times 16}{36} = 4}$$

$$I \doteq \int_0^{\pi/2} \operatorname{sen} x \, dx$$

$$\int_0^{\pi/2} \operatorname{sen} x \, dx = [-\cos x]_0^{\pi/2} = 1$$

$$p_\xi(x) \equiv 2/\pi$$

$$p_\xi(x) = 8x/\pi^2$$

$$\xi = \pi\gamma/2.$$

$$I \approx \frac{\pi}{2N} \sum_{j=1}^{N} \operatorname{sen} \xi j$$

$$a < x < b$$

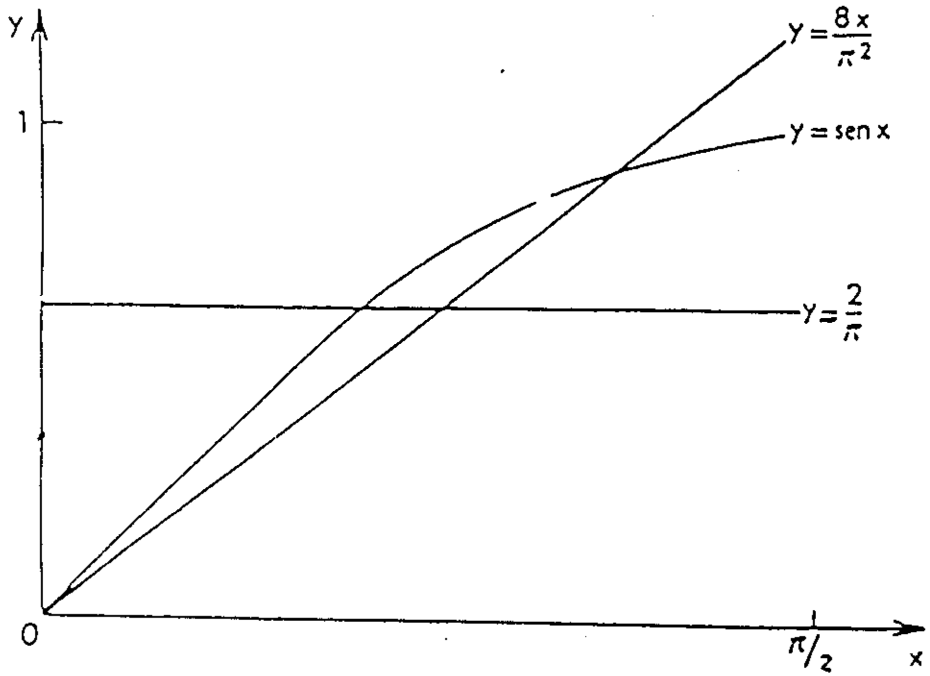
$$I = \int_a^b g(x)\,dx$$

$$\eta = g(\xi)/p_\xi(\xi).$$

$$M\eta = \int_a^b [g(x)/p_\xi(x)]\,p_\xi(x)\,dx = I$$

j	1	2	3	4	5	6	7	8	9	10
γ_j	0,865	0,159	0,079	0,566	0,155	0,664	0,345	0,655	0,812	0,332
ξ_j	1,359	0,250	0,124	0,889	0,243	1,043	0,542	1,029	1,275	0,521
sen ξ_j	0,978	0,247	0,124	0,776	0,241	0,864	0,516	0,857	0,957	0,498

$$I \approx 0{,}952.$$



$$p_\xi(x) = 8x/\pi^2.$$

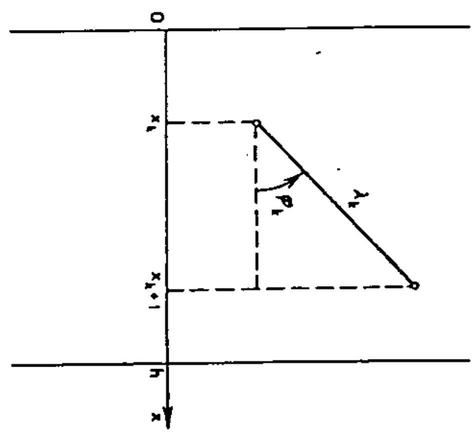
$$\int_0^\xi (8x/\pi^2)\,dx = \gamma;$$

$$\xi = \frac{\pi}{2}\sqrt{\gamma}.$$

$$I \approx \frac{\pi^2}{8N}\sum_{j=1}^{N}\frac{\operatorname{sen}\xi_j}{\xi_j}.$$

j	1	2	3	4	5	6	7	8	9	10
γ_j	0,865	0,159	0,079	0,566	0,155	0,664	0,345	0,655	0,812	0,332
$\breve{\xi}_j$	1,461	0,626	0,442	1,182	0,618	1,280	0,923	1,271	1,415	0,905
$\dfrac{\operatorname{sen} \breve{\xi}_j}{\breve{\xi}_j}$	0,680	0,936	0,968	0,783	0,937	0,748	0,863	0,751	0,698	0,868

$$I \approx 1{,}016.$$



$$\Sigma = \Sigma_c + \Sigma_s$$

$$\Sigma_c / \Sigma$$

$$p(x) = \Sigma e^{-\Sigma x}$$

$$R = \int_0^{\lambda} p(x')\, dx'$$

$$\lambda = -\frac{1}{\Sigma} \ln \gamma.$$

$$\mu = 2\gamma - 1.$$

$$\lambda_k = -(1/\Sigma) \ln \gamma$$

$$x_{k+1} = x_k + \lambda_k \mu_k.$$

$$x_{k+1} > h.$$

$$x_{k+1} < 0.$$

$$\gamma < \Sigma_c / \Sigma .$$

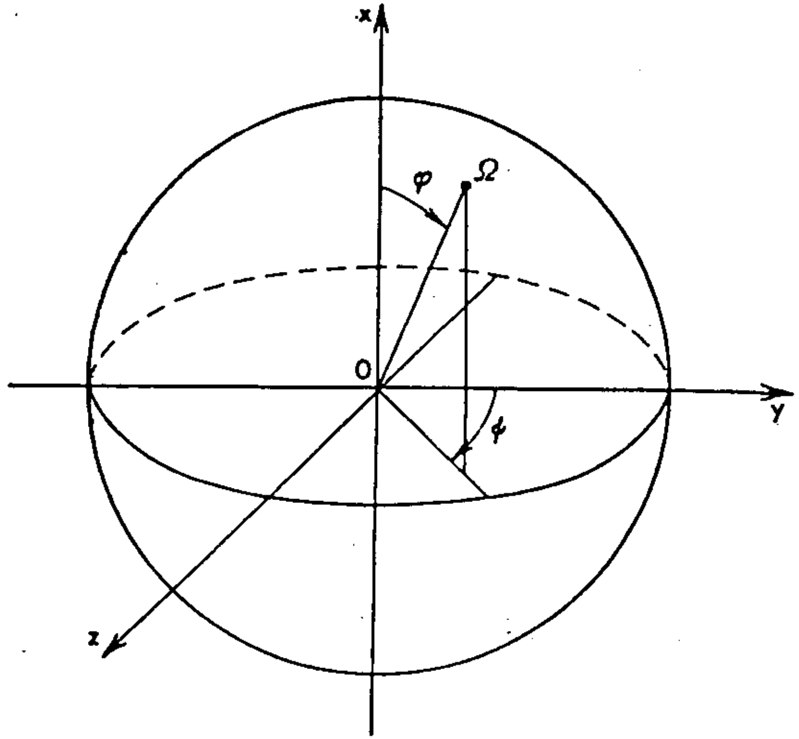
$$\mu_{k+1} = 2\gamma - 1$$

$$x_0 = 0 \text{ y } \mu_0 = 1.$$

$$p^+ \approx \frac{N^+}{N},$$

$$p^0 \approx \frac{N^0}{N},$$

$$p^- \approx \frac{N^-}{N}$$



$$dS = \operatorname{sen} \varphi \, d\varphi \, d\psi$$

$$0 \leqslant \varphi \leqslant \pi, \; 0 \leqslant \psi < 2\pi.$$

$$p(\varphi, \psi) \, d\varphi \, d\psi = dS/4\pi$$

$$p(\varphi, \psi) = \frac{\operatorname{sen} \varphi}{4\pi}$$

$$p_\varphi(\varphi) = \int_0^{2\pi} p(\varphi, \psi) \, d\psi = \frac{\operatorname{sen} \varphi}{2}$$

$$p_\psi(\psi) = \int_0^{\pi} p(\varphi, \psi) \, d\varphi = \frac{1}{2\pi}$$

$$p(\varphi, \psi) = p_\varphi(\varphi) \, p_\psi(\psi)$$

$$\psi = 2\pi\gamma$$

$$\frac{1}{2} \int_0^{\varphi} \operatorname{sen} x \, dx = \gamma$$

$$\cos \varphi = 1 - 2\gamma$$

The cross section for photoelectric effect in K shell of an atom with atomic number Z for a photon of energy hν is

$$\sigma_P = \phi_0 4\sqrt{2}\left(\frac{1}{\alpha}\right)^{7/2}\frac{Z^5}{137^4};\phi_0 = \frac{8}{3}\pi r_0^2.$$

where $r_0^2 = \left(e^2/mc^2\right)^2 = 7.94\times\ 10^{26}\mathrm{cm}^2$,

$\alpha = h\nu/mc^2$ is the energy in units of mc^2

Total Compton Cross Section

$$\sigma_C = 2\pi r_0^2\left\{\frac{1+\alpha_0}{\alpha_0^2}\left[\frac{2(1+\alpha_0)}{1+2\alpha_0}-\frac{\ln(1+2\alpha_0)}{\alpha_0}\right]+\frac{\ln(1+2\alpha_0)}{2\alpha_0}-\frac{1+3\alpha_0}{(1+2\alpha_0)^2}\right.$$

The expression for α as a function of α_0 and of the angle of photon scattering θ is given by

$$\alpha = \alpha_0/[1+\alpha_0(1-\cos\theta)].$$

$$\sigma = \sigma_c + \sigma_p$$

Probability that it will be a compton process $P(c) = \frac{\sigma_c}{\sigma}$

Probability that it will be a Photo Eletric Process

$$P(p) = \frac{\sigma_p}{\sigma} \quad \text{or} \quad P(p) = 1 - P(c)$$

The mean free path is

$$\lambda = \frac{1}{\sigma\rho}$$

The mean free path is

$$\lambda = \frac{1}{\sigma\rho}$$

The distance x travel before collision have a probability

$$p(x) = \lambda\, e^{-\lambda x}$$

The distance x travel before collision have a probability

$$p(x) = \lambda e^{-\lambda x}$$

So

$$R = \int_0^x \lambda e^{-\lambda x'} dx'$$

$$= 1 - e^{-\lambda x}$$

$$\text{or } x = -\frac{1}{\lambda} \ell n(1-R)$$

1 - R has the same distribution as R so

$$x = -\frac{1}{\lambda} \ell n R$$