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Period-Dependence of Q in the Earth Mantle

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1.Introduction

Estimations of the Q-factor in the Earth are based mainly on surface wave data,on long-period S waves and on free oscillations of the Earth. Q proved to be independent of the period in a band ranging from about 10 sec to 1000 sec. Recent studies however (Der and McElfresh, 1977; Sipkin and Jordan, 1979; Der et al, 1982; Bache et al, 1985) provided evidence that short-period body waves require higher Q-values. The theoretical background for such period-dependence of Q is based on the absorption band concept (Liu et al, 1976, Kanamori and Anderson, 1977),according to which Q has a minimum and approximately constant value within the absorption band, and increases rapidly outside the band.

The knowledge of the period-dependence of Q is essential for the restitution of the source radiation spectrum in the period range corresponding to seismic broad-band observations, mainly for the periods less 2-4s. For large periods, (>20 s) the effect of anelastic absorption is negligible, so that so that a difference in Q-models practically does not affect on estimates of seismic moment.

Various models of the period-dependence of Q at short periods have been proposed (Liu et al., 1976; Anderson & Given, 1982; Bock & Clements, 1982; Bache et al, 1985; Douglas, 1992). Though based on the concept of an absorption band, they differ in the location of the high-frequency boundary. The Q-model, which is practised on a large scale, was proposed by Liu et al.,1976.

In this model the high-frerquency boundary of the absorption band is characterized by the parameter τ: at the period $T=2\pi\tau$ Q reaches twice its lowest value. Most estimates of the parameter range in the limits from 0 05 s up to 0.5 s. Douglas (1992) even finds that Q does not reveal practically any period dependence in the frequency range 1-8 Hz, which leads to at value even lower than 0.05 s.

A difficulty in estimation of the parameter τ is related with the fact that observed spectra of seismic waves are affected by two factors: source radiation spectrum and anelastic absorption. In the present study Q(T) is estimated for P-waves in a way analogous to that of Bock&Clements(1985), and Douglas, (1992), who assumed a shape of signal in the source. Incidentally, periods, at which the period- dependence of Q is expected to be noticeable, are lower than the corner period for the earthquakes recorded at teleseismic distances. We assumed that at the periods shorter than the corner period the radiated spectra comply on the average with the ω-square model. Spectral amplitudes at periods not longer than the corner period are obtained from band-pass seismograms of a large number of earthquakes. From the comparison of the observed spectral amplitudes with spectral amplitudes calculated for different Q(T) models, the best model is obtained by way of an optimization procedure.

2. A ground for absorption band model

The most general reology is described by the equation of state for standard linear body:

$$\sigma + \tau_\sigma \dot{\sigma} = M_P(\varepsilon + \tau_\varepsilon \dot{\varepsilon}) \qquad (1)$$

where σ(t) and ε(t) are stress and strain, respectively, M_R - relaxation modulus, determining a ratio of stress to strain after relaxation, τ_σ is relaxation time of stress under constant strain, τ_ε is relaxation time of strain under constant stress. For harmonic oscillations with frequency ω a relationship between stress and strain is linear

$$\sigma = \mu\varepsilon$$

where μ is complex. It is easy to see that for standard linear body $Q^{-1} = Im\mu / Re\mu$ is expressed as follows:

$$Q^{-1} = \frac{\omega(\tau_\varepsilon - \tau_\sigma)}{1 + \omega^2 \tau_\varepsilon \tau_\sigma} \qquad (2)$$

However this mechanism cannot explain the observed constancy of Q in a wide range of periods. According to (2) the function $Q^{-1}(\omega)$ should have a peak at the frequency $\omega = \frac{1}{\sqrt{\tau_\varepsilon \tau_\sigma}}$. But if a medium is characterized by a continuous spectrum of relaxation times τ_ε within an interval (τ_1, τ_2), and $\tau_\sigma/\tau_\varepsilon$=const, then

$$Q^{-1}(\omega) = Q_m^{-1}\left[\frac{2}{\pi}\tan^{-1}\frac{\omega(\tau_2 - \tau_1)}{1 + \omega^2 \tau_1 \tau_2}\right] \qquad (3)$$

If τ_2 is large, and τ_1 is small, Q is practically constant within the interval $\tau_1^{-1} >> \omega >> \tau_2^{-1}$. This is schematically shown in fig 1.

If τ_2 is large (for the Earth's material it is of the order 10000 s), for high frequencies formula (3) may be written approximately :

$$Q^{-1}(\omega) = Q_m^{-1}\left[\frac{2}{\pi}\tan^{-1}\frac{1}{\omega\tau_1}\right] \qquad (4)$$

So a dependance Q(ω), or Q(T), is determined by a single parameter $\tau = \tau_1$, and the problem of determination of the function Q(T) is reduced to estimation of the only one parameter τ.

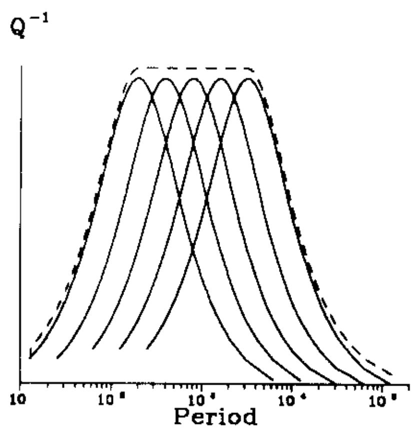

Fig. 1.

3. Method

Broadband seismograms of selected events of 1978-1986 obtained at the Central Seismological Observatory at Erlangen have been recalculated earlier into sets of one-octave bandpass seismograms (Kaiser & Duda, 1986, Chowdhury & Duda 1986, Vorbeck et al, 1987, Duda et al, 1989). Though P-wave spectra are the most suitable data for solving the problem stated above, the bandpass seismograms were utilized, due to their immediate availability. In this case however an additional correction needs to be introduced for the variable absolute bandwidth corresponding to the one-octave bandpass seismograms. Digital broadband records yield the ground velocity with practically no amplitude distortion in a period range which exceeds the range of periods of teleseismic P-waves. The one-octave digital filters have been applied to the broadband records.

Evidently, for one-octave filters the relation holds $\Delta f = f_m / \sqrt{2}$, i.e. the bandwidths Δf are inversely proportional to the mid-band periods T_m. Thus, with $A(\Delta,T)$ - the spectral density of displacement at the epicentral distance Δ, the amplitudes of bandpass seismograms $\overline{A}_T$ are proportional to $\frac{A(\Delta,T)}{T}\Delta f = \frac{\pi\sqrt{2}}{T^2}A(\Delta,T)$, where T is the prevailing period from the one-octave bandpass seismogram. The period is in practice close to the mid-band period T_m of the respective filter.

The above reasoning is correct for bandpass filters having relatively narrow widths. But it becomes progressively incorrect, as the bandwidths increase, i.e. as the mid-band periods decrease. Thus, in general, at the epicentral distance Δ the amplitudes of the bandpass seismograms A are rather proportional to $B(T)A(\Delta,T)/T^2$, where $B(T)$ is constant for narrow bandwidths (large periods) and decreasing, as the bandwidths increase. A way to estimate the multiplier $B(T)$ will be shown later.

The amplitude spectrum of the displacement at the epicentral distance Δ is represented as $A(\Delta,T)=S(T)R(\Delta,T)$, where $S(T)$ is the displacement spectrum of P-waves at the earthquake focus, and $R(\Delta,T)$ is the attenuation factor which combines the effects of geometrical spreading and of the absorption of P-waves. The dependence of R on the source depth may be neglected, as only earthquakes with shallow foci are considered in the investigation. If the radiated spectrum complies with the ω-square model, for the asymptote of the spectrum at periods shorter than the corner period the following relation holds: $S(T) \sim T^2$. Thus, the amplitudes of the bandpass seismograms at the corresponding periods are proportional to $B(T)R(\Delta,T)$. Consequently, if the measured amplitudes $\overline{A}_T$ are divided by the multiplier $B(T)$, the resulting variation of the amplitudes with period at the given seismic station will be due to the anelastic attenuation, i.e. to the absorption only.

The attenuation factor $R(\Delta,T)$ may be calculated for a given velocity and absorption model of the Earth, with an accuracy up to a constant factor, the latter reflecting the intensity of the source. The amplitude-period variation may be calculated for different absorption models, notably for models differing from each other in the parameter τ. A comparison with the amplitude-period variation determined from observations allows one to select the appropriate $Q(T)$ model. The procedure is as follows.

The Q-model for the Earth is represented as $Q(r,T)=Q_m(r)Q(T)$. (Here we assume that period dependence of Q is one and the same for all depths in the mantle). The function $Q_m(r)$ is assumed to be known: in the present study the PREM model is accepted. The function Q(T) is equal to 1 within the absorption band, and is to be estimated at shorter periods. This function is parametrized by a set of parameters, e.g. $s_1, s_2, .., s_n$. In particular, if the Liu&Anderson model is accepted, only one parameter, τ, describes the model. Thus, for any given values of the parameters the attenuation factor $R(\Delta,T; s_1, s_2, .., s_n)$ can be calculated.

The amplitudes from the bandpass seismogram $\overline{A}_T$ at periods not longer than the corner period are divided by the bandwidth multiplier B(T). The corrected amplitudes $\hat{A}_i$ satisfy the relation

$$\hat{A}_i = MR(\Delta,T_i) \qquad (5)$$

where M is a constant related to the source intensity, and i is the bandpass number. From (5)

$$\log\hat{A}_{ij} = C_j + q(T_i,\Delta)$$

The function $q(T_i,\Delta)$ depends on the parameters s_k (k=1,2,..,n), which may may be estimated jointly with the unknown intensity factors C_j for all the earthquakes (j=1,2,..,N) by minimizing the functional

$$\Sigma_{ij}(\log\hat{A}_{ij} - C_j - q(T_i,\Delta_j;s_k))^2$$

Evidently,

$$C_j = \sum_{i \text{ for fixed } j}(\log\hat{A}_{ij} - q(T_i,\Delta_j;s_k))/N_j$$

where N_j is the number of bandpass seismograms with measurable amplitudes at periods not longer than the corner period, for the j-th earthquake. The problem reduces itself then to the estimation of the s_k-values.

The function $Q(T)$ was parametrized by its values in the points corresponding to the mid-band periods T_i. As model parameters the values $s_i = Q^{-1}(T_i)$ are adopted. Thus, the total number of the parameters is equal to the maximum number of passbands which can be used. With this approach the Liu-Anderson model can be verified, and in case of a reasonable agreement, the parameter τ may be estimated.

4. Data

Though, as it was mentioned above, it would be more appropriate to use directly the amplitude spectra of the ground velocity obtained from the broad-band recordings - in this case there would be no need to introduce the factor *B(T)*, - the band-pass seismograms calculated earlier for a total of 147 shallow earthquakes have been utilized as being easily available.

The maximum amplitudes of P-waves in the time window up to 40 s after the first arrival are determined. The epicentral distances of the earthquakes range from 9° up to 90°. The measurements for a given earthquake are made only for the mid-band periods shorter than the one corresponding to the bandpass seismogram with the largest amplitude. The latter mid-band period is assumed to correspond to the corner period . The amplitudes of P-waves with period near 0.25s are small. They are found not measurable in a number of cases. Therefore the parameters s_1 (for the mid-band period of 0.25s), and also s_2 (for 0.5 s) to some extent, are generally found to be less reliable than the remaining ones. The largest period less than the corner period turned out to be equal to 4s, so the total number of parameters was equal to 5.

The multiplier *B(T)* for the correction for the bandwidth has been estimated by numerical modelling. Due to different phase shifts between spectral components in different signals, no exact relation exists between the spectral density and the bandpass amplitude. However, this relation may be estimated for different synthetic signals satisfiying assumed models of the radiation spectrum.

The amplitude spectrum of ground displacement for the ω-square model is:

$$S(\omega) \approx \frac{\sin \omega X}{\omega X} \frac{\sin \frac{\omega t_0}{2}}{\frac{\omega t_0}{2}}$$

with t_o - the rise time, and X depending on the rupture velocity, the wave velocity, and the relative orientation of the rupturing direction and the direction of the ray (Aki & Richards, 1980). Here, a linear rise of the seismic moment within the time t_o , as well as unidirectional rupture along the fault is assumed.

The ground velocity corresponding to this model of displacement spectrum is shown in fig.2.

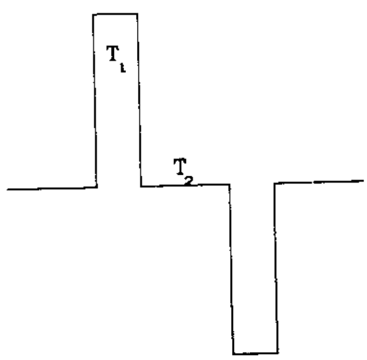

Fig.2

parameters T_1 and T_2 are related to the parameters of the spectrum t_o and X. The signals were simulated for a random selection of the parameters T_1 and T_2, and their amplitude spectra as well as bandpass seismograms were calculated. The maximum amplitudes of the bandpass seismograms A_{max} divided by the respective bandwidth $2\Delta f$ are designated A(f). Also, the spectra were averaged over the same one-octave intervals, the average being designated S(f).

Mean values of the ratio S(f)/A(f) were calculated for different combinations of the parameters T_1 and T_2. Indeed, it increases with frequency, though not too strongly: for the shortest period 0.25 s it is by ~30% higher than for the periods 2-4 s. In logarithmic scale this difference is about 0.1.

5. Results

The coefficients s_1 -s_5 corresponding the periods 0.25, 0.5, 1., 2., 4 s were determined by a method of steepest descent. The coefficients are not independent: one of them may be chosen arbitrarily. Indeed, for each passband the amplitude A_k may be expresses as

$$A(\Delta)\exp(-s_k t^* / T_k)$$

where t^* corresponds to $Q_m(r)$. Since the data used represent relative rather than absolute values of amplitudes, only the differences

$$\frac{S_i}{T_i} - \frac{S_{i-1}}{T_{i-1}}$$

ant not the absolute values of the parameters s may be estimated. This means that one coefficient may be chosen arbitrarily. A variation of this coefficient leads to corresponding adjustment of the other coefficients. However, the variations are limited by the condition

$$S_i > S_{i-1}$$

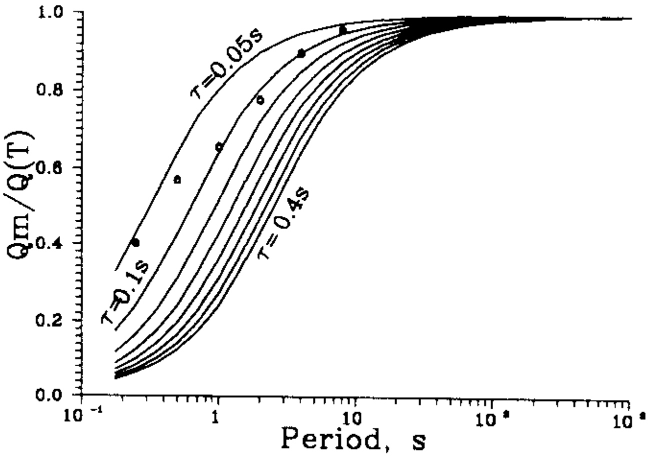

Fig.3

The adopted solution for the parameters s_i is shown in fig.3. The parameter s_5 was chosen as the arbitrary one. It was adjusted so that the other parameters would lay on a smooth curve.

In the same fig.3 the curves Q_m /Q(T) corresponding to the Liu-Anderson model are drawn for different values of the parameter τ. It may be concluded that the data are in a good agreement with the curve corresponding to τ=0.1s. As was noted above, the results at the shortest periods are not reliable due to relatively small amount of amplitude data for these periods, so that the discrepancy should not be attached greater importance.

6. Applications

The selected Q(T) model was used for calculation of calibrating curves for restitution of source radiation spectrum from ground velocity spectrum of P wave recorded by a broadband recording system. Such curves were calculated for different source depths and periods in the range from 0.25s to 32 s with a step of one octave. The curves are utilized in a software PASTA (Program for Amplitude Spectra Treatement and Analysis) , which is a supplement to IASPEI software PITSA. Amplitude spectrum of ground velocity corrected for instrument responce is calculated in PITSA, and the result is saved in a ASCII file. This file is used as input for PASTA, in which magnitude spectrum, spectral magnitudes (averaged values of the magnitude spectrum), moment rate spectrum $S_M(\omega)$, and scalar seismic moment M_0 as asymptotic value of $S_M(\omega)$ at low frequencies, are calculated. The moment rate spectrum is used for calculation of P-wave energy by the formula

$$E = \frac{1}{8\pi^2 \rho_0 \alpha_0^5} \int_0^\infty |S(\omega)|^2 \omega^2 d\omega$$

Also moment magnitude M_w is calculated according the formula

$$\log M_0 = 1.56 M_w + 16.1$$

Similarly moment magnitude the magnitude based on the total energy is introduced as

$$M_e = 0.64 \log E - 6.64$$

This relationship was obtained proceeding from empirical correlation between energy and seismic moment:

$$\log E = \log M_0 - 5.78$$

It is obvious that moment magnitude characterizes intensity of source radiation at low frequencies, whereas the ‘energy’ magnitude relates to the intensity at high frequencies, around the corner frequency.

7. Conclusions

From the analysis of P-wave spectra at UKAEA arrays (EKA, YKA, GBA and WRA) from explosions in eastern Kazakhstan, Bache et al (1985, 1986) have shown that the intrinsic attenuation can be represented by an absorption band model with $0.04<\tau<0.1$, while $0.5<t^*<1.0$, where t^* corresponds to Q at low frequencies, i.e. to Q_m. The epicentral distances of the Kazakhstan test site to the UKAEA arrays range from 36° to 85°.

In our calculations we used the PREM-model for Q (r), and if the same distance range as above is used, t^* changes from 0.95 to 1.25, i.e. its average exceeds only slightly the value of 1. According to Bache et al.(1985) such value of t^* correspond to the values of τ about 0.1s (see above), which is in a good agreement with the results of the present study.

So the proposed method for estimation of Q(T) from broad-band data provides the acceptable results in spite of inaccuracy and scantiness of the used data, espesially at the highest frequencies. The use of digital broad-band records and their amplitude spectra instead of measurements of amplitudes at band-pass records would lead to improvement of the Q(T) model.

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