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Incorporation of Azimuth Anomalies into Surface Wave Tomography

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1.Introduction

The data which are widely used in tomographic reconstruction of lateral velocity variations are travel times of seismic waves. This problem can be easily linearized: time delays respectively a properly chosen starting velocity model are represented as linear functionals of the unknown velocity variations. Thus the tomography problem is reduced to a system of linear equations. However, another data exist, which also depend on velocity variations, and can be obtained from seismological observations easily enough: such data are polarization anomalies, which are related to anomalies of directions of wave propagation.

The data on surface wave polarization allow to determine the azimuth of a wave arriving to a station, and consequently the *azimuth anomaly,* which is a deviation of the observed azimuth from that corresponding to a great circle path. Since a path of surface wave is governed by phase rather than group velocity, the azimuth anomalies are related to variations of phase velocities. So we may expect that incorporation of azimuth anomalies to the tomography problem for surface waves would give an additional information about phase velocity distribution.

Observations of surface waves show marked azimuth anomalies in some cases (Lander, 1984; Lerner-Lam & Park,1989; Nesterov & Yanovskaya, 1988; Levshin et al.,1992) . However, up to present time these anomalies were interpreted only qualitatively. A reason, why they have not been used in tomographic reconstruction (separately or jointly with travel time data) is that it is difficult to construct a linear functional of velocity variations for azimuth anomalies similar to that for time delays. Following Hu and Menke (1992) we shall show how to construct a procedure for determining a relationship between polarization anomalies and velocity variations for general case. It will be seen that the procedure is too much complicated, and therefore it is invalid for inversion of large body of data. Then it will be shown how to simplify this procedure for 2D case if the velocity in the starting model may be assumed to be constant. Obviously, this case is applicable for surface wave data.

2. General case

The problem is to derive a relationship between perturbation of slowness vector and velocity variation. Let the velocity in the starting model be $V_0(\mathbf{x})$, the slowness vector in the starting model $\mathbf{p}_0$, the velocity and slowness vector in a perturbed model $V(\mathbf{x})$ and $\mathbf{p}$ respectively. The ray tracing differential equations are following:

$$\frac{d\mathbf{x}}{ds} = \mathbf{p}V, \qquad \frac{d\mathbf{p}}{ds} = \nabla\left(\frac{1}{V(\mathbf{x})}\right) \tag{1}$$

where $\mathbf{x}$ is the position vector along a ray, $\mathbf{p} = \mathbf{t} / V(\mathbf{x})$, $\mathbf{t}$ is unit vector tangent to the ray, s is arc length of the ray.

For perturbation of the ray $\delta\mathbf{x}$ and $\delta\mathbf{p}$ we have the following differential equations:

$$\frac{d\delta\mathbf{x}}{ds} = V(\mathbf{x})\delta\mathbf{p} + \mathbf{p}_0(\delta V(\mathbf{x}_0) + \frac{\partial V}{\partial \mathbf{x}}\delta\mathbf{x}) \tag{2}$$

$$\frac{d\delta\mathbf{p}}{ds} = \nabla\left(\delta V^{-1}\right) + \frac{\partial}{\partial \mathbf{x}}\nabla V^{-1}\delta\mathbf{x} \tag{3}$$

The equations (2),(3) are to be solved with the initial conditions $\delta\mathbf{x}_s=0$ and $\delta\mathbf{x}_r=0$, where $\mathbf{x}_s$ and $\mathbf{x}_r$ are positions of source and receiver, respectively. So to determine $\delta\mathbf{p}_r$ in the receiver it is necessary to solve the two-point ray tracing problem. Hu & Menke (1992) reduced this problem to one-point ray tracing.

In the approach proposed by Hu & Menke the velocity model is described by a finite set of parameters $\mathbf{m}$, and δV is replaced by a vector $\delta\mathbf{m}$. Also it is convenient to replace the variable s by travel time τ. Then the equations (1) can be written in general form

$$\frac{d\mathbf{x}}{d\tau} = \mathbf{F}(\mathbf{x},\mathbf{p},\mathbf{m}), \qquad \frac{d\mathbf{p}}{d\tau} = \mathbf{G}(\mathbf{x},\mathbf{p},\mathbf{m}) \tag{4}$$

and the equations (2),(3) in the form

$$\frac{d\delta\mathbf{x}}{d\tau} = \frac{\partial\mathbf{F}}{\partial\mathbf{x}}\delta\mathbf{x} + \frac{\partial\mathbf{F}}{\partial\mathbf{p}}\delta\mathbf{p} + \frac{\partial\mathbf{F}}{\partial\mathbf{m}}\delta\mathbf{m} \tag{5}$$

$$\frac{d\delta\mathbf{p}}{d\tau} = \frac{\partial\mathbf{G}}{\partial\mathbf{x}}\delta\mathbf{x} + \frac{\partial\mathbf{G}}{\partial\mathbf{p}}\delta\mathbf{p} + \frac{\partial\mathbf{G}}{\partial\mathbf{m}}\delta\mathbf{m} \tag{6}$$

If we integrate these equations from the source to the receiver, we have to take into account that in the perturbed model the ray and the travel time between the source and the receiver are to be perturbed. Thus we obtain

$$\delta\mathbf{x}_r = \int_0^T (\frac{\partial\mathbf{F}}{\partial\mathbf{x}}\delta\mathbf{x} + \frac{\partial\mathbf{F}}{\partial\mathbf{p}}\delta\mathbf{p} + \frac{\partial\mathbf{F}}{\partial\mathbf{m}}\delta\mathbf{m})d\tau + \mathbf{F}\delta T = 0 \tag{7}$$

$$\delta\mathbf{p}_r = \delta\mathbf{p}_s + \int_0^T (\frac{\partial\mathbf{G}}{\partial\mathbf{x}}\delta\mathbf{x} + \frac{\partial\mathbf{G}}{\partial\mathbf{p}}\delta\mathbf{p} + \frac{\partial\mathbf{G}}{\partial\mathbf{m}}\delta\mathbf{m})d\tau + \mathbf{G}\delta T \tag{8}$$

Our aim is to obtain the linear relationship between $\delta\mathbf{p}_r$ and $\delta\mathbf{m}$ in the form

$$\delta\mathbf{p}_r = \frac{\partial\mathbf{p}_r}{\partial\mathbf{m}}\delta\mathbf{m} \tag{9}$$

The matrix $\frac{\partial \mathbf{p}_r}{\partial \mathbf{m}}$ can be obtained as $\frac{\delta \mathbf{p}_r}{\delta \mathbf{m}}$, where $\delta \mathbf{p}_r$ is calculated from differential analogue of (7),(8) for a given $\delta \mathbf{m}$. It is clear from (7),(8) that the variations $\delta \mathbf{p}_r$ and $\delta \mathbf{x}_r$ result from variations of the model parameters $\delta \mathbf{m}$, of the initial ray direction in the source $\delta \mathbf{t}_s$, and of the travel time between the source and the receiver δT:

$$\delta \mathbf{x}_r = \delta \mathbf{x}_r^m + \delta \mathbf{x}_r^p + \delta \mathbf{x}_r^T$$
$$\delta \mathbf{p}_r = \delta \mathbf{p}_r^m + \delta \mathbf{p}_r^p + \delta \mathbf{p}_r^T$$

The variations $\delta \mathbf{x}_r^m$ and $\delta \mathbf{p}_r^m$ can be obtained from the differential equations (5),(6) with the initial conditions $\delta \mathbf{x}_s = 0$ and $\delta \mathbf{p}_s = -\frac{\mathbf{p}_s}{V(\mathbf{x}_s)} \frac{\partial V(\mathbf{x}_s)}{\partial \mathbf{m}} \delta \mathbf{m}$ by numerical integration from τ=0 to τ=T.

The variations $\delta \mathbf{x}_r^p$ and $\delta \mathbf{p}_r^p$ can be obtained by numerical integration of the differential equations

$$\frac{d\delta \mathbf{x}^p}{d\tau} = \frac{\partial \mathbf{F}}{\partial \mathbf{x}} \delta \mathbf{x}^p + \frac{\partial \mathbf{F}}{\partial \mathbf{p}} \delta \mathbf{p}^p$$
$$\frac{d\delta \mathbf{p}^p}{d\tau} = \frac{\partial \mathbf{G}}{\partial \mathbf{x}} \delta \mathbf{x}^p + \frac{\partial \mathbf{G}}{\partial \mathbf{p}} \delta \mathbf{p}^p$$

with the initial conditions $\delta \mathbf{x}_s = 0$ and $\delta \mathbf{p}_s = \frac{\delta \mathbf{t}_s}{V(\mathbf{x}_s)}$. For the variation of the travel time δT only, we have

$$\delta \mathbf{x}_r^T = \mathbf{F}(\mathbf{x}_r)\delta T, \quad \delta \mathbf{p}_r^T = \mathbf{G}(\mathbf{x}_r)\delta T$$

So we can calculate the matrices

$$\mathbf{M} = \frac{\delta \mathbf{x}_r^m}{\delta \mathbf{m}}, \quad \mathbf{N} = \frac{\delta \mathbf{p}_r^m}{\delta \mathbf{m}}, \quad \mathbf{P} = \frac{\delta \mathbf{x}_r^p}{\delta \mathbf{p}_s^p}, \quad \mathbf{Q} = \frac{\delta \mathbf{p}_r^p}{\delta \mathbf{p}_s^p}$$

and express the total variations of $\mathbf{x}_r$ and $\mathbf{p}_r$ in terms of these matrices:

$$\delta \mathbf{x}_r = \mathbf{M}\delta \mathbf{m} + \mathbf{P}\delta \mathbf{p}_s + \mathbf{F}_r \delta T$$

$$\delta \mathbf{p}_r = \mathbf{N}\delta \mathbf{m} + \mathbf{Q}\delta \mathbf{p}_s + \mathbf{G}_r \delta T$$

Since $\delta \mathbf{x}_r$=0, the variations $\delta \mathbf{p}_s$ and δT can be expressed through $\delta \mathbf{m}$:

$$\delta \mathbf{p}_s = \mathbf{R}\delta \mathbf{m}, \quad \delta T = \mathbf{S}\delta \mathbf{m}$$

Consequently

$$\delta \mathbf{p}_r = (\mathbf{N} + \mathbf{Q}\mathbf{R} + \mathbf{G}_r \mathbf{S})\delta \mathbf{m}$$

and

$$\frac{\partial \mathbf{p}_r}{\partial \mathbf{m}} = \mathbf{N} + \mathbf{Q}\mathbf{R} + \mathbf{G}_r \mathbf{S}$$

This is the matrix relating a variation of slowness vector in the receiver and variations of model parameters. The inconvenience of this approach is obvious: to construct the matrix $\frac{\partial \mathbf{p}_r}{\partial \mathbf{m}}$ it is necessary to integrate the ray tracing system n+3 times (for variations of n model parameters and of 3 components of the vector $\delta \mathbf{p}_s$). This procedure should be performed for each source-receiver pair. For the problems with large number of model parameters this procedure is useless.

3. Approximate functional for a starting model with constant velocity

We shall treat 2D model, assuming velocity in the starting model to be constant. Surface wave velocity corresponding to a fixed period satisfy to this assumption: in fact, lateral variations of phase velocities are small, and in the first approximation surface wave propagate along great circle paths, or along straight lines in a plane case. For simplicity we shall consider only the plane case. So the problem can be formulated as follows:

in linear approximation to derive a relationship between azimuth anomaly $\delta\alpha$ and lateral variation of phase velocity $\delta c(x,y)=c(x,y)-c_0$.

It is convenient to write the ray tracing system in the form

$$\frac{d\mathbf{x}}{dq} = S\mathbf{t} \qquad (10a)$$

$$\frac{d\mathbf{t}}{dq} = -S\mathbf{n}\frac{(\nabla c, \mathbf{n})}{c} \qquad (10b)$$

where $\mathbf{t}$ and $\mathbf{n}$ are unit vectors tangent and orthogonal to the ray, respectively, S is length of the ray, q is a parameter varying from 0 to 1, so that ds=Sdq, where ds is an element of the ray length..

A system for the variations $\delta\mathbf{x}$ and $\delta\mathbf{t}$ can be written as follows (assuming $\nabla c_0=0$):

$$\frac{d\delta\mathbf{x}}{dq} = \mathbf{t}_0\delta S + S_0\delta\mathbf{t} \qquad (11a)$$

$$\frac{d\delta\mathbf{t}}{dq} = -S_0\frac{(\nabla \delta c, \mathbf{n}_0)}{c_0}\mathbf{n}_0 \qquad (11b)$$

The values of $\nabla\delta c$ should be taken on the ray corresponding to the model c(x,y), i.e. in the points $\mathbf{x}=\mathbf{x}_0+\delta\mathbf{x}$, so that

$$\nabla\delta c(\mathbf{x}) = \nabla\delta c(\mathbf{x}_0) + \left(\frac{\partial \nabla c}{\partial \mathbf{x}}\right)_0 \delta\mathbf{x} \qquad (12)$$

The last term in the right-hand side of (12) in general is of the second order, because $\frac{\partial \nabla c}{d\mathbf{x}} = \frac{\partial \nabla \delta c}{d\mathbf{x}}$, however it is important in the case, when $\nabla\delta c(\mathbf{x}_0)$ is close to zero, while the second derivatives of the velocity are sufficiently large. This is the case, when the ray is directed along a ravine or a ridge of the function $\delta c(x,y)$. However, in tomographic problems, when the data for a large amount of paths are used, such cases are rare, though they would result in additional errors in the data.

The equation (11b) can be integrated from the receiver to the source:

$$\delta\mathbf{t}(q) = \delta\mathbf{t}_r - S_0 \int_0^q \frac{(\nabla\delta c, \mathbf{n}_0)}{c_0} \mathbf{n}_0 dq' \qquad (13)$$

where $\delta\mathbf{t}_r$ is a variation of the unit vector $\mathbf{t}$ in the receiver.

Substitute (13) into the equation (11a) and integrate from the receiver to the source, i.e. from q=0 to q=1:

$$\delta\mathbf{x} = \mathbf{t}_0\delta S + S_0 \int_0^1 \delta\mathbf{t}_r dq - S_0^2 \int_0^1 dq \int_0^q \frac{(\nabla\delta c, \mathbf{n}_0)}{c_0} \mathbf{n}_0 dq' \qquad (14)$$

Taking into account that in the source $\delta\mathbf{x}=0$, and turning to integration along the undisturbed ray ($ds=S_0dq$) we obtain

$$S_0\delta\mathbf{t}_r = \int_0^{S_0} ds \int_0^s \frac{(\nabla\delta c, \mathbf{n}_0)}{c_0} \mathbf{n}_0 ds' - \mathbf{t}_0\delta S \qquad (15)$$

The second term in the right-hand side of (15) should be at least of the second order of smallness: a variation of a unit vector should be orthogonal to it, so that in the linear approximation

$$(\delta\mathbf{t}_r, \mathbf{t}_0) = 0, \qquad |\delta\mathbf{t}_r| = (\delta\mathbf{t}_r, \mathbf{n}_0)$$

Consequently,

$$|\delta\mathbf{t}_r| = \int_0^{S} ds \int_0^{s} \frac{(\nabla\delta c, \mathbf{n}_0)}{S_0 c_0} \mathbf{n}_0 ds' \qquad (16)$$

The double integral in (16) can be transformed to a single integral by exchange of the order of integration:

$$\int_0^S ds \int_0^s f(s')ds' = \int_0^S f(s')ds' \int_{s'}^S ds = \int_0^S (S-s')f(s')ds'$$

It is easy to see that $|\delta\mathbf{t}_r|$ is equal to a deviation of azimuth in the receiver, so that

$$\delta\alpha = \int_0^{S} (S_0 - s)\frac{(\nabla\delta c, \mathbf{n}_0)}{S_0 c_0} ds = \int_0^{S} \frac{(S_0 - s)}{S_0 c_0}\left(\frac{\partial\delta c}{\partial x} n_{0x} + \frac{\partial\delta c}{\partial y} n_{0y}\right) ds \qquad (17)$$

Thus the azimuth anomaly turns out to be expressed as a linear functional of spatial derivatives of the velocity variation δc. The relationship (17) may be used for inversion of a set of azimuth anomalies $\delta\alpha_i$ ($i=1,2,\dots N$) into the velocity distribution $c(x,y)$.

4. Results of testing

Since deriving the formula (17) we assumed that a value of the velocity gradient is to be taken at the initial rather than at the disturbed ray, it is necessary to estimate an error related to this assumption. It should be expected that the error would be large if $\nabla\delta c$ varies too fast. This error was estimated on some numerical examples: for some velocity models the azimuth anomalies were calculated exactly by numerical integration of the ray tracing system, and by the use of the approximate formula (17). The velocity models 1 and 2 are shown in figs.1a,b. The models contain positive and negative velocity anomalies, respectively. The velocity anomaly in the both models consists 10%. The rays in these models are shown in figs.2a,b. The azimuth anomalies were calculated at the both ends of rays - receiver and source were exchanged. Figs.3a,b show the azimuth anomalies for these models. It is seen that the error (deviation of the approximate azimuth anomaly from that calculated exactly) is not too large. However, if the velocity anomaly increases, the deviation increases rapidly, which indicates to a strong nonlinearity of the problem.

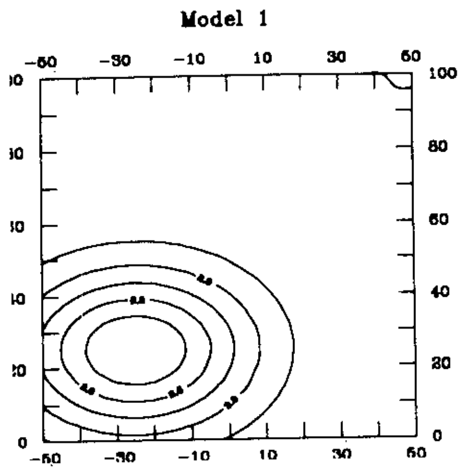


Fig.1a .

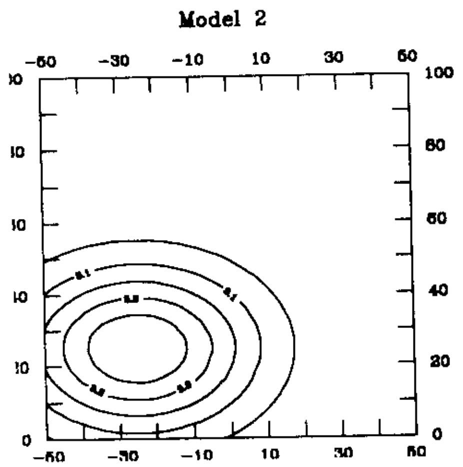


Fig.1b.

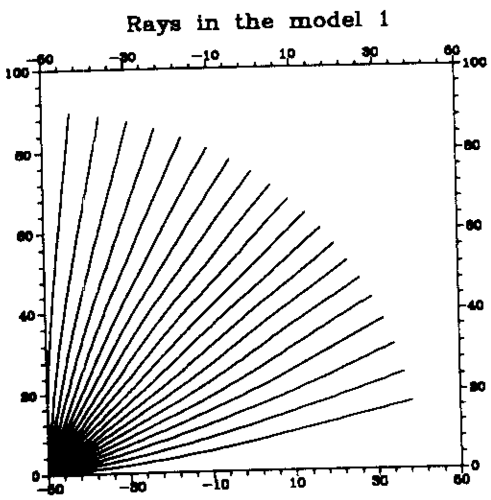


Fig.2a

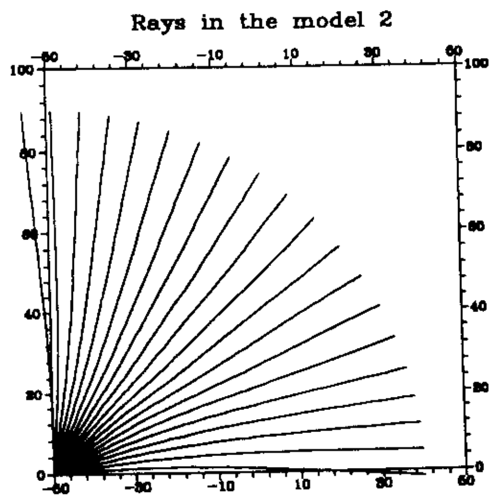
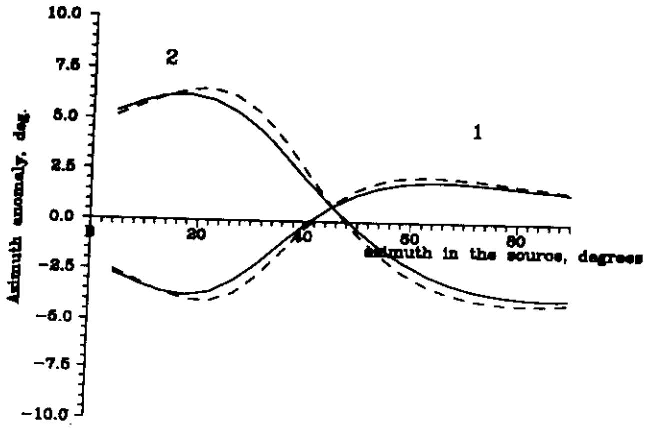


Fig.2b



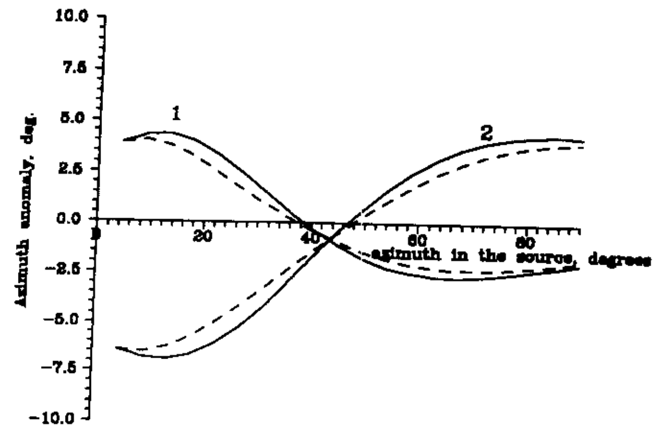


Fig.3a Azimuth anomalies in the model 1

Fig.3b Azimuth anomalies in the model 2

Solid line - exact solution, dashed line - approximate solution

Curve 1 - azimuth anomaly in the source, 2 - in the receiver

5. Tomographic reconstruction: model example

Velocity reconstruction from the azimuth anomaly data (azimuthal tomography) differs from the well-known time delay tomography, because the azimuth anomalies are related to the velocity gradients, whereas travel times (or time delays) to the velocity distribution directly. Formula (17) represents the azimuth anomaly in the receiver as a functional of two velocity derivatives $\partial\delta c/\partial x$ and $\partial\delta c/\partial y$. It is obvious that these derivatives are not independent: to determine them both it is sufficient to know either $\delta c(x,y)$, or the absolute value of the velocity gradient, and values of δc at one boundary of the region, for instance, $\delta c(x,y_0)$, Therefore it is necessary to transform the functional (17) so that it would contain only independent unknown functions. For the tomography problem it is convenient to take the velocity disturbance $\delta c(x,y)$. This function should be represented either in parametric form, or by a set of values in discrete cells. In the latter case (17) is reduced to a linear system of equations respectively $m_k=\delta c_{ij}/c_0$ $(k=(i-1)M+j)$:

$$\mathbf{Am=b} \tag{18}$$

where $b_q=\delta\alpha_q$. This system should be solved under some apriori assumptions on the vector of the unknows **m**.

Also it is obvious that it is also necessary to accept a condition, which fixes absolute values of velocity disturbances: the equation (18) determines only the differences on the adjacent cells: for example, the equation holds if one and the same constant value is added to all values of δc_{ij} . If a mean value of the velocity is known, such condition is equality to zero of a sum of the velocity variations in all cells:

$$\sum_{ij} \delta c_{ij} = 0 \qquad (19)$$

Another condition can be constructed if travel time along one or several rays is known.

The method was tested on the following example. Fig.4 shows the velocity model with two anomalies - positive and negative. For this model azimuth anomalies have been calculated for 40 rays, schematically shown in fig5. For solving the tomography problem the area was divided into 100 cells (10×10), and the condition (19) was used. The linear system of 41 equations with 100 unknowns was solved by SVD method. The result is shown in fig.6. Though the solution differs from the real velocity distribution in details, the main anomalies are revealed distinctly. It should be noted that magnitudes of the velocity anomalies are close to those in the real model, which is not always proved to be in the time delay tomography.

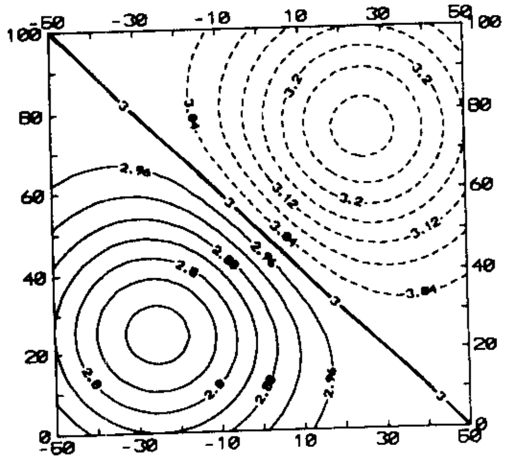


Fig.4: the model

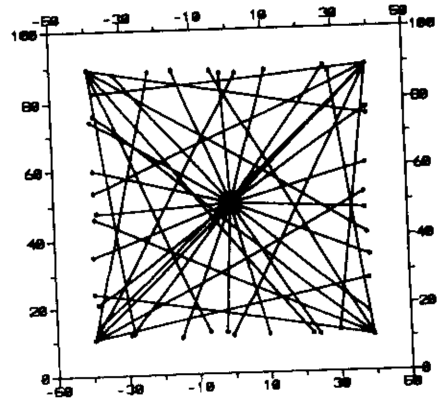


Fig.5: the rays

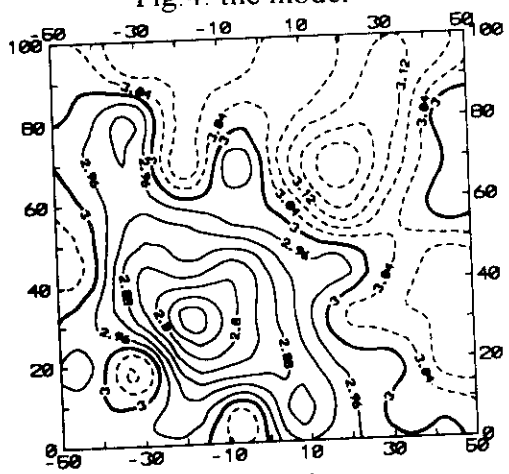


Fig.6: the solution

References

Hu G. and Menke W. 1992. Formal inversion of laterally heterogeneous velocity structure from P-wave polarization data. Geophys.J.Int., v.110, .63-69.

Lander A.V. 1984. Surface wave anomalous occurences in Northeast Eurasia and their relation to the Momsky rift area. In: Computational seismology, 16, 127-155.(in Russian)

Lerner-Lam A.L., Park J.J. 1989. Frequency-dependent refraction and multipathing of 10-100 second surface waves in the western Pacific//Geophys.Res.Lett., 16, 527-530.

Levshin A.L. Ratnikova L.I., J.Berger. 1992. Peculiarities of surface wave propagation across Central Eurasia. BSSA, 82, No.6, 2464-2493.

Nesterov A.N., Yanovskaya T.B. 1988. Lateral lithosphere inhomogeneities in South-Eastern Europe from surface wave observations. Izv.AN SSSR, Fizika Zemli, 11, 3-15.(in Russian)