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Excitation and Propagation of Tsunami as Surface Waves

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Introduction

Two alternative hypotheses on tsunami generation exist : one explains tsunami as a result of a sudden displacement of the sea bottom and a consequent change of a volume of the water basin; according to another one tsunami is regarded as an analogue of surface wave caused by gravitational force in the water layer. The latter hypothesis does not require finite displacements of the sea bottom, and explains tsunami from earthquakes with hypocenters below the bottom. Indeed, strong tsunami are caused by earthquakes with source depth of about 40 km and larger.

In most theoretical studies on tsunami the source was assumed as a dislocation over a fault area (Pod'yapolsky, Gusyakov). Though such approach allows tsunami wave fields to be calculated for any orientation and size of the fault, this approach is inconvenient for drawing conclusions on relation between parameters of earthquake and tsunami. In surface wave studies the approach based on representation of a source as spacially concentrated forces is widely used , and it seems to be helpful in tsunami studies as well.

Here we apply the approach developed in surface wave theory to outline a theory of tsunami waves caused by point sources, and to determine a relation between tsunami fields and parameters of earthquakes.

Notations

c_f - velocity of acoustic wave in the liquid layer
ρ_f - density of the liquid
H - thickness of the liquid layer
λ,μ - Lame coefficients in elastic half-space
ρ - density of the elastic medium
a,b - velocities of P and S waves respectively
$\mathbf{u}_i=(u_i,v_i,w_i)$ - displacement vector (i=1 - liquid layer, i=2 - elastic half-space)
g - gravitational acceleration
ω - angular frequency
τ_{xz},τ_{zz} - stress components
z=0 - undisturbed surface of the liquid layer.

Formulation of the problem

Equations of motion in the liquid layer and the elastic half-space are following:

$$c_f^2\nabla div\mathbf{u}_1 - g\mathbf{e}_z div\mathbf{u}_1 = \frac{\partial^2\mathbf{u}_1}{\partial t^2} \qquad (1)$$

$$(\lambda+2\mu)\nabla div\mathbf{u}_2 - \mu rot rot\mathbf{u}_2 = \rho\frac{\partial^2\mathbf{u}_2}{\partial t^2} \qquad (2)$$

These equations should be solved under the following boundary conditions:
- pressure at the free surface of liquid layer $z=w_1$ vanishes;
- normal components of displacement and traction at z=H are continuos;
- tangential component of traction in half-space at z=H vanishes.

The boundary conditions are written as follows:

$$c_f^2 \mathrm{div}\mathbf{u}_1 - g w_1 = 0 \qquad \text{at } z=0 \qquad (3a)$$

$$w_1 = w_2 \qquad \text{at } z=H \qquad (3b)$$

$$\tau_{zz}^{(2)}(H) = \rho_f [c_f^2 div\mathbf{u}_1 - g w_1(0) - g\int_0^H div\mathbf{u}_1 dz] \qquad (3c)$$

$$\tau_{xz}^{(2)}(H) = 0 \qquad (3d)$$

$$\tau_{yz}^{(2)}(H) = 0 \qquad (3e)$$

where $\tau_{xz}^{(2)}(H), \tau_{xy}^{(2)}(H), \tau_{zz}^{(2)}(H)$ are expressed in terms of parameters of the elastic medium:

$$\tau_{xz}^{(2)} = \mu\left[\partial u_2 / \partial z + \partial w_2 / \partial x\right]$$

$$\tau_{yz}^{(2)} = \mu\left[\partial v_2 / \partial z + \partial w_2 / \partial y\right]$$

$$\tau_{zz}^{(2)} = \lambda div\mathbf{u}_2 + 2\mu\frac{\partial w_2}{\partial z}$$

At $z \to \infty$ displacement $\mathbf{u}_2$ tends to zero.

Solution of equations of motion for plane wave

Solution of equations (1),(2) in a form of plane stationary wave propagated along x-axis may be written as follows:

$$\mathbf{u}_i(x,z,\omega,t) = \mathbf{U}_i(z,\omega)\exp[i\omega(t-x/c)] \qquad (4)$$

where $\mathbf{u}_i=(u_i,0,w_i)$

Substituting (4) into (1),(2), and taking into account the condition at $z \to \infty$ we obtain the following expressions for amplitudes as functions of depth:

$$U_1(z,\omega) = \frac{ic_f^2}{\omega c}[-B\exp(-\eta_2 z / c_f) + C\exp(\eta_1 z / c_f)] \qquad (5a)$$

$$W_1(z,\omega) = \frac{c_f}{\omega^2}[\eta_1 B\exp(-\eta_2 z / c_f) - \eta_2 C\exp(-\eta_1 z / c_f)] \qquad (5b)$$

where

$$\eta_1 = -\omega\gamma - g / 2c_f$$

$$\eta_2 = \omega\gamma - g / 2c_f$$

$$\gamma^2 = \frac{c_f^2}{c^2} - 1 + \frac{g^2}{4c_f^2\omega^2}$$

$$U_2(z,\omega) = \frac{ia^2}{\omega c}D\exp[-\omega\alpha(z-H)/a] - \frac{i\beta b}{\omega}F\exp[-\omega\beta(z-H)/b] \qquad (5c)$$

$$W_2(z,\omega)=\frac{\alpha a}{\omega}D\exp[-\omega\alpha(z-H)/a]-\frac{b^2}{\omega c}F\exp[-\omega\beta(z-H)/b] \qquad (5d)$$

where

$\alpha^2=a^2/c^2-1,\ \beta^2=b^2/c^2-1.$

The coefficients B,C,D,F are to be determined from boundary conditions, which lead to a linear system of equations with zero right-hand sides. The system has non-zero solution if its determinant is equal to zero. The condition of solvability gives the dispersion equation in the following form:

$$\begin{aligned}&m\left[-\gamma ch(\omega\gamma H/c_f)+Psh(\omega\gamma H/c_f)\right]\{-a^2(1-2b^2/c^2)^2+4b^3a\alpha\beta/c^2\}-\\&c_f a\alpha\left[Qch(\omega\gamma H/c_f)-Rsh(\omega\gamma H/c_f)-Q\exp[-gH/2c_f^2]\right]+\\&\frac{ga\alpha}{\omega(1-c_f^2/c^2)}\left[\gamma ch(\omega\gamma H/c_f)+Psh(\omega\gamma H/c_f)-\gamma\exp[-gH/2c_f^2]\right]=0\end{aligned} \qquad (6)$$

where

$m=\rho/\rho_f,$

$P=g(2c_f^2/c^2-1)/2c_f\omega,$

$Q=g\gamma/c_f\omega$

$$R=1-\frac{g^2}{2c_f^2\omega^2}$$

It is easy to see that for $H\to 0$ this equation transfers to the Rayleigh equation for homogeneous elastic half-space: two last terms vanish, and the equation is satisfied if the second multiplier in the first term is equal to zero. With H continuously increasing from 0 this solution remains though the velocity c becomes to be dependent on frequency. But in this case a new root arises which corresponds to a gravitational wave in the liquid layer. If we assume $a\to\infty$, $b\to\infty$, and $c_f\to\infty$, which corresponds to incompressible liquid and absolutely rigid half-space, the equation is reduced to the following:

$$th(\omega H/c)=\omega c/g$$

For thin layer, i.e. for small ωH, the velocity does not depend on frequency: $c=\sqrt{gH}$.

Tsunami waves due to a point source

Since the solution (5),(6) is analogous to that for surface waves in elastic half-space with the exception that the velocity is determined by another root of the dispersion equation, we may use the inferences in the surface wave theory to determine tsunami wave field from a point source.

Displacemenet in stationary surface waves in laterally homogeneous half-space exited by a point source is determined by the formulaa (Keilis-Borok,ed,1989):

$$\mathbf{u}(r,\varphi,\omega,t)=\frac{\exp(-i\pi/4)}{\sqrt{8\pi}}\,\frac{\exp[i\omega(t-r/c)]}{\sqrt{\omega r/c}}\,\frac{\mathbf{U}(z,\omega)}{\sqrt{cuI_0}}\,\frac{Q(h,\varphi,\omega)}{\sqrt{cuI_0}} \qquad (7)$$

where

r - horizontal distance from the source,

φ - azimuth of a point of observation,

I_0 - integral of kinetic energy: $I_0=\int\rho[\,U(z,\omega)^2+W(z,\omega)^2]dz$,

h - source depth,

$Q(h,\varphi,\omega)$- function of source radiation: $Q(h,\varphi,\omega)=m_{rs}(\omega)\,B_{rs}(h,\varphi,\omega)$,

where $m_{rs}(\omega)$ is spectrum of seismic moment tensor.

The second term in (7) describes wave propagation, the third and fourth factors express the effects of the receiver and source, respectively. In the half-space with weak lateral variations they correspond to the structures in the vicinity of these two points.

Properties of tsunami waves

a) Velocity

Phase velocity of tsunami is determined as a root of the dispersion equation (6). For the model consisting of water layer of 4 km thickness and elastic half-space with parameters corresponding to the upper mantle the dispersion curves of phase and group velocities are shown in fig.1. For periods T>300 s c and u do not practically depend on T and are close to the value corresponding to a thin layer over rigid half-space, i.e. to $\sqrt{gH}$. For smaller periods c decreases rapidly with period decreasing. The interval of periods, within which the phase velocity does not depend on T, corresponds to tsunami waves.

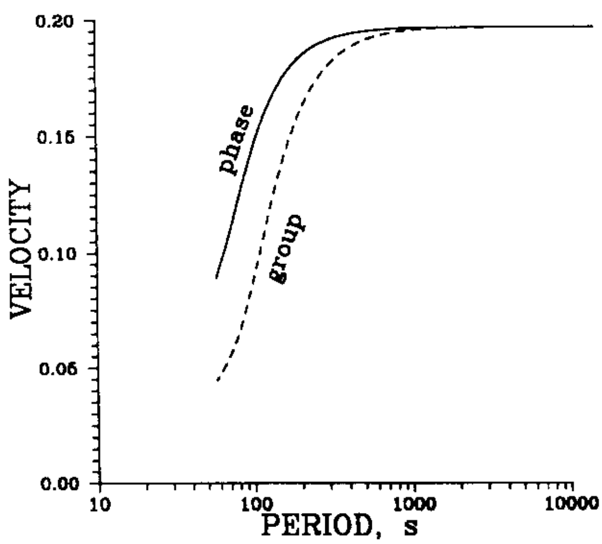

Fig.1

b) Amplitude-frequency responce

Amplitudes of vertical (W) and horizontal (U) displacement of the free surface normalized by $\sqrt{I_0}$ are shown in fig.2 as a function of period. It is clear that at large periods horizontal displacement dominates, and polarization of tsunami waves becomes linear. At smaller periods polarization is elliptic, tending to be circular at the periods smaller than those of tsunami waves. The similar amplitude-frequency responce at depth 30 km from the sea bottom is shown in fig.3. The vertical amplitudes corresponding to tsunami period range at h=30km are much smaller than at the surface. For instance, at T=900 s, the amplitude at h=30 km is ~ 15000 times smaller than at the surface. This means that if amplitude at h=30 km is 30 μm, which is reasonable displacement from strong earthquakes, the vertical surface displacement achieves to 0.5 m.

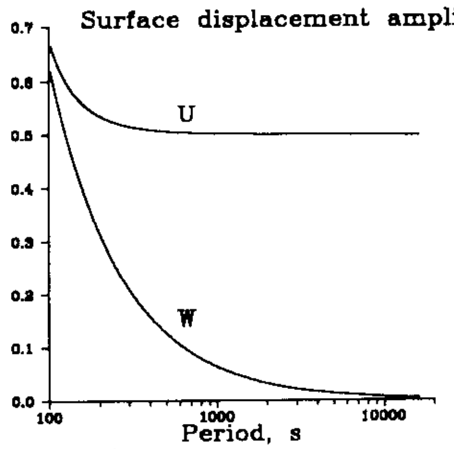

Fig.2

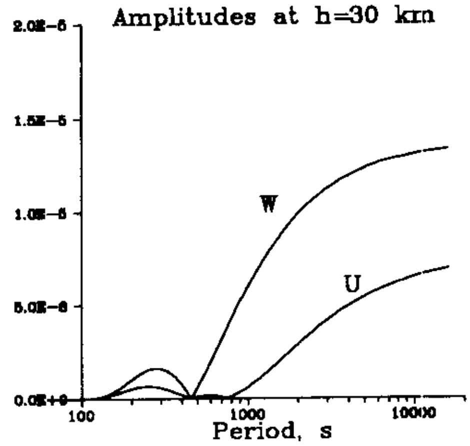

Fig.3

c) Amplitude-depth distribution

Amplitudes of horizontal and vertical components of tsunami waves $U(z,\omega)$ and $W(z,\omega)$ normalized by $\sqrt{I_0}$ for T=600 s and T=900 s are shown in fig.4,5.

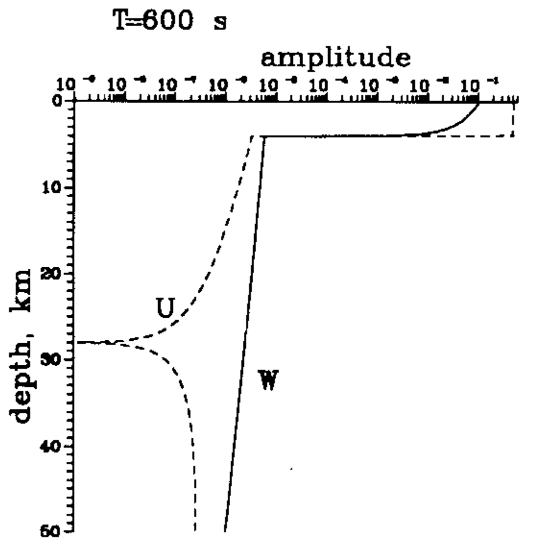

Fig.4

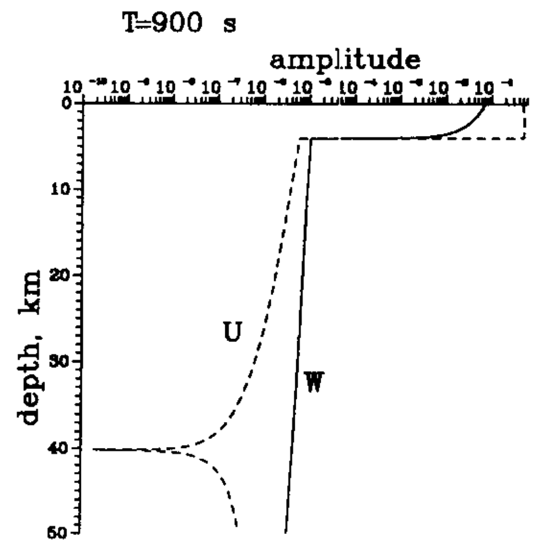

Fig.5

d) Source radiation functions

We shall analyse source functions for two particular source mechanisms: dip slip and strike slip on a vertical fault.

DIP SLIP.

Let the fault plane be orthogonal to the x-axis, and displacement is vertical. For this case only two components of the seismic moment tensor differ from zero:

$$m_{xz} = m_{zx} = M_0 F(\omega)$$

where M_0 is scalar seismic moment.

The corresponding components of tensor **B** are as follows (Keilis-Borok ed.,1989):

$$B_{xz} = B_{zx} = \frac{i\cos\varphi}{2}[\xi W_2 + dU_2 / dz]$$

where $\xi=\omega/c$ is wave number.

Consequently

$$Q(h,\omega,\varphi) = iM_0 \cos\varphi[\xi W_2 + dU_2 / dz]F(\omega)$$

STRIKE SLIP

The fault is assumed to be the same as in the former case, but slip occurs in the direction of y-axis. For this case non-zero components of the moment tensor are

$$m_{xy} = m_{yx} = M_0 F(\omega)$$

and the corresponding components of the tensor B are as follows:

$$B_{xy} = B_{yx} = -\frac{1}{2}\xi \sin 2\varphi U_2$$

The source function Q is following:

$$Q(h,\omega,\varphi) = -M_0 \xi \sin 2\varphi U_2 F(\omega)$$

To estimate the source radiation function we have to assume a value of seismic moment M_0 and the spectrum $F(\omega)$. We shall estimate the radiation function up to the factor M_0 and for the azimuth, in which radiation is maximum. So for dip slip we shall calculate

$$q_d(h,\omega) = [\xi W_2 + dU_2 / dz]F(\omega) / \sqrt{cuI_0},$$

and for strike slip

$$q_s(h,\omega) = \xi U_2 F(\omega) / \sqrt{cuI_0}.$$

For seismic moment spectrum it is reasonable to assume the function

$$F(\omega) = \frac{1}{i\omega(i\omega\tau + 1)}$$

where τ is rise time.

The functions $q_d(\omega)$ and $q_s(\omega)$ are shown in fig.6 for h=10,20 and 40 km (τ =30s). As is expected, dip slip exites much more intensive tsunami than strike slip. The radiation functions increase with period, though this does not mean that such large periods should be expected in tsunami waves: vertical component of the third term in (7) decreases with period (see fig.2). Fig.7 shows a product of the third and the fourth terms in (7) : vertical component of surface displacement from dip slip. From this figure we may conclude that dominated period in tsunami wave increases with focal depth. This explains different periods in observed tsunami waves - from 5 min. up to 1 hour.

On the basis of fig.7 it is easy to estimate amplitudes of tsinami waves from an earthquake with a given seismic moment at a given distance from the source. For example, amplitude of the wave at Δ=300 km from the earthquke at h=10 km with $M_0=10^{29}$ dyn.cm is about 30 cm, which is a realistic value.

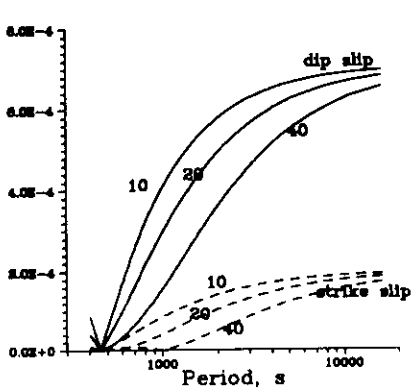

Fig.6

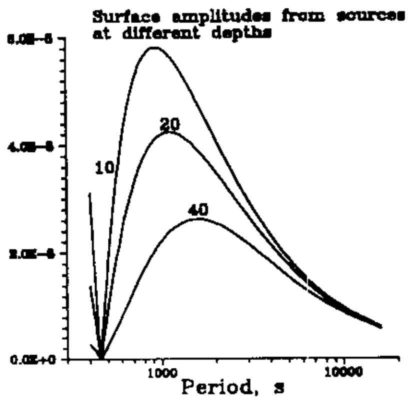

Fig.7

Energy of tsunami wave

Energy of tsunami wave is estimated as the total energy flux across a cylindric surface:

$$E = \frac{1}{2\pi} \iint_s r d\varphi dz \int_0 \frac{\rho\omega^2 |\mathbf{u}(r,\varphi,\omega)|^2}{2} u d\omega \qquad (8)$$

Substituting (7) to (8) and taking into account that $I_0 = \int \rho [U(z,\omega)^2 + W(z,\omega)^2] dz$, we can write the formula for the energy as follows:

$$E = \frac{1}{32\pi^2} \int_0 \omega d\omega \int_0^{2\pi} \frac{Q^2(h,\varphi,\omega)}{c u I_0} d\varphi \qquad (9)$$

Practically integration over ω in (9) should be performed up to the highest frequency of tsunami wave ϖ.

For the source mechanisms treated above this integral is reduced to the following:

$$E = \frac{M_0^2}{32\pi} \int_0^{\varpi} \omega q^2(h,\omega) d\omega$$

For numerical estimates we assumed ϖ corresponding to the smallest period equal to 200 s. For $M_0=10^{29}$ dyn.cm the tsunami energy estimated by this formula is about 2.10^{21}erg, which is in agreement with independent estimates of the tsunami energy (Iida,1963)

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