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Second Workshop on
Three-Dimensional Modelling of Seismic Waves
Generation, Propagation and their Inversion

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Earthquake Faulting: The Inverse Problem

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References

Das, S. (1992) Reactivation of an oceanic fracture by the Macquarie Ridge earthquake of 1989, *Nature*, **357**, 150-153, 1992.

Das, S. (1993) The Macquarie Ridge earthquake of 1989, Geophys. J. Intl., 115, 778-798.

Das, S. and B.V. Kostrov (1990) Inversion for seismic slip rate and distribution with stabilising constraints: Application to the 1986 Andreanof Islands earthquake, *J. Geophys. Res.*, **95**, 6899-6913, 1990.

Das, S. and Kostrov, B. V. (1994) Diversity of solutions of the problem of earthquake faulting inversion. Application to SH waves for the great 1989 Macquarie Ridge earthquake, *Phys. Earth Planet. Int.*, **85**, 293-318.

Kostrov, B. V. and Das, S. (1988) Principles of earthquake source mechanics, *Appl. Math. Mech. Ser.*, 286 pp. Cambridge University Press, New York.

Press, W.H., B.P. Flannery, S.A. Teukolsky and W.T. Vetterling (1986) Numerical recipes. The Art of Scientific Computing, Cambridge Univ. Press, N.Y., 818 pp.

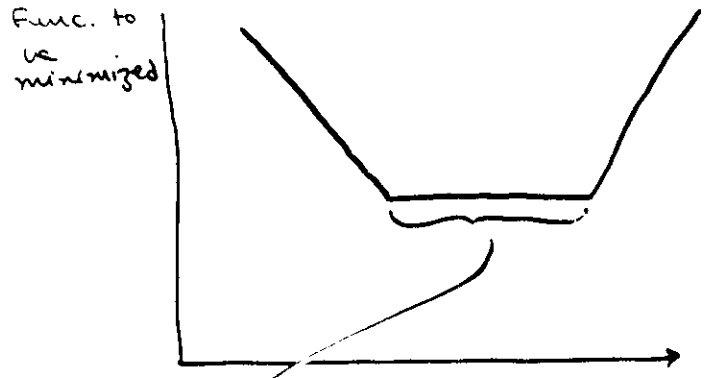
Definition of some common terms used in inverse theory.

STABILITY is defined as the property that a solution is insensitive to small random errors in the data.

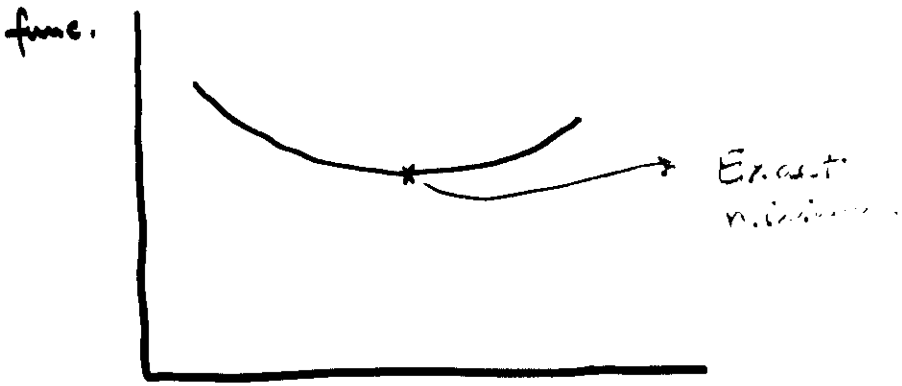
ROBUSTNESS is the property that a solution is insensitive w.r.t. a small # of big error (outliers) in the data.

NON-UNIQUENESS is the property that more than one solution (almost) equally well-fitting the data exists.

A pathological example of non-uniqueness.



Every point here is a valid minimum as shown by the scan. is [illegible]



In geophysics, the data itself contains noise + the models are themselves approximations.

The inverse problem of obtaining the details of the rupture process from an analysis of teleseismically recorded seismograms is an UNSTABLE problem.

Note that the problem is unstable even in the imaginary case of a continuous distribution of seismic stations over the surface of the earth. From the computational point of view, the instability is equivalent to non-uniqueness of the solution.

The real situation is even worse because the # of stations with appropriate records is very limited (10-20 stns, even today - cf. 1989 Macquarie Ridge eq.)

Hence, one needs some additional constraints on the soln. (i.e. in addition to the requirement of fitting the seismograms). These constraints should be based on the physics of the earthquake faulting problem.

But knowledge of this physics is rather limited.

<u>Possible physical constraints</u>

① Limit on fracture propagation speed

② Less physically founded: Direction of slip vector on fault is parallel to direction of the average stress drop → leads to the "no back-slip" constraint.

③ Max. slip rate on fault can be constrained based on fracture mechanics results from lab. expts. on rocks.

In order for a particular slip distribution to be an acceptable solution to the inverse problem it must satisfy the following three conditions.

1. The solution must explain the data.
2. The solution must be physically reasonable.
3. If more than one solution fits the data equally well, additional information must be supplied to uniquely define which solution is being obtained **and such constraints should be explicitly stated and not hidden in the method of inversion**.

Displacement at a station is given by

$$u_K(\bar{x}_1, t_1) = \int_0^{t_1} dt_1 \iint_\Sigma K_{ik}(\bar{x}_1, \bar{x}; t_1, t)\, a_i(\bar{x}, t)\, dS$$

where $u_K \rightarrow$ k th comp. of displacement at $(\bar{x}_1, t_1)$

$K_{ik} \rightarrow$ impulse response of medium

$a_i(\bar{x}, t) \rightarrow$ slip at $(\bar{x}, t)$ on fault

$\Sigma \rightarrow$ fault area.

After some simple algebra

(to move derivative from K to a in convolut.

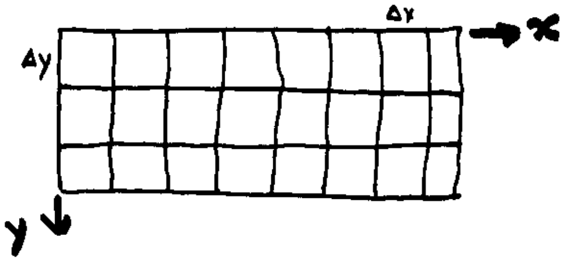
we get

$$S_j(t_1) = \int_0^{t_1} dt_1 \iint_\Sigma W_j(\xi, t_1 - t)\, \dot{a}(\xi, t)\, dS$$

$j(t_1) \rightarrow$ Seismogram at j th stn. at time t_1 $\rightarrow$ DATA

$l_j \rightarrow$ impulse response ; direction of slip is taken to be constant + $\parallel^{l}$ to the applied stress direction $\rightarrow$ known

$\dot{} \rightarrow$ SLIP RATE on fault (UNKNOWN)



Fault is discretiz.

In each cell, $\dot{a}$ is a linear function

W_j is discretized by integrating over cells

The source time function is discretized by taking a fixed time step Δt and assuming slip rate varies linearly with time.

Then, in discrete form we get the system of linear eqns

$$\underbrace{A}_{\text{discretized impulse response } W_j}\ \underbrace{x}_{\text{unknown slip rates } \dot{a}} \approx b \longrightarrow \text{data from Seismograms}$$

\# of eqns = (# of grids along x).
(# of grids along Y).
(# of source time steps)

CONSTRAINTS

1. No back-slip i.e. $\bar{x} \geq 0$ always

2. Causality condn: grids not allowed to slip until first wave from point of rupture initiation arrives at it
(STRONG CAUSALITY CONSTRAINT)

3. Seismic moment condn.

The seismic moment resulting from the solution must be consistent with that obtained from long-period data (e.g. the CMT solution M_o or geodetically obtained moment).

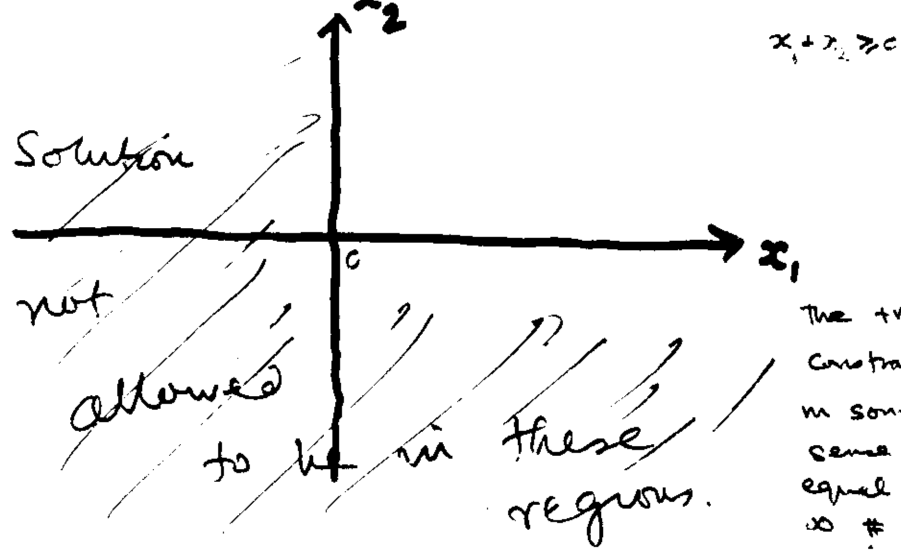
$\Sigma c_i x_i \approx M_o$, summed over fault with c_i containing rigidity modulus, etc.

So we need to solve a CONSTRAINED linear set of equations.

Please forget, at this point, EVERYTHIN you ever learned about solving un constrained linear system as they are quite IRRELEV

One example: In un constrained system $Ax = b$, the # of equations should be $\geq$ # of unknowns.

this is not relevant to the present probl as the constraints provide additional conditions e.g.



The +ve constrai in some sense, equal to ∞ # of

Let $\bar{r} = \bar{b} - A\bar{x}$ be the vector of residuals, which we have to minimize. In order to use the linear-programming approach which is very convenient for solving constrained systems, we shall minimize the l_1-norm of $\bar{r}$:

i.e. minimize $(|r_1| + |r_2| + \cdots + |r_m|)$

where m = # of eqns.

The method of soln. is described in standard text books:

Inverse problem theory by A. Tarantola

Numerical recipes by Press et al.

1. No Back-slip condn. is found to be essential (Explain why).

2. Seismic moment constraint may be needed if data is very poor. For the 1986 Andr. Is. eq. (M_w = 8.1) it was needed. For the 1989 Macquarie eq., the soln. gave results consistent with CMT moment & the constraint did not need to be applied.

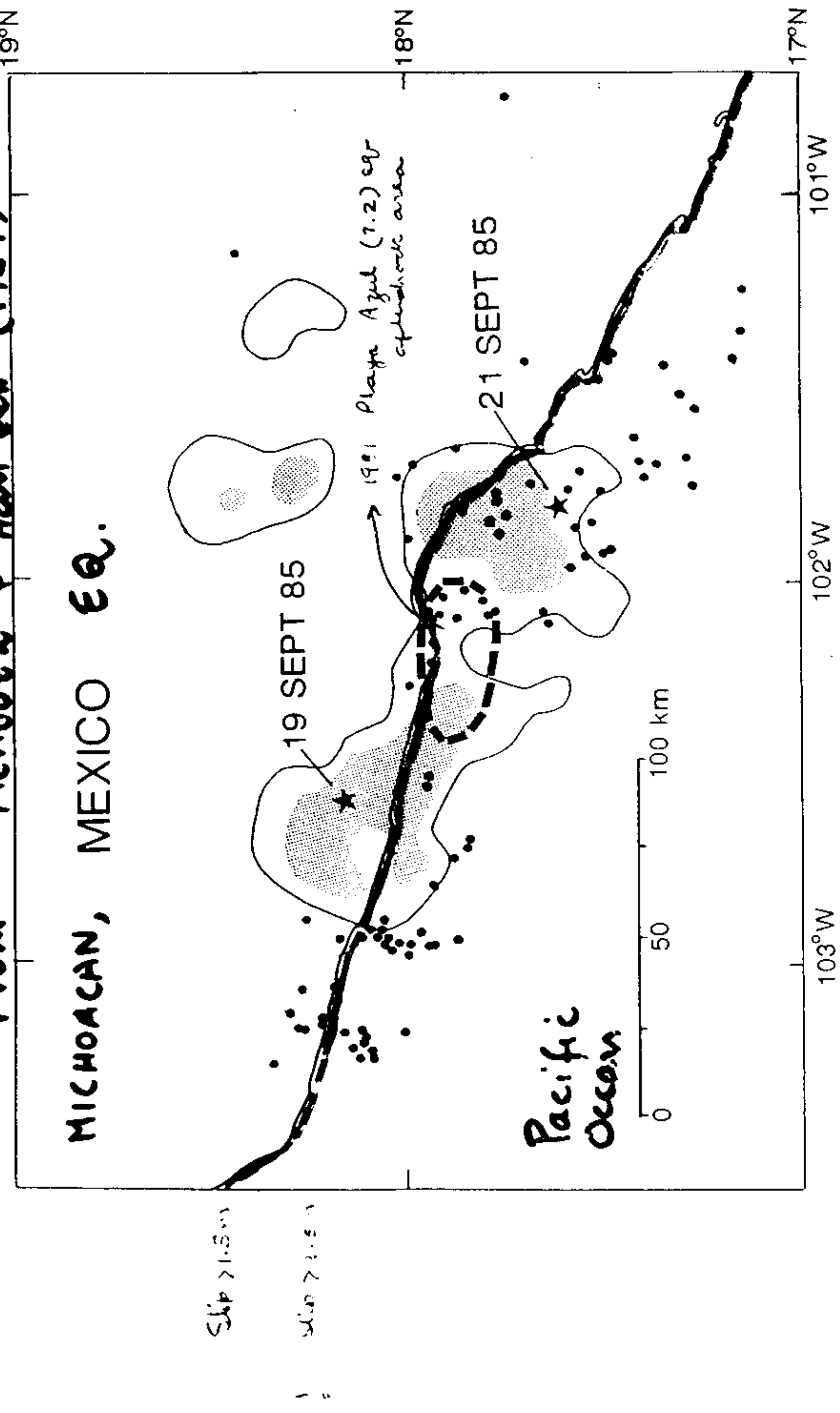
In order to be confident about physical conclusions relating to the fault rupture process, such as the relation between areas of high moment release and aftershock locations, we must examine the reliability of the moment release pattern obtained. (The reliability of the aftershock locations is a separate issue). This problem has not been sufficiently studied for the problem of determination of the earthquake slip rate history and distribution on a fault from inversion of observed seismograms. For the problem of determination of earth structure from seismic observations, the problem has been discussed in the now-classic papers of Backus and Gilbert (1968, 1970) and the same ideas are applicable here.

The inverse problem is unstable and in order to stabilize it one has to introduce stabilizing constraints. This can be done either by reducing the number of unknown being solved for by making a priori assumptions about the solution or by introducing additional constraints based on physical considerations. Das and Kostrov (1990) have discussed why the latter approach is preferable. Once the solution has been stabilized, the question of resolution arises and the well-known trade-off between stability and resolution (Backus and Gilbert, 1968) has to be considered. That is, stabilizing the solution too much may result in poor resolution.

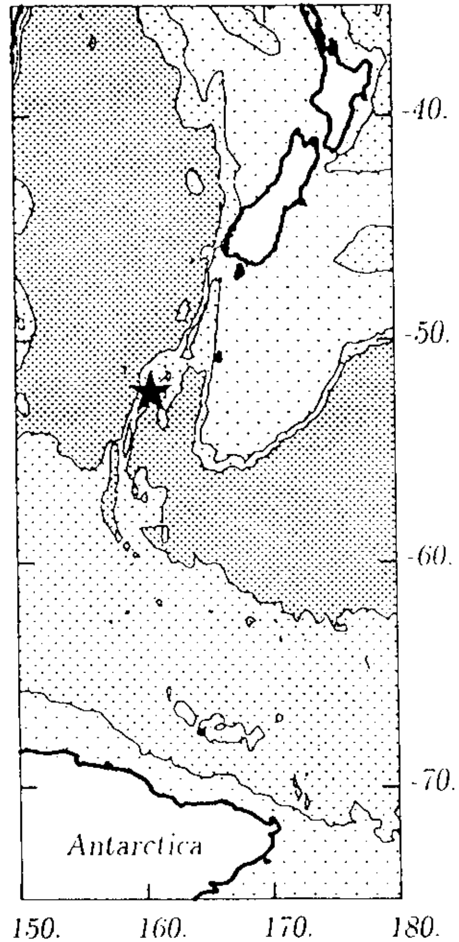
In the present context, the question comes down to this: by how much can we displace the areas of high and low moment and by how much can we change their areal extent and the amount of moment released and yet obtain almost equally good fits to the seismograms. We shall stabilize the solution by introducing additional constraints and examine their effect on the solution.

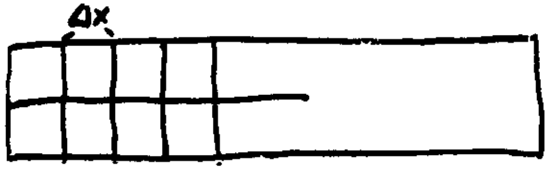
From Mendoza + Hartzell (1989)

MICHOACAN, MEXICO EQ.



The 1989 Macquarie Ridge eq.
(Mw = 8.2)





$\Delta x = 10$ Km   fault length = 450 Km

$\Delta \tau = 3$ secs (source step time interval)

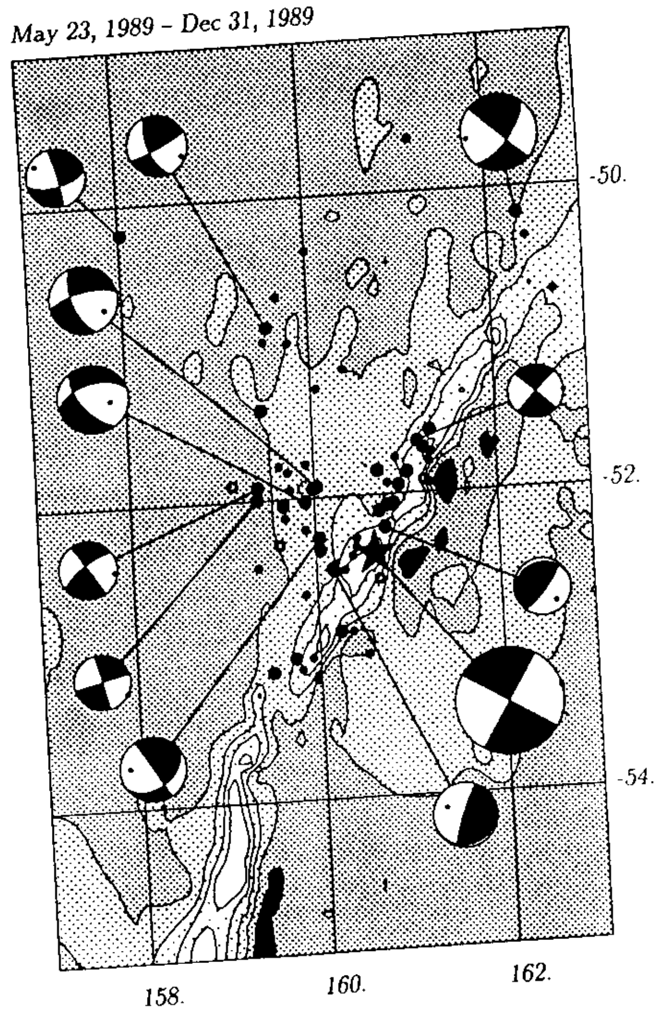
Source time = 63 secs.

We find that even with a large # of unknowns, the amplitude at some stations are always poorly fit (other stations fit very well).

This suggests that the instument responses at some stns. may have some inconsistencies. In order to overcome this, we include a station scaling factor for the problem stations & solve for these scale factors as part of the soln.

It was found that the scale factor was often close to 2. or ½ suggesting a missing scale factor in the instrument response!

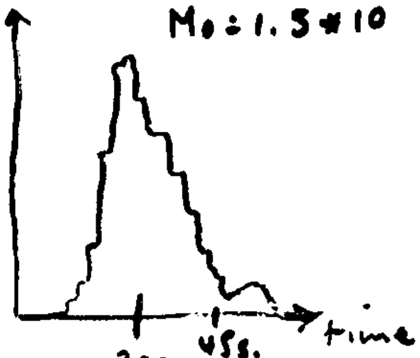
A <u>weak</u> causality condn. is used, viz. we assume that slip rate is zero at a cell and time step which would produce a signal before the first arrival at any station from the hypocentral grid.



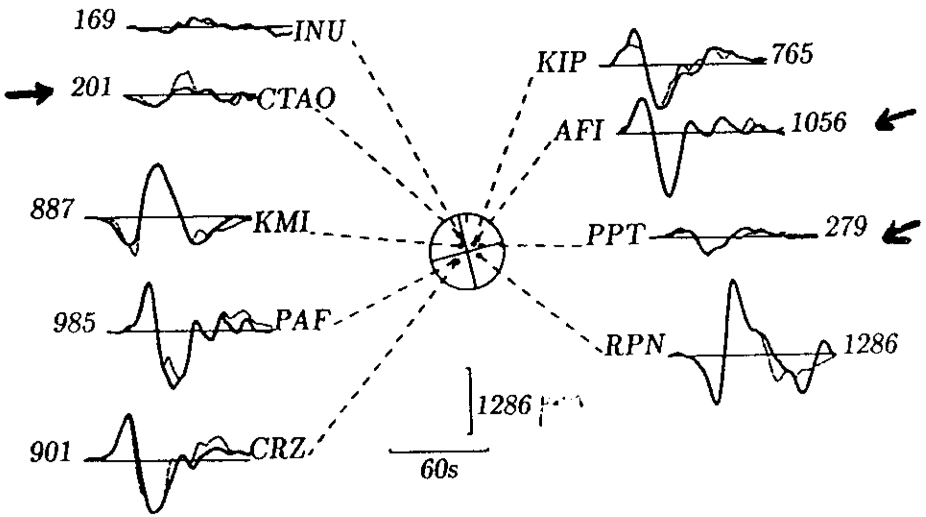
Source func.

Mo : 1.5 * 10^{21} Nm (constraint)

(Very stable for all inversions & shows no significant moment release after ~ 45 seconds.)



CASE A



SH - waves
(SH nodal planes shown)

l_1 error of fit = .28 (I use $\sqrt{\ }$ in the error of fit formula whereas most people don't use $\sqrt{\ }$ so that in other people's terms error = $.28^2$ = .08 !)

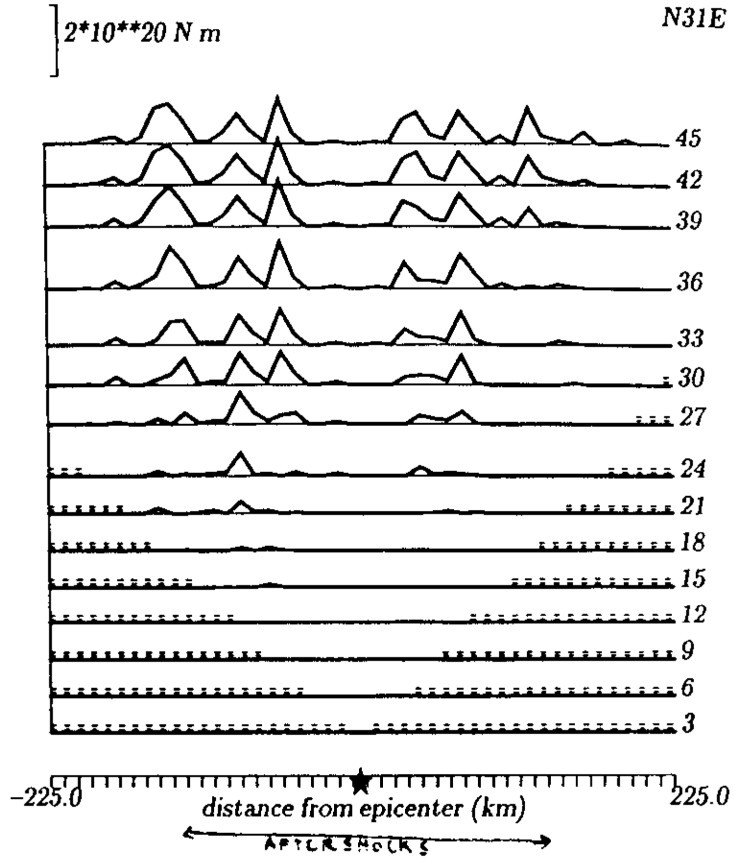
Fig 5

Note: Hypocentral region is a region of low moment release.

Slightly more moment released in the SW direction from the hypocenter than in NE direc.

CASE A

(Moment)



|| Moment spreads along fault BEYOND area delineated by aftershocks!

... not allowed to slip.

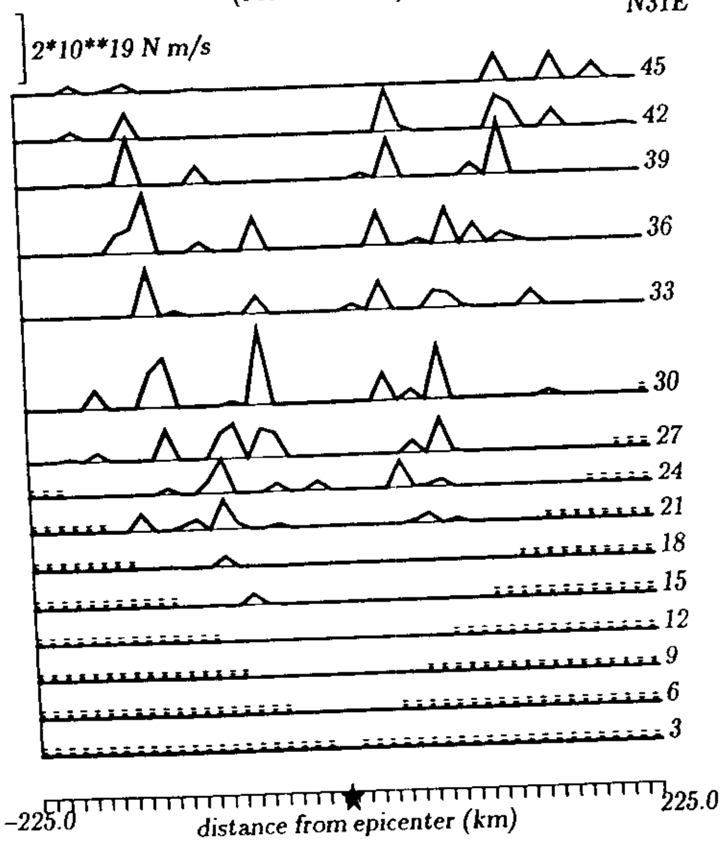
No simple rupture propagation ...

Looks like "asperities" breaking but is this an artefact?

Solution constrained to have most uniform moment rate distribution, with the error of misfit being allowed to increase by 10% of previous case(A).

CASE A

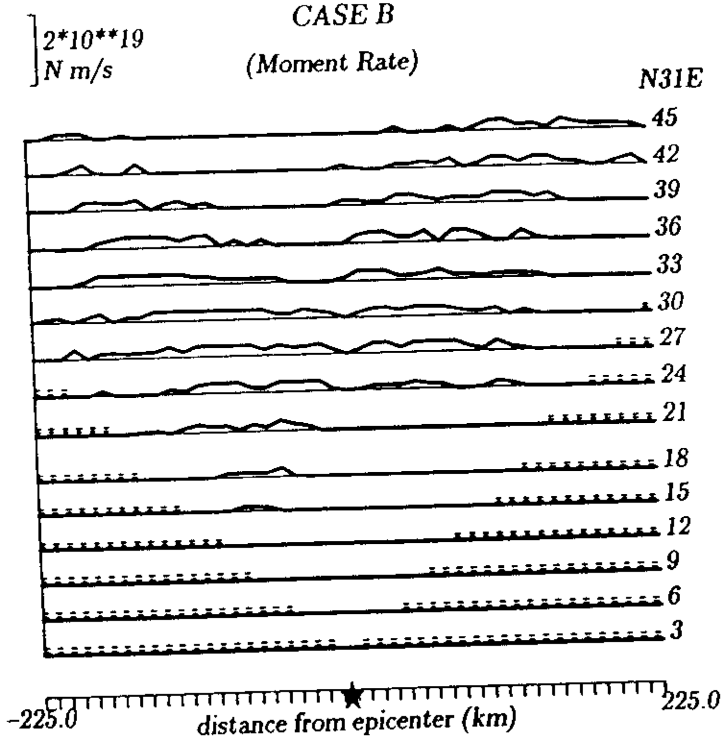
(Moment Rate)

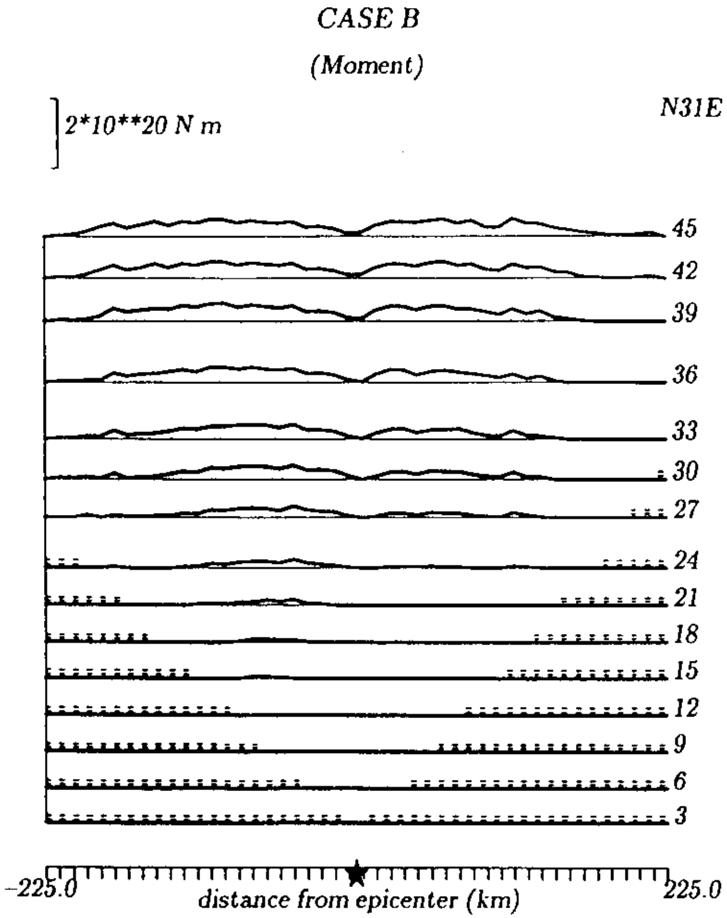


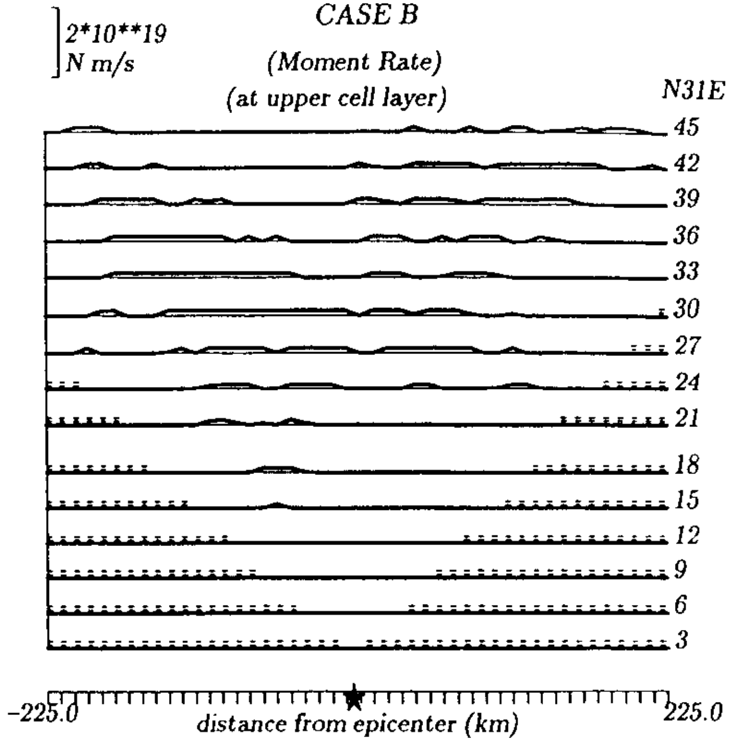
Acausal regions (constraint)

CASE B

(Moment Rate)




CASE B
(Moment)
2*10**20 N m
N31E
45
42
39
36
33
30
27
24
21
18
15
12
9
6
3
-225.0
distance from epicenter (km)
225.0


2*10**19
N m/s
CASE B
(Moment Rate)
(at upper cell layer)
N31E
45
42
39
36
33
30
27
24
21
18
15
12
9
6
3
-225.0
distance from epicenter (km)
225.0

Fig
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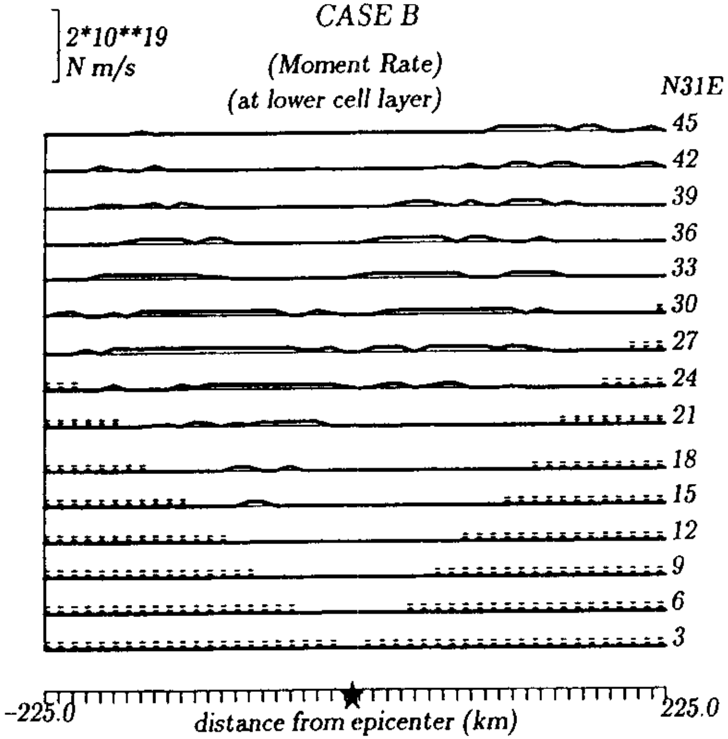
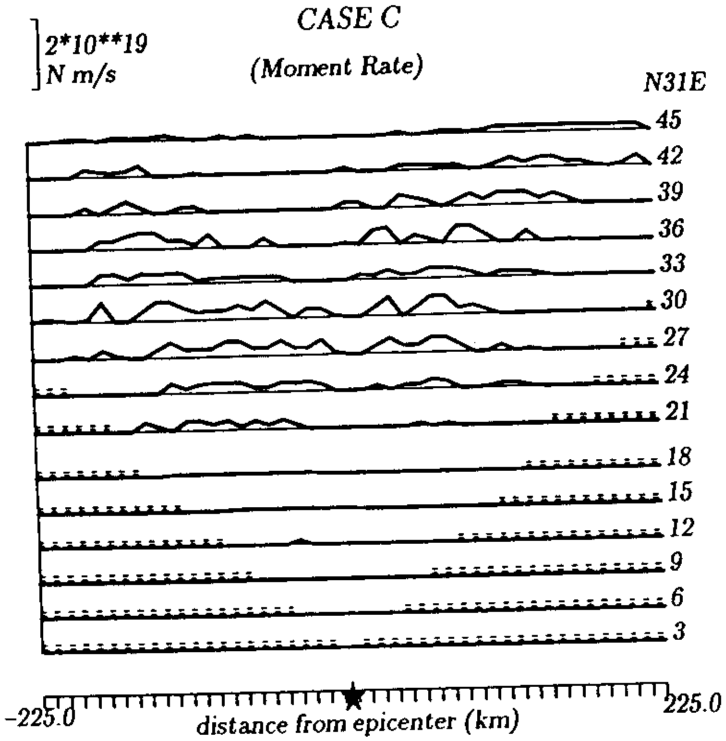


Fig 14

Case B shows that the distribution of moment rate is still non-uniform (with moment rates over the fault having comparable values — unlike what is expected for a propagating crack, say, where much higher values of slip (/moment) rates would be seen near the leading rupture edge).

Thus, the soln. cannot be made completely uniform + still satisfy the data.

Solution constrained to be uniform in space only.



Note very little moment release at last few time steps.

Fig 15

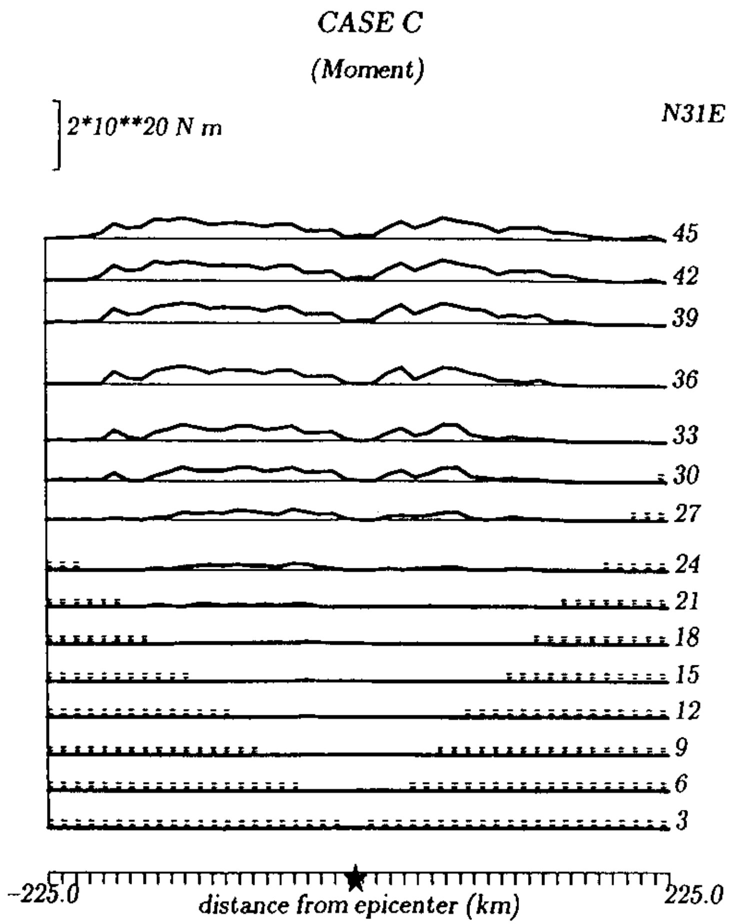
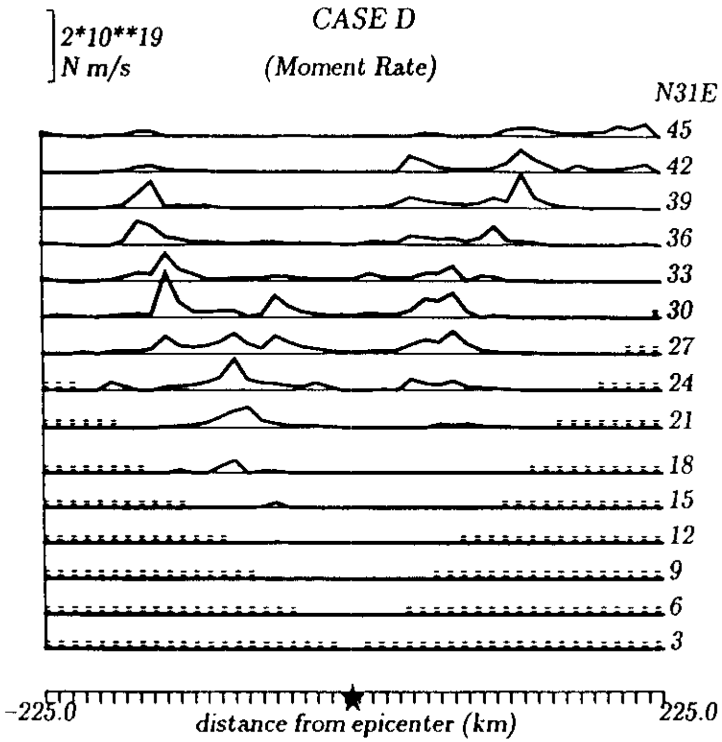


Fig 16

Solution constrained to be as smooth as possible (+ yet satisfy data) along fault strike — i.e. we minimize the sum of moduli of the <u>second difference</u> of slip rate along strike.



Soln. is smoother than that in Case A.

Fig

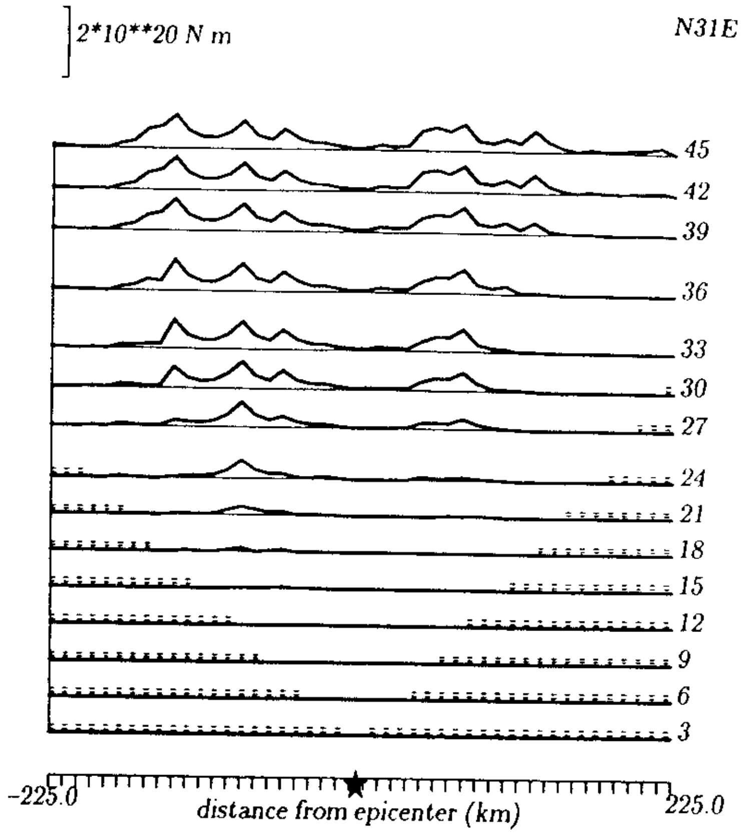
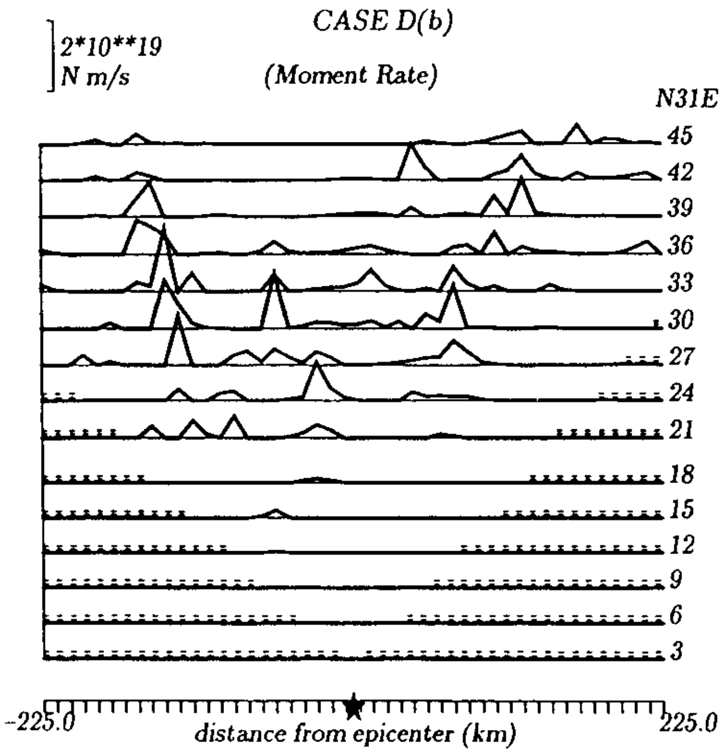


Fig 18

Example of incomplete optimisation

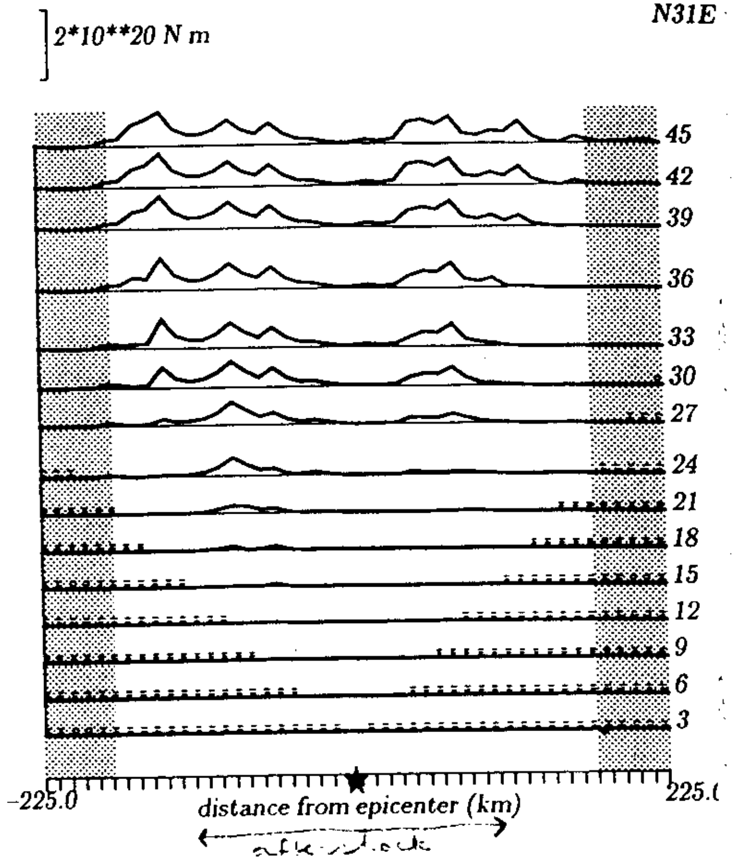


Search for the smoothest soln. was stopped too soon without reaching the minimum & shows that the smooth soln. indeed was not reached!

To check if the moment on the ... aftershock zone is truly there, we minimize the moment release in the shaded areas, after obtaining the smoothest soln. (previous Case D).

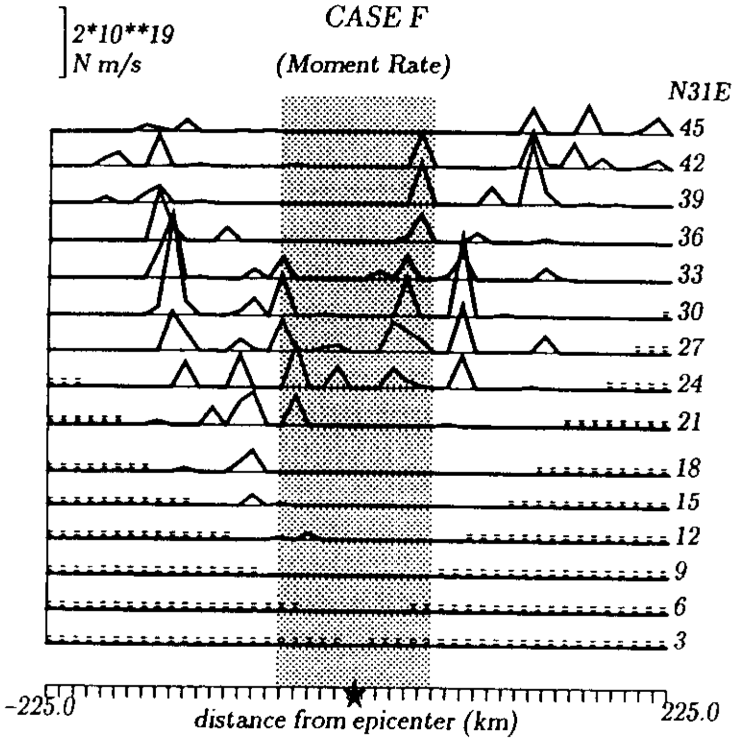
CASE E

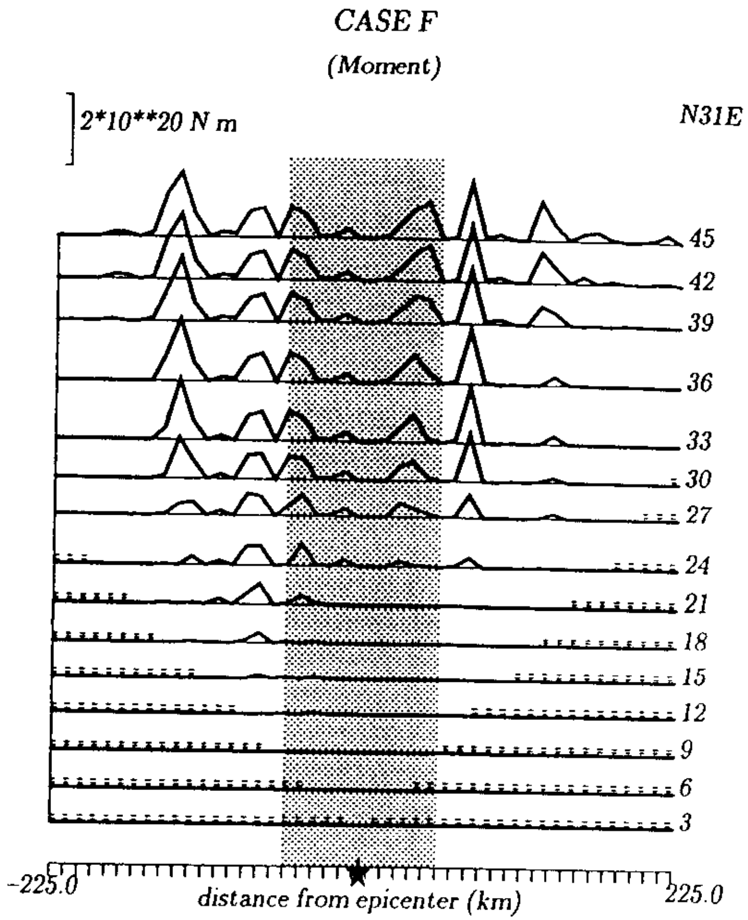
(Moment)



Result: We are unable to remove moment outside aftershock area & still fit seismograms well. We even increase tolerance of fit to 30% (instead of 21 10%) & were still unable to suppress moment outside aftershock area.

| to Increase moment at 11 central grids along strike.





Result: We do increase moment in central area (comp. Case D) but hypocentral area is still region of low moment release.

Fig 23

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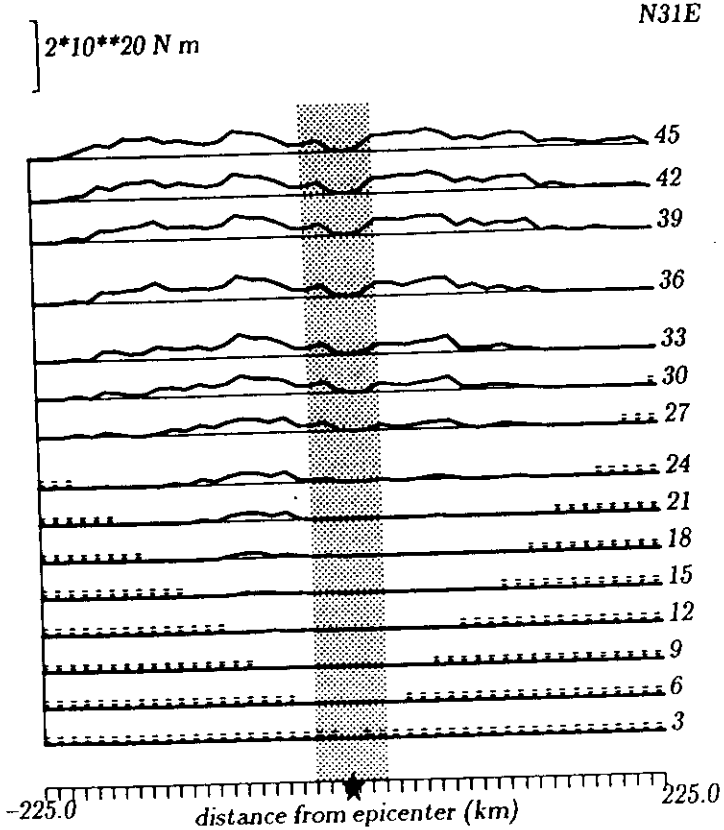
Try to maximise moment in central 5 grids but starting from most uniform soln. (case c).

CASE G

(Moment)



again: unable to remove moment outside aftershock area!

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