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"Toxicant-Induced Mortality in Models of *Daphnia* Populations"

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TOXICANT-INDUCED MORTALITY IN MODELS OF *DAPHNIA* POPULATIONS

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Abstract—A method to determine the mortality effects of a hydrophobic chemical on a population is proposed. The ecotoxicological protocol is based on individual organism response and is derived from the static theory of "survival of the fattest." This study, focusing upon effects of mortality and the effects of toxicant stress on population succession, examines the static assessment survival of the fattest in a dynamic population model. A premise in this approach is that risk assessment should not be based solely upon chemical properties of the toxicant and that the biology of the exposed organisms is an important factor in the determination of effects.

Keywords—Effects *Daphnia* populations Lipophilic chemicals Narotics QSARs

INTRODUCTION

The relationships between toxic chemicals and their effects on populations are intricate, complex and frequently poorly understood. The first steps in current chemical assessment procedures to delineate these relationships are generally based upon quantitative structure-activity relations (QSARs). QSARs are mathematical expressions that relate biological activity (molar concentrations causing quantal effects) to descriptors of molecular properties of a sequence of chemical compounds. These approaches are based on properties of the chemicals and ignore a most important part of the problem—biological properties of the exposed organisms. It is our opinion that the present theoretical basis for determining effects of chemicals on populations is inadequate primarily because past developments do not encompass sufficient biological detail. This inadequacy is magnified considerably when it is noted that an improper investigative focus at the population level is usually employed. These deficiencies have hindered development in ecotoxicology. Such hindrances are especially restrictive regarding creation of a foundation for the determination of effects of toxicants on a biological system. The theme of this article is that a proper focal level in ecotoxicology must consider initially the individual, organismic response. Chemical impact occurs at the level of the individual, not at the population level. Even though the target site of a chemical may be specific tissues, the exposed, affected individual is the appropriate reference point for extrapolation to the population level.

In the classical ecological organizational scheme, the individual is special and unique. In the hierarchy from the cell to tissue to individual organism to population, the levels below the individual are sets of genetically identical elements while the population is structured by genetic variation [1]. It is individual variation that is often missing or suppressed in studies of the effects of chemicals on populations. Consideration of this variability is needed to develop properly the appropriate theoretical basis for ecotoxicology. Variation in the distribution of genetic, physiologic and physical characteristics of individuals in a population together with the biogeochemical environment of the population determines the characteristics of the effects resulting from chemical exposure.

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Population susceptibility intrinsically implies the existence of variation – variation that is viewed here as structure in the population. Susceptibility to a chemical, an individual property, is not static but is a variable related to the dynamics of the individual. Dynamic susceptibility of individuals must be reflected in the distribution of susceptibility for the population. A basic premise of this article is that effects of a chemical at the population level result entirely from effects of the chemical on the individuals that compose the population.

For investigations of effects, it is essential to note distinctions between individual properties and population properties. Individual properties include physiological variables, such as body size and composition, as well as tolerance or susceptibility to a toxicant. Population properties include distributions of individual properties, such as distribution of tolerances or susceptibility, and moments of these distributions.

For modeling purposes, techniques exploring the role of the individual in determining population dynamics are a relatively recent phenomenon but are now evolving at a rapid rate [2]. Most of the original works in mathematical ecology do not take individual properties into account. Aggregation into a population state variable was necessary for computational reasons. Studies at the population or higher organizational levels have not proved to be successful in ecotoxicology because individual variation, including a population's tolerance distribution, is lost in these representations. Recent progress in the analysis of individual-based population models is encouraging because there have been significant developments in the area of accessible computing power. This allows one to track large numbers of individuals in a reasonable computational time period. We shall utilize an approach that first develops a physiologically structured population model and then performs the analysis by numerical techniques.

This article presents a theoretical study of the effects of a lipophilic narcotic on a dynamic *Daphnia* population. The work is theoretical because data, facilities and techniques are not presently available to generate the information needed for corroboration of the basic hypotheses or outcomes. Both the assumptions and the conclusions obtained are conjectures that must be tested; however, because the foundations of our model formulations are solidly grounded in the biological and toxicological literature, we feel that they merit the efforts necessary to check their consistency. The effects literature is sparse because theoretical efforts in ecotoxicology have virtually ignored organismic biology. These efforts represent our initial attempt to include physiological biology in a population assessment procedure.

Effects are limited for the present discussion to mortality in the population. This restriction is not necessary but it is sufficient to illustrate the procedure that we suggest. The rudiments of the underlying theory for mortality are given in Lassiter and Hallam [3], where a theory, intuitively nicknamed "survival of the fattest," is developed for acute chemical exposures and a static population. It analyzes an effect of toxic exposure by relating the *n*-octanol/water partition coefficient to the partition coefficient of the fat and aqueous phases of the animal, by hypothesizing equilibration within the body, and by employing quantitative structure–activity relationships as a component of the biological response assessment. Because the chemical is assumed to be lipophilic, a known distribution of lipid in the population is necessary to apply this static theory. We are aware of only two distributions of lipid in an aquatic population (Brockway [4] and J. Clark, personal communication). These distributions, for static fish populations, indicate that there can be much variation in lipid content of fish of the same class. We assume that the same is true for daphnids.

The dynamic behavior of individuals coupled with the possibility of chronic or multiple acute toxicant exposures requires a dynamic perspective. To our knowledge, expressions for the dynamic distribution of lipid in any aquatic population are nonexistent at the present time. Nondestructive sampling methods are currently being developed and it is hoped they will ultimately lead to progress in this area. For present purposes, however, it is necessary to obtain dynamic lipid distributions by methods other than experimentation. The approach espoused here, to focus at the biological/chemical interface of the individual, necessitates development of a dynamic representation of an individual organism. The specific individual representation employed is one developed for the purpose of determining the effects of a chemical on an individual daphnid. The individual model, based upon energetics and described in detail in Hallam et al. [5], is an important part of the population model. A brief summary of the individual model is presented below so that its role in the dynamics of the population can be understood.

Exposure and uptake of chemical also must be

modelled to determine the effects of a toxicant on a population. We utilize a modification of the uptake model, FGETS, developed by Barber et al. [6]. GETS and FGETS were formulated for fish and must be appropriately modified to be used for other organisms such as *Daphnia*. Required changes are discussed briefly below.

Population dynamics are a compilation of all individual dynamics. We model population dynamics by employing a partial differential equation that incorporates individual dynamics explicitly in the representation and describes the behavior of the population in terms of a density that is a function of individual physiological model variables and time. The population model, its behavior in the absence of the chemical and the behavior in the presence of the toxicant will be discussed later in this work.

This paper indicates theoretical developments formulated to explore the effects of a chemical on a dynamic *Daphnia* population when toxic exposure is allowed through both the environmental and the food chain pathways.

MODEL OF INDIVIDUAL DAPHNID DYNAMICS

The dynamics of an individual daphnid are well documented in the literature. Goulden and Hornig [7] and Tessier et al. [8] explore the role of lipids and their dynamics in *Daphnia* populations. Kooijman and Metz [9] employ a representation of the von Bertalanffy type to model the energetics of a daphnid. They also apply their model to the assessment of effects of a chemical on a *Daphnia* population. Kooijman [10] extends these results to develop a theoretical formulation that accounts for a generic storage compartment. Philosophically, our model is closely related to these efforts; however, the specifics of both the individual and the population models are considerably different than those of Kooijman.

Any modeling project must be consistent with its objectives; hence, our individual model must allow for relevant interaction with the chemical and must be able to account for its toxicity. Appropriate model components must be chosen with the specific chemical and type of exposure in mind.

Most industrial chemicals and many chemicals of environmental concern are non-ionic, are nonreactive, induce baseline narcosis and are, to some degree, lipophilic [11]. The chemicals employed in our illustrations are assumed to have these characteristics although the procedure only requires lipophilicity and nonreactivity. A method of determining the toxicity effects threshold, such as application of a QSAR, is also needed. The lipid in an individual buffers the action of a lipophilic chemical, allowing larger body burdens in fatter individuals than in less fat organisms to elicit equivalent biological response. Thus, the additional lipid in an individual results in an extension of the toxicity effect thresholds for an acute exposure. Lipid storage provides protection against toxic stress from transient exposures only if the organism is not forced to rapidly mobilize quantities of stored lipids. When an organism rapidly utilizes stores of lipid in a situation where high body burdens have been obtained, internal release of the chemical can lead to toxicity effects under conditions where there is no change in the external environmental concentration of a chemical. These considerations indicate that, for the class of chemicals under consideration, a dynamic lipid compartment is necessary in any individual model that is utilized to represent the biological/chemical interaction.

MATHEMATICAL MODEL OF A DAPHNID

We now summarize the model of a life history of a daphnid. More detail and background information can be found in [5]. Figure 1 is the conceptual model listing compartments and the flow chart for an adult female daphnid. The model assumes that the only inputs to the lipid and protein compartments are obtained from a decoupling of the food lipid and structure, that is, no synthesis of fat can occur from the carbohydrates and proteins of the resource. This is not a valid hypothesis for most higher tropic level organisms but may be for *Daphnia*. This decoupling hypothesis is a simplifying factor in the individual model and consequently in the population model.

Let m_L and m_S denote the mass of the lipid and mass of the structure, respectively, in an individual organism. The dimensions, units, estimated values of parameters and the variables associated with the individual are listed in Table 1. Structure is regarded as primarily protein and carbohydrates. Each of these components is assumed to have both labile and nonlabile portions. The nonlabile portion of the structure is viewed as protein and carbohydrates bound in soma and is designated in the model by m_{PS}, the mass of the protected structure. The labile portion of the structure component is represented by $m_S - m_{PS}$. The nonlabile lipid is assumed to be proportional to m_{PS} and in the

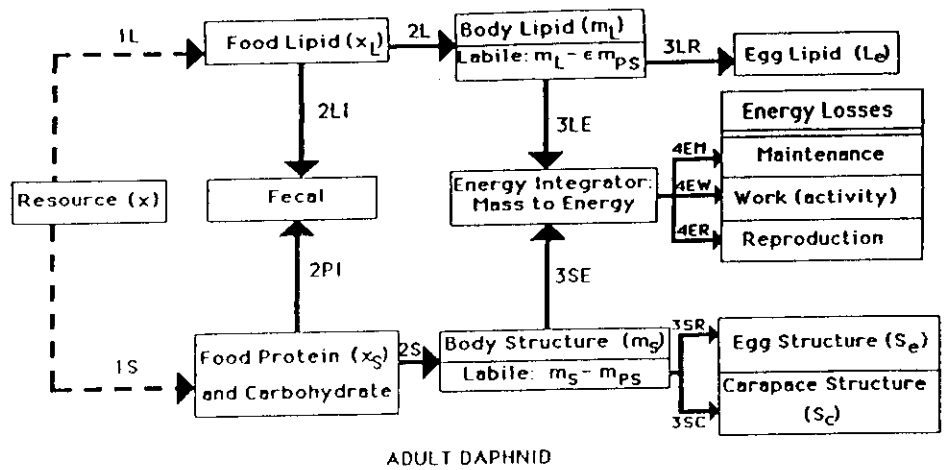


Fig. 1. Compartment and flow diagram for the individual model.

Table 1. Individual compartment labels and related interpretations and parameters

Symbol	Interpretation	Dimension
m_S (variable)	Mass of structural material	mg
m_L (variable)	Mass of lipid	mg
x (parameter)	Density of resource	mg/mm^3
x_S (parameter)	Density of resource structure (fraction of x)	mg/mm^3
x_L (parameter)	Density of resource lipid (fraction of x)	mg/mm^3
L_e (variable)	Total lipid in eggs	mg
S_e (variable)	Total structural material in eggs	mg
S_c (variable)	Total structural material in carapace	mg
m_{PS} (variable)	Mass of protected structure (nonlabile)	mg
ϵ (parameter)	Fraction lipid associated with nonlabile structure	mg lipid/mg structure
Input flows: $2L$, $2S$		
A_{0L} (parameter)	Assimilation rate/ingestion rate	nondimensional
A_{0S} (parameter)	Assimilation rate/ingestion rate	nondimensional
A_1 (parameter)	Reciprocal of constant allometrically relating maximal filtering rate to organism length squared	mg$^{2/3}$ day mm^{-3}
A_2 (parameter)	Reciprocal of constant allometrically relating maximal ingestion rate to organism length squared	day
Output flows		
E_S (variable)	Maximum number of eggs made from labile structure	egg
e_L (parameter)	Minimal lipid content per egg	mg egg^{-1}
e_M (parameter)	Maximal lipid content per egg	mg egg^{-1}
TED (variable)	Power demand	joules day^{-1}
AE (variable)	Power supply	joules day^{-1}
A_3 (parameter)	Labile lipid mobilization rate	day^{-1}
A_4 (parameter)	Labile structure mobilization rate	day^{-1}
A_5 (parameter)	Multiplier representing energetic costs for activity due to viscous forces	joules mg$^{-1/3}$ day^{-1}
A_6 (parameter)	Multiplier representing energetic costs for activity due to inertial forces	joules mg$^{-2/3}$ day^{-1}
A_7 (parameter)	Energy required to maintain one mg lipid per day	joules mg^{-1} day^{-1}
A_8 (parameter)	Energy required to maintain one mg structure per day	joules mg^{-1} day^{-1}
A_9 (parameter)	Differentiation and growth rate of lipid while in brood pouch	day^{-1}
A_{10} (parameter)	Differentiation and growth rate of structure while in brood pouch	day^{-1}

model is represented by ϵm_{PS}; hence, the labile lipid is $m_L - \epsilon m_{PS}$. The density of the resource is denoted by x and we assume that $x = x_L + x_S$ where x_L and x_S are the lipid and structural portions of the resource density, respectively. The resource is assumed to be utilized for growth according to a hyperbolic uptake law [2,12]. The losses of energy for maintenance and the activity are assumed to operate on a continuous time scale. Hence, in intervals when there is no reproductive loss the daphnid is represented by the differential equations (see Table 1 for definition of symbols):

$$\frac{dm_L}{dt} = g_L = \frac{A_{0L} x_L m_S}{A_1 m_S^{1/3} + A_2 x} - \begin{cases} A_3(m_L - \epsilon m_{PS}) & TED > AE \\ A_3(m_L - \epsilon m_{PS}) \dfrac{TED}{AE} & TED \le AE, \end{cases} \quad (1)$$

where available energy is

$$AE = 37.68 A_3(m_L - \epsilon m_{PS}) + 16.75 A_4(m_S - m_{PS})$$

and the total energy demand is activity energy plus maintenance energy:

$$TED = A_5(m_L + m_S)^{1/3} + A_6(m_L + m_S)^{2/3} + A_7 m_L + A_8 m_S.$$

The differential equation for m_S is

$$\frac{dm_S}{dt} = g_S = \frac{A_{0S} x_S m_S}{A_1 m_S^{1/3} + A_2 x} - \begin{cases} A_4(m_L - \epsilon m_{PS}) & TED > AE \\ A_4(m_L - \epsilon m_{PS}) \dfrac{TED}{AE} & TED \le AE, \end{cases} \quad (2)$$

In the brood pouch, a juvenile is assumed not to have access to food; thus, individual dynamics are represented here by $dm_L/da = -A_9 m_L$, $dm_S/da = -A_{10} m_S$.

Each of the parameters A_i, $i = 0, 1, \ldots, 10$ is assumed to be constant. Representative values employed in our study along with sources are listed in Table 2. The reproductive losses are assessed at the discrete times of reproduction. It is clear that the processes of carapace formation and allocation of biomass to eggs occur over a continuous time frame, but because the time scales are small compared to the population evolution scale (and there is little information on the specific time scales of these processes), we treat them as discrete events. The reproductive losses include biomass allocation to eggs, the energy required to deposit this mass in the eggs, allocation of structure to carapace and the energy required to make the carapace. These operations, as well as the mechanism employed to determine the number of eggs produced, are described in detail in [5].

Table 2. Parameter values used in model analysis

Symbol	Value	Dimension	Source used in computation
x	5×10^{-7}	mg mm^{-3}	Lynch et al. [21]
x_L	1.25×10^{-7}	mg mm^{-3}	Blazka [22], Paloheimo et al. [23]
x_S	3.75×10^{-7}	mg mm^{-3}	Blazka [22], Paloheimo et al. [23]
ϵ	0.12	–	Kooijman and Metz [9], Tessier et al. [8]
A_{0L}	1	–	Blazka [22], Paloheimo et al. [23]
A_{0S}	0.8	–	Blazka [22], Paloheimo et al. [23]
A_1	5.5×10^{-7}	mg$^{2/3}$ day mm^{-3}	Lassiter [12]
A_2	0.08	day	Lassiter [12]
A_3	10	day^{-1}	Created
A_4	8	day^{-1}	Created
A_5	2.9×10^{-8}	joules mg$^{-1/3}$ day^{-1}	Gerritsen [24]
A_6	3.5×10^{-10}	joules mg$^{-2/3}$ day^{-1}	Gerritsen [24]
A_7	0.4	joules mg^{-1} day^{-1}	Kooijman [10]
A_8	0.5	joules mg^{-1} day^{-1}	Kooijman [10]
A_9	10	day^{-1}	Created
A_{10}	8	day^{-1}	Created

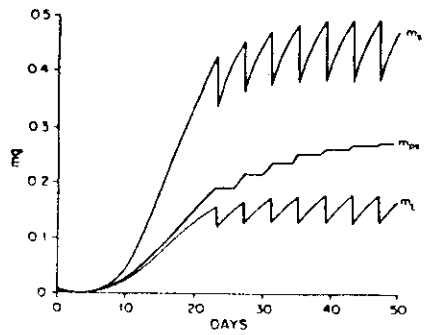


Fig. 2. Lipid (m_L) and structure (m_S) cycles in the individual model. The dynamics illustrate the decrease of size while in the brood pouch, exponential growth as a juvenile and the molt cycle. The nonlabile structure, m_{PS} is a nondecreasing function of age.

Examples of the numerical solution of the individual model are given in Figures 2 and 3. A typical individual is presented in Figure 2. The organism grows until it reaches reproductive size. The allocation of lipid and structural mass to reproduction is indicated by the instantaneous decreases in the component masses. After reaching reproductive maturity, the female is assumed to reproduce in a periodic manner. The gestation period is a species property that influences population dynamics.

UPTAKE IN AQUATIC ANIMALS

The uptake model that is employed in connection with the individual model above is a modification of FGETS [6]. This model, based upon thermodynamic potential, represents the chemical exchange between fish and aqueous environment that occurs across gill membranes and the chemical exchange that occurs across gut walls from ingestion of the contaminated resource. The model in its simplest form without food uptake is of the classical bioaccumulation form; however, an advantage of the approach of [6] is that the parameters of the uptake model are represented in considerable detail and they include many factors that can influence internal concentrations. These detailed features include the fractions of the organism that are lipid (P_L), aqueous (P_A) and structure (P_S); the partition coefficients that indicate the affinity of the chemical towards lipid (K_L) and structure (K_S); the conductance of the exposed membrane (k_W); the total weight of the organism (W_T), and the active (effective) exposure area (S). The general form of the model is

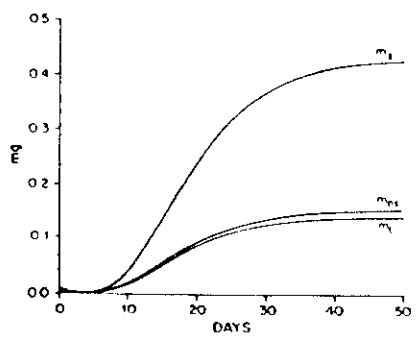


Fig. 3. Individual growth where reproductive size is not attained over an individual's life span.

$$\frac{dB_T}{dt} = S \cdot k_W \cdot C_W + C_F \cdot F - \frac{B_T}{W_T \cdot BCF}[S \cdot k_W + E \cdot k_E]. \quad (3)$$

In Equation (3) B_T represents the total toxicant in the organism; C_W and C_F represent the concentrations of toxicant in the environment and in the food, respectively; F and E are the mass fluxes of food and feces, respectively; BCF is the bioconcentration factor (total concentration in the organism/C_W); and k_E is the partition coefficient of chemical to excrement (C_E/C_A, where C_A and C_E are the concentrations of the chemical in the aqueous portion of the organism and its feces, respectively.) The dimensions, units and estimated values of parameters used in Equation (3) are indicated in Table 3. The unit conductance, k_W, may be calculated from the molecular weight of the chemical and the n-octanol/water partition coefficient. To compute k_W for fish, Barber et al. [6] employ fluid flow characteristic parameters such as the Sherwood number and the characteristic dimension of interlamellar channels and the toxicant's diffusion coefficient, which is a function of the molecular weight of the chemical. For *Daphnia*, we assume k_W is dependent upon the diffusivity of the chemical through the carapace.

A seemingly natural approach to model the uptake of chemical from food would be to employ hypotheses similar to those imposed for cuticular uptake. The complications of this and other options for modeling the uptake from contaminated food are discussed in [13].

Table 3. Toxicant uptake model variables and parameters

Symbol	Interpretation	Dimension
B_T	Total toxicant burden in whole organism	mg
S	Active exposure area	mm^2
k_W (parameter)	Conductance: exposure tissue	mm/d
k_E (parameter)	Conductance: intestinal tissue	mm/d
C_W (parameter)	Toxicant concentration in ambient water	mg/L
C_A	Toxicant concentration in aqueous portion of the organism	mg/L
C_I	Toxicant concentration in intestinal contents	mg/L
E	Egestive flux	mg/d
F	Feeding flux	mg/d
BCF	Bioconcentration factor	Dimensionless
W_T	Weight	mg

A basic assumption of this model representation, that of equilibration of chemical between the organism's body and the gut contents, is, of course, not necessarily true. It has been demonstrated [14] that this is a worst case assumption during increasing body concentration when exposure is to contaminated food, that is, no more chemical could be taken up under any thermodynamically consistent assumption than would be taken up when food and body equilibrate. During depuration, however, this assumption leads to predicted minimum depuration times, that is, any other thermodynamically consistent assumption would lead to longer depuration times. For toxicity evaluations, this would usually not be considered the worst case scenario.

EFFECTS OF TOXICANTS ON INDIVIDUALS

The basic ideas employed to assess the effects of chemicals on an individual and on a static population are given in [3]. We review these ideas to set the stage for this study on the effects of a chemical on a population. Again, the particular effect focus is on mortality of the individual but sublethal effects could be considered by the same methods.

The assessment of mortality due to chemical action is implemented by utilizing QSARs. The procedure consists of combining the following: (a) the differential Equations (1) and (2) for evolution of the individual dynamics; (b) the differential Equation (3) for the total concentration of toxicant in the organism; (c) the determination of mortality from QSARs.

The first two of these steps couple the dynamics of individuals with the chemical uptake through the weight terms in Equations (1) and (2). Step (c), the assessment of mortality, utilizes results of [11] and [15]; see Figure 4. These bioassays were developed for baseline narcotic chemicals and relate a chemical property, K_{ow}, to mortality of the individual. Most of the data in Figure 4 are for fish. Some *Daphnia* data [16] appears analogous to this and, indeed, overlaps with it. We are not aware of a complete *Daphnia* QSAR. To illustrate our approach, we extrapolate and utilize the composite QSAR function obtained from both fish and *Daphnia*. There are numerous QSARs in the literature for modes of action other than baseline narcosis. For example, the modes of polar narcosis [17] and uncoupling of oxidative phosphorylation [18] have been documented. A common feature of each of these modes of action is that lipophilicity of the chemical, as measured by K_{ow}, is important. We do not present modes of action other than narcosis in detail. Because each of the derived QSARs is based, at least in part, upon K_{ow}, it appears that assessment of risk after exposure to chemicals with these other modes of action also must utilize lipid as an individual model component.

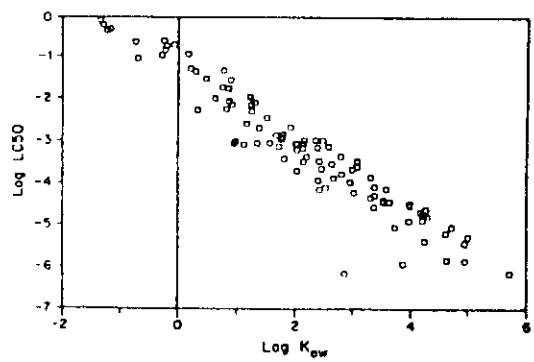
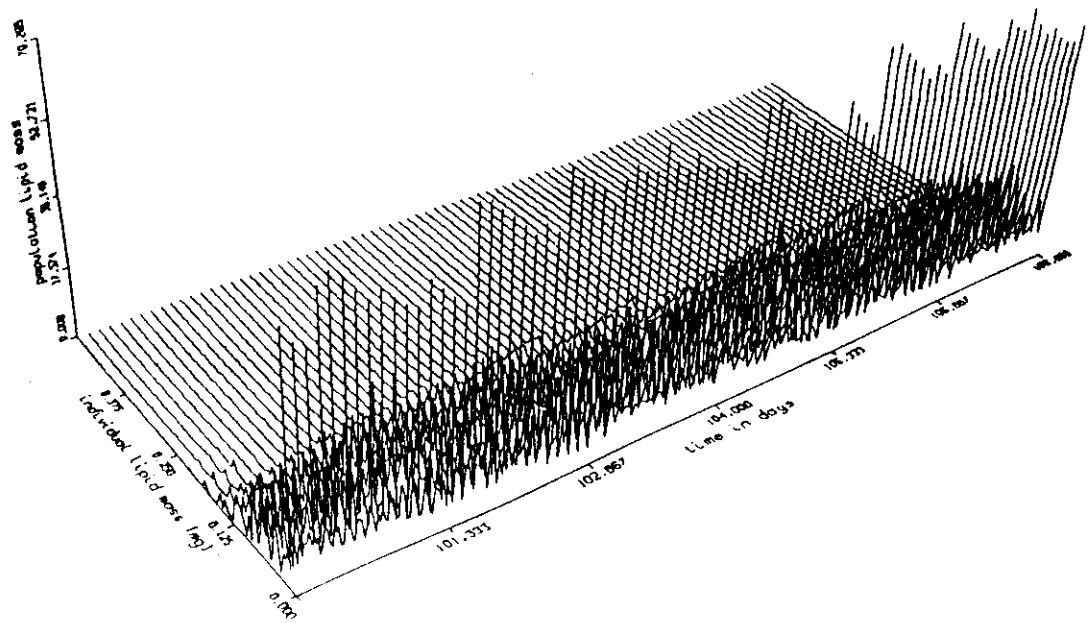


Fig. 4. Relationships between log LC50 and log K_{ow}. Fitted result is $\log(LC50) = -0.8\text{-}\log(K_{ow})$. Data from [15–17].



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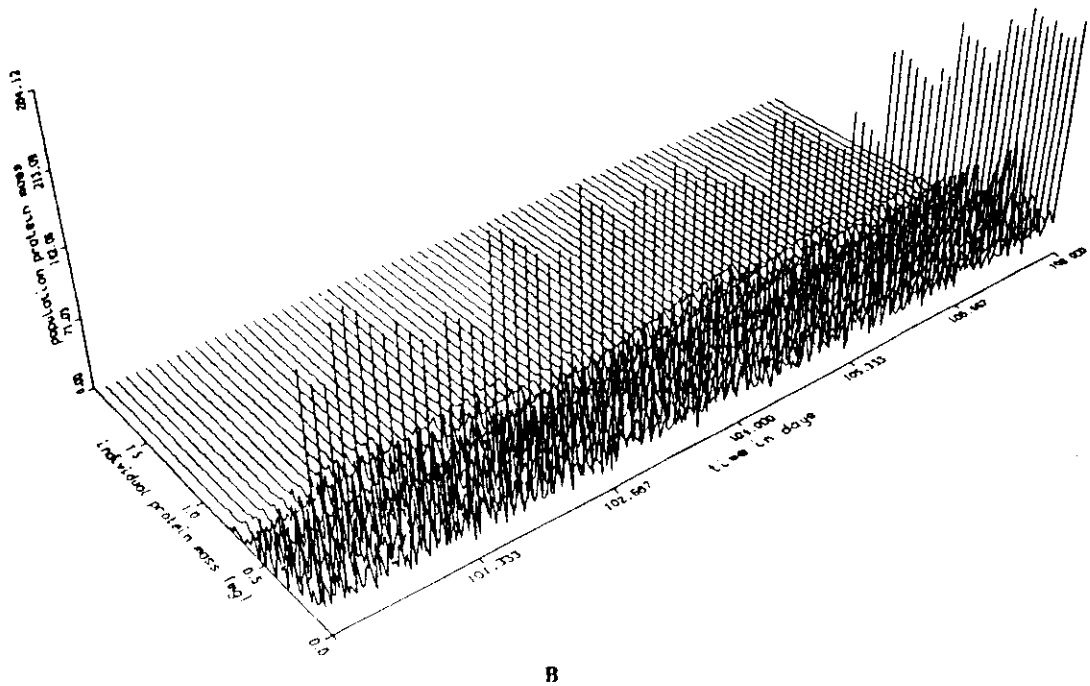
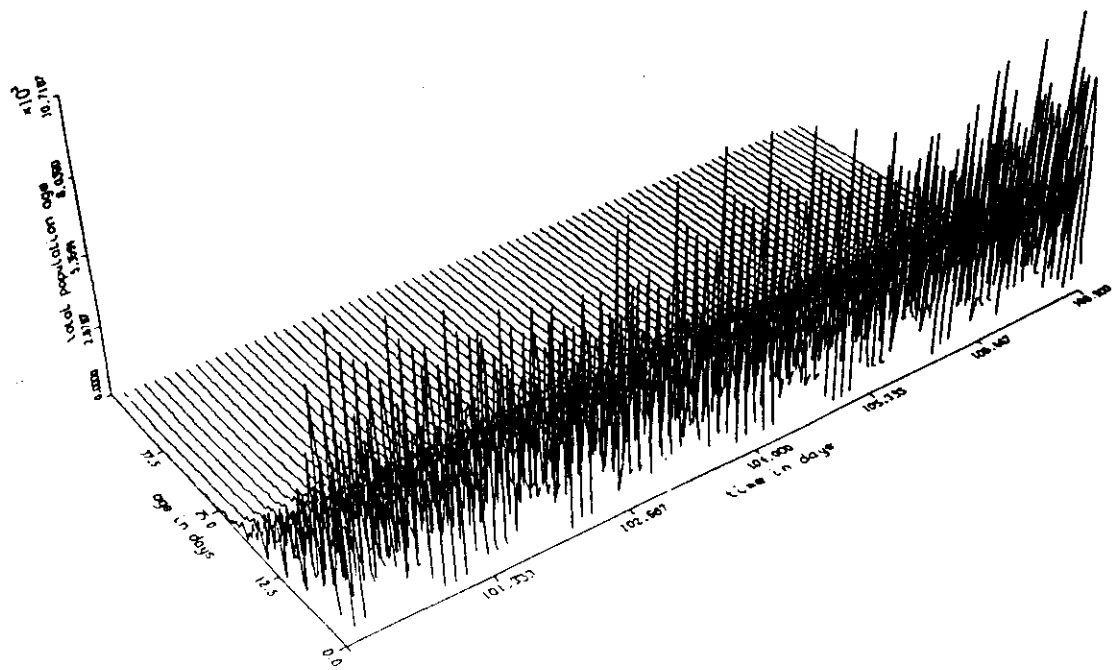


B

Fig. 5. (A) The scaled lipid density in a *Daphnia* population modeled by Equation (5) as a function of time. Output is indicated for each computation point on the interval 100 to 108 d. Note that there are short-term fluctuations in the population associated with the species reproductive period (assumed here to be 4 d); (B) the scaled protein density for the population of (A). (*Continued on facing page.*)



C

Fig. 5 continued. (C) The scaled age density for the population of (A).

AN ACUTE EXPOSURE-STATIC POPULATION THEORY: SURVIVAL OF THE FATTEST

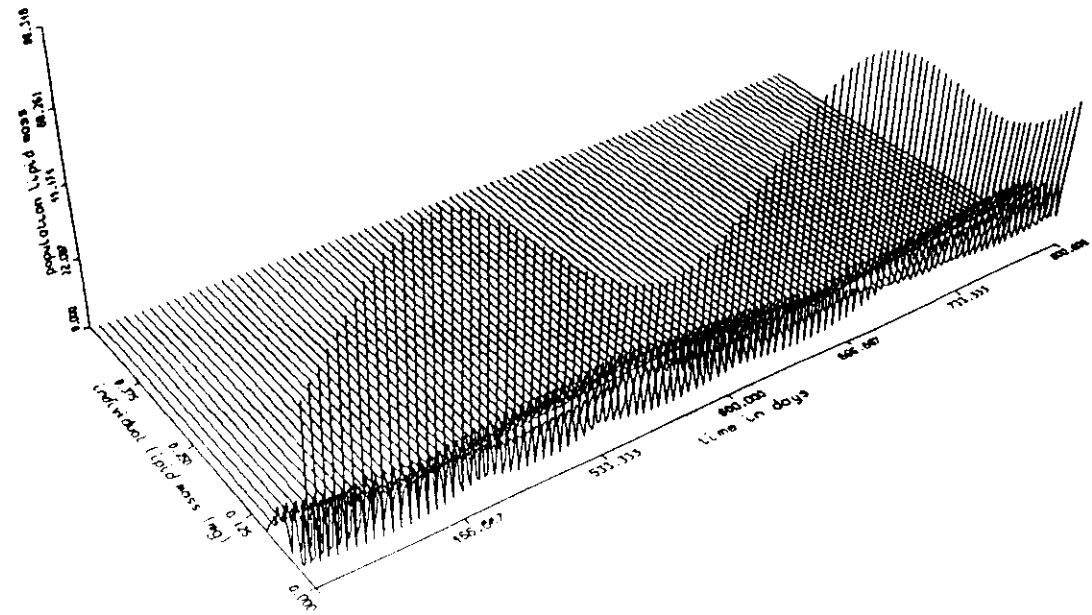
In a recent article [3], we developed an approach that utilizes individual variation to structure a population and to explain the effects of an acute toxicant exposure from a lipophilic narcotic on that population. The basic idea is that lipid provides a buffer against toxic stress and this factor must be utilized in any consideration of effects. According to this theory, in an assessment of mortality, an individual with a smaller lipid fraction body content will die before another individual with a larger lipid fraction given equal exposure [3]. The hypothesis for this theory is directly related to the static state of the population. This static state allows only acute toxicant exposures. The pathway of exposure is apparently not important for effects on static populations.

DYNAMICS THEORY: EFFECTS OF TOXICANTS ON POPULATIONS

Methods

Our prototype dynamic population model for *Daphnia* is based on individual response so that effects can be determined directly and the cumulative effect at the population level ascertained. Assessment of effects of toxic exposure involves nonlinear processes at the individual level—through the dynamics of components such as lipid and size—and at the population level—through density-dependent representations such as mortality. The toxicant-population model is formulated so that chronic as well as acute exposures may be investigated. First, we will sketch the approach used to model the population. Then, we will discuss the effects of the toxicant on the population.

The dynamic population model. An approach that allows incorporation of individual dynamics into a dynamic population formulation is the McKendrick–von Foerster equation [2]. This partial differential equation explicitly represents physiological variables as they are used to determine the dynamics of individuals. It also keeps track of the total population through the population density function. We have developed the prototype model for *Daphnia* because they are classical aquatic bioassay test species and have a dynamic lipid cycle [8]. The assessment of effects of a lipophilic narcotic mandates that the individual model should



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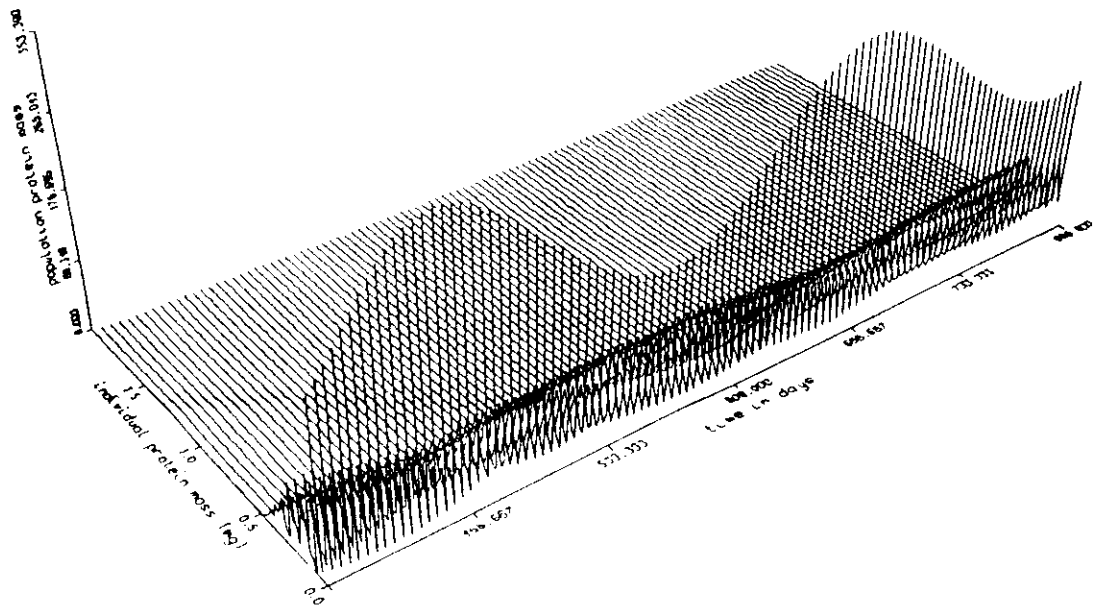
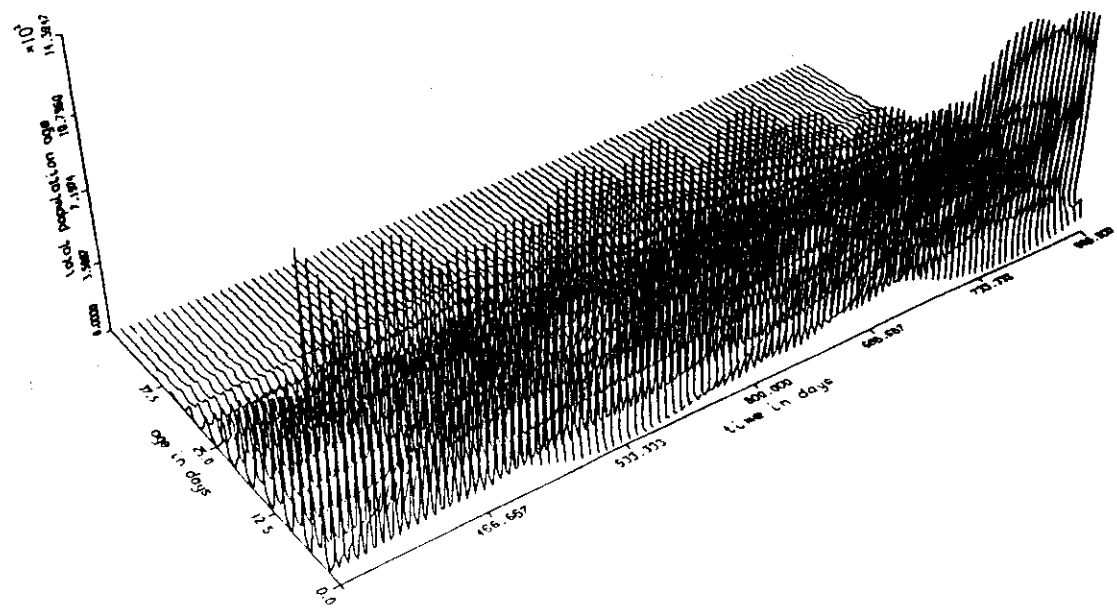


B

Fig. 6. (A) The scaled lipid density of a dynamic population as modeled by Equation (5) represented as a function of time over the interval 400 to 800 d. Note the existence of long-term fluctuations on scales much larger than the species periodic reproductive time; (B) the scaled protein density for the population of (A). (*Continued on facing page.*)



C

Fig. 6 continued. (C) The scaled age density for the population of (A).

minimally include some measures of the physiological variables of age, lipid and structure. Age is necessary to reflect the life history of an individual. Lipid, a bioconcentration site for the toxicant, is necessary to assess the effects of the chemical on the individual. Structure is necessary to measure weight and, subsequently, length of the organism.

If $\rho = \rho(t, a, m_L, m_S)$ is the population density function that depends upon time t and the physiological variables a representing age, m_L representing the mass of the lipid and m_S representing the mass of the structure compartment and g_L and g_S are the growth rate of the lipid and structural components of an individual as given by the Equations (1) and (2), respectively, then an equation that incorporates these physiological variables into a population scheme is

$$\rho_t + \rho_a + (\rho \cdot g_L)_{m_L} + (\rho \cdot g_S)_{m_S} = -\mu\rho. \qquad (4)$$

The subscripted terms represent partial derivatives with respect to the variable indicated by the subscript. The birth process for the population is represented by a boundary condition and the mortality rate is given explicitly in the differential equation as μ, which generally is a nonlinear function of the density ρ. The particular form of the birth process may be written in several equivalent representations, one of which is

$$\rho(t, 0, m_{L_0}, m_{S_0}) = \iiint \beta(t, a, m_{L_0}, m_{S_0}, m_L, m_S) \times \rho(t, a, m_L, m_S) da\, dm_L\, dm_S,$$

where m_{L_0} is lipid mass and m_{S_0} is structure mass at age = 0; β is the birth function that represents the number of eggs with lipid content m_{L_0} and structure content m_{S_0} born to an individual of age a with lipid content m_L and structure content m_S at time t.

Several different types of mortality are represented in our numerical model formulation. We include formulations for age-dependent mortality, size-dependent mortality and density-dependent mortality. The age-dependent mortality is assessed uniformly along cohorts, whereas the density-dependent mortality is assessed uniformly across the population (Table 4). The size-dependent mortality is viewed as possibly caused by predation and is determined by weight of the individual. The population model (4) is, in general, nonlinear but it would be linear if the density-dependent mortality term were omitted. Some restriction on mortality

is generally needed to prevent the population from growing beyond reasonable size. The specific forms of these mortality functions are generic and can be structured to test processes. Examples of mortality functions are given in Table 4.

This population model is a first-order hyperbolic partial differential equation that may be represented in an alternate manner by the method of characteristics. In this method the partial differential equation is reduced to a set of ordinary differential equations that are valid along certain special curves called characteristics. Specifically, in this

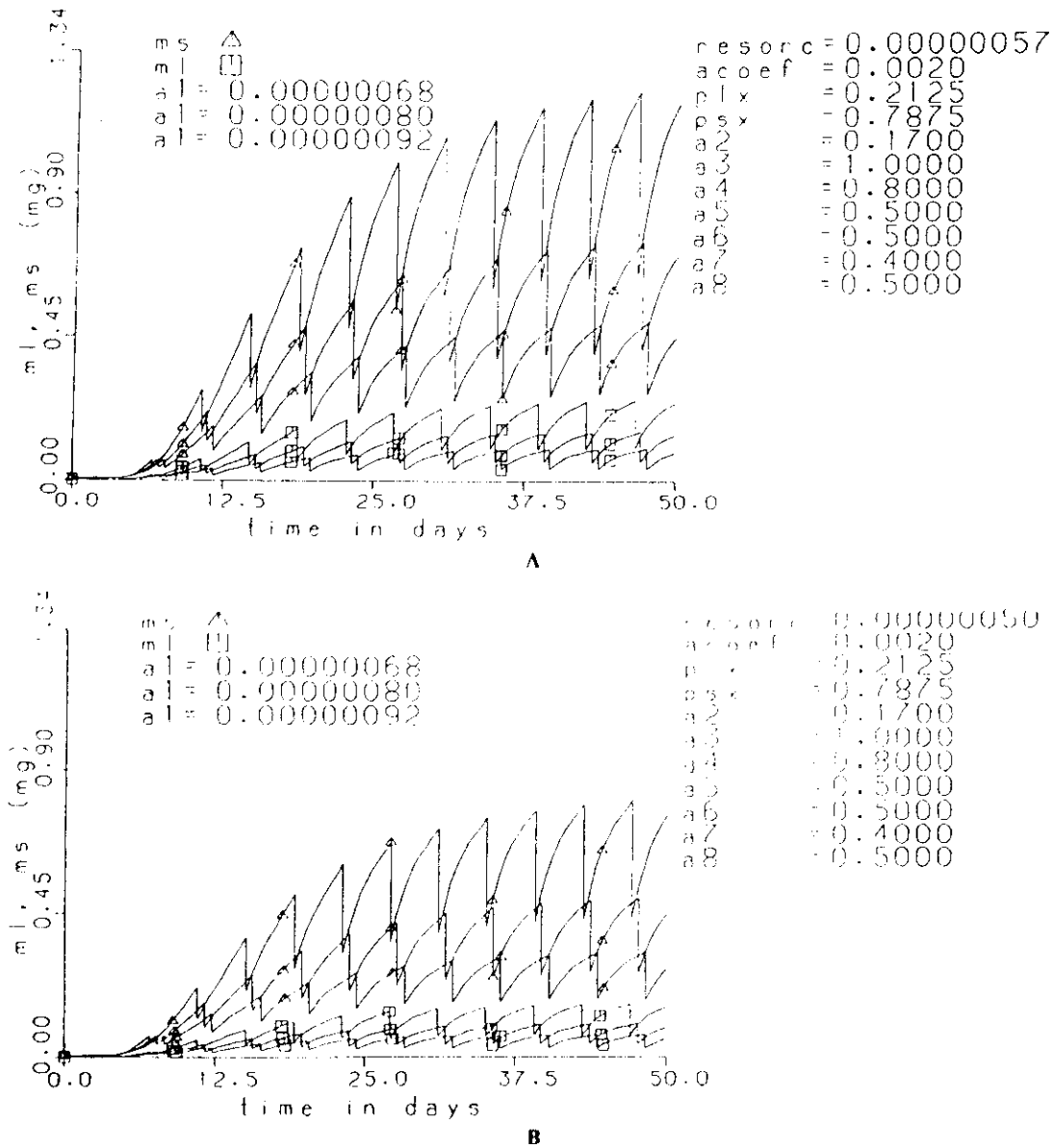


Fig. 7. The graphs of the first nine ecotypes listed in Table 5. The size ranges of the structure and lipid compartments for the ecotypes vary considerably; hence, there is variation in the population in both size and age structure. (A) Ecotypes 1 (largest m_S), 2 (middle m_S), 3 (smallest m_S); (B) ecotypes 4 (largest m_S), 5 (middle m_S), 6 (smallest m_S). (*Continued on facing page.*)

model an equivalent representation for the partial differential Equation (4) is the following system of five ordinary differential equations.

$$\frac{da}{d\lambda} = 1,$$

$$\frac{dt}{d\lambda} = 1,$$

$$\frac{dm_L}{d\lambda} = g_L,$$

$$\frac{dm_S}{d\lambda} = g_S,$$

$$\frac{d\rho}{d\lambda} = -(\mu + (g_L)_{m_L} + (g_S)_{m_S})\rho, \qquad (5)$$

where λ is the characteristic variable.

If the function ρ, as described by (5), is computed along characteristics, it is not a strict density function in the sense that its dimensional units are numbers (per volume); however, ρ can be converted to a density function with units numbers (per volume) by including the Jacobian of the transformation induced by (4). Let $n = \rho h$ where

$$h_a + h_t = [(g_L)_{m_L} + (g_S)_{m_S}]h$$

Table 4. Mortality functions used in population model

Individual mortality

Weight-dependent mortality:

$\mu_W = \gamma_W \cdot \mu_{IW} = \gamma_W \cdot \mu_{IW}(W)$, W = weight (mg)

$$\mu_{IW} = \begin{cases} V_0, & W = 0 \\ V_C, & W \in [W_1, W_2] \\ V_F, & W = W_3 \\ \text{continuous and linear,} & \text{elsewhere} \end{cases}$$

Age-dependent mortality:

$$\mu_A = \begin{cases} \kappa_a, & 0 \le a \le 50 \\ \infty, & a > 50 \end{cases}$$

Population mortality

Density-dependent mortality:

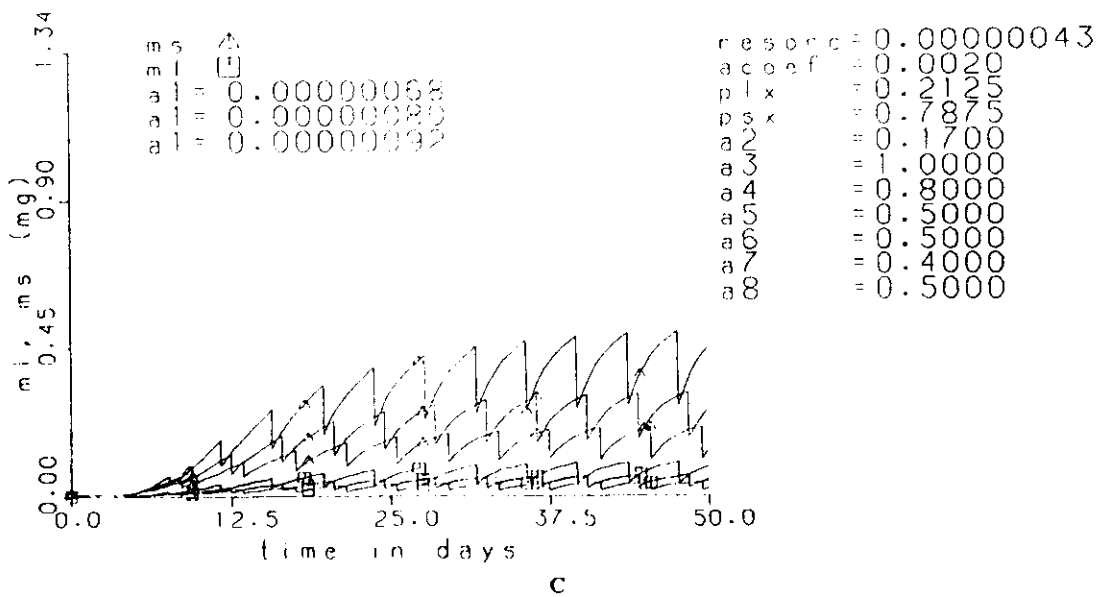
$\mu_D = \mu_D(P_B)$, P_B = total population biomass (Frank et al. [25])

$$\mu_D = \begin{cases} D_M, & P_B \in [0, P_T] \\ D_0, & P_B = P_0 \\ 2D_M, & P_B \ge P_C \\ \text{continuous and linear,} & \text{elsewhere} \end{cases}$$

and

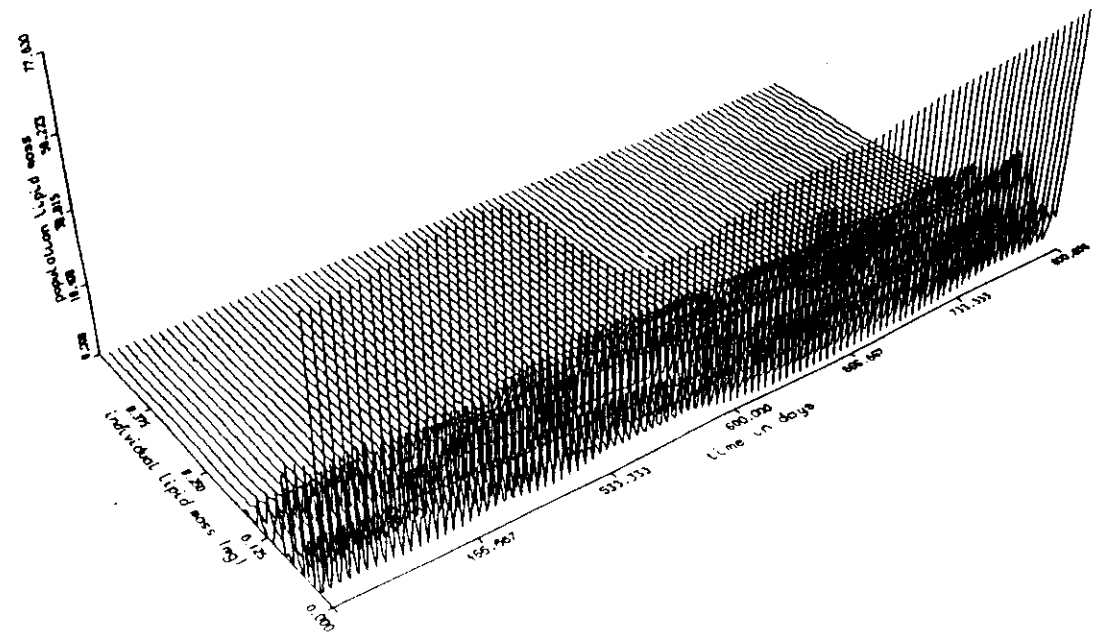
$$n_a + n_t = -\mu n.$$

In each of these equations subscripts indicate the partial derivative with respect to the subscripted variable. Along characteristics n is a density func-

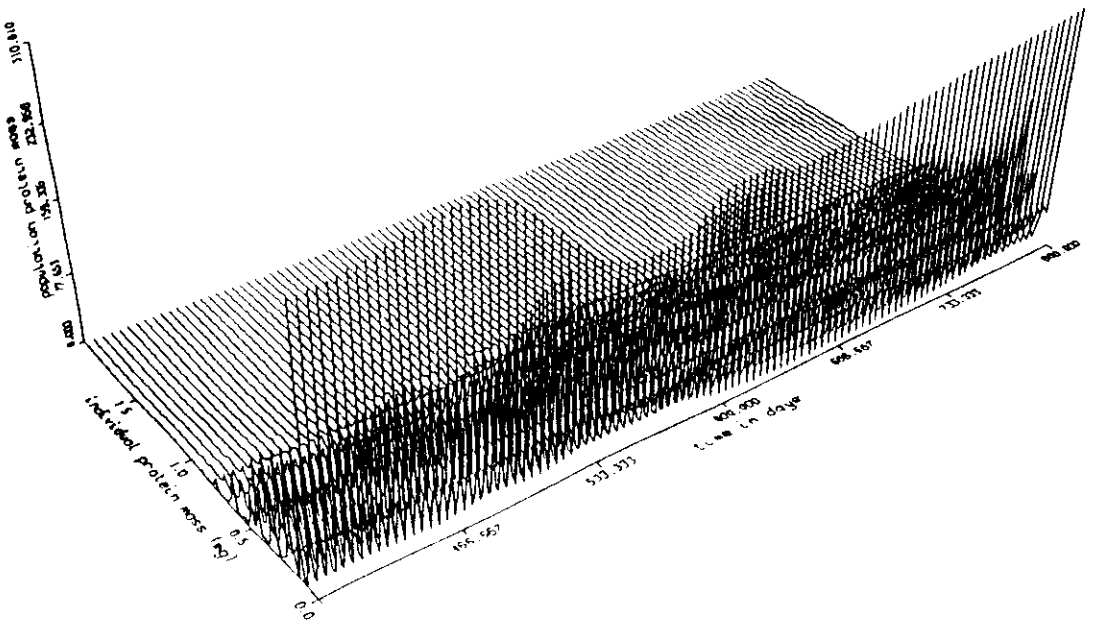


C

Fig. 7 continued. (C) Ecotypes 7 (largest m_S), 8 (middle m_S), 9 (smallest m_S).

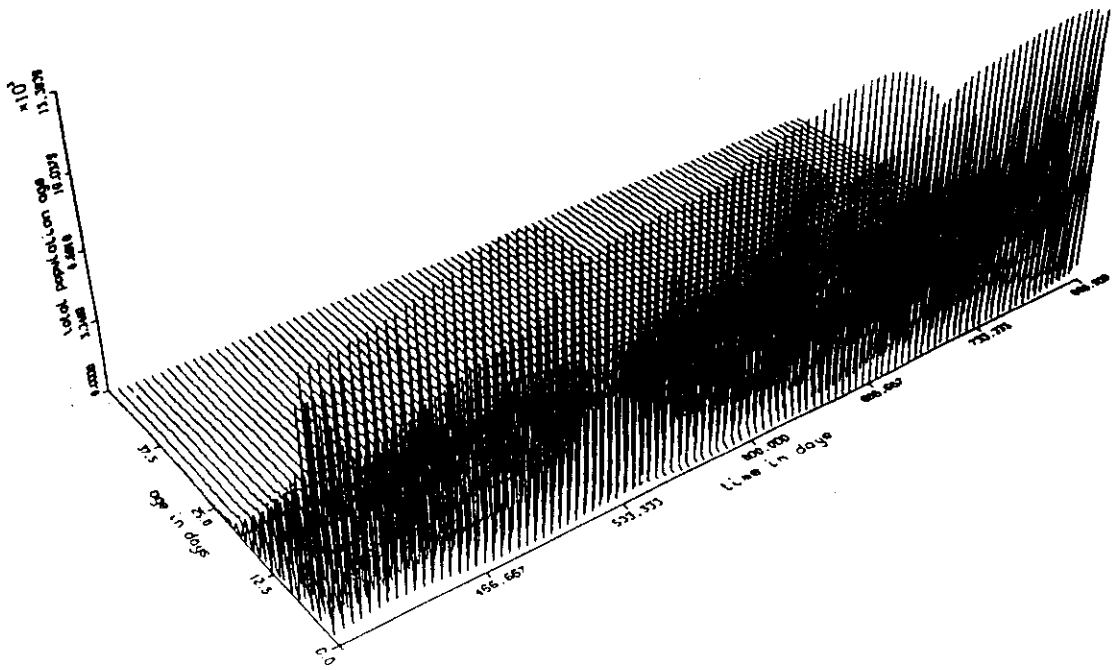


A



B

Fig. 8. The climax population. The figure illustrates the unstressed dynamic population as viewed through its lipid density, its structure density and its age density. (A) The dynamic lipid density of the climax population on the interval 400 to 800 d; (B) the dynamic structural mass density of the climax population on the interval 400 to 800 d. (*Continued on facing page.*)



C

Fig. 8 continued. (C) The dynamic age density of the climax population on the interval 400 to 800 d.

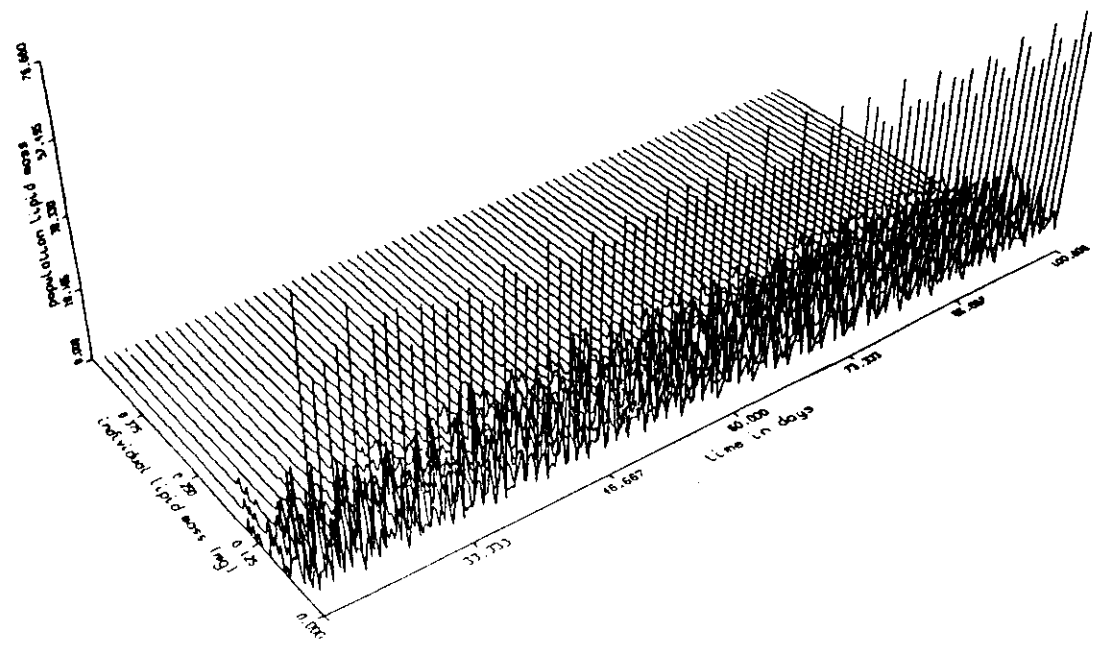
tion that has units numbers (per volume) and satisfies

$$\frac{dn}{d\lambda} = -\mu n. \tag{6}$$

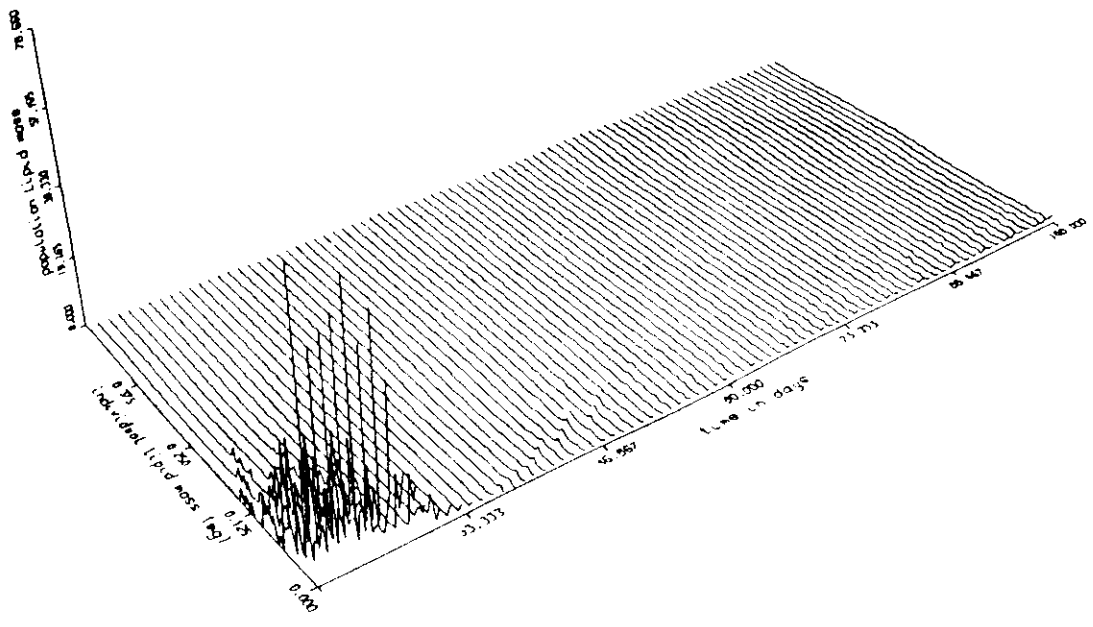
In (6), mortality is the only force acting along characteristics. Our approach is to solve this system of ordinary differential Equations (5) numerically with the ρ equation replaced by (6). Examples of graphical representations of some numerical solutions are given in Figures 5 and 6 for several situations, including shorter time scales where output at each computation time is presented (Fig. 5) and longer time scales where many computational outputs are omitted (Fig. 6).

In the absence of a toxicant, the behavior of the population is oscillatory on several scales. There are oscillations on the time scale of the periodic reproductive time or gestation period of the species (Fig. 6). There are longer-term oscillations that do not seem to be related to mortality but rather to other factors such as the rate at which the organisms grow and the birth process. This longer-term oscillation apparently is not due to the dynamics of the resource, which is assumed to be at constant density. In particular, this is not a typical predator-prey oscillation where both predator and prey have oscillatory behavior. This oscillatory behavior of a consumer when the resource is at a relatively constant level is characteristic, however, of some *Daphnia* populations [19], which do oscillate in the presence of a nonoscillatory algal resource. Our analyses, based upon numerical studies of the population model, indicate that the cause of the longer-term fluctuations is apparently not a result of the assumed density-dependent regulation, since oscillations still occur even when the density dependence is removed. We mention this because the folklore in rudimentary nonstructured models indicates that oscillations are often caused by inclusion of density dependence (c.f. [20]).

Our numerical procedure essentially follows cohorts of individuals along characteristics. This allows effects of toxicant exposures to be assessed at the individual level, while the overall effects on the population can still be determined by an accumulation of individual effects. The addition of a toxicant leads to another type of mortality assessed at the organism level according to lipid content of the

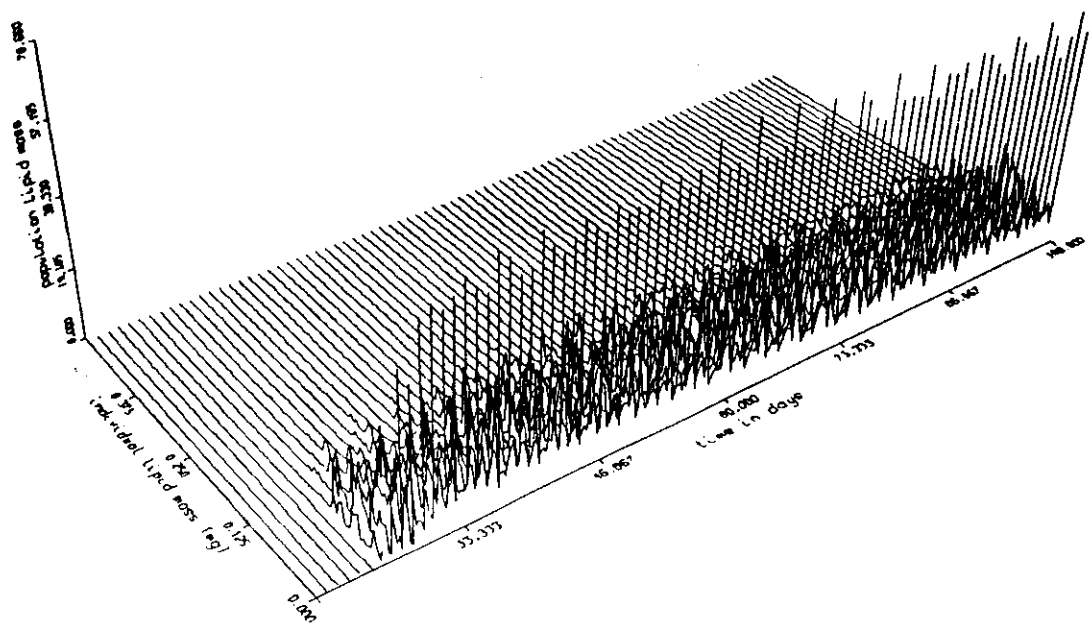


A

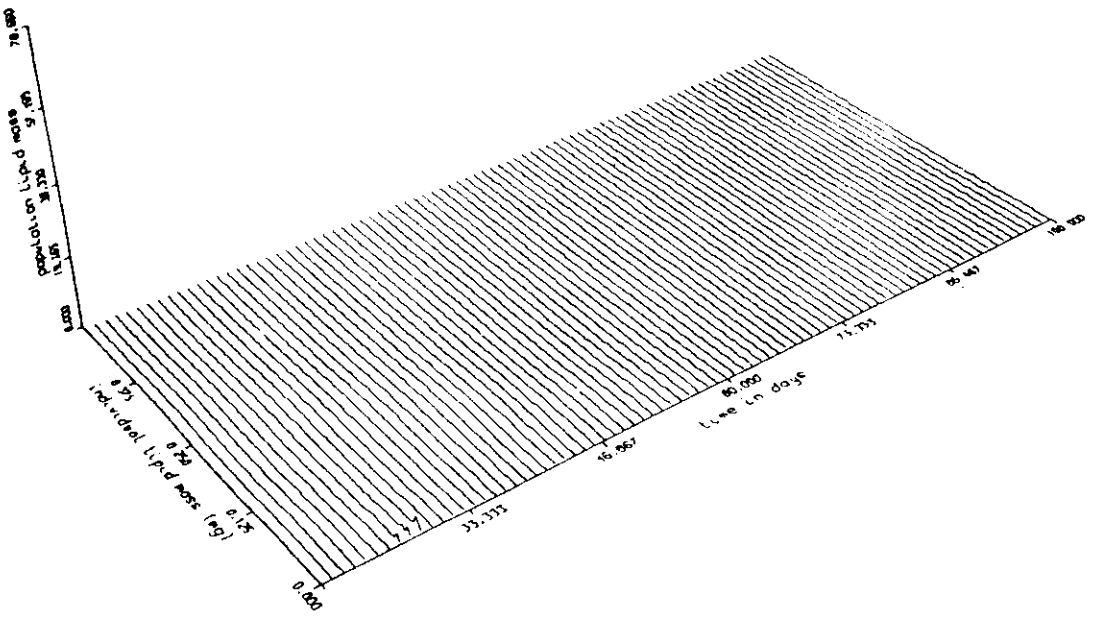


B

Fig. 9. Chemical stress case I (see text for details). The figure illustrates the changes in the lipid and age densities of the population due to toxicant impact. The initial time of exposure is chosen so that there is considerable variation among individuals in the population. The nominal population dynamics and the toxicant stressed population dynamics are presented. The stressed population dynamics are examined from both the depression and stimulation perspectives; (A) The lipid density of the model population in the absence of the toxicant on the interval 20 to 100 d; (B) the lipid density of the model population with a toxic exposure through both environmental and food pathways initiating at day 24 and terminating at day 31. (*Continued on the following two pages.*)

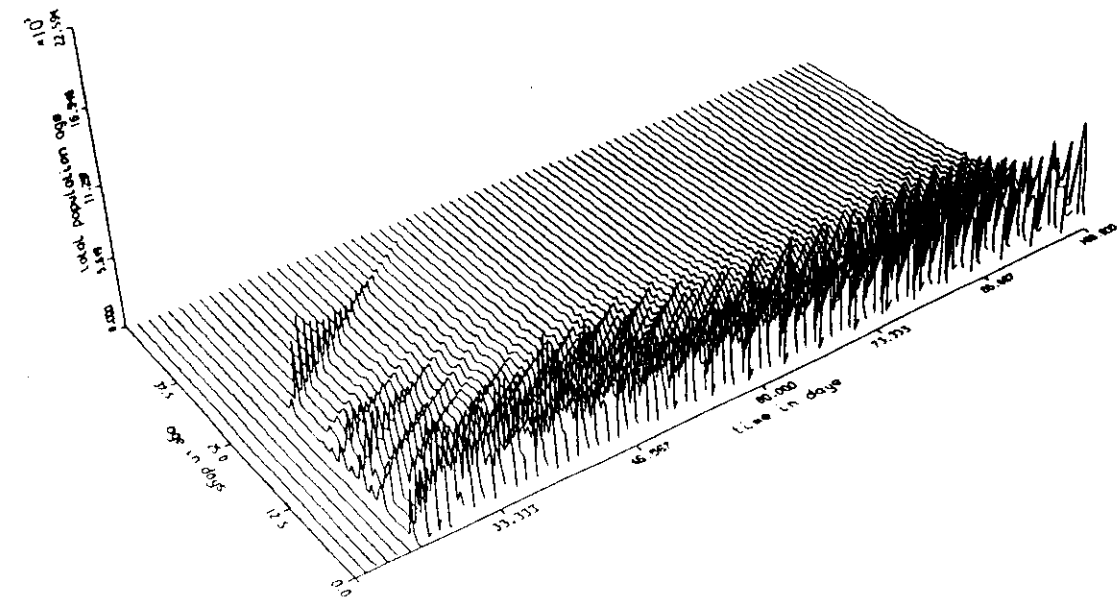


C

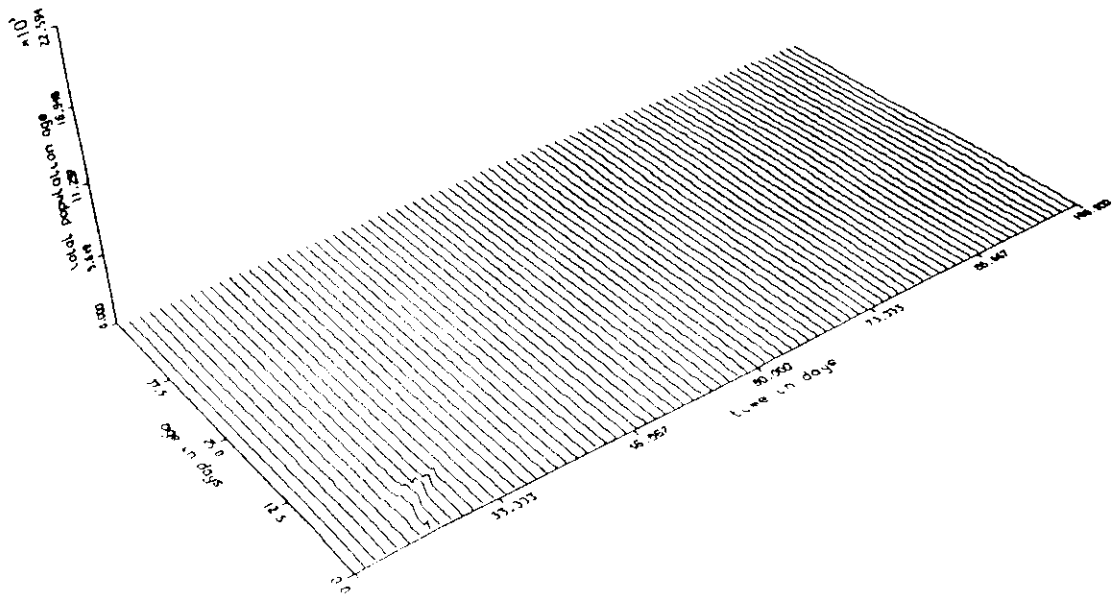


D

Fig. 9 continued. (C) Depression effects on lipid density function caused by the toxicant; (D) stimulation effects on lipid density function caused by the toxicant.



E



F

Fig. 9 continued. (E) Depression effects on age density function caused by the toxicant; (F) stimulation effects on age density function caused by the toxicant.

individual. Hence, in the model there are four essentially distinct causes of death: mortality due to the physiological process of aging, mortality due to size (as, for example, determined by predation), mortality due to density dependence (determined by total population biomass) and mortality due to toxicity (determined by the lipid distribution in the population and toxicant exposure concentration and duration).

The population structure and analysis as re-

Table 5. Parameters and ecotypes of individuals used

Ecotype no.	Parameter values	Age at first brood	Structure mass (mg) after last reproduction	Lipid mass (mg) after last reproduction	No. of eggs in last brood
1	$x = 0.57500E-06$ $A_1 = 0.68000E-06$ $x_L = 0.12219E-06$	6.65	0.64310	0.09678	65
2	$x = 0.57500E-06$ $A_1 = 0.80000E-06$ $x_L = 0.12219E-06$	7.15	0.40030	0.05954	40
3	$x = 0.57500E-06$ $A_1 = 0.92000E-06$ $x_L = 0.12219E-06$	7.65	0.26210	0.03801	27
4	$x = 0.50000E-06$ $A_1 = 0.68000E-06$ $x_L = 0.10625E-06$	7.05	0.42890	0.06388	43
5	$x = 0.50000E-06$ $A_1 = 0.80000E-06$ $x_L = 0.10625E-06$	7.65	0.26100	0.03801	27
6	$x = 0.50000E-06$ $A_1 = 0.92000E-06$ $x_L = 0.10625E-06$	8.30	0.17420	0.02510	18
7	$x = 0.42500E-06$ $A_1 = 0.68000E-06$ $x_L = 0.09031E-06$	7.65	0.26100	0.03801	27
8	$x = 0.42500E-06$ $A_1 = 0.80000E-06$ $x_L = 0.09031E-06$	8.45	0.16940	0.02514	16
9	$x = 0.42500E-06$ $A_1 = 0.92000E-06$ $x_L = 0.12219E-06$	9.45	0.10870	0.01560	11
10	$x = 0.57500E-06$ $A_1 = 0.68000E-06$ $x_L = 0.14375E-06$	6.80	0.60810	0.12980	60
11	$x = 0.57500E-06$ $A_1 = 0.80000E-06$ $x_L = 0.14375E-06$	7.25	0.37050	0.07777	38
12	$x = 0.57500E-06$ $A_1 = 0.92000E-06$ $x_L = 0.14375E-06$	7.80	0.24620	0.05116	25
13	$x = 0.50000E-06$ $A_1 = 0.68000E-06$ $x_L = 0.12500E-06$	7.20	0.40040	0.08438	40
14	$x = 0.50000E-06$ $A_1 = 0.80000E-06$ $x_L = 0.12500E-06$	7.80	0.24620	0.05116	25
15	$x = 0.50000E-06$ $A_1 = 0.92000E-06$ $x_L = 0.12500E-06$	8.50	0.16960	0.03533	16
16	$x = 0.42500E-06$ $A_1 = 0.68000E-06$ $x_L = 0.10625E-06$	7.80	0.24620	0.05116	25
17	$x = 0.42500E-06$ $A_1 = 0.80000E-06$ $x_L = 0.10625E-06$	8.65	0.15830	0.03291	15
18	$x = 0.42500E-06$ $A_1 = 0.92000E-06$ $x_L = 0.10625E-06$	9.65	0.10420	0.02145	10

Continued

Table 5 continued.

Ecotype no.	Parameter values	Age at first brood	Structure mass (mg) after last reproduction	Lipid mass (mg) after last reproduction	No. of eggs in last brood
19	$x = 0.57500E-06$ $A_1 = 0.68000E-06$ $x_L = 0.16531E-06$	6.90	0.56480	0.16040	57
20	$x = 0.57500E-06$ $A_1 = 0.80000E-06$ $x_L = 0.16531E-06$	7.40	0.35480	0.09936	35
21	$x = 0.57500E-06$ $A_1 = 0.92000E-06$ $x_L = 0.16531E-06$	8.00	0.23610	0.06562	23
22	$x = 0.50000E-06$ $A_1 = 0.68000E-06$ $x_L = 0.16531E-06$	7.35	0.37250	0.10480	38
23	$x = 0.50000E-06$ $A_1 = 0.80000E-06$ $x_L = 0.16531E-06$	8.00	0.23610	0.06562	23
24	$x = 0.50000E-06$ $A_1 = 0.92000E-06$ $x_L = 0.16531E-06$	8.70	0.15960	0.04404	15
25	$x = 0.42500E-06$ $A_1 = 0.68000E-06$ $x_L = 0.12219E-06$	8.00	0.23610	0.06562	23
26	$x = 0.42500E-06$ $A_1 = 0.80000E-06$ $x_L = 0.12219E-06$	8.85	0.14990	0.04130	14
27	$x = 0.42500E-06$ $A_1 = 0.92000E-06$ $x_L = 0.12219E-06$	9.90	0.10340	0.02830	9

ported here is based upon 27 different types of individuals. These individual ecotypes are determined by the constant level of resource on which they feed, the quality of that resource as indicated by its lipid content and the filtering rate of the organism. Each of these three individual characteristics ranges through three levels for the total of 27. Any number of individuals can be employed in the model; however, the computations become more burdensome as diversity increases. The present number gives considerable diversity to the population. Figure 7 indicates the graphs of some of the ecotypes of individuals. The parameter values used to generate the ecotypes are listed in Table 5.

The population model records the time and age dynamics of cohorts of individuals in the population, assesses mortality and indicates births. Organisms are assumed to be clones of their parent; this is a proper hypothesis for *Daphnia*, which in nonstressed conditions reproduce parthenogenetically.

Our objective is to indicate the processes that impact studies on population effects of toxicants. Although many environmental conditions are important for ecological systems, factors such as seasonality, temperature and pH were not included in process formulations since these variables affect only the parameter values in process representations and not the formulations themselves.

Results: Toxic effects at the population level

An *effect* on a population is defined here as any deviation from the population behavior that is caused by a toxicant. It is possible to have positive and negative effects, i.e., stimulation and depression. If ρ is the density function of the unstressed (nominal) population and ρ_S is the density function of the stressed population, the depression effects are represented by $(\rho - \rho_S)_+$ and the stimulation effects are represented by $(\rho_S - \rho)_+$. (Here, y_+ is y if y is positive; it is equal to zero if y is negative or equal to zero.) The total effects of the chemical on the population are thus given by the absolute value of $\rho - \rho_S$.

Dynamic populations and chronic exposures. Persistence of a dynamic population after chronic exposure is determined not only by the lipid distribution, which is fundamental to response to acute exposures, but also by the growth of the individuals in the population and population processes such as birth and mortality rates. The population model was run through several iterations in order that the effect of the initial distribution be diminished and that biological diversity and succession be attained. The 27 ecotypes described above were coupled with a fixed set of mortality parameters to delineate characteristics of the *Daphnia* population. The population model, when run for a sufficiently long period, evolves to a single ecotype—the fastest growing organism among the ecotypes. The climax population consists of individuals of various ages all of ecotype 1. The dynamics of this population are presented in Figure 8(A)-(C), representing the dynamic lipid distribution in the population, the dynamic structural mass distribution in the population and the dynamic age distribution in the population. All other population graphs are scaled by a physiological variable size to allow viewing of the larger individuals in the population. Otherwise, the numbers of smaller individuals would completely dominate the graphics.

To demonstrate that chemical stress can affect population structure, the theoretical population was exposed to several toxic chemicals and the time evolution of the stressed population studied. The characteristic dynamics of a stressed population can be significantly different from the reference population.

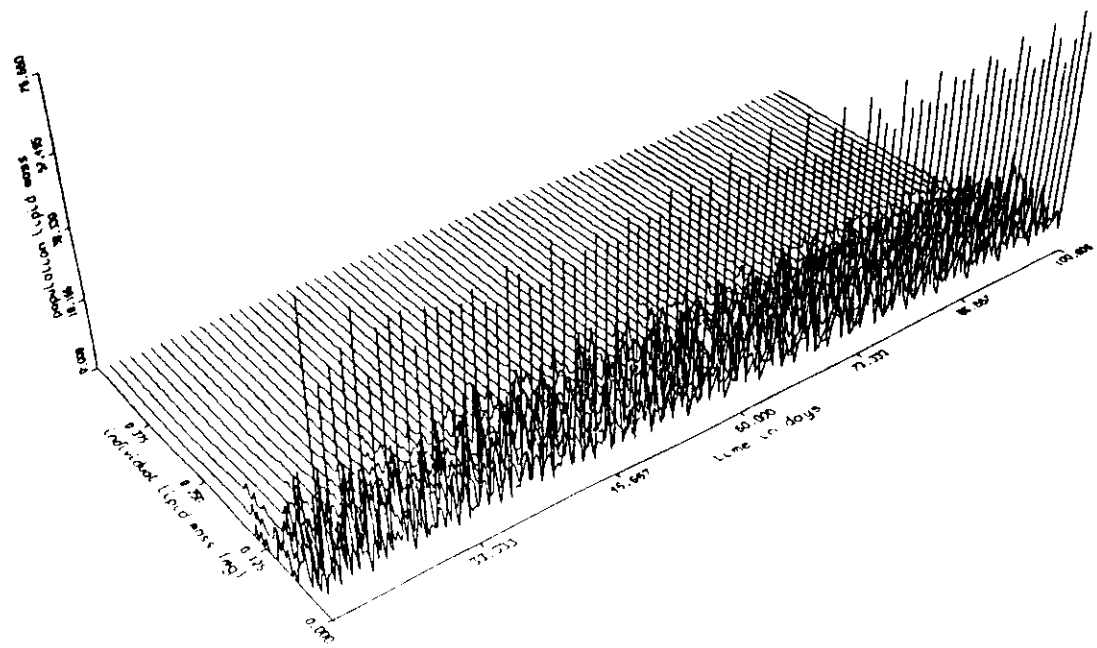
Chemical stress case 1. A 7-d exposure to a 16 ppm aqueous concentration, initiated on day 24, to a chemical having an octanol/water partition coefficient of 10^4 decreases the population to two cohorts of ecotype 27, which filters at the lowest rate and feeds at the lowest resource level with the lowest lipid content. These individuals are the slowest growers and the leanest of any of the ecotypes. The toxic stress has completely reversed the ecotypic succession (where the fastest growers formed the climax population) and also violates survival of fattest to the greatest opposite extreme (the leanest individuals survive the chronic stress). The effects of the toxicant exposure are illustrated in Figure 9(A)-(F). The lipid distributions in the unstressed population and the stressed population are shown in Figures 9(A) and 9(B), respectively; Figures 9(C) and 9(D) represent the depression effects and the stimulation effects, respectively. Figures 9(E) and 9(F) illustrate the effects of the toxicant on the age distribution of the same respective populations. Because of the assumed density dependent mortality, this population became extinct after several generations.

Chemical stress case 2. Model simulations showed that exposure to a different chemical can result in a completely different population structure. A 7-d exposure initiated at day 24 to a chemical with an octanol/water partition coefficient of 4.5×10^6 at concentration 4×10^{-6} (4 ppm) results in a population composed of a mixture of ecotypes 3, 5 and 7. These individuals, almost identical in size and lipid content and reproductive capability, are intermediate growth ecotypes. They are neither the faster nor slower growing organisms in our population and are determined by a trade-off between filtering rate and resource level. Neither the fattest nor the climax ecotype remain in the population after toxic exposure. Figures 10(A)-(D) illustrate the effects of this toxicant on the population.

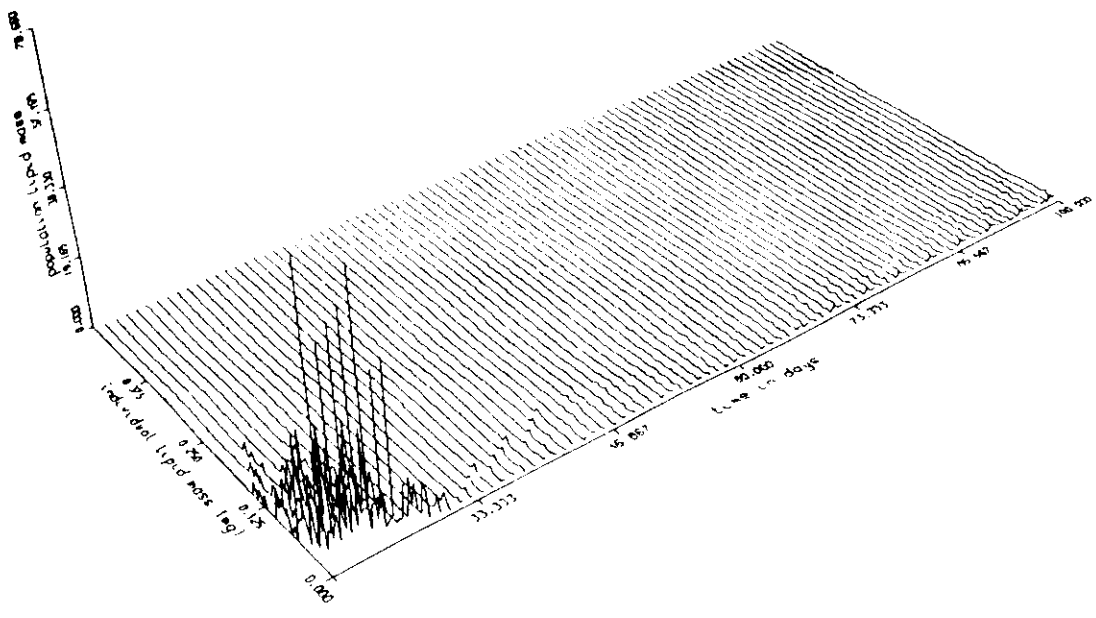
Chemical stress case 3. Another interesting situation results from an 8-d exposure, initiated at day 24, to a chemical having an octanol/water partition coefficient of 2×10^4 at concentration 8×10^{-6} (8 ppm). This leads to a population consisting of many ecotypes. When the simulation is continued after termination of toxicant release, the population is ultimately dominated by the fastest growing ecotype of the surviving ecotypes. This climax individual ecotype dominates only after a long time period because of the pressure of another similarly fast-growing ecotype. At the end of the exposure period, the two ecotypes that ultimately dominate the population were a very minor part of the population. When the time of exposure is extended an additional 0.5 d, the two cohorts of the dominant ecotypes are eliminated so that they are now no longer a part of the stressed population.

SUMMARY

To demonstrate the feasibility of a theoretical approach to study the effects of a chemical on a population, a model population structured by several different environmental and physiological characteristics was selected and then exposed to different chemicals. This reference population was stressed by a chronic exposure to a point close to extinction wherein only a small number of cohorts survived at the end of the exposure. The particular ecotypes of the surviving cohorts of individuals were determined by both biological succession in

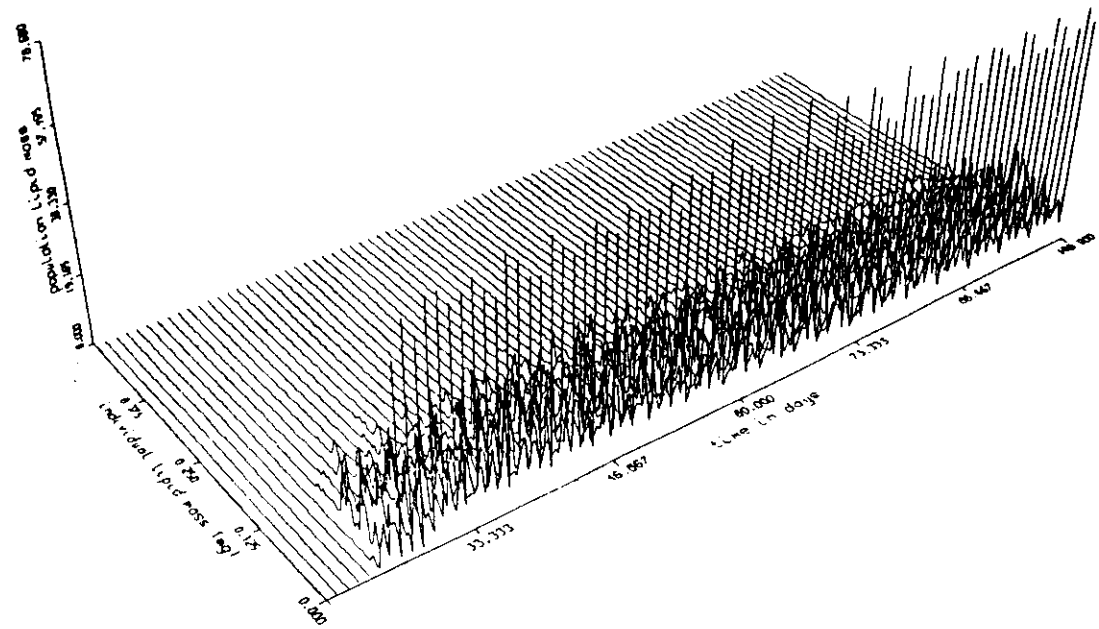


A



B

Fig. 10. Chemical stress case 2 (see text for details). (A) The lipid density of the model population in the absence of the toxicant on the interval 20 to 100 d; (B) the lipid density of the model population with a toxic exposure through both environmental and food pathways initiating at day 24 and terminating at day 32. (*Continued on facing page.*)



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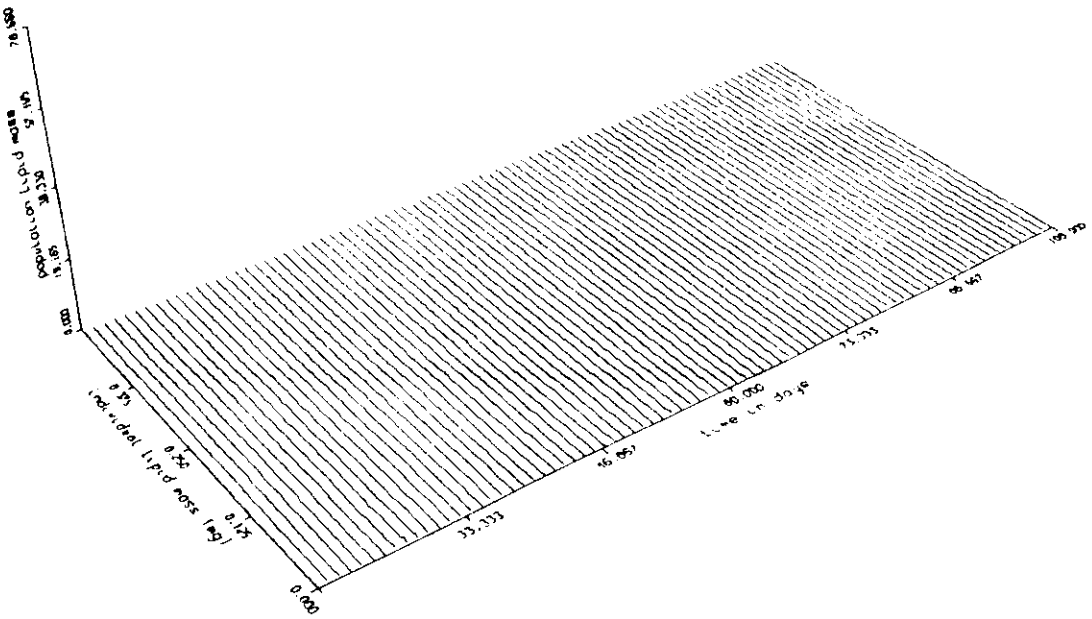

D

Fig. 10 continued. (C) Depression effects on lipid density function caused by the toxicant; (D) stimulation effects on lipid density function caused by the toxicant.

the population and toxicological aspects of the chemical. The important biological features for ecotype persistence are related to the intrinsic oscillations of the population, to the assessment of mortality and to the reproductive characteristics of the ecotypes. Toxicological features of importance besides the physical/chemical properties include the length of exposure, the initial time of exposure and the dose of the toxicant.

It is clear that in a dynamic setting survival of the fattest is not generally a valid theory. In some cases analyzed here a setting was produced where the surviving individuals were not the fattest of all possible ecotypes, but could be at the extremes of the ecotypes, such as the faster or the slower growers or even those growing at an intermediate rate.

That exposure may come from both the environmental and food chain pathways complicates determination of cause of death or, equivalently, reason for survival in dynamic populations. When exposure is via the food chain pathway, population persistence can be governed by amount of toxicant uptake from food. Many larger individuals can literally eat themselves to death by accumulating lethal doses of chemicals from food. This is illustrated in Figure 8, where an exposed population ultimately decreases to a few individuals having similar ecotype characteristics. They feed at the lowest resource level, the quality of their food is the lowest from the available lipid perspective because it is at the lowest fat level (although the higher resource structure material implies greater growth for the organism) and they have the lowest rate of filtering.

Although our investigations of this complex model are incomplete at present, an implication of the current study is clear: Risk assessment should not be based solely upon attributes of the toxic chemical. The biology of the exposed organisms is fundamental to the determination of the effects of the toxicant on a population.

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