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Mathematical Modelling of Active Vibroseismic Monitoring of a Seismic-prone Zone

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One of the most ptomising recent lines in searching for physical precursors of earthquakes in seismic-prone regions is active vibroseismic monitoring. The idea of utilizing artificial vibrational oscillations in studying slow variations of physical properties of the geological environment was first put forward by Russian geophysicists [Alekseev, Nikolaev, Chichinin, 1974], and since then this idea has been undergoing intensive development both in Russia and abroad [Clymer, McEvily, 1981].

Among the most significant practical achievements in this field one should note the recent publication [Karageorgi, Clymer, McEvily, 1992] concluding a three-year-long study of the variations of seismic waves in the region of Parkfield, one of the possible focuses of an imminent strong earthquake in California.

Here we won't touch upon technical aspects of the creation of powerful vibrators, the system control of them as well as recording and transformation of sweep signals in the impulse form.

I would like to say that the most powerful in the world seismic vibrators and, also, the unique recording and analyzing equipment we have developed on the test site near Novosibirsk.

The experiments carried out on this test site demonstrate a high degree of coincidence between the repeated vibro-effect events at distances over 300 km. Let us note that a sweep-signal was within the frequency band 0.5-7 Hz. Thus, using the vibroseismic monitoring it is possible to study time-dependent variations of the tensely-deformed state of rocks in seismic-prone regions.

However, here arises a question: What sort of variations in the dynamic characteristics of seismic waves can serve as precursors of the possible earthquake?

At the present time, a certain number of the earthquake precursors are known. For example, a change in the velocity ratio of P- and S- waves in the seismic-prone zone a few months before the earthquake; rotation of compression axes in this zone, as well as changes in the electroconductivity of the medium, and so on. The above-mentioned and other effects can be explained in terms of dilatancy of rocks. As known, the dilatancy means a nonelastic increase of the volume of rocks due to the appearance of cracks and their growth. In this case, the rock becomes anisotropic, and we deal with the so-called extensive-dilatancy anisotropy. We'll take this physical model as a basis for construction of a mathematical model.

As you can find in a number of theoretical papers [Hoening (1979), Hudson (1980), Nishizawa (1982), Thomsen (1988)], a medium containing aligned allipsoidal cracks can be replaced by a homogeneous transversely isotropic medium.

The transverse isotropy is a special case of anisotropy and is described by five independent elastic constants, whereas anisotropy, in general, is described by up to 21 independent constants. A medium having an axis of rotational symmetry, which may be oriented in any direction, show a transverse isotropy. If this axis is not vertical, this type of anisotropy is called the "azimuthal anisotropy". This kind of anisotropy is described by the system of elastic equations for the anisotropical model.

Now, we must calculate these efficient modules so as to take into account orientation and density of cracks, aspect ratio of the cracks, changes of the crack-fluid content.

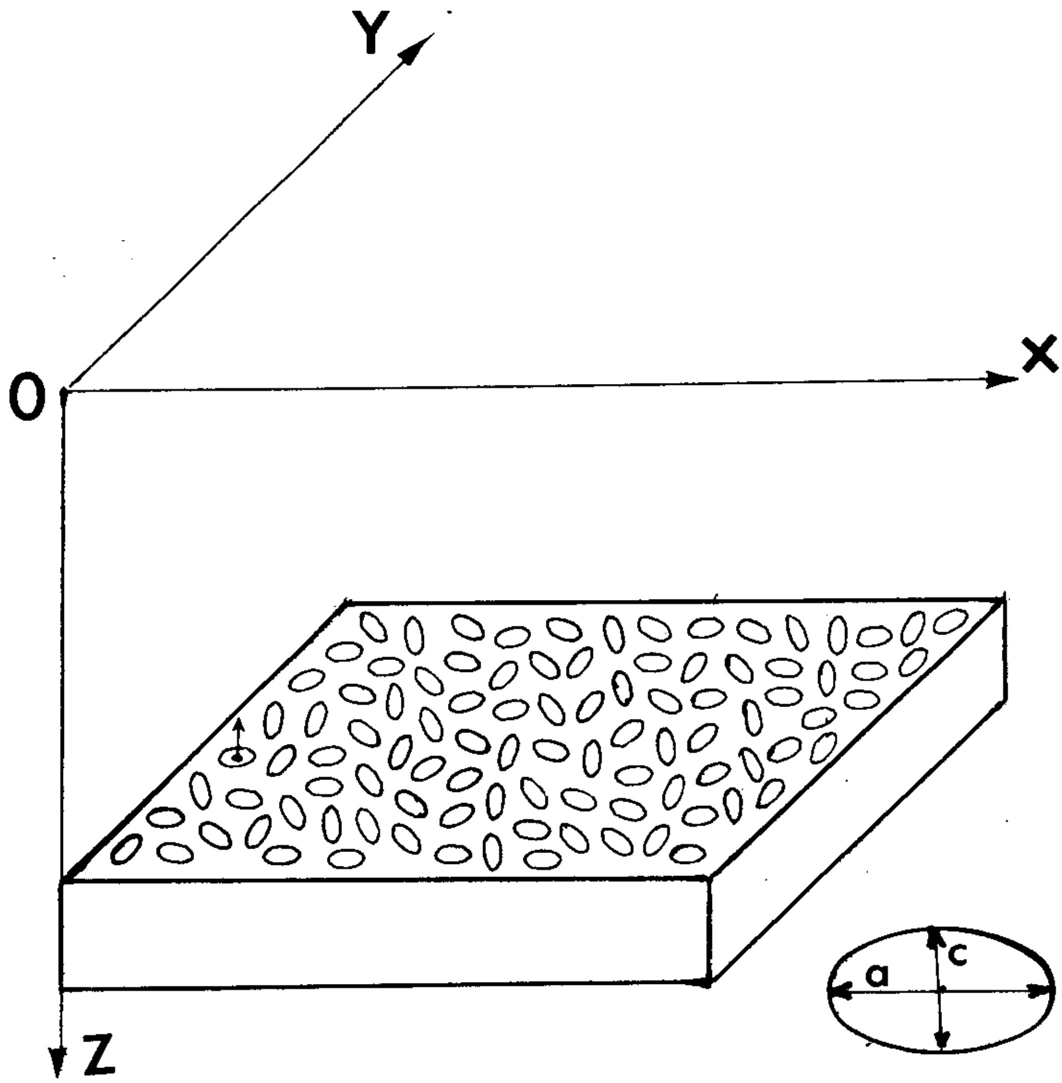
Several theories have been developed to calculate the effective elastic constants of media containing aligned ellipsoidal cracks [Nishizawa (1982), Hudson (1980, 1981)]. They are all based on the scattering of waves at the cracks. These theories have been used to analyze the wave propagation in cracked media and to explain the observed anisotropy. The basic assumptions of these theories are that the dimensions of the cracks are small with respect to the seismic wavelength, the cracks are in the dilute concentration and have small aspect ratios. Here we mean the ratio between the length of an ellipsoidal crack and its width. The physical model study shows a good coincidence of experimental results with theoretical conclusions according to the Hudson model [Ass'ad and etc.(1993)].

A theory that is valid for large concentrations of cracks has been proposed by Thomsen (1988) and is based on the work by Hoenig (1979).

Let us consider the physical model of a crack medium that was submitted by Hoenig (1979). It is the planar transverse isotropy in which the cracks are randomly distributed in the plane parallel to the plane YOX.

Physical Model
of Crack - Induced Anisotropy

(Hoening 1979)



aspect ratio : $\alpha = c/a$.

$$
\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_{23} \\ \varepsilon_{31} \\ \varepsilon_{12} \end{pmatrix} = \begin{pmatrix} 1/E_s & -\upsilon_s/E_s & -\upsilon_s/E_s & 0 & 0 & 0 \\ & 1/E_s & -\upsilon_s/E_s & 0 & 0 & 0 \\ & & 1/\overline{E} & 0 & 0 & 0 \\ & & & 1/\overline{\mu} & 0 & 0 \\ & & & & 1/\overline{\mu} & 0 \\ & & & & & 1/\mu_s \end{pmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{pmatrix} \quad (1)
$$

Thomsen (1989)

$$
\upsilon_s = \frac{E_s}{2\mu_s} - 1 = \frac{1}{2} - \frac{E_s}{6K_s}
$$

Hoening (1979)

$$
\overline{\varepsilon}_{ij} = \overline{M}_{ijpq} \cdot \overline{\sigma}_{pg} =
$$

$$
M^s_{ijpq} \cdot \overline{\sigma}_{pg} + \left(\delta_{ir}\delta_{js} - K_f M^s_{ijkk}\delta_{ir}\right)\phi_c \left\langle \varepsilon^c_{rs} \right\rangle \quad (2)
$$

$$
\phi_c = \frac{3}{4\pi(c/a)e}
$$

$$
\frac{1}{\overline{\mu}} = \frac{1}{\mu_s}\left[1 + \frac{16}{3}\frac{(1-\upsilon_s)}{(2-\upsilon_s)}e\right] \quad (3)
$$

$$
\frac{1}{\overline{E}} = \frac{1}{E_s}\left[1 + \frac{16}{3}\left(1 - \frac{K_f}{K_s}\right)\left(1-\upsilon_s^2\right)D_c e\right] \quad (4)
$$

where: $D_c^{-1} = 1 - \frac{K_f}{K_s} + \frac{K_f}{K_s} \cdot \frac{\beta_3}{c/a} = 1 - \frac{K_f}{K_s} \cdot \frac{16\left(1-\upsilon_s^2\right)}{9(1-2\upsilon_s)} \cdot \frac{e}{\phi_c}$

$$
\beta_3 = \frac{4}{\pi}\left(1-\upsilon_s^2\right)
$$

$$\overline{\varepsilon}_{ij} = \overline{M}_{ijpq}\overline{\sigma}_{pq} \tag{5}$$

$$\begin{array}{lll} (11) \leftrightarrow 1, & (22) \leftrightarrow 2, & (33) \leftrightarrow 3, \\ (23) = (32) \leftrightarrow 4, & (31) = (13) \leftrightarrow 5, & (12) = (21) \leftrightarrow 6 \end{array} \tag{6}$$

$$\overline{\varepsilon}_i = \overline{M}_{ij}\overline{\sigma}_j \tag{7}$$

$$\overline{\sigma}_j = \overline{M}_{ij}^{-1}\overline{\varepsilon}_i \tag{8}$$

$$\overline{\sigma}_j = \overline{c}_{ij}\overline{\varepsilon}_i \tag{9}$$

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ & c_{11} & c_{13} & 0 & 0 & 0 \\ & & c_{33} & 0 & 0 & 0 \\ & & & c_{44} & 0 & 0 \\ & & & & c_{44} & 0 \\ & & & & & c_{66} \end{pmatrix} \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_{23} \\ \varepsilon_{31} \\ \varepsilon_{12} \end{pmatrix} \tag{10}$$

$$\overline{M}'_{mnks} = M_{ijpq}\, l_{mi}\, l_{nj}\, l_{kp}\, l_{sq} \tag{11}$$

$$\begin{aligned} \frac{\partial \sigma_1}{\partial x} + \frac{\partial \sigma_{12}}{\partial y} + \frac{\partial \sigma_{13}}{\partial z} + X &= \rho \frac{\partial^2 U_x}{\partial t^2} \\ \frac{\partial \sigma_{12}}{\partial x} + \frac{\partial \sigma_2}{\partial y} + \frac{\partial \sigma_{23}}{\partial z} + Y &= \rho \frac{\partial^2 U_y}{\partial t^2} \\ \frac{\partial \sigma_{13}}{\partial x} + \frac{\partial \sigma_{23}}{\partial y} + \frac{\partial \sigma_3}{\partial z} + Z &= \rho \frac{\partial^2 U_z}{\partial t^2} \end{aligned} \tag{12}$$

Lamb's problem for the anisotropic inhomogeneous half-space with the horizontal axis of symmetry.

$$\rho\frac{\partial^2 U}{\partial t^2}=a_{33}\frac{\partial^2 U}{\partial x^2}+a_{13}\frac{\partial^2 V}{\partial x\partial y}+a_{13}\frac{\partial^2 W}{\partial z\partial x}+a_{44}\left(\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 V}{\partial x\partial y}\right)+\frac{\partial}{\partial z}\left(a_{44}\left(\frac{\partial W}{\partial x}+\frac{\partial U}{\partial z}\right)\right)+F_x(x,y,z,t) \quad (1)$$

$$\rho\frac{\partial^2 V}{\partial t^2}=a_{44}\left(\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 U}{\partial x\partial y}\right)+a_{13}\frac{\partial^2 U}{\partial x\partial y}+a_{11}\frac{\partial^2 V}{\partial y^2}+a_{12}\frac{\partial^2 W}{\partial z\partial y}+\frac{\partial}{\partial z}\left(a_{66}\left(\frac{\partial V}{\partial z}+\frac{\partial W}{\partial y}\right)\right)+F_y(x,y,z,t) \quad (2)$$

$$\rho\frac{\partial^2 W}{\partial t^2}=a_{44}\left(\frac{\partial^2 W}{\partial x^2}+\frac{\partial^2 U}{\partial x\partial z}\right)+a_{66}\left(\frac{\partial^2 V}{\partial y\partial z}+\frac{\partial^2 W}{\partial y^2}\right)+\frac{\partial}{\partial z}\left(a_{13}\frac{\partial U}{\partial x}+a_{12}\frac{\partial V}{\partial y}+a_{11}\frac{\partial W}{\partial z}\right)+F_z(x,y,z,t) \quad (3)$$

$$\sigma\big|_{z=0}=a_{13}\frac{\partial U}{\partial x}+a_{12}\frac{\partial V}{\partial y}+a_{11}\frac{\partial W}{\partial z}=0 \quad (4)$$

$$\tau_{xz}\big|_{z=0}=a_{44}\left(\frac{\partial W}{\partial x}+\frac{\partial U}{\partial z}\right)=0 \quad (5) \qquad \tau_{yz}\big|_{z=0}=a_{66}\left(\frac{\partial V}{\partial z}+\frac{\partial W}{\partial y}\right)=0 \quad (6)$$

$$\frac{\partial U}{\partial t}\bigg|_{t=0}=U\big|_{t=0}=0, \qquad \frac{\partial V}{\partial t}\bigg|_{t=0}=V\big|_{t=0}=0, \qquad \frac{\partial W}{\partial t}\bigg|_{t=0}=W\big|_{t=0}=0 \quad (7)$$

$$S(z,n,m,t)=\int_0^a\int_0^b U(z,x,y,t)\cos\left(\frac{n\pi}{b}y\right)\sin\left(\frac{m\pi}{a}x\right)dydx \tag{8}$$

$$U(z,x,y,t)=\frac{2}{a}\sum_{n=1}^{\infty}\left(\frac{1}{b}S(z,n,0,t)+\frac{2}{b}\sum_{m=1}^{\infty}S(z,n,m,t)\cos\left(\frac{n\pi}{b}y\right)\sin\left(\frac{m\pi}{a}x\right)\right) \tag{9}$$

$$H(z,n,m,t)=\int_0^a\int_0^b V(z,x,y,t)\sin\left(\frac{n\pi}{b}y\right)\cos\left(\frac{m\pi}{a}x\right)dydx \tag{10}$$

$$V(z,x,y,t)=\frac{1}{a}\sum_{m=1}^{\infty}H(z,0,m,t)+\frac{2}{a}\sum_{n=1}^{\infty}\left(\sum_{m=1}^{\infty}H(z,n,m,t)\sin\left(\frac{n\pi}{b}y\right)\right)\cos\left(\frac{m\pi}{a}x\right) \tag{11}$$

$$R(z,n,m,t)=\int_0^a\int_0^b W(z,x,y,t)\cos\left(\frac{n\pi}{b}y\right)\cos\left(\frac{m\pi}{a}x\right)dydx \tag{12}$$

$$W(z,x,y,t)=\frac{1}{ab}R(z,0,0,t)+\frac{2}{ab}\sum_{m=1}^{\infty}R(z,0,m,t)\cos\left(\frac{m\pi}{b}y\right)+$$

$$\frac{2}{a}\sum_{n=1}^{\infty}\left(\frac{1}{b}R(z,n,0,t)+\frac{2}{b}\sum_{m=1}^{\infty}R(z,n,m,t)\cos\left(\frac{n\pi}{b}y\right)\right)\cos\left(\frac{m\pi}{a}x\right) \tag{13}$$

$$U\Big|_{\substack{x=0\\x=a}}=\frac{\partial V}{\partial x}\Big|_{\substack{x=0\\x=a}}=\frac{\partial W}{\partial x}\Big|_{\substack{x=0\\x=a}}=0 \quad (14) \qquad \frac{\partial U}{\partial y}\Big|_{\substack{y=0\\y=b}}=V\Big|_{\substack{y=0\\y=b}}=\frac{\partial W}{\partial y}\Big|_{\substack{y=0\\y=b}}=0 \quad (15)$$

$$\rho\frac{\partial^2 S}{\partial t^2}=\frac{\partial}{\partial z}\left(a_{44}\left(\frac{\partial S}{\partial z}-k_x R\right)\right)-a_{13}k_x\frac{\partial R}{\partial z}-\left(a_{33}k_x^2+a_{44}k_y^2\right)S-(a_{13}+a_{44})k_x k_y H+f_1 \quad (16)$$

$$\rho\frac{\partial^2 H}{\partial t^2}=\frac{\partial}{\partial z}\left(a_{66}\left(\frac{\partial H}{\partial z}-k_y R\right)\right)-a_{12}k_y\frac{\partial R}{\partial z}-\left(a_{44}k_x^2+a_{11}k_y^2\right)H-(a_{44}+a_{13})k_x k_y S+f_2 \quad (17)$$

$$\rho\frac{\partial^2 R}{\partial t^2}=\frac{\partial}{\partial z}\left(a_{11}\frac{\partial R}{\partial z}-a_{13}k_x S+a_{12}k_y H\right)+a_{66}k_y\frac{\partial H}{\partial z}+a_{44}k_x\frac{\partial S}{\partial z}-\left(a_{44}k_x^2+a_{66}k_y^2\right)R+f_3 \quad (18)$$

$$a_{44}\left(\frac{\partial S}{\partial z}-k_x R\right)\Big|_{z=0}=0 \quad (19)$$

$$a_{66}\left(\frac{\partial H}{\partial z}-k_y R\right)\Big|_{z=0}=0 \quad (20)$$

$$\left(a_{66}\frac{\partial R}{\partial z}+a_{13}k_x S+a_{12}k_y H\right)\Big|_{z=0}=0 \quad (21)$$

$$U|_{t=0}=\frac{\partial S}{\partial t}\Big|_{t=0}=0,\qquad H|_{t=0}=\frac{\partial H}{\partial t}\Big|_{t=0}=0,\qquad R|_{t=0}=\frac{\partial R}{\partial t}\Big|_{t=0}=0 \quad (22)$$

$$k_x=\frac{m\pi}{a},\qquad k_y=\frac{m\pi}{b},\qquad (m,n=1,2,3,\dots)$$

$$\hat{\rho}_i \frac{S_i^{j+1}-2S_i^j+S_i^{j-1}}{\tau^2}=\left(a_{i+1}^{44}\frac{\left(S_{i+1}^j-S_i^j\right)}{h_i\hbar_i}-a_i^{44}\frac{\left(S_i^j-S_{i-1}^j\right)}{h_{i-1}\hbar_i}\right)-\frac{k_x}{2\hbar_i}\left(a_{i+1}^{44}\left(R_{i+1}^j+R_i^j\right)-a_i^{44}\left(R_i^j+R_{i-1}^j\right)\right)-$$

$$\frac{k_x}{2}\left(a_{i+1}^{13}\frac{\left(R_{i+1}^j-R_i^j\right)}{\hbar_i}+a_i^{13}\frac{\left(R_i^j-R_{i-1}^j\right)}{\hbar_i}\right)-\left(\hat{a}_i^{33}k_x^2+\hat{a}_i^{44}k_y^2\right)S_i^j-k_xk_y\left(\hat{a}_i^{13}+\hat{a}_i^{44}\right)H_i^j+f_{1,i}^j \qquad (2$$

$$\hat{\rho}_i \frac{H_i^{j+1}-2H_i^j+H_i^{j-1}}{\tau^2}=\left(a_{i+1}^{66}\frac{\left(H_{i+1}^j-H_i^j\right)}{h_i\hbar_i}-a_i^{66}\frac{\left(H_i^j-H_{i-1}^j\right)}{h_{i-1}\hbar_i}\right)-\frac{k_y}{2\hbar_i}\left(a_{i+1}^{66}\left(R_{i+1}^j+R_i^j\right)-a_i^{66}\left(R_i^j+R_{i-1}^j\right)\right)-$$

$$\frac{k_y}{2}\left(a_{i+1}^{12}\frac{\left(R_{i+1}^j-R_i^j\right)}{\hbar_i}+a_i^{12}\frac{\left(R_i^j-R_{i-1}^j\right)}{\hbar_i}\right)-\left(\hat{a}_i^{44}k_x^2+\hat{a}_i^{11}k_y^2\right)H_i^j-k_xk_y\left(\hat{a}_i^{13}+\hat{a}_i^{44}\right)S_i^j+f_{2,i}^j \qquad (2$$

$$\hat{\rho}_i \frac{R_i^{j+1}-2R_i^j+R_i^{j-1}}{\tau^2}=\left(a_{i+1}^{11}\frac{\left(R_{i+1}^j-R_i^j\right)}{h_i\hbar_i}-a_i^{11}\frac{\left(R_i^j-R_{i-1}^j\right)}{h_{i-1}\hbar_i}\right)+\frac{k_x}{2\hbar_i}\left(a_{i+1}^{13}\left(S_{i+1}^j+S_i^j\right)-a_i^{13}\left(S_i^j+S_{i-1}^j\right)\right)+$$

$$\frac{k_y}{2}\left(a_{i+1}^{66}\frac{\left(H_{i+1}^j-H_i^j\right)}{\hbar_i}+a_i^{66}\frac{\left(H_i^j-H_{i-1}^j\right)}{\hbar_i}\right)+\frac{k_y}{2\hbar_i}\left(a_{i+1}^{12}\left(H_{i+1}^j+H_i^j\right)-a_i^{12}\left(H_i^j+H_{i-1}^j\right)\right)+$$

$$\frac{k_x}{2}\left(a_{i+1}^{44}\frac{\left(S_{i+1}^j-S_i^j\right)}{\hbar_i}+a_i^{44}\frac{\left(S_i^j-S_{i-1}^j\right)}{\hbar_i}\right)-\left(\hat{a}_i^{44}k_x^2+\hat{a}_i^{66}k_y^2\right)R_i^j+f_{3,i}^j \qquad (2$$

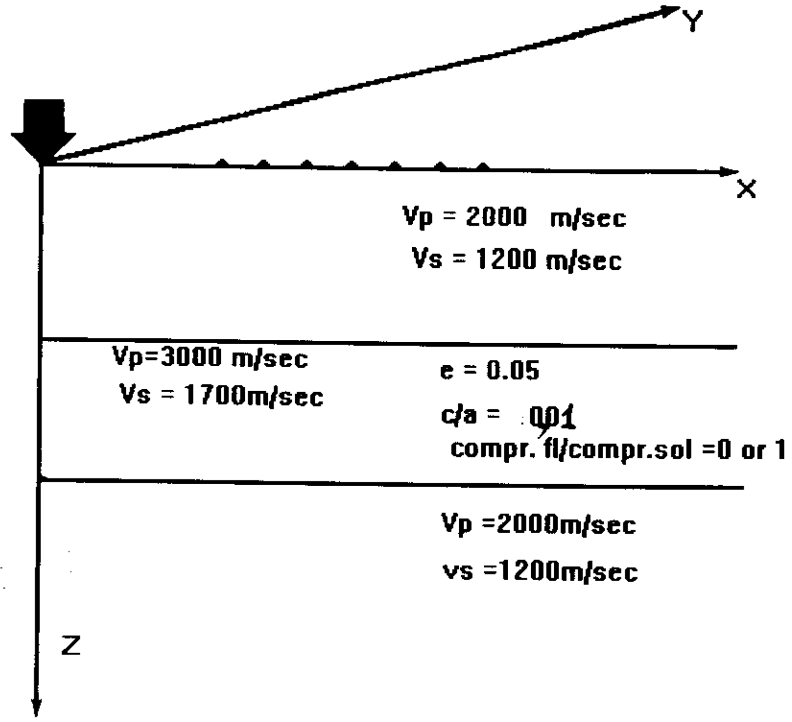
where:

$$\hat{\rho}_i = \frac{\rho_{i+1} h_i + \rho_i h_{i-i}}{h_i + h_{i-1}}; \qquad \hat{a}_i^{11} = \frac{a_{i+1}^{11} h_i + a_i^{11} h_{i-1}}{h_i + h_{i-1}};$$

$$\hat{a}_i^{44} = \frac{a_{i+1}^{44} h_i + a_i^{44} h_{i-1}}{h_i + h_{i-1}}; \qquad \hat{a}_i^{33} = \frac{a_{i+1}^{33} h_i + a_i^{33} h_{i-1}}{h_i + h_{i-1}};$$

$$\hat{a}_i^{13} = \frac{a_{i+1}^{13} h_i + a_i^{13} h_{i-1}}{h_i + h_{i-1}}; \qquad \hat{a}_i^{66} = \frac{a_{i+1}^{66} h_i + a_i^{66} h_{i-1}}{h_i + h_{i-1}};$$

$$\hbar_i = \frac{h_i + h_{i-1}}{2}$$



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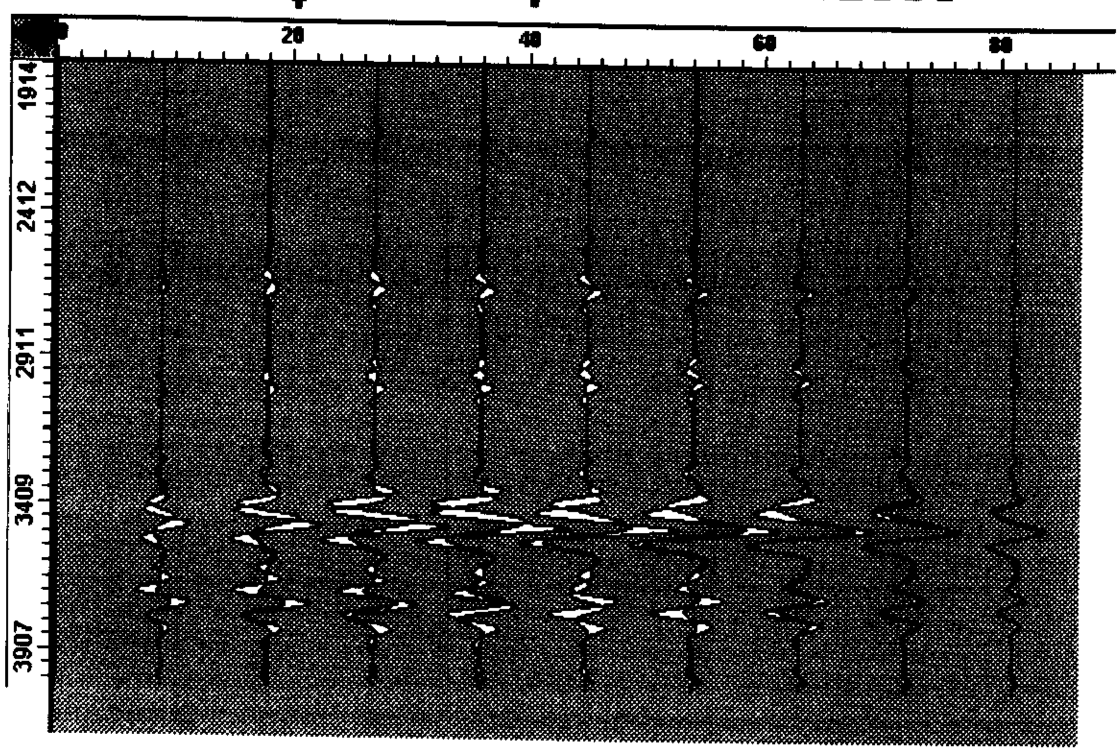

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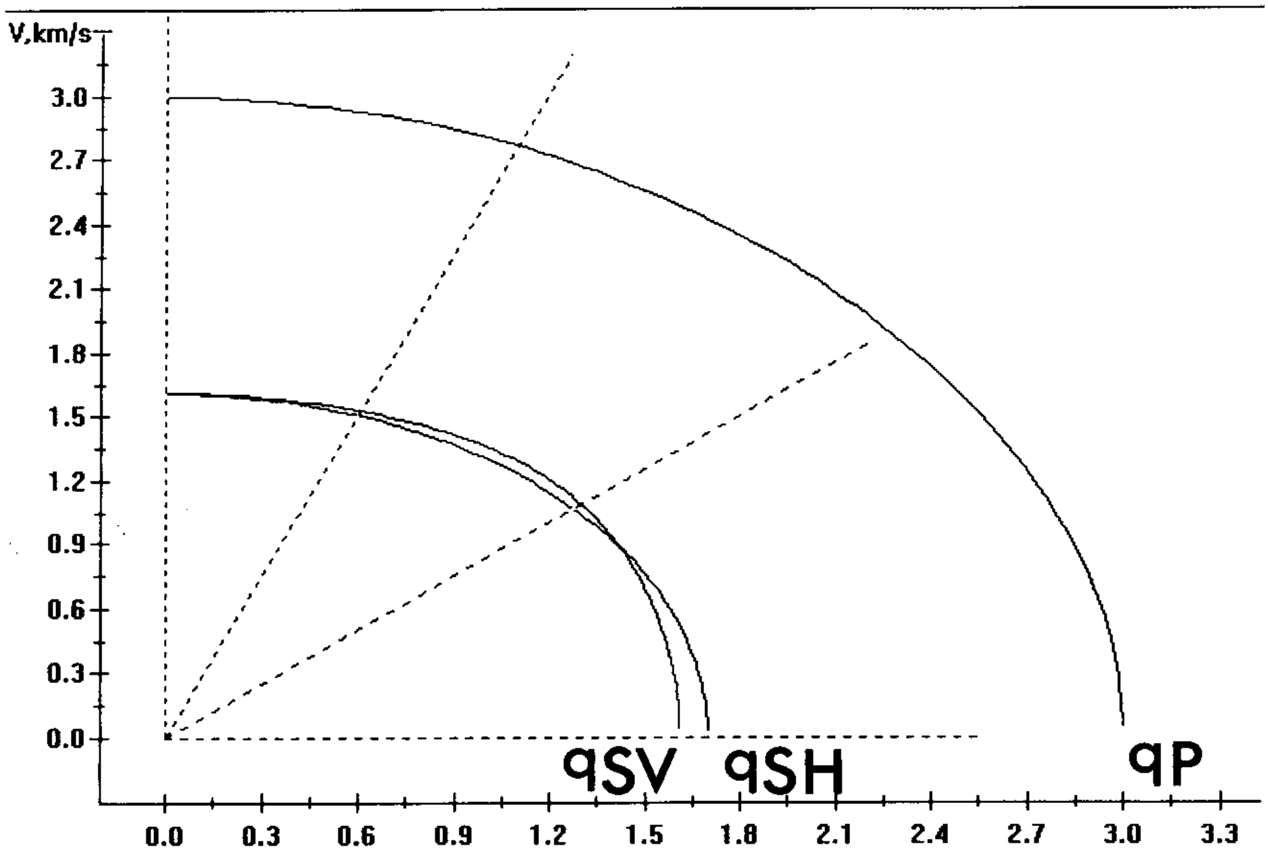

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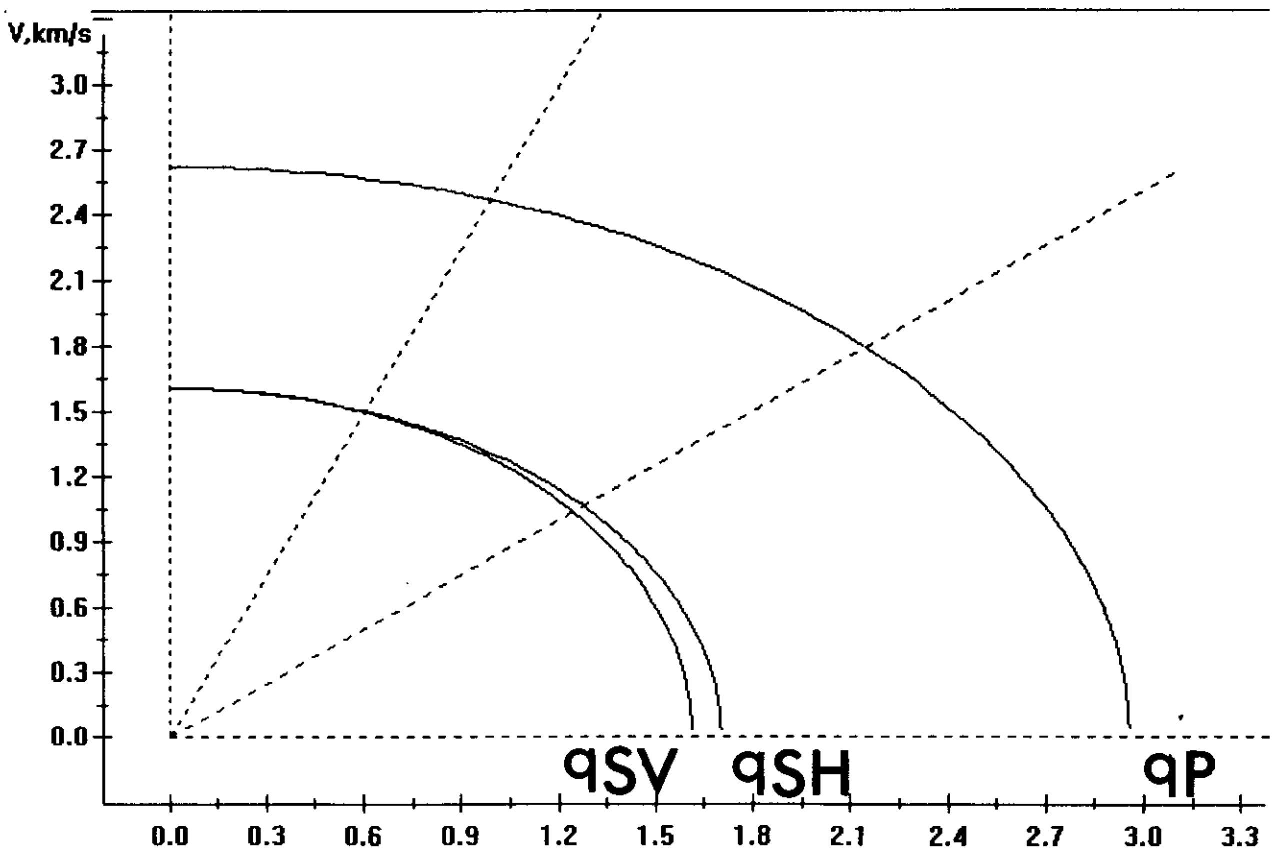

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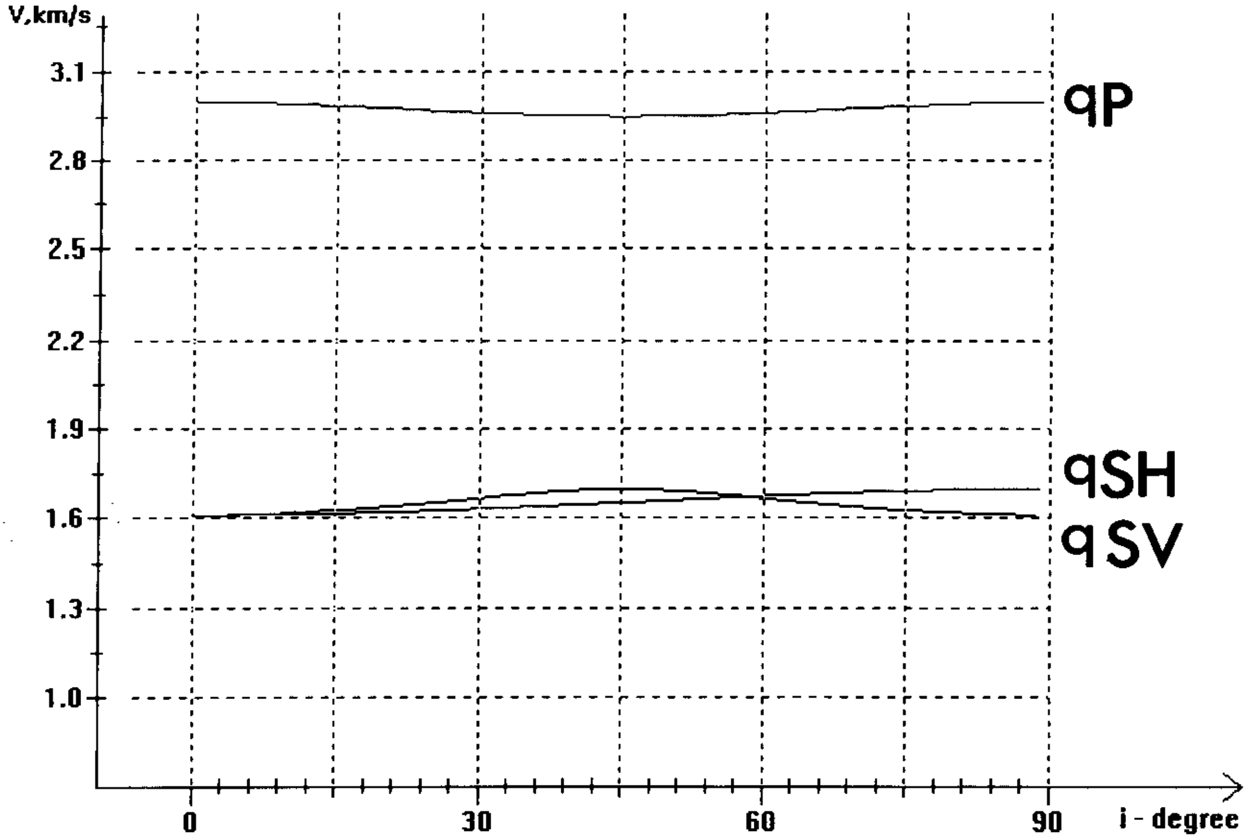
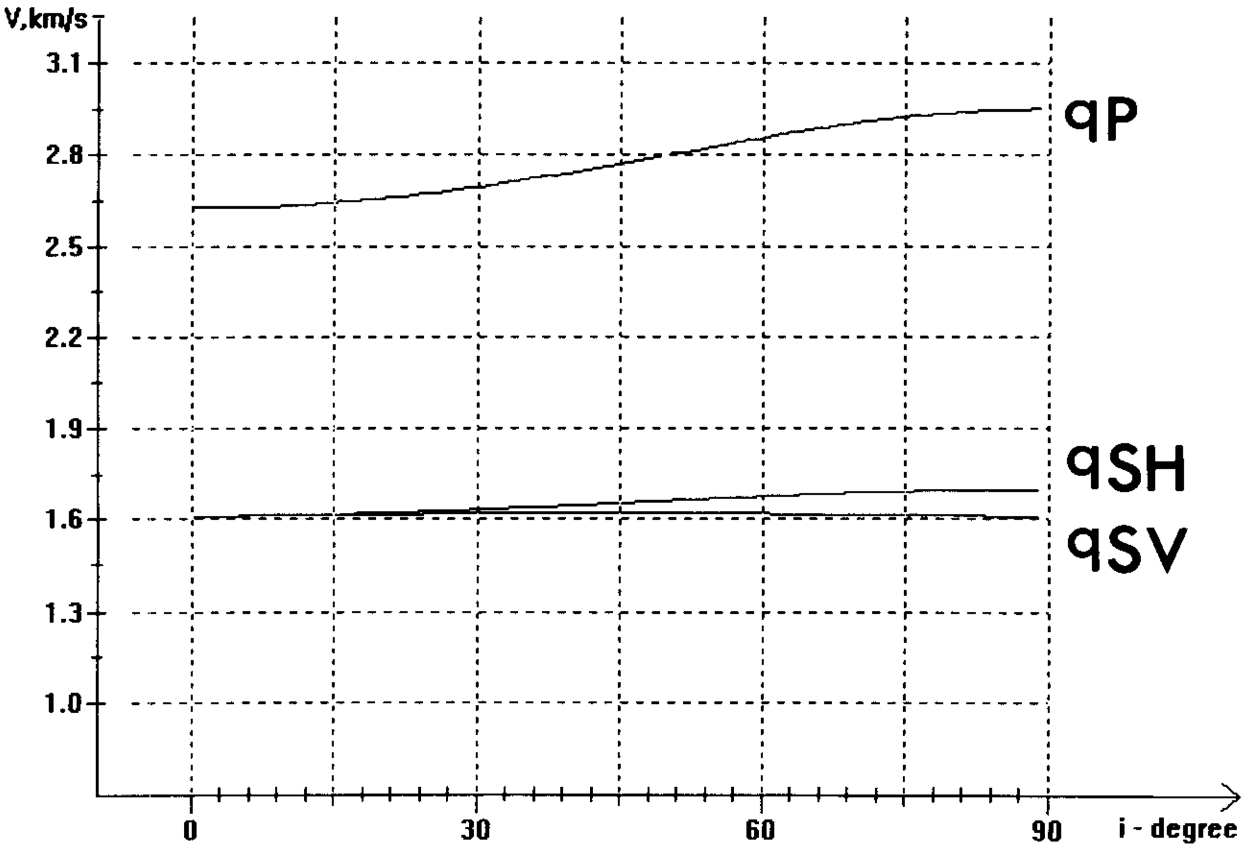
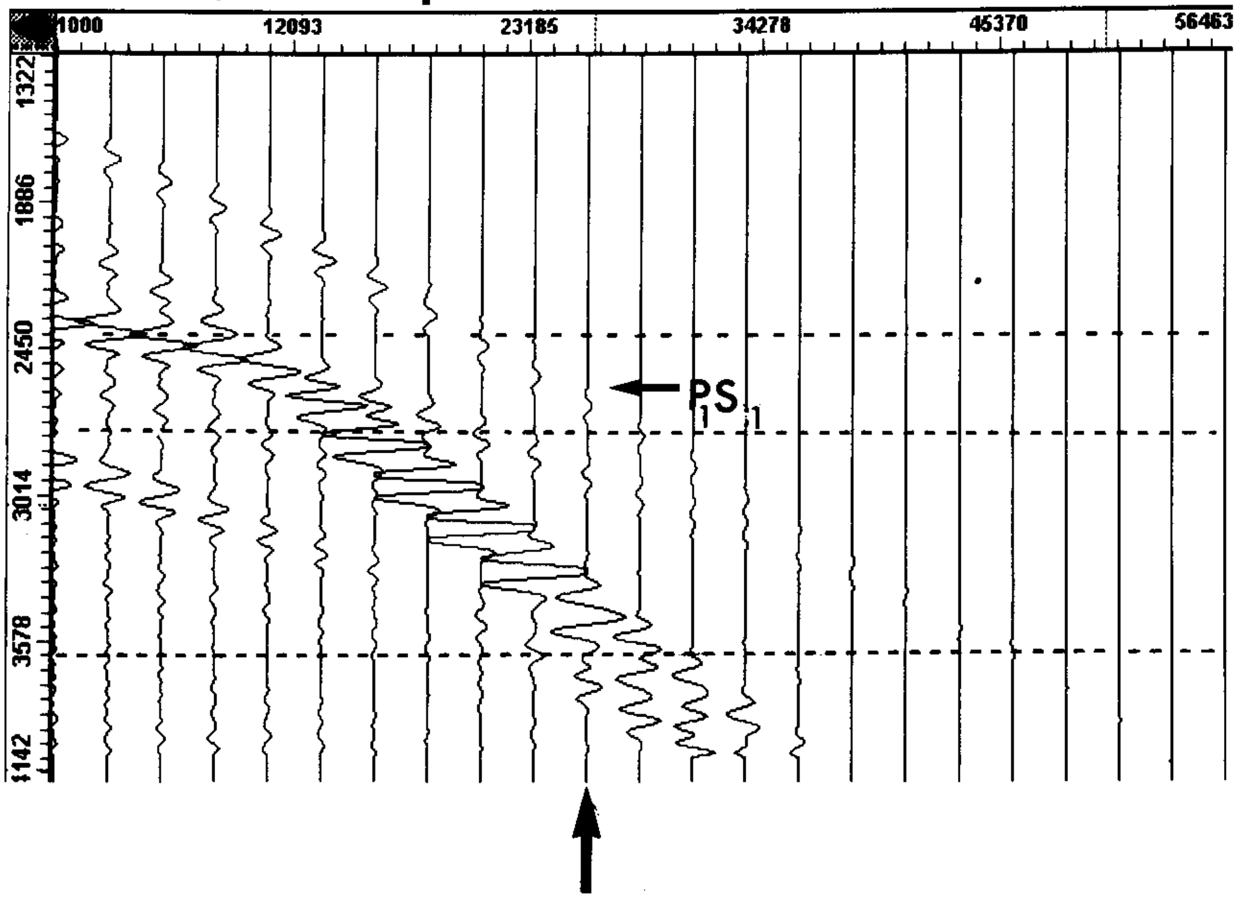


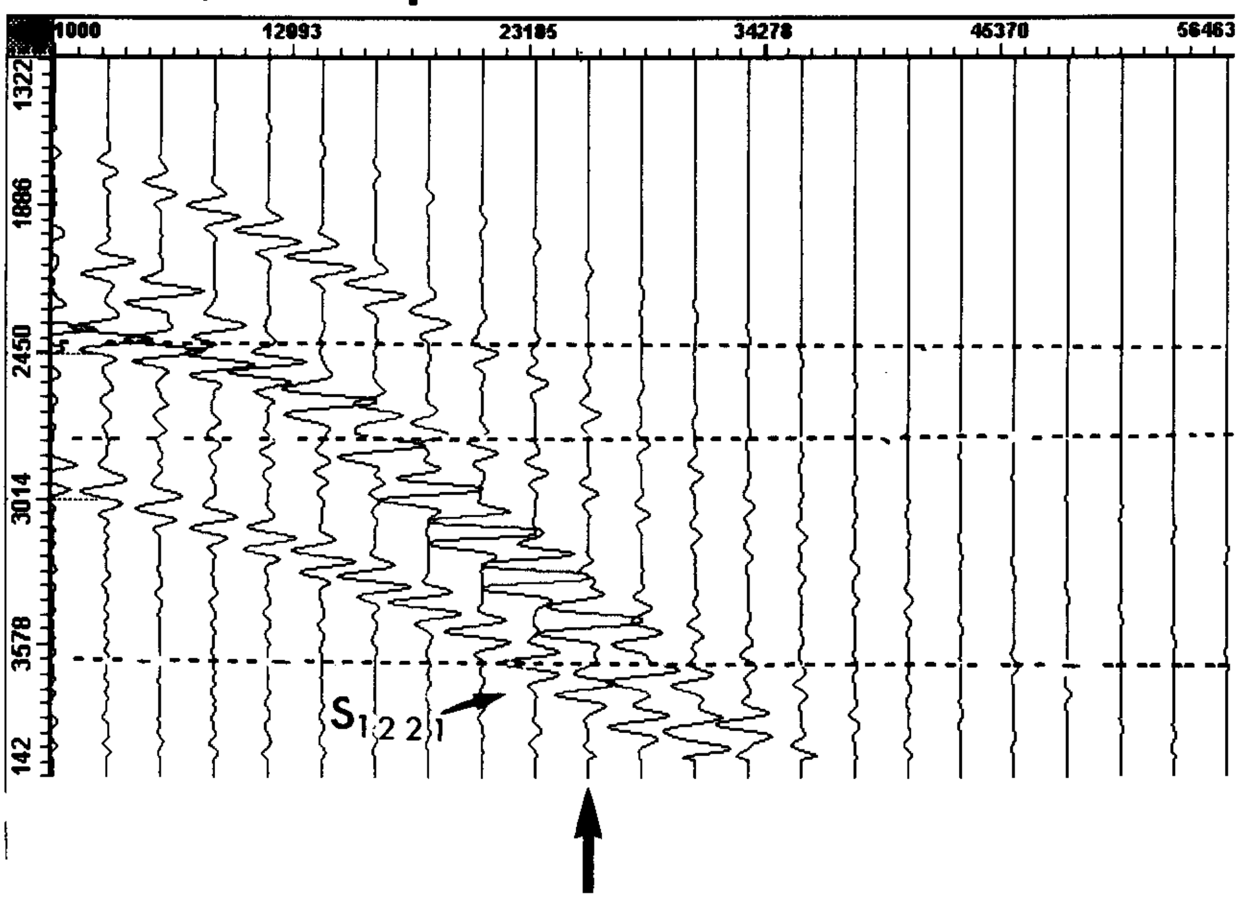
Ray indic. compr. fluid = 0



Uφ compr. fl =1 φ =45°



Uφ compr. fl = 0 φ =45°



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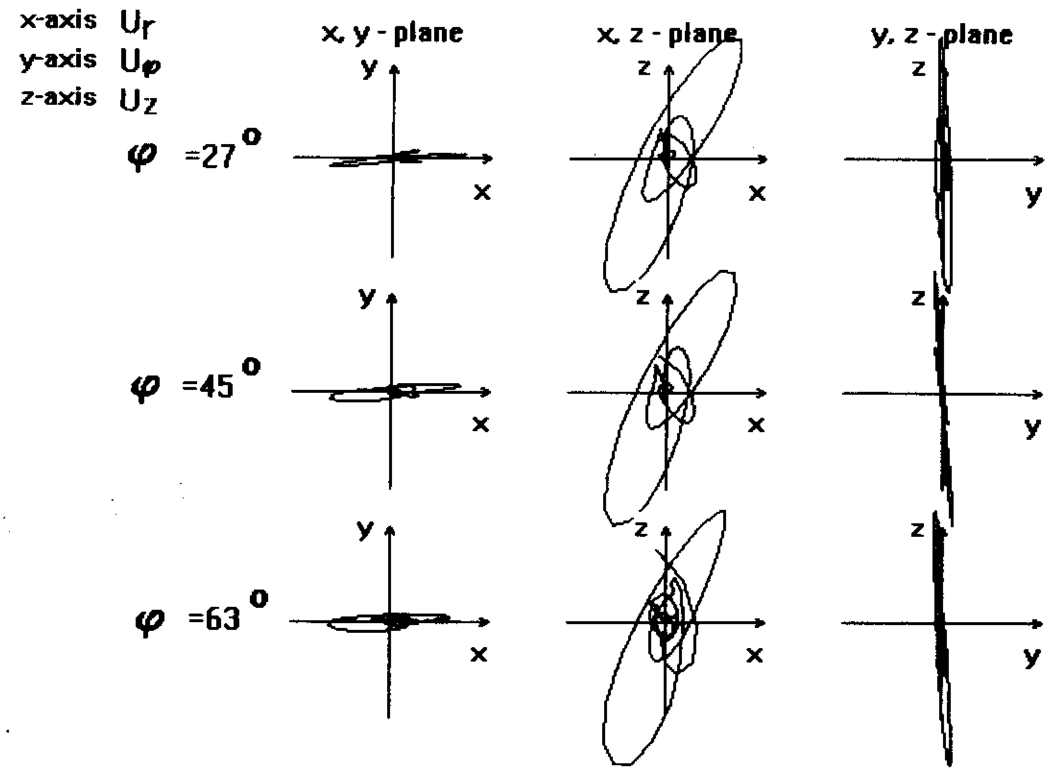

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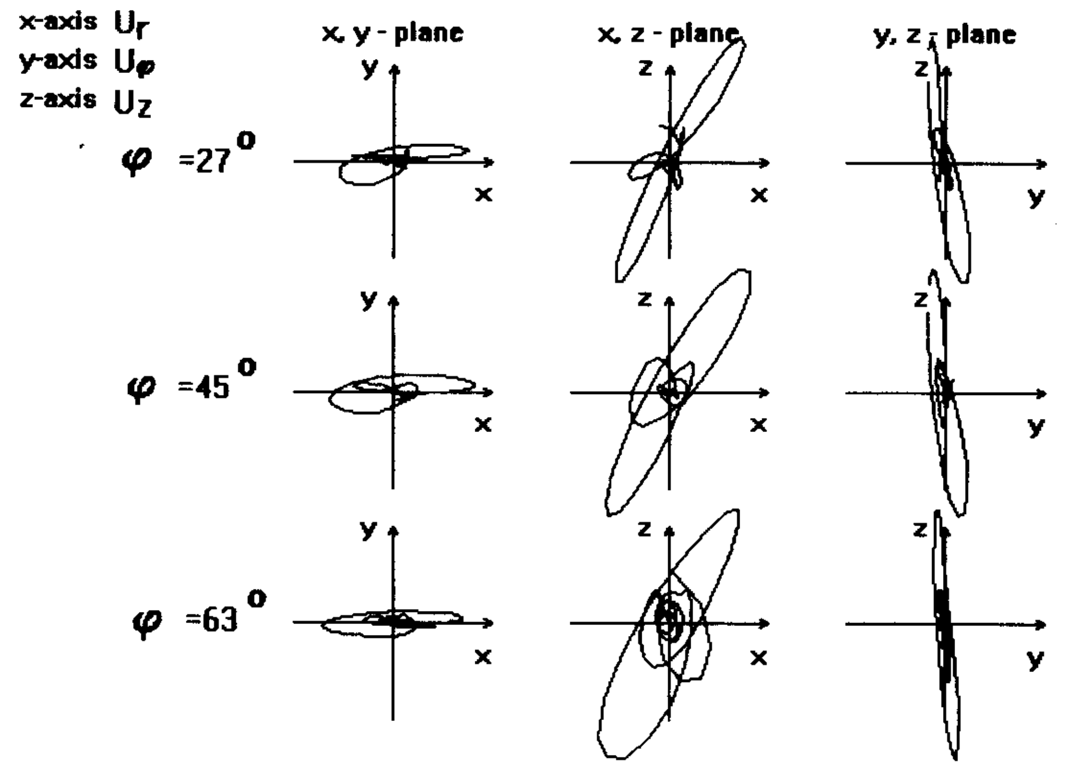

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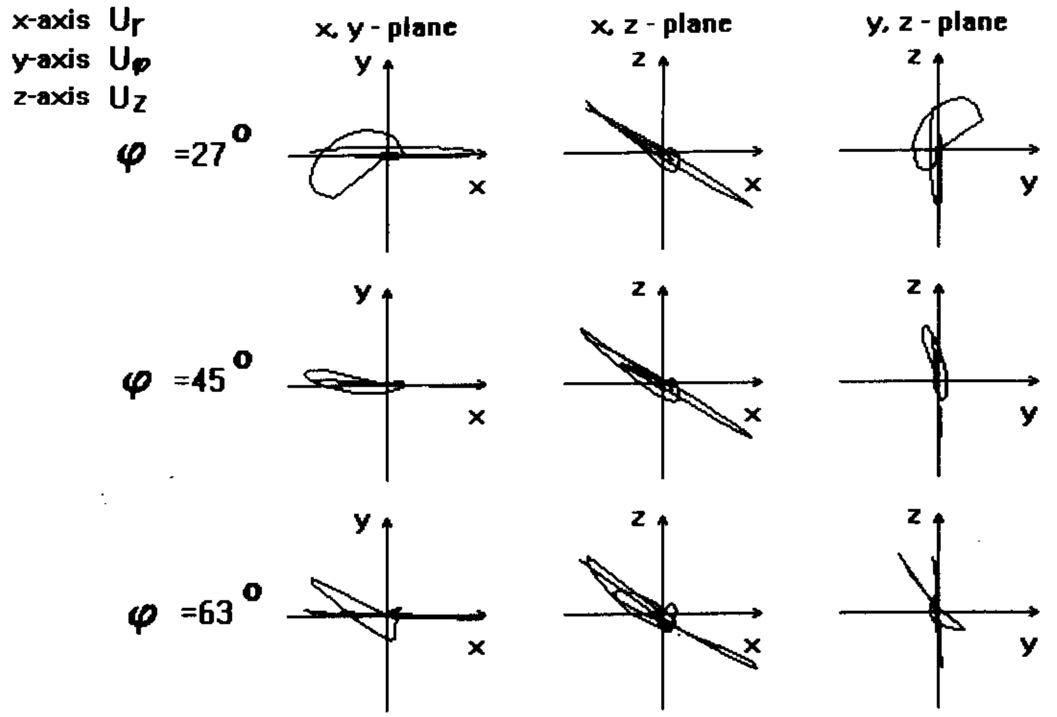

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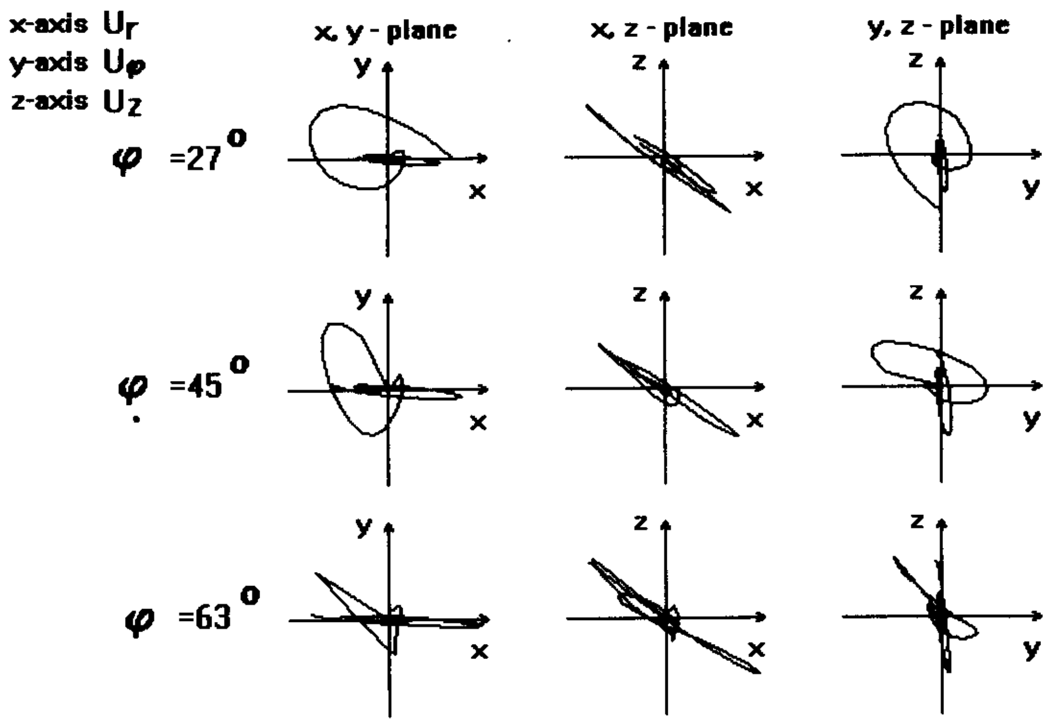
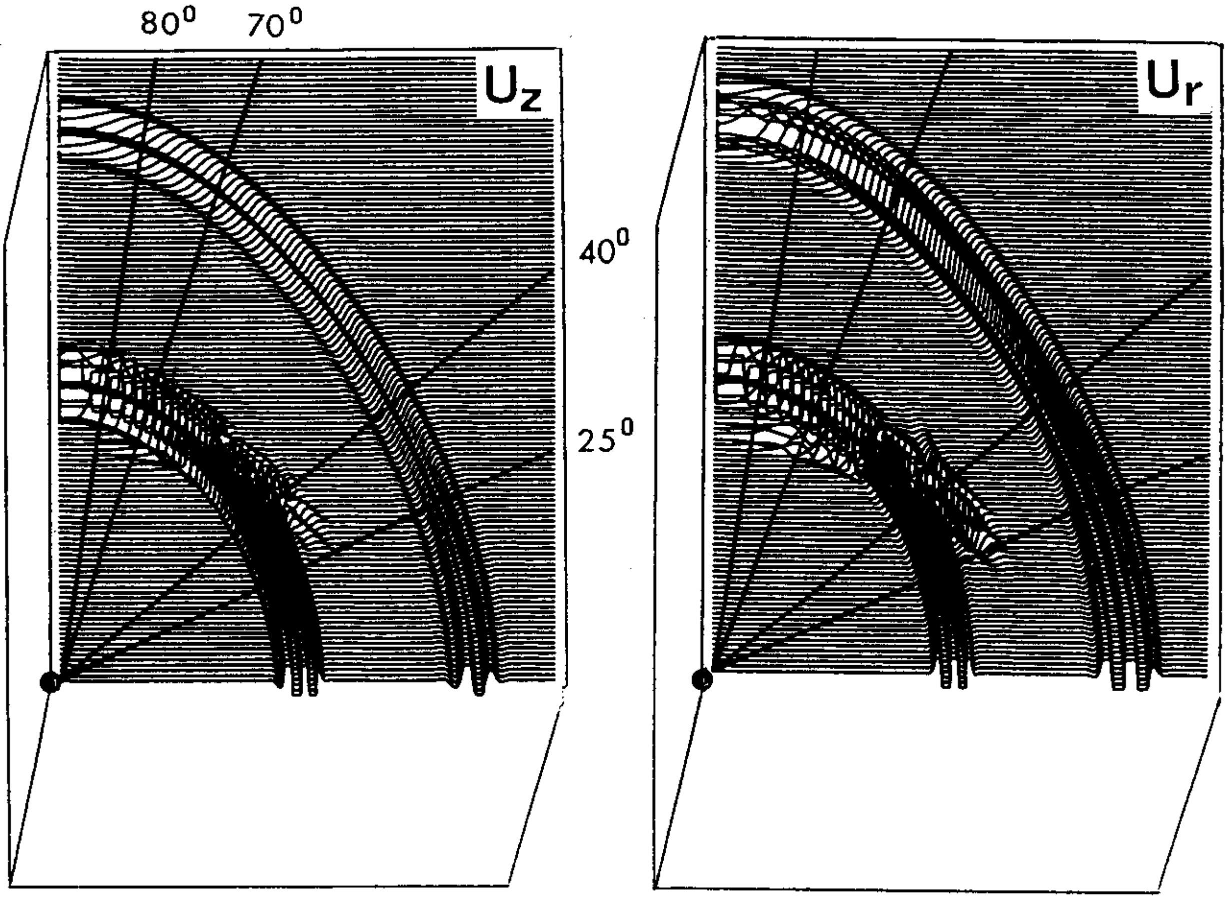
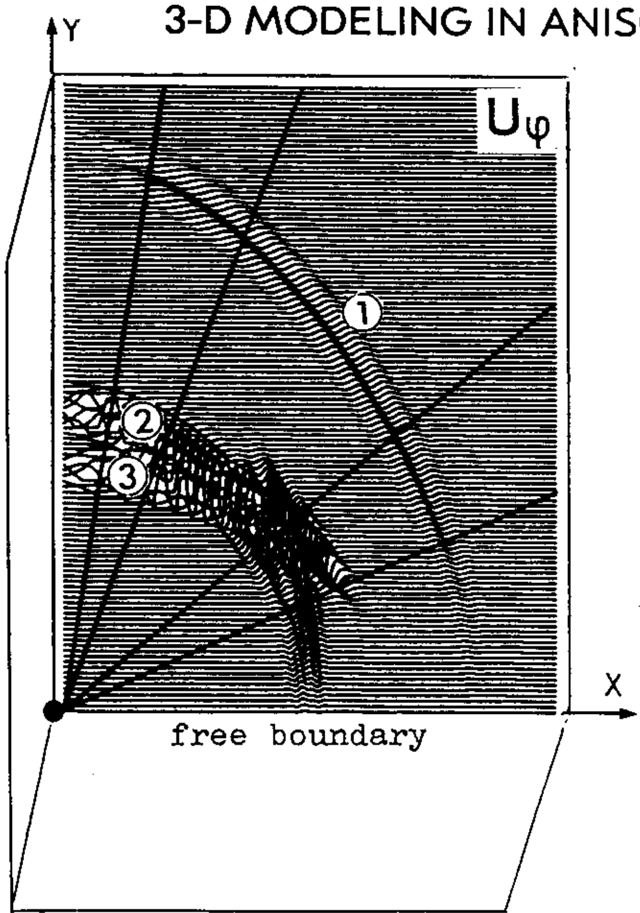


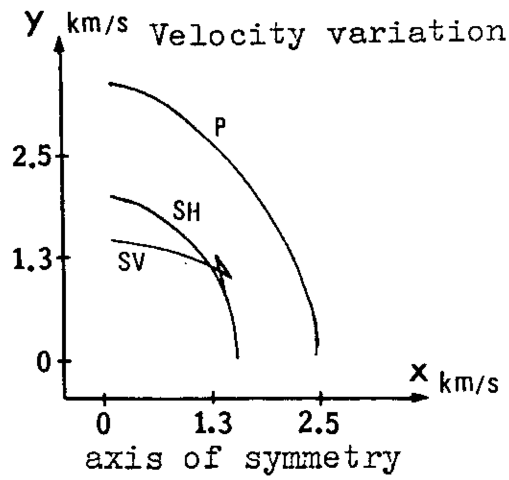


3-D MODELING IN ANISOTROPIC MEDIUM



Model of the medium

$c_{11} = 11.7 \cdot 10^{10}$ dyne/cm^2
$c_{33} = 6.25 \cdot 10^{10}$ dyne/cm^2
$c_{44} = 2.28 \cdot 10^{10}$ dyne/cm^2
$c_{66} = 4.06 \cdot 10^{10}$ dyne/cm^2
$c_{13} = 2.0 \cdot 10^{10}$ dyne/cm^2



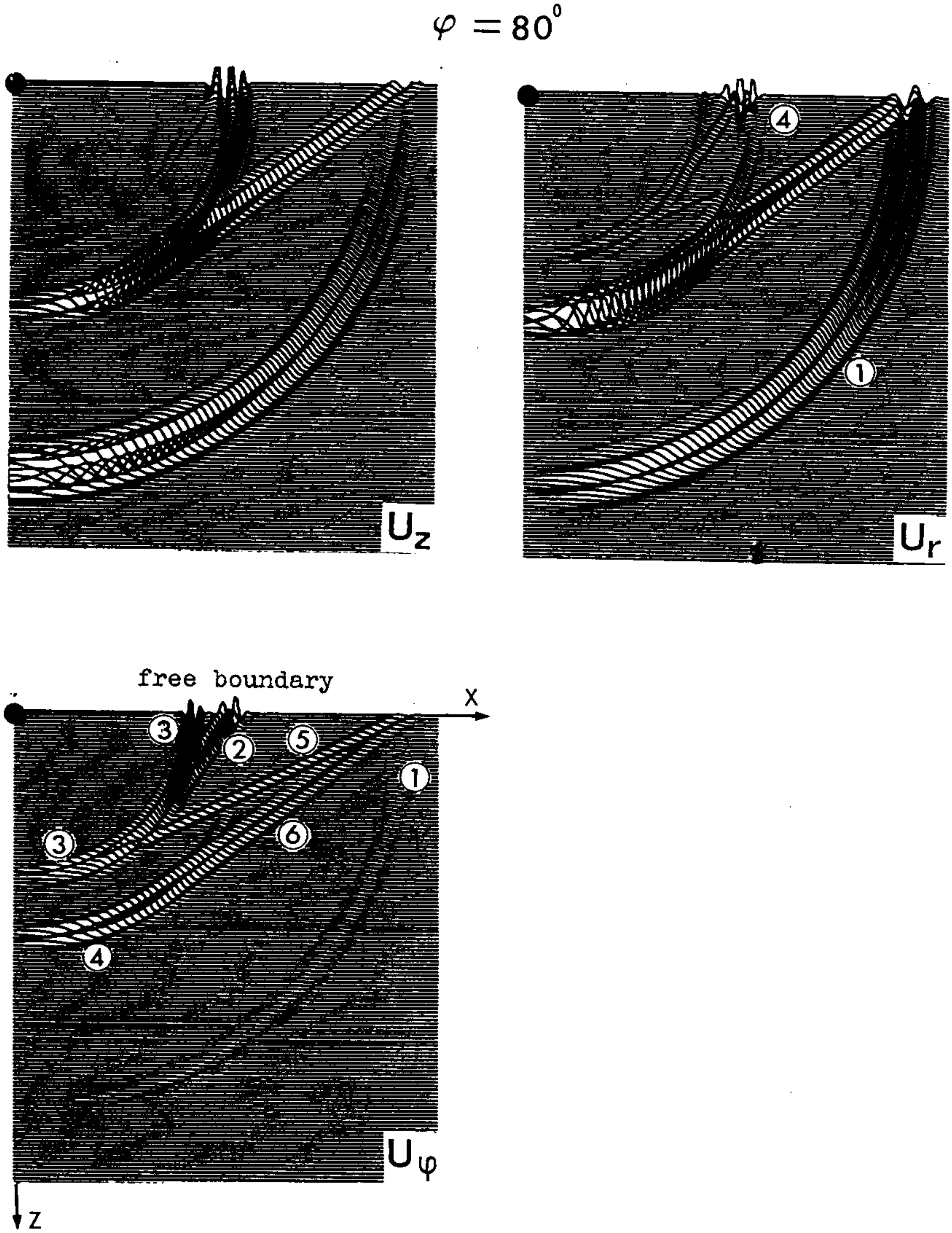


Fig. Snapshots of the vertical plane (φ = 80°) of three displacement components at the fixed moment of time for the anisotropic model of the medium.