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Conference on
Ultrafast Transmission Systems in Optical Fibres

13 - 17 February 1995

The Sliding Frequency Soliton Laser

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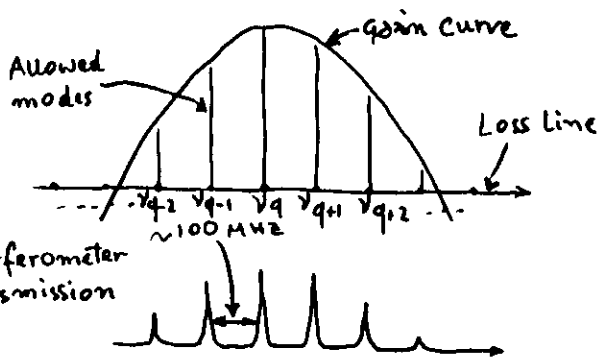
OUTLINE OF TALK

- BACKGROUND
  - --- Mode-Locked versus Frequency-Shifted Feedback Method for Pulse Generation
- SOLITON REGIME OF FIBER FSF LASERS

Including
  - --- Fiber Nonlinear Refractive Index
  - --- Group Velocity Dispersion
  - --- Bandwidth Limited Amplification

- ANALOGY BETWEEN FREQUENCY SLIDING FILTER AND SIGNAL METHODS
  - --- Demonstration of Soliton Transmission Stabilization by Sliding Frequency Signal Method
- ANALYSIS OF SOLITARY WAVE STABILITY IN SFS LASERS
  - --- Noise-Induced Time Jitter
  - --- Soliton-Soliton Interaction
  - --- Role of Group-Velocity Dispersion
  - --- Nonlinear Polarization Rotation
  - --- Third Order Dispersion
  - --- Continuous Wave Injection
- ANALYSIS OF SELF-STARTING DYNAMICS

MODE LOCKING of LASERS



...illation Frequencies in a Cavity of Length L

$$\omega_{q+1} - \omega_q = \frac{\pi c}{L n} \equiv \omega$$

...sulting Multimode Electric Field

$$E(t) = \sum_n E_n e^{-i[(\omega_0 + n\omega) + \phi_n]} = E(t+T)$$

$$T = \frac{2\pi}{\omega} = \frac{2Ln}{c}$$

...ode-locking, eg. for $\phi_m \equiv 0$

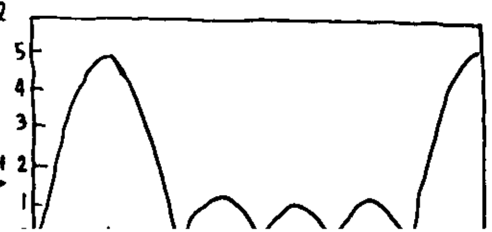
$$E(t) = \sum_{-(N-1)/2}^{(N-1)/2} e^{-i(\omega_0 + n\omega)t} = e^{-i\omega_0 t} \frac{\sin(N\omega t/2)}{\sin(\omega t/2)}$$

...eld Amplitude

$\sqrt{P} = |E|$

...ith

= 5



VOLUME 17, NUMBER 8 APPLIED PHYSICS LETTERS 15 OCTOBER 1970

ANALYSIS OF A DYE LASER TUNED BY ACOUSTO-OPTIC FILTER

William Streifer* and John R. Whinnery†
Microwave Laboratory, *Stanford University*, *Stanford*, *California* 94305

(Received 27 May 1970; in final form 10 July 1970)

A model of a dye laser electronically tuned by an acousto-optic filter is analyzed. Chirping modes are superposed to yield a steady-state periodic solution which predicts a shift of the spectrum to the high end of the filter band and appreciable spectral narrowing. For the example considered, the theory predicts a narrowing factor of 100.

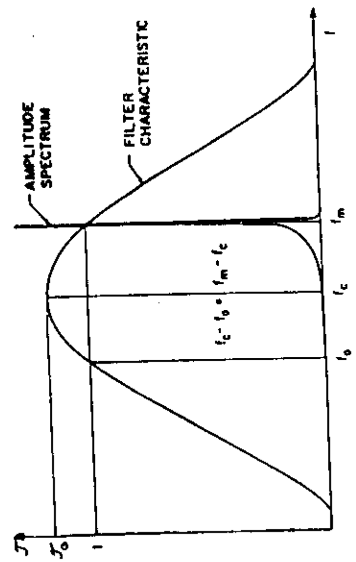


FIG. 2. Schematic of the laser spectrum.

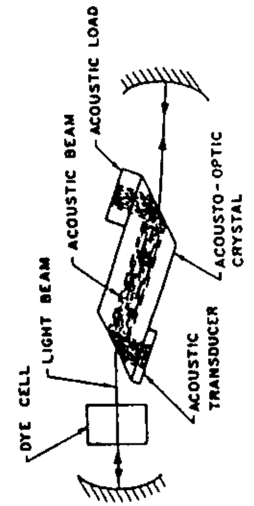


FIG. 1. Schematic of the tunable laser. One mirror is aligned to the ordinary wave, the other to the extraordinary.

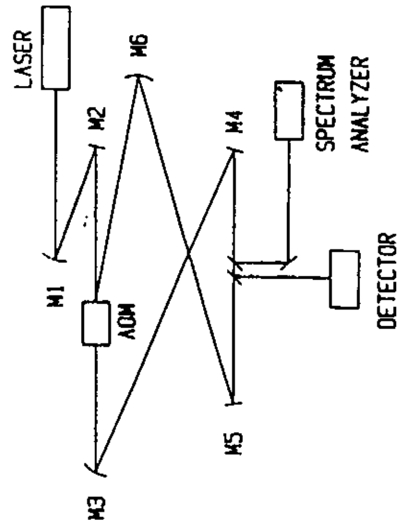
Pulse generation with an acousto-optic frequency shifter in a passive cavity

F. V. Kowalski, J. A. Squier,[a)] and J. T. Pinckney
Department of Physics, Colorado School of Mines, Golden, Colorado 80401

(Received 24 October 1986; accepted for publication 21 January 1987)

A train of pulses is generated by coupling the cw output of a single frequency He-Ne laser into a passive ring "cavity" which contains an acousto-optic frequency shifter. Each cavity pass shifts the frequency of the wave by 80 MHz. This results in an intensity pattern which is an Airy function in time. For a single frequency input, the output pulse width is 2 ns with a repetition rate of 80 MHz. Data are also presented on the output of this device for multimode cw input.

Appl. Phys. Lett. 50 (12), 23 March 1987



We now calculate the total electric field E_T a distance aL from the AOM, where $a<1$. This corresponds, for instance, to the electric field at the output coupler. For the arrangement in Fig. 1 we get

$$E_T = E_0\tau e^{i\delta_0} + E_0\tau r e^{i\delta_1} + E_0\tau r^2 e^{i\delta_2} + E_0\tau r^3 e^{i\delta_3} + \cdots ,$$

where

$$\delta_0 = \omega_0 aL/c - \omega_0 t ,$$

$$\delta_1 = \omega_0 L/c + [(\omega_0+\Omega)/c]aL - (\omega_0+\Omega)t ,$$

$$\delta_2 = \frac{\omega_0 L}{c} + (\omega_0+\Omega)\frac{L}{c} + (\omega_0+2\Omega)\frac{aL}{c} - (\omega_0+2\Omega)t, \text{etc.}$$

Factoring $E_0\tau e^{i\delta_0}$ from all the terms and noting that the cavity length was set so that it satisfied $\Omega L/c = 2\pi$ we get

$$E_T = E_0\tau e^{i\delta_0}[1 + re^{i\Delta} + r^2e^{i2\Delta} + r^3e^{i3\Delta} + \cdots] , \qquad (1)$$

where $\Delta = \omega_0 L/c + \Omega aL/c - \Omega t$. Solving for the intensity we get

$$I = I_0 \frac{\tau^2}{[1-r]^2}\,\frac{1}{1+4r\sin^2(\Delta/2)/(1-r)^2} \qquad (2)$$

This is standard form for the intensity in a Fabry–Perot interferometer. However our phase Δ is now time dependent. The intensity as a function of Δ is the Airy function.

TEMPORAL AND SPECTRAL DEVELOPMENT OF THE SFSL FIELD

$\delta t = 2\pi/\delta\omega = 157$ (0.8 ns); COHERENT PUMP

$Z_L = 650\,m$

$z_0 = 2\,Z_L$

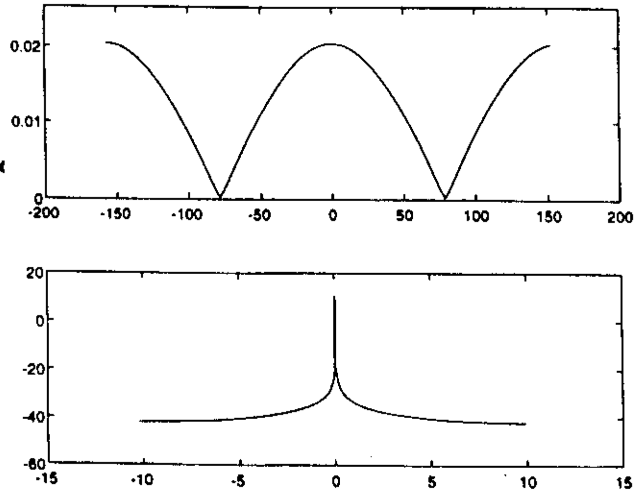
) = 15 ps/nm.km
anomalous)

$\delta\nu = 1.27$ GHz
(for computational help)

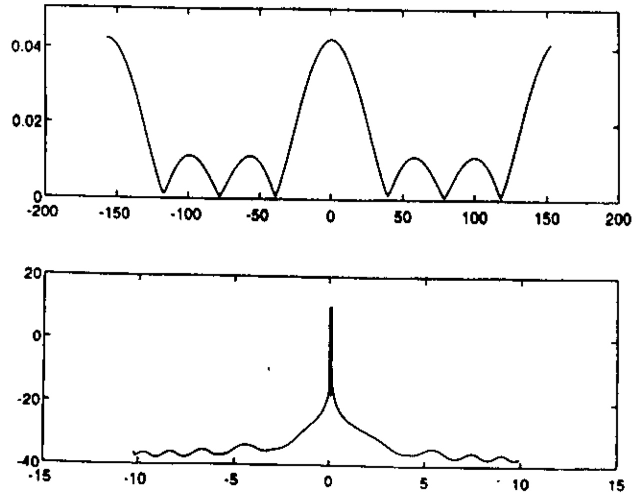
$\delta\Omega_{Filter} = 164$ GHz

or $\Delta\lambda_{Filter} = 1.3$ nm

TRANSIT NUMBER N = 1



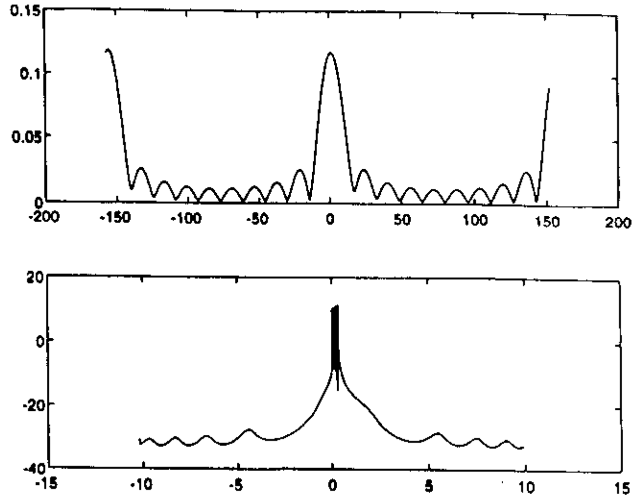
N = 3



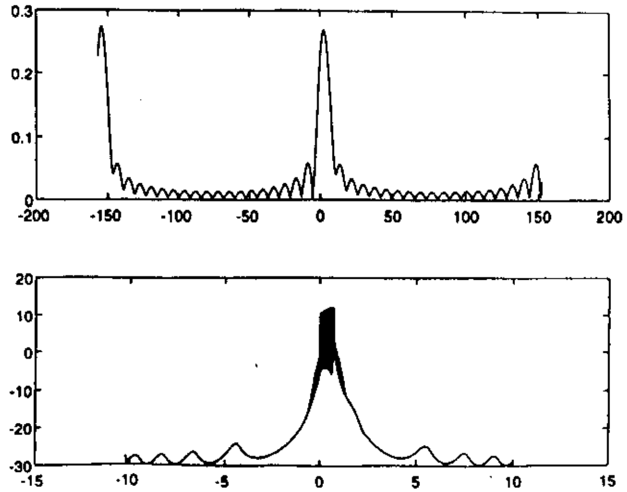
TEMPORAL AND SPECTRAL DEVELOPMENT OF THE SFSL FIELD

COHERENT PUMP

TRANSIT NUMBER N = 10



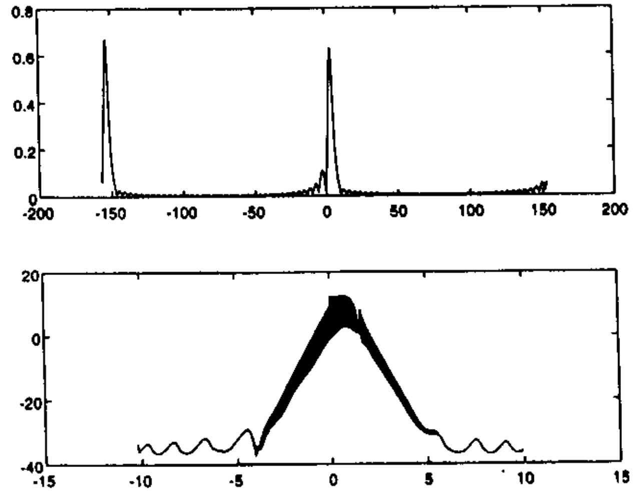
N = 20



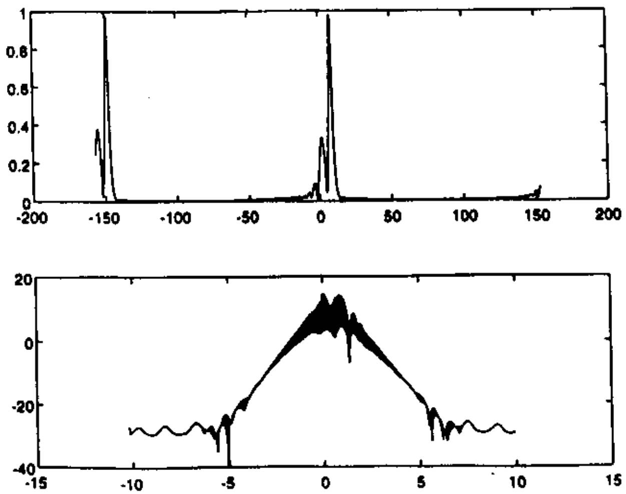
TEMPORAL AND SPECTRAL DEVELOPMENT OF THE SFSL FIELD

COHERENT PUMP

TRANSIT NUMBER N = 40



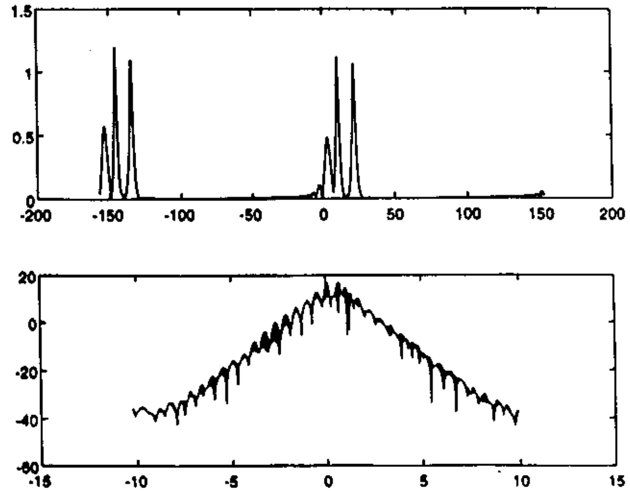
N = 80



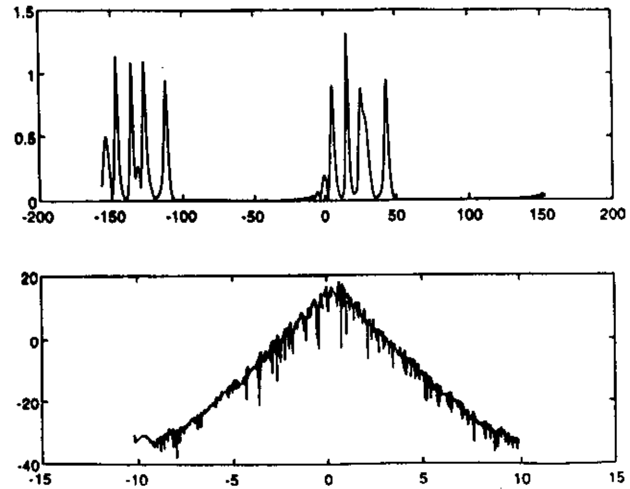
TEMPORAL AND SPECTRAL DEVELOPMENT OF THE SFSL FIELD

COHERENT PUMP

TRANSIT NUMBER N = 160



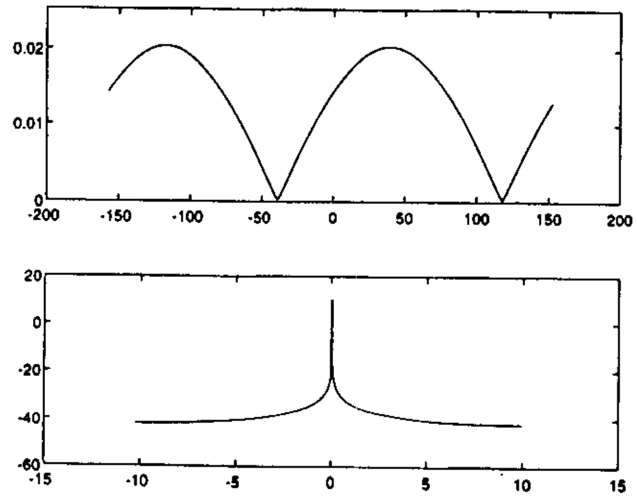
N = 640



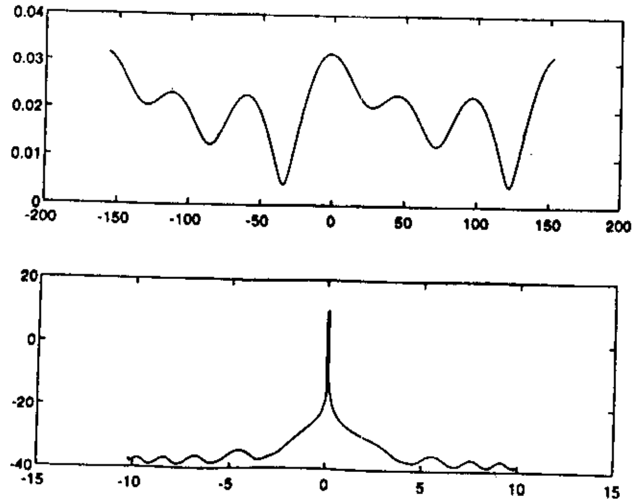
TEMPORAL AND SPECTRAL DEVELOPMENT OF THE SFSL FIELD

INCOHERENT PUMP

TRANSIT NUMBER N = 1



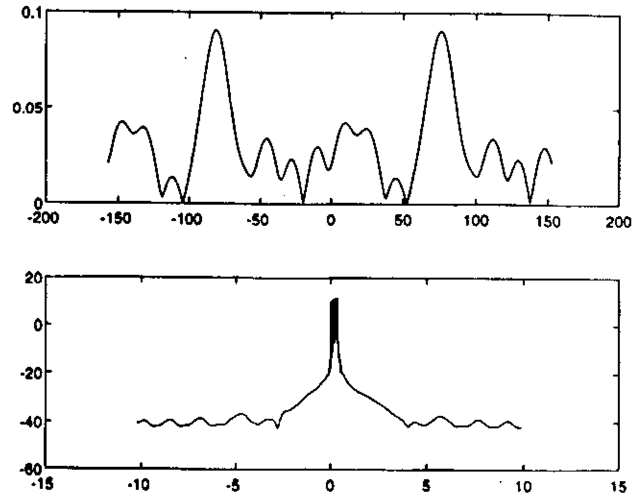
N = 3



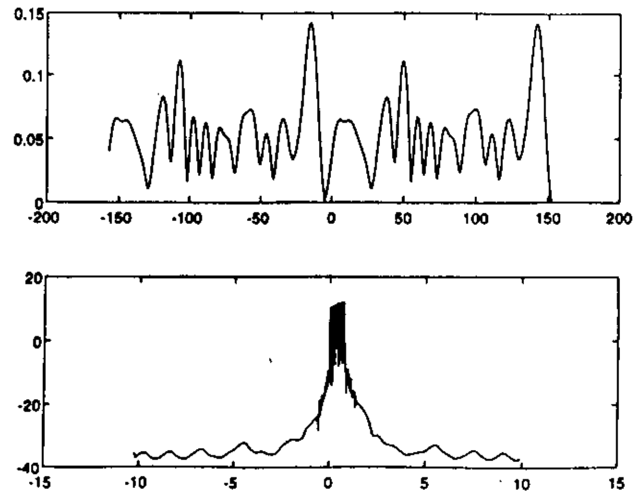
TEMPORAL AND SPECTRAL DEVELOPMENT OF THE SFSL FIELD

INCOHERENT PUMP

TRANSIT NUMBER N = 10



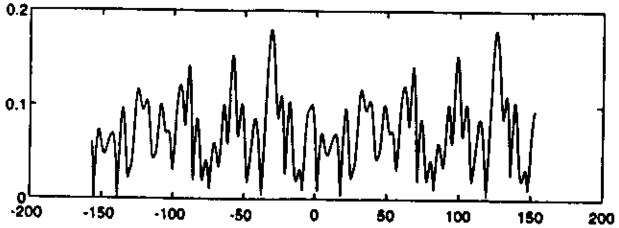
N = 20

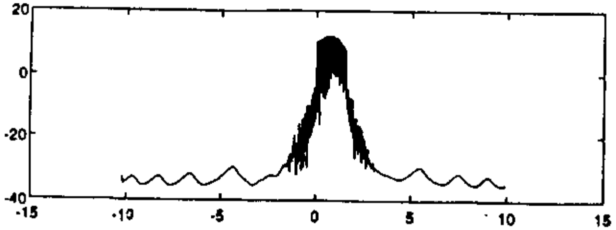


TEMPORAL AND SPECTRAL DEVELOPMENT OF THE SFSL FIELD

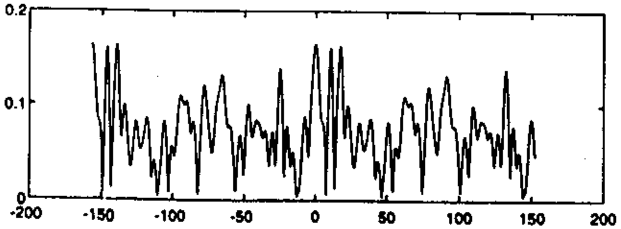
INCOHERENT PUMP

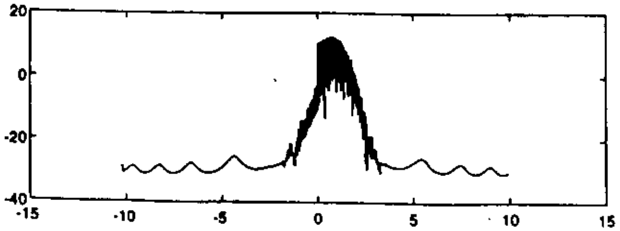
TRANSIT NUMBER N = 40





N = 80

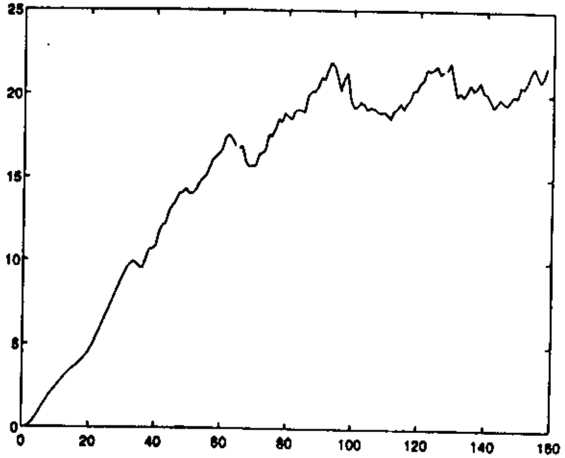




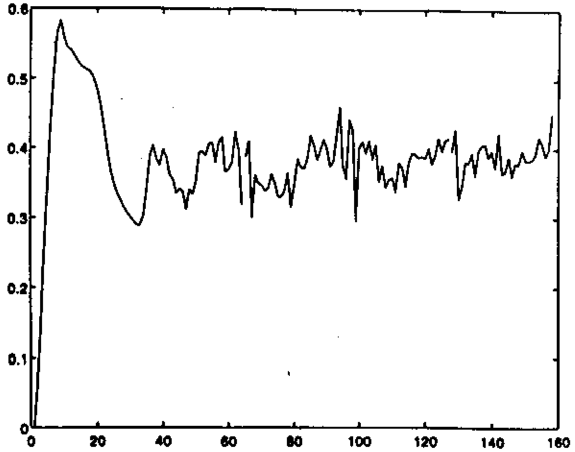
ENERGY AND FREQUENCY EVOLUTIONS

With Coherent Seeding

--- Energy vs Transit Number



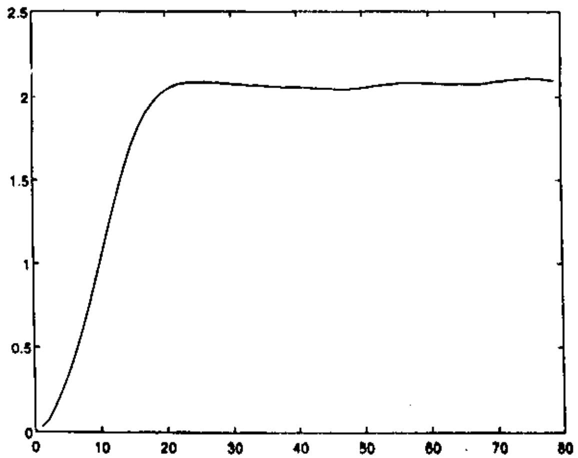
--- Average Frequency vs Transit Number



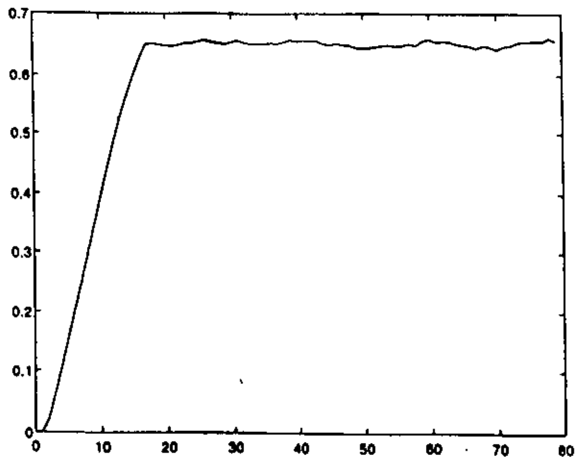
ENERGY AND FREQUENCY EVOLUTIONS

With Incoherent Seeding

--- Energy vs Transit Number



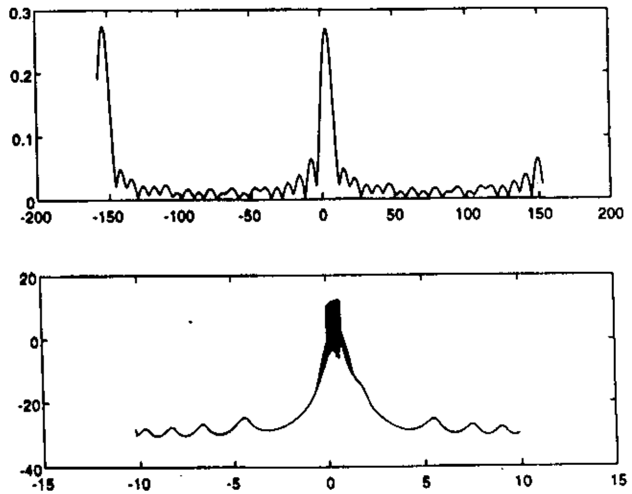
--- Average Frequency vs Transit Number



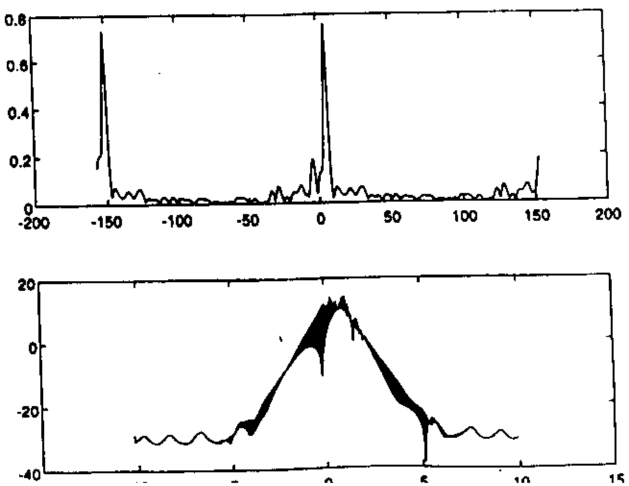
TEMPORAL AND SPECTRAL DEVELOPMENT OF THE SFSL FIELD

PARTIALLY COHERENT PUMP

TRANSIT NUMBER N = 20



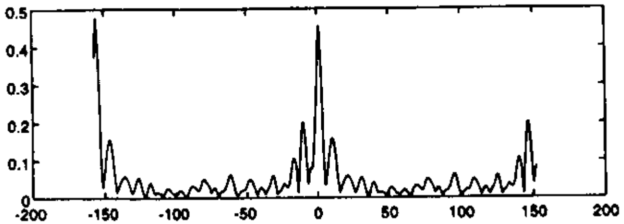
N = 40

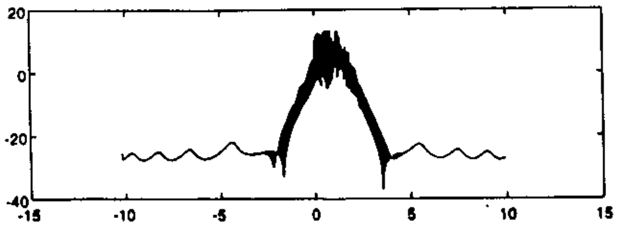


TEMPORAL AND SPECTRAL DEVELOPMENT OF THE SFSL FIELD

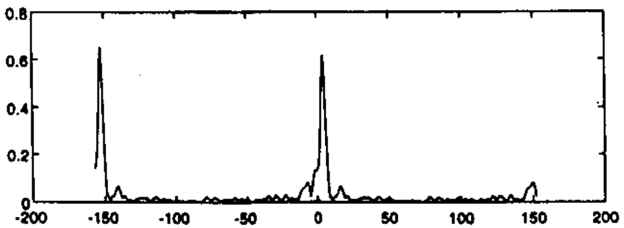
PARTIALLY COHERENT PUMP

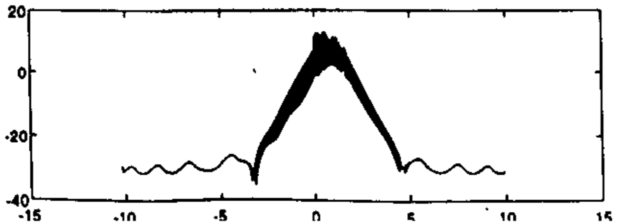
TRANSIT NUMBER N = 80, RANDOM SAMPLE 1





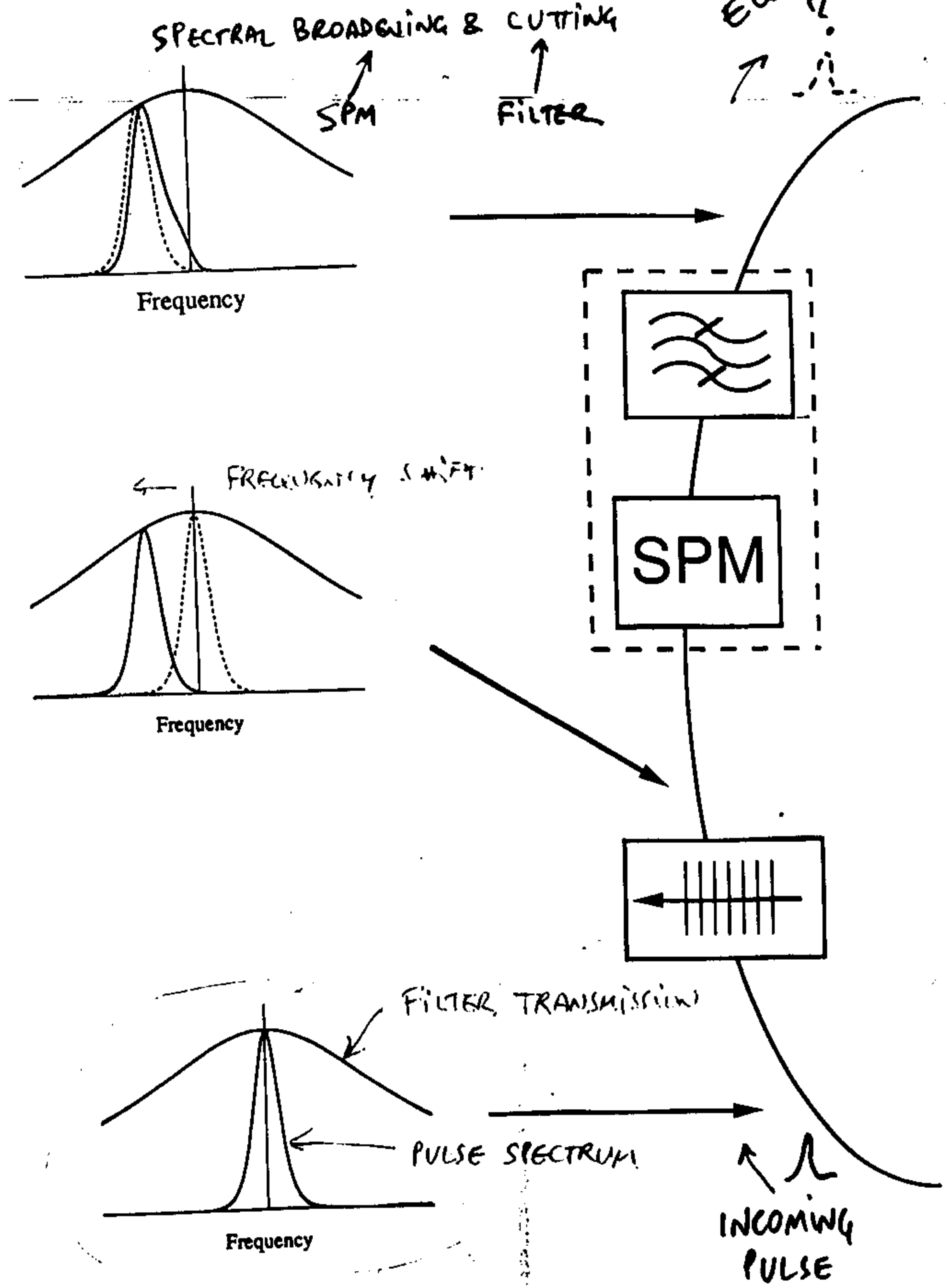
TRANSIT NUMBER N = 80, RANDOM SAMPLE 2



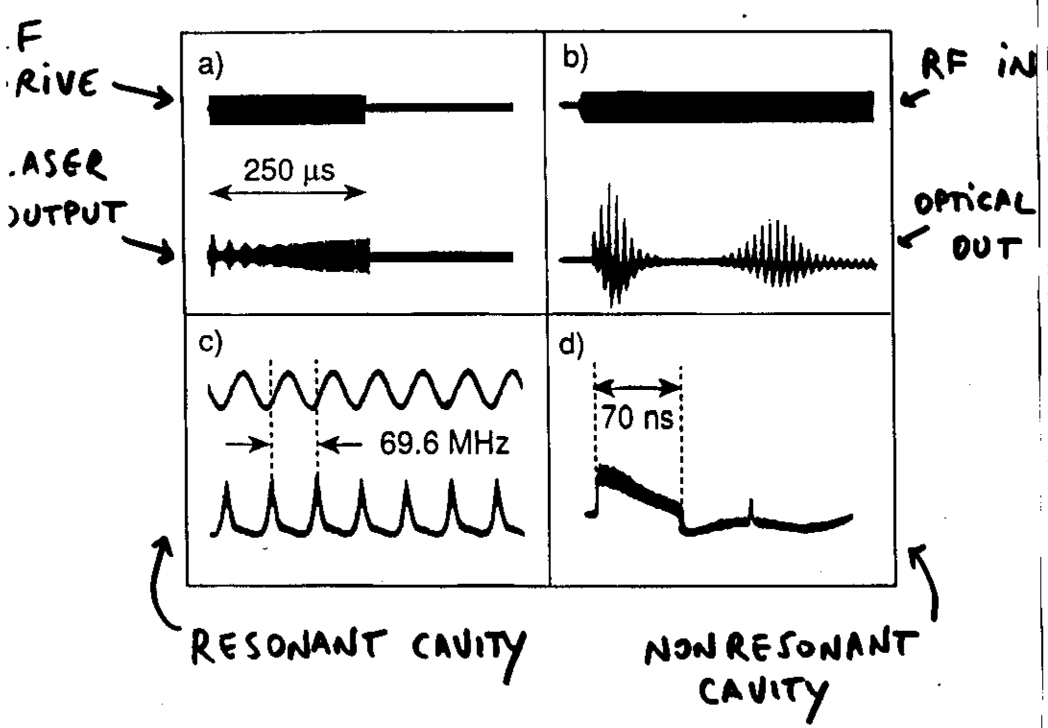


HAND-WAVING EXPLANATION

Sliding-Frequency Soliton Laser



DYNAMICS OF SFS LASER START-UP



The Sliding-Frequency (SF) soliton laser

$$u_Z = \frac{i}{2}u_{TT} + i|u|^2u + \delta u + \beta u_{TT} - i\alpha_0 T u$$

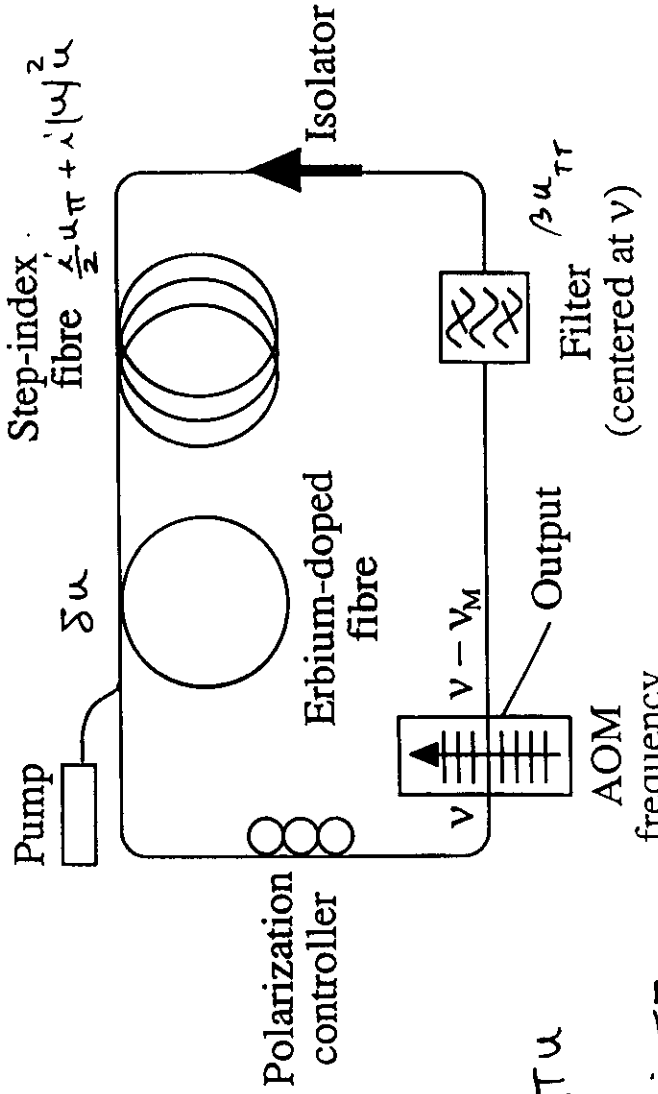


Fig. 1

$$u_Z = i\alpha_0 T u$$

$$\downarrow \quad i\alpha_0 T Z$$

$$u = u_0 e^{i\alpha_0 T Z}$$

CONTINUOUS FREQUENCY SHIFT

THE EQUIVALENCE BETWEEN THE SLIDING FREQUENCY AND SLIDING FILTER METHODS

Whenever the center filter frequency reads

$$\omega_f = \alpha_0 Z$$

$$P = i\delta u + i\beta(\partial_T + i\omega_f(Z))^2 u$$

alternative to sliding the filters is to keep the filters fixed and continuously slide the frequency of the signal

Formally, $u(T, Z) = U(T + \alpha_0 Z^2/2, Z) exp[-i\alpha_0 ZT - i\alpha_0^2 Z^3/3]$

equation for U where $P = i\delta U + i\beta U_{T'T'} - \alpha_0 T' U$

$$T' = T + \alpha_0 Z^2/2$$

SOLITON PERTURBATION THEORY (Karpman & Maslov, 1977)

Ansatz: $\quad q_1(T, Z) = \eta_1 \mathrm{sech}[\eta_1 T - \sigma_1(Z)] \exp[-i\kappa_1 T + i\theta_1(Z)].$

In the presence of a perturbation R, one obtains from inverse scattering theory

$$\tau = \eta_1(T - \xi_1)$$

$$\varphi = -\kappa_1 T + \theta_1.$$

$$\frac{d\eta_1}{dZ} = \int_{-\infty}^{\infty} \mathrm{Re}\{R[q, q^*]e^{-i\varphi}\} \,\mathrm{sech}\,\tau \, d\tau ,$$

$$\frac{d\kappa_1}{dZ} = -\int_{-\infty}^{\infty} \mathrm{Im}\{R[q, q^*]e^{-i\varphi}\} \,\mathrm{sech}\,\tau \,\tanh\tau \, d\tau ,$$

$$\frac{d\xi_1}{dZ} = \frac{1}{\eta_1}\mathrm{Im}\{K(2\zeta_1)\} + \frac{1}{\eta_1^2}\int_{-\infty}^{\infty} \mathrm{Re}\{R[q, q^*]e^{-i\varphi}\} \,\tau\,\mathrm{sech}\,\tau \, d\tau ,$$

$$\frac{d\theta_1}{dZ} = \mathrm{Re}\{K(2\zeta_1)\} + \xi_1 \frac{d\kappa_1}{dZ} + \frac{1}{\eta_1}\int_{-\infty}^{\infty} \mathrm{Im}\{R[q, q^*]e^{-i\varphi}\} \,\mathrm{sech}\,\tau \,(1 - \tau\tanh\tau) \, d\tau ,$$

$$\partial_Z V - \frac{i}{2}\partial_{\tau\tau}V - i|V|^2V - i\alpha_0\tau V = \delta V + \beta\partial_{\tau\tau}V$$

the soliton solution with $\delta = \beta = 0$ reads

$$V(\tau, Z) = \eta sech[\eta(\tau - \xi(Z))]exp[i\kappa(Z)\tau + i\psi(Z)]$$

where $\xi = \alpha_0 Z^2/2 + \kappa_0 Z$, $\kappa = d\xi/dZ$, and $d\psi/dZ = (\eta^2 - \kappa^2)/2$.

In order to analyse the effect of the right hand side, we employ the perturbation method with the one soliton ansatz and $\eta = \eta(Z)$. One obtains

$$\frac{d\kappa}{dZ} = \alpha_0 - 4\beta\eta^2\kappa/3,$$

$$\frac{d\eta}{dZ} = 2\delta\eta - 2\beta\eta(\eta^2/3 + \kappa^2)$$

for the soliton frequency κ and amplitude η

attracting fixed point solution

$\kappa_0 = 3\alpha_0/(4\beta\eta^2)$ (i.e., $\kappa \to \kappa_0$ for $Z \to \infty$)

and $\eta \to \eta_0 = 1$ for $\kappa_0^2 = (\delta/\beta - 1/3)$.

$$\partial_Z V - \frac{i}{2}\partial_{\tau\tau}V - i|V|^2V - i\alpha_0\tau V = \delta V + \beta\partial_{\tau\tau}V,$$

by inserting a low intensity radiative field $V = f(\tau)exp(i\Omega Z)$ one obtains that in the absence of a soliton (or away from it) f obeys

$$\frac{d^2 f}{d\hat{\tau}^2} - \hat{\tau} f = O(\delta, \beta).$$

where $\hat{\tau} = (-2\alpha_0)^{1/3}(\tau - \Omega/\alpha_0)$.

Without the filter, the solution is known as the Airy function $f(\hat{\tau}) = Ai(\hat{\tau})$

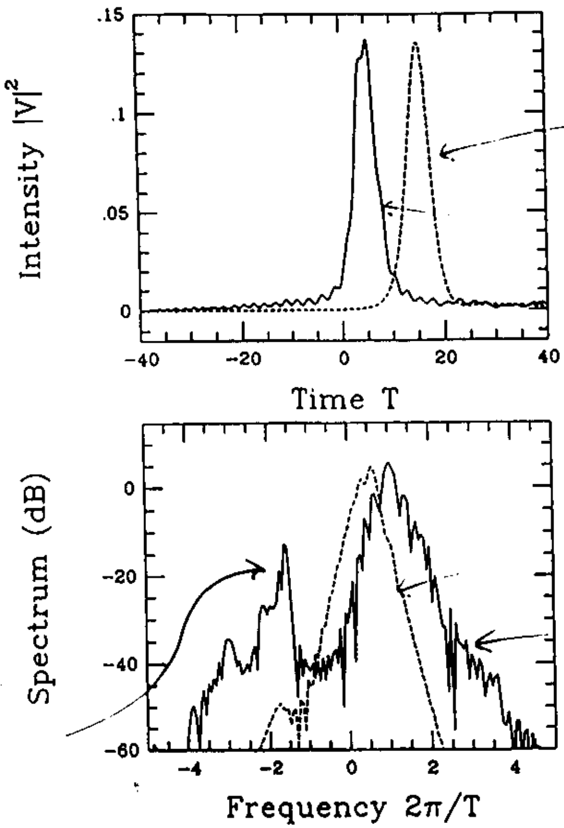
For large $|\hat{\tau}|$, and $\hat{\tau} > 0$, f has the asymptotic expression

$$f(-\hat{\tau}) \simeq (\hat{\tau})^{-1/4}(\pi)^{-1/2}sin(2\hat{\tau}^{3/2}/3 + \pi/4).$$

The comparison between the Fourier spectrum of $f(\hat{\tau})$ and that of the soliton clearly shows that the soliton and the radiation tend to separate in the frequency domain as Z grows larger.

SEPARATION BETWEEN SOLITON AND RADIATION BY SLIDING FILTERS

$$V(Z=0,T) = A\,\mathrm{sech}(T) \quad ; A = 0.7$$



with filter

without filter

with filter

without filter

radiation peak

$\alpha = -0.02$ $\delta = 0.024$; $\beta = 0.15$

The perturbed nonlinear Schrödinger equation

$$\frac{\partial u}{\partial Z} - \frac{i}{2}\frac{\partial^2 u}{\partial T^2} - i|u|^2 u - i\alpha T u = \delta u + V_f \frac{\partial u}{\partial T} + \beta \frac{\partial^2 u}{\partial T^2}$$

α sliding frequency rate

δ the excess gain

V_f filter induced group velocity

β filter dispersion

Solitons are stable solutions of this perturbed NLS

The excess gain (laser threshold) is minimized for

$$\frac{\alpha}{\beta} = \sqrt{\frac{8}{27}} \quad \rightarrow \alpha = \alpha_c$$

(CRITICAL SLIDING RATE FOR STABLE TRAPPING BY THE FILTER)

and the soliton equilibrium frequency is

$$\kappa = \sqrt{\frac{1}{6}}$$

and introducing dimensional units

$$\beta = \frac{2}{\Delta\Omega^2 Z_A \beta_2}, \qquad \alpha = \frac{(2\pi \cdot 10^{-6})\Delta f \tau^3}{Z_A \beta_2}$$

we determine the pulse duration

$$\tau_{FWHM} = arcosh(3)\sqrt{\frac{2}{3}}\sqrt[3]{\frac{1}{\pi \cdot 10^{-6}\Delta f \Delta\Omega^2}}.$$

The pulsewidth is independent from GVD and laser length.

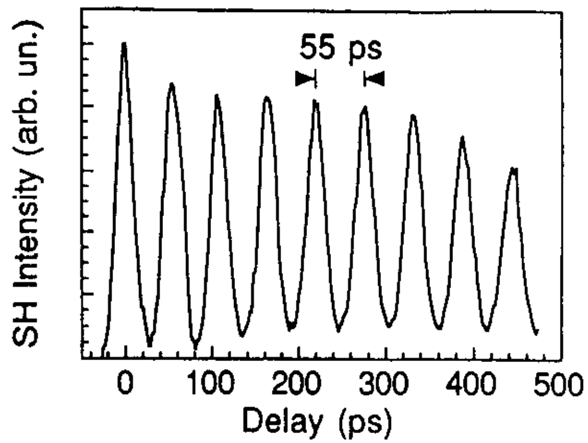
The soliton frequency detuning at the equilibrium is

$$\Delta\nu = \sqrt{\frac{1}{6}}\frac{1}{2\pi\tau}.$$

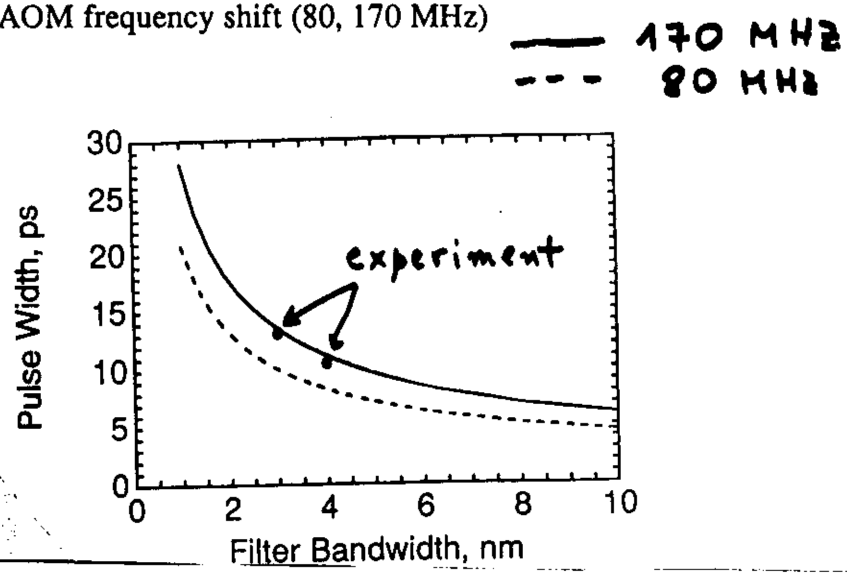
The experimental values are

Δf(MHz) $\rightarrow$ AOM frequency shift
$\Delta\Omega$(2π THz) $\rightarrow$ filter bandwidth
τ(ps) $\rightarrow$ soliton width
Z_A(km) $\rightarrow$ laser length
β_2(ps^2/km) $\rightarrow$ group-velocity dispersion

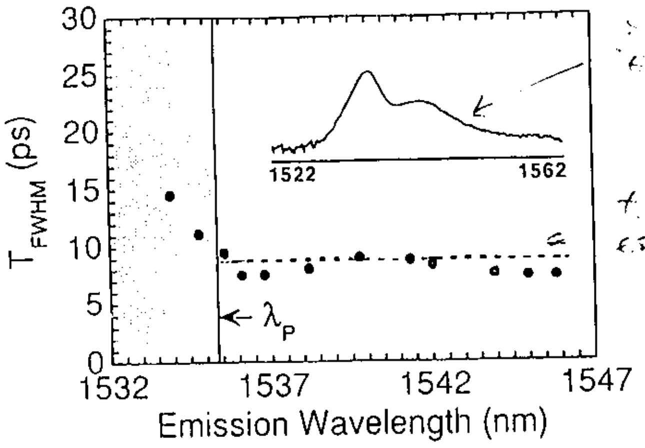
Autocorrelation trace of the 19 GHz train



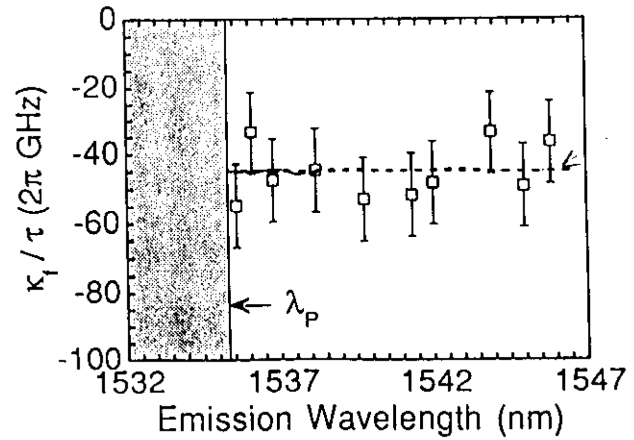
Pulse duration vs filter bandwidth for two values of the AOM frequency shift (80, 170 MHz)

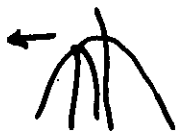


MINIMUM PULSE WIDTH VS. EMISSION WAVELENGTH

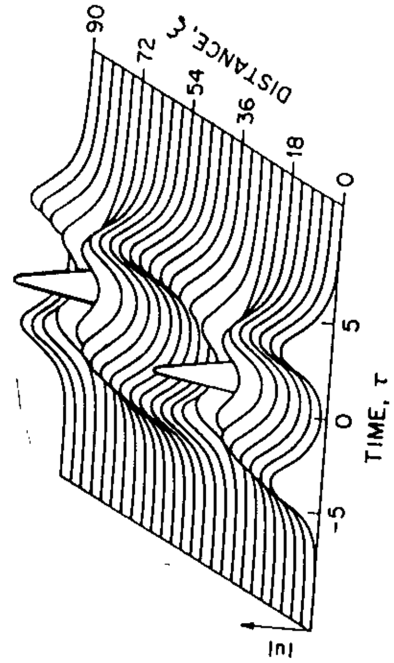


EQUILIBRIUM SOLITON FREQUENCY DOWNSHIFT FROM CENTER WAVELENGTH OF FILTER





A LIMITATION TO THE MAXIMUM TRANSMISSION DISTANCE IS GIVEN BY THE COALESCENCE OF ADJACENT IN-PHASE SOLITONS



One obtains for the initial separation Δ (ps) the maximum distance Ls (km), where τ is the input pulsewidth (ps) and D is the dispersion (ps²/km)

$$\Delta = \frac{8}{7}\tau \ln\left(\frac{L_s D}{0.12\tau^2}\right)$$

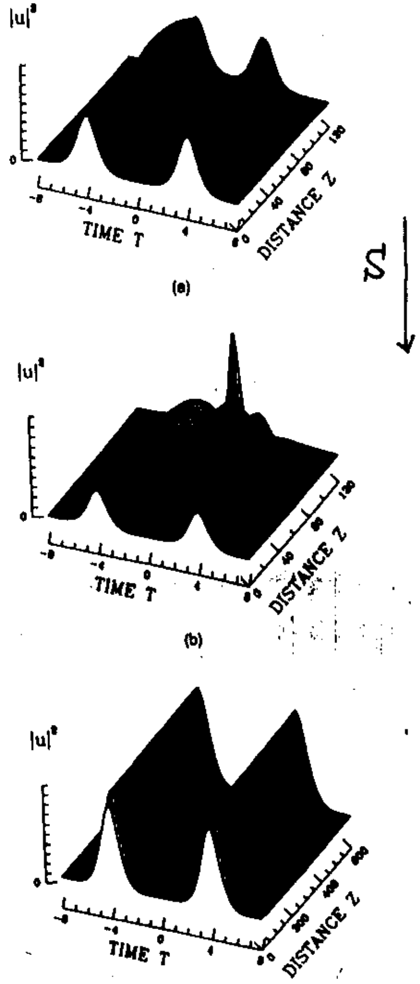
SOLITON STABILIZATION BY CONTINUOUS WAVE PHASE-SENTITIVE PUMPING

$P = (i\delta + \Delta)u + iS,$

$\delta < 0$ is average loss,

Δ is phase detuning between the pump of amplitude S and the field u

suppression of soliton interactions



phase-locking of the solitons to the pump

Synchronous Amplitude and Phase Modulation

$$P = (\alpha_r + i\alpha_i)cos(\Omega T + \Psi)u + i\delta u + i\beta u_{TT}$$

$\alpha_{r(i)}$ is a phase (amplitude) modulation coefficient

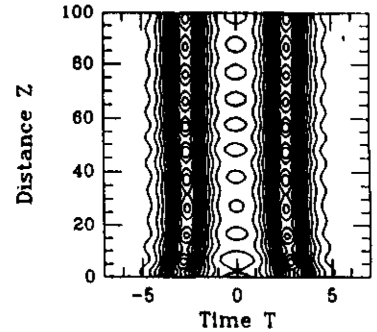
δ is the excess gain

β is the averaged amplification bandwidth

$$\beta = \frac{2}{\Delta\Omega^2 z_a D},$$

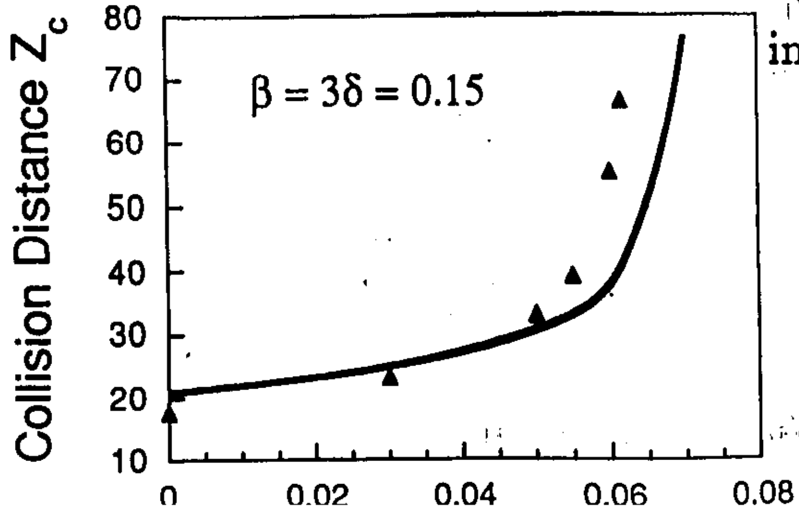
periodic potential

$V(T) = \alpha cos(\Omega T + \Psi)$

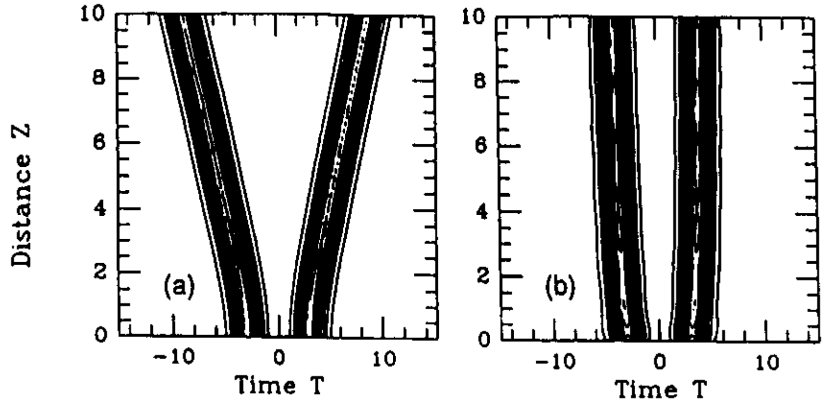


suppression of soliton interactions

increase of collision distance



REDUCTION OF SOLITON INTERACTIONS WITH BLA -
OUT - OF - PHASE SOLITONS
$\phi = \pi$



WITHOUT FILTERS Fig. (9) WITH FILTERS

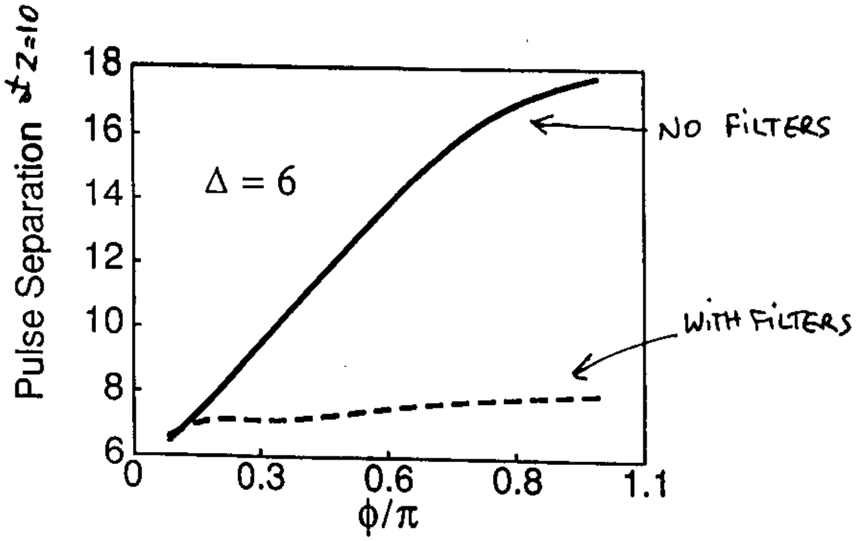
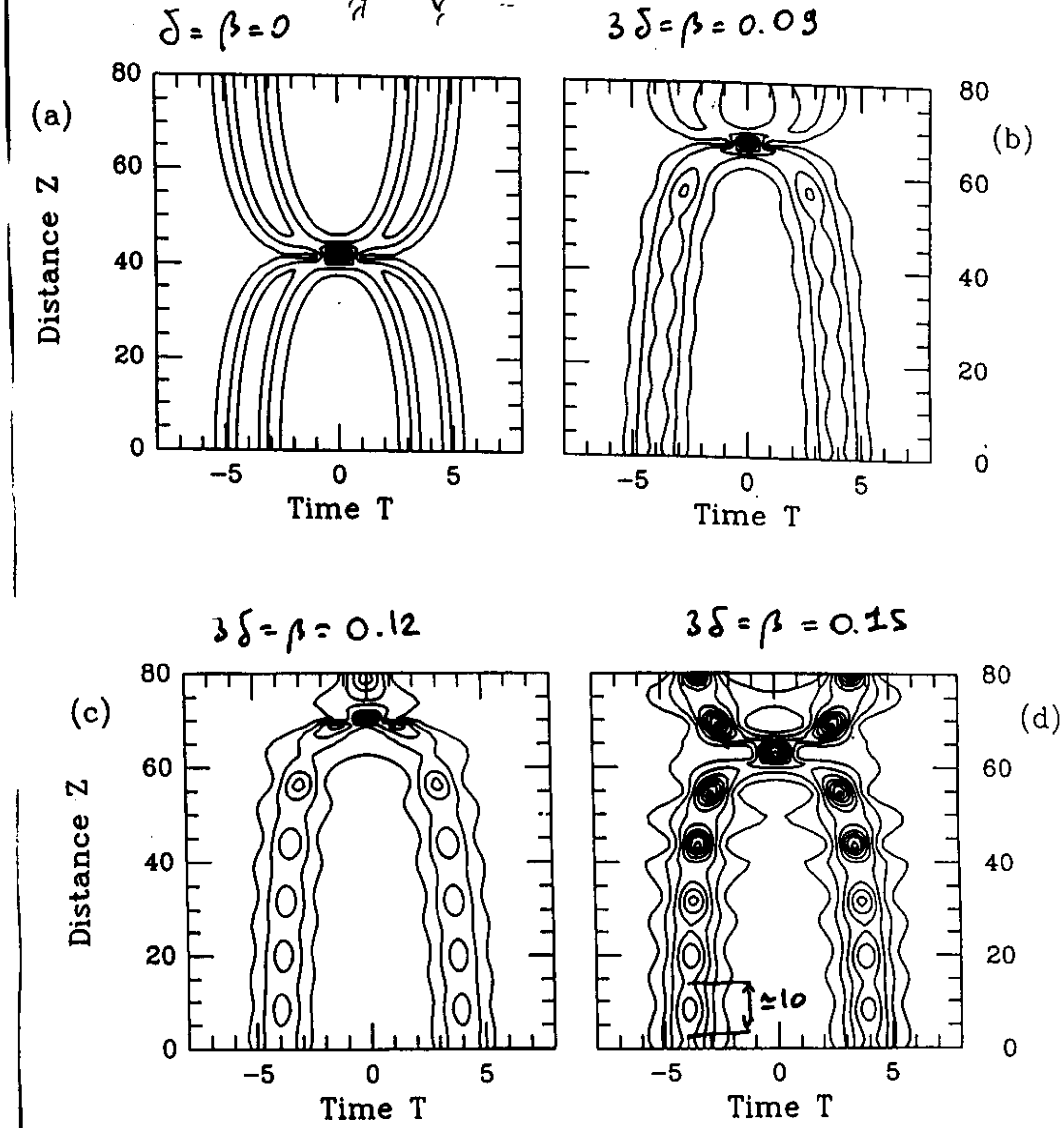


Fig. (10)

REDUCTION OF INTERACTION FORCES WITH BLA

(here $\sigma = 8$)



REDUCTION OF INTERACTION FORCES IS BLOCKED BY SOLITON INSTABILITY FROM AMPLIFICATION OF BACKGROUND

ADIABATIC TWO-SOLITON INTERACTION THEORY (Karpman & Solov'ev, 1981)

Quasi particle approach

$$q(T,Z) = q_1(T,Z) + q_2(T,Z) \ ,$$

Ansatz

$$q_1(T,Z) = \eta_1 \mathrm{sech}[\eta_1 T - \sigma_1(Z)] \exp[-i\kappa_1 T + i\theta_1(Z)].$$

Write

$$|q|^2 q = (|q_1|^2 q_1 + q_1^2 q_2^* + 2|q_1|^2 q_2) + (|q_2|^2 q_2 + q_2^2 q_1^* + 2|q_2|^2 q_1):$$

Obtain

$$\frac{\partial q_l}{\partial Z} - \frac{i}{2}\frac{\partial^2 q_l}{\partial T^2} - i|q_l|^2 q_l = R[q_l, q_l^*, q_{\bar{l}}, q_{\bar{l}}^*] \equiv (q_l^2 q_{\bar{l}}^* + 2|q_l|^2 q_{\bar{l}}) \ ,$$

where $l = 1,2,$ and $\bar{l} = 3 - l.$

TWO-SOLITON INTERACTION DESCRIBED BY THE INTEGRABLE SET OF ODE's

$$\frac{d\eta}{dZ} = 0, \frac{d\kappa}{dZ} = 0, \qquad \frac{dq}{dZ} = -4\eta^3 e^{-\eta\Delta}\cos(\Psi)$$

$$\frac{dp}{dZ} = 4\eta^3 e^{-\eta\Delta}\sin(\Psi) \qquad \frac{d\Delta}{dZ} = 2q, \frac{d\Psi}{dZ} = 2p\eta.$$

Where

$q = (\kappa_2 - \kappa_1)/2, \ p = (\eta_2 - \eta_1)/2,$

$\kappa = (\kappa_1 + \kappa_2)/2, \ \ \eta = (\eta_1 + \eta_2)/2,$

$\Psi = \theta_2 - \theta_1 - \kappa\Delta$

$\Delta = \xi_1 - \xi_2 > 0,$

it was pointed out that soliton interactions could be effectively suppressed by sliding the filters. In order to explain this effect, we employ here a two soliton perturbation approach (see V.I. Karpman & V.V. Solov'ev, Physica 3D, 481 (1981))

Consider the two interacting pulses

$$V_j(Z,\tau) = \eta_j sech(\eta_j(\tau - \xi_j))exp[i\kappa_j(\tau - \xi_j) + \psi_j], j = 1,2.$$

one obtains (Y.K. & S.W., 1993)

$$\frac{dq}{dZ} = 4\eta^3 e^{-\eta\Delta} cos(\sigma) - \frac{4}{3}\beta q \eta^2 - \frac{8}{3}\beta\kappa\eta p,$$

$$\frac{dp}{dZ} = 4\eta^3 e^{-\eta\Delta} sin(\sigma) + 2p\{\delta - \beta(\eta^2 + \kappa^2)\} - 4\beta\eta\kappa q,$$

$$\frac{d\kappa}{dZ} = \alpha_0 - \frac{4}{3}\beta\kappa\eta^2,$$

$$\frac{d\Delta}{dZ} = -2q,$$

$$\frac{d\sigma}{dZ} = 2p\eta - \frac{4}{3}\beta\kappa\eta^2\Delta,$$

$$\frac{d\eta}{dZ} = 2\delta\eta - 2\beta\eta\left(\frac{\eta^2}{3} + \kappa^2\right)$$

where $q = (\kappa_2 - \kappa_1)/2$ is the frequency difference,

$p = (\eta_2 - \eta_1)/2$ is the amplitude difference

$\kappa = (\kappa_1 + \kappa_2)/2$, $\eta = (\eta_1 + \eta_2)/2$,

$\sigma = \Delta\kappa + \Psi$, $\Psi = \psi_2 - \psi_1$ is the phase difference,

$\Delta = \xi_1 - \xi_2 > 0$ is the pul

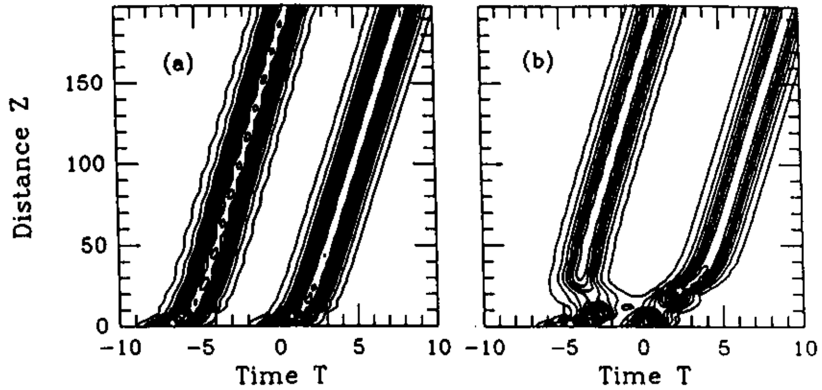


Fig. (19)

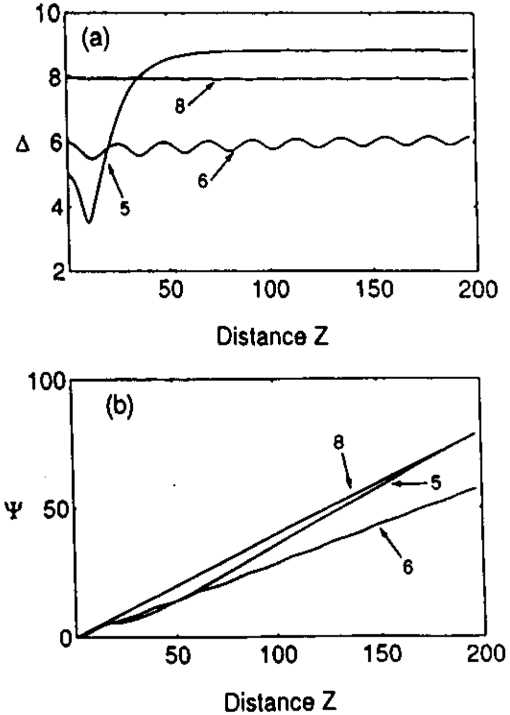
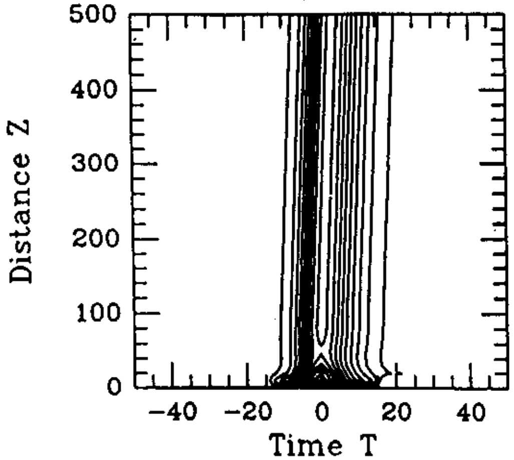


Fig. (20)

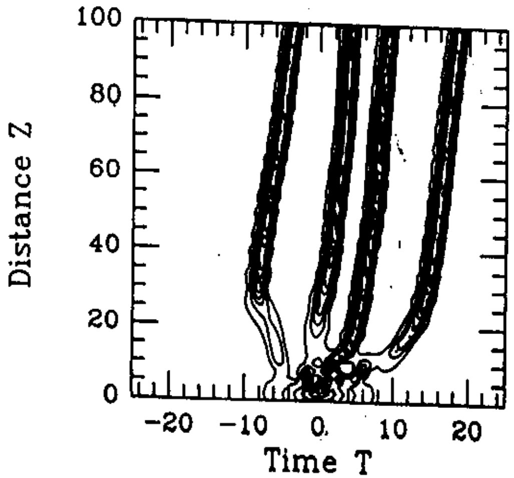
Normal versus anomalous dispersion

high dispersion cases:
saturable absorber + frequency shifter



$\beta_2 = +53\ ps^2/km$

asymmetric temporal envelope

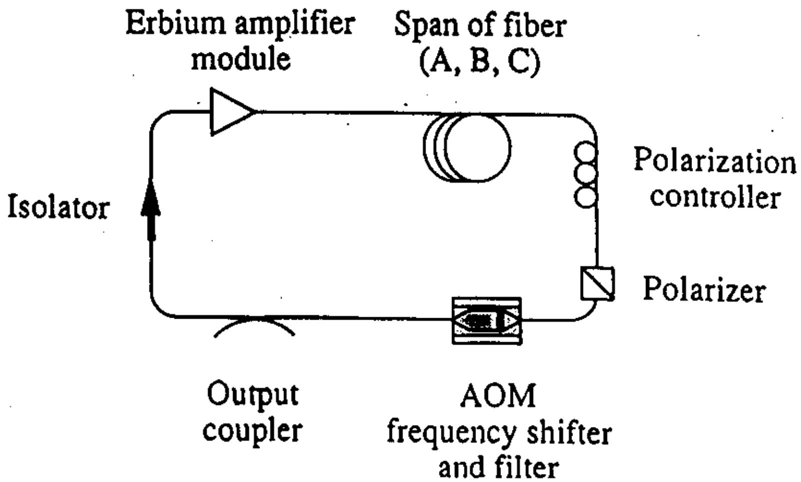


$\beta_2 = -53\ ps^2/km$

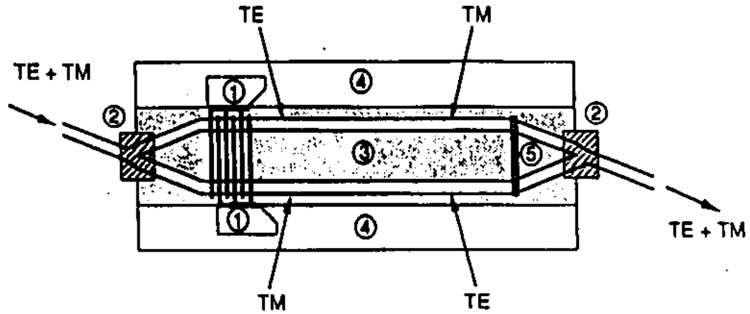
fundamental soliton generation

SCHEMATIC OF THE SFSL

Fiber (a): L = 110 m, β_2 = -20 ps^2/km
Fiber (b): L = 58 m, β_2 = 0 @ 1550 nm
Fiber (c): L = 40 m, β_2 = +83 ps^2/km



SCHEMATIC OF THE AOMF



(1) Unidirectional transducer
(2) Polarization beam splitter
(3) Acoustic waveguide
(4) Ti indiffused region
(5) Acoustic absorber

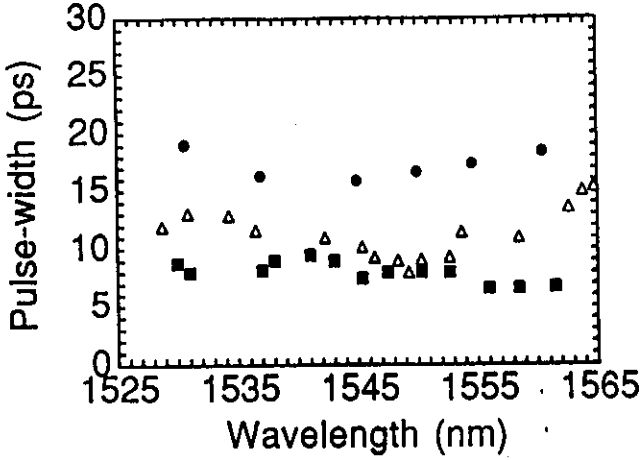
Tunability of the SFS laser

AVERAGE CAVITY GVD $\longrightarrow$

■ $\beta_2 = -15.3$ ps²/km

△ $\beta_2 = -1$ ps²/km @ $\lambda = 1550$ nm

● $\beta_2 = +51.5$ ps²/km



- ROLE OF AVERAGE GVD -

THEORY (NORMAL / ANOMALOUS DISPERSION)

The propagation equation is

$$i\frac{\partial u}{\partial z} - \frac{1}{2} D \frac{\partial^2 u}{\partial t^2} + |u|^2 u = -\alpha_0 t u + i\delta u + i\beta\frac{\partial^2 u}{\partial t^2} + i\gamma|u|^2 u + i\sigma|u|^4 u$$

D average ring disperison $\begin{cases} D>0 & \text{normal dispersion} \\ D<0 & \text{anomalous dispersion} \end{cases}$

γ, σ saturable absorber

When $\alpha_0 = \sigma = 0$ there is an exact solution [1]

$$u(t,z) = A \,\mathrm{sech}\left(\frac{t}{\tau}\right)^{1-i\nu} e^{i\Gamma z}$$

$\Rightarrow$ a chirped soliton solution with

$$\boxed{\nu = -a \pm \sqrt{a^2+2}}$$ CHIRP PARAMETER

$$a = \frac{3(\gamma\beta + D/2)}{\beta - D\gamma/2}$$

$$\Gamma = \frac{D\nu^2/2 - D/2 - 2\beta\nu}{\tau^2}$$

[1] O. E. Martinez, R. L. Fork, and J. P. Gordon, "Theory of Passively Mode-Locked Lasers for the Case of a Nonlinear Complex-Propagation Constant," *J. Opt. Soc. Am.*, **B2**, pp. 753–760, 1985.

PULSE STABILITY

$\delta > 0$

The low intensity background is unstable $\Rightarrow$ the solitary wave decays owing to its interaction with exponentially growing noise

$\delta < 0$

It has been shown [1] that, also with $\gamma \neq 0$ the solitary wave is unstable

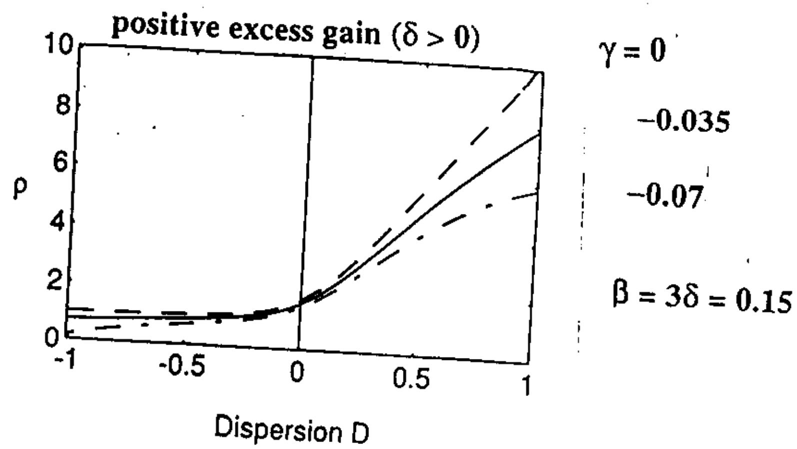
$\Rightarrow$ to describe the pulsed laser operation it is essential to introduce some stabilizing elements

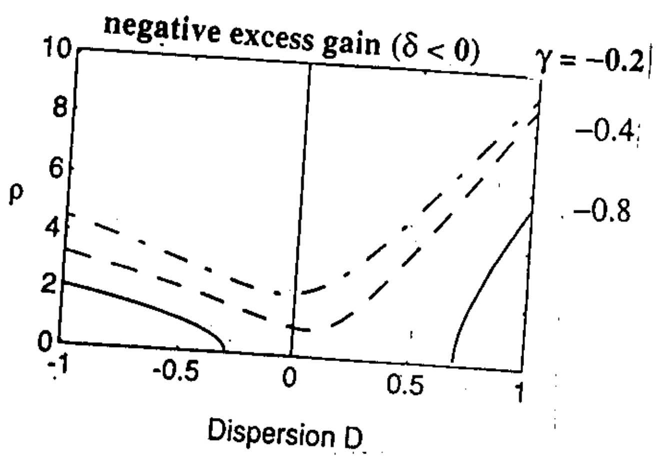
— Sliding frequency ($\alpha_0 \neq 0$)

— Higher order nonlinear gain saturation terms ($\sigma \neq 0$)

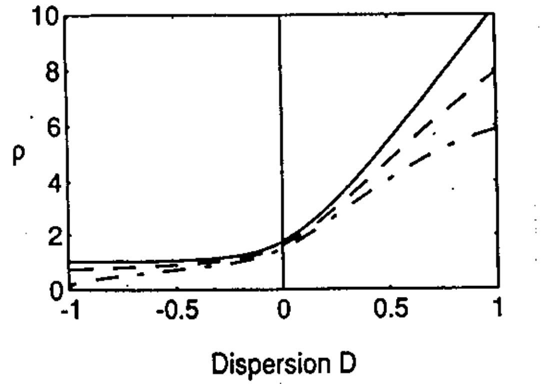
[1] C. Pare, L.Gagnon, P. A. Belanger, *Opt. Commun.* **74**, pp. 228, 1989

Pulsewidth versus GVD

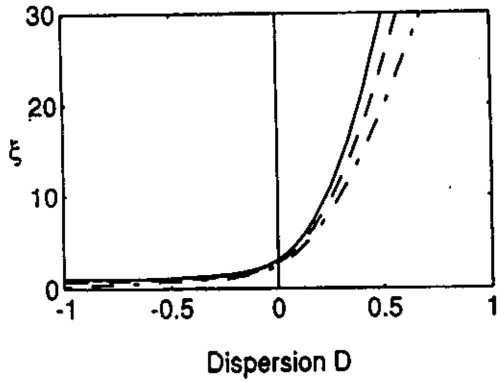




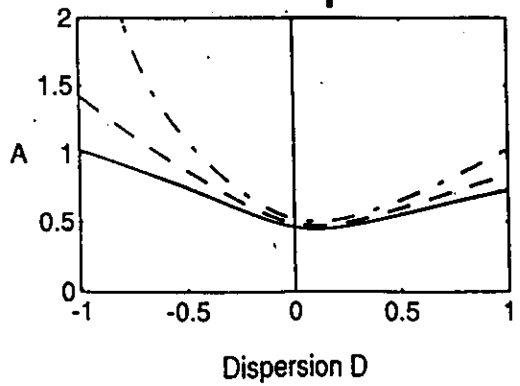
Pulse Time Width ($\delta > 0$)



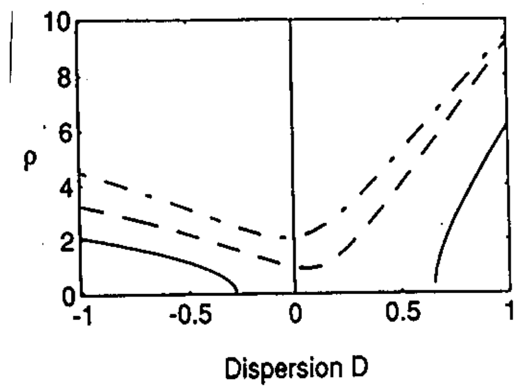
Pulse Spectral Width



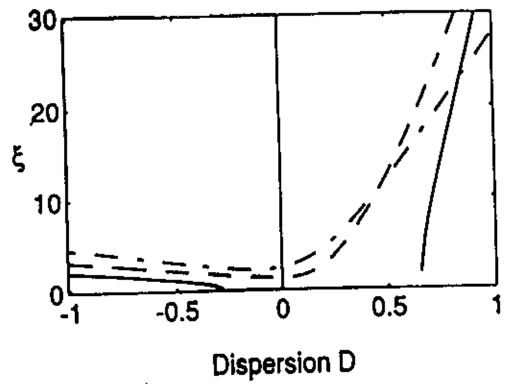
Pulse Amplitude



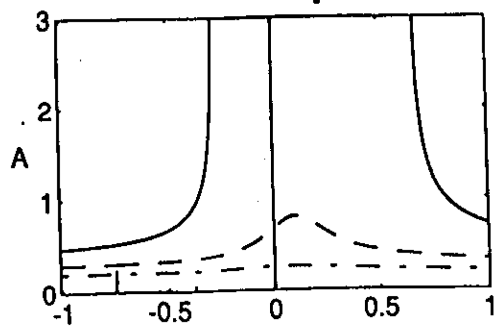
Pulse Time Width ($\delta < 0$)



Pulse Spectral Width

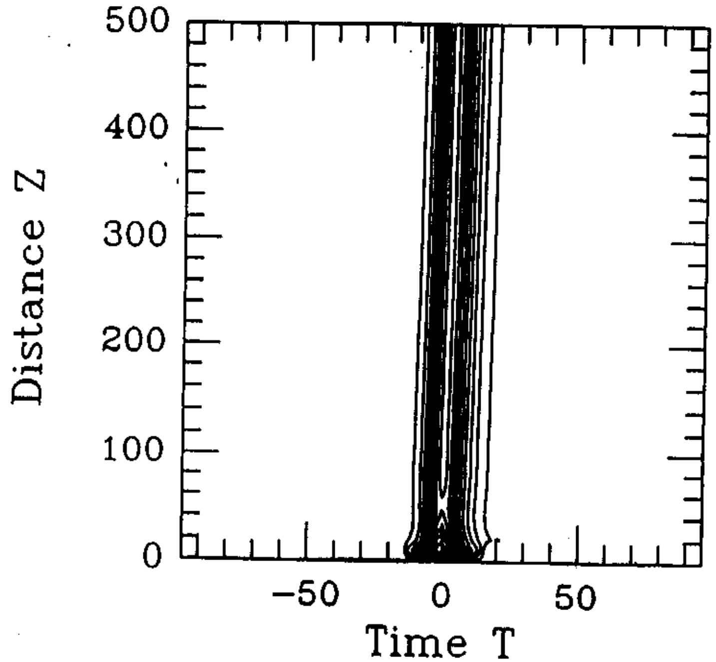


Pulse Amplitude



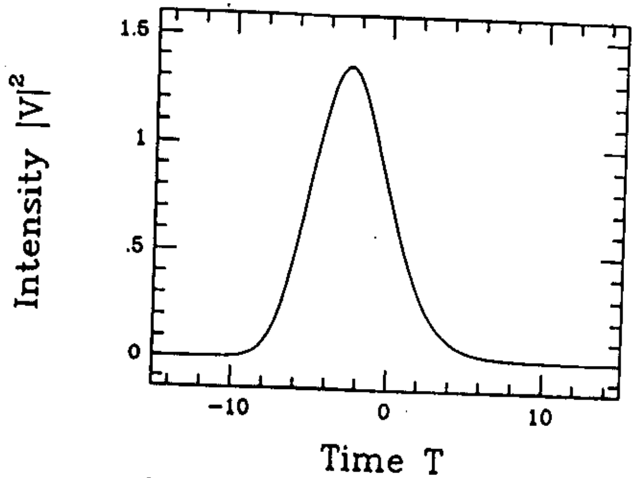
Normal dispersion SFS laser

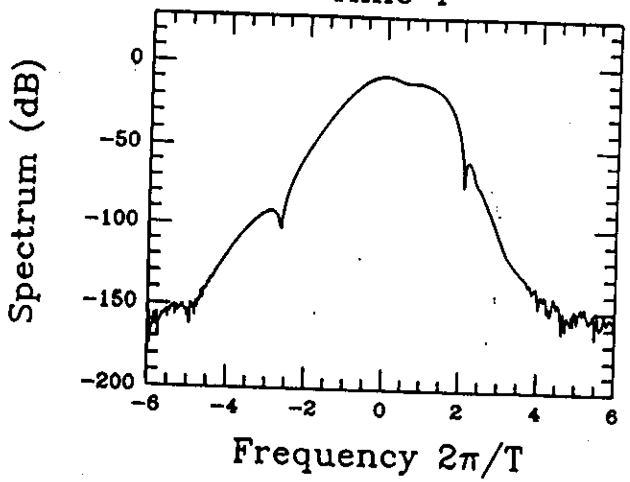
$\Delta f = 170$ MHz $\rightarrow$ AOM frequency shift
$\Delta\Omega = 2\pi\, 0.3$ THz (2.3 nm) $\rightarrow$ filter bandwidth
$Z_A = 70$ m ($2\,Z_0$) $\rightarrow$ laser length
$\beta_2 = +53$ ps^2/km $\rightarrow$ average GVD
$\tau = 1.35$ ps $\rightarrow$ time unit ($T_{Z=0} = 5\tau - T_{Z=500} = 10.5\,\tau$)



Normal dispersion

low dispersion case: $\beta_2 = +14$ ps^2/km, AOM frequency shift = 820 MHz

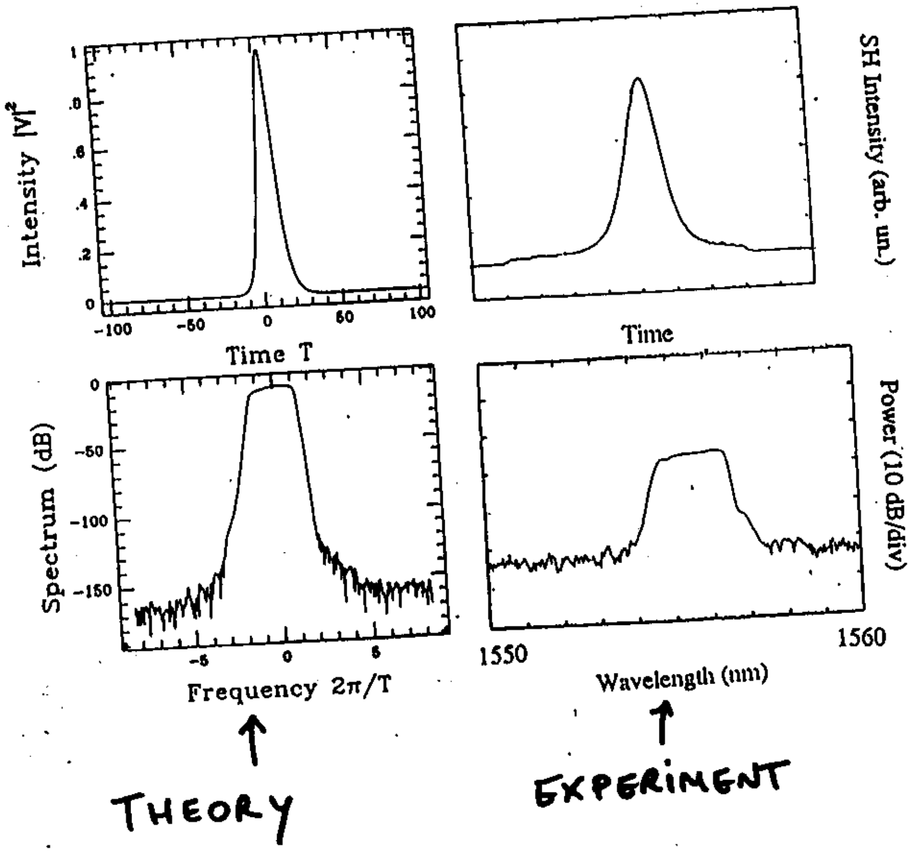




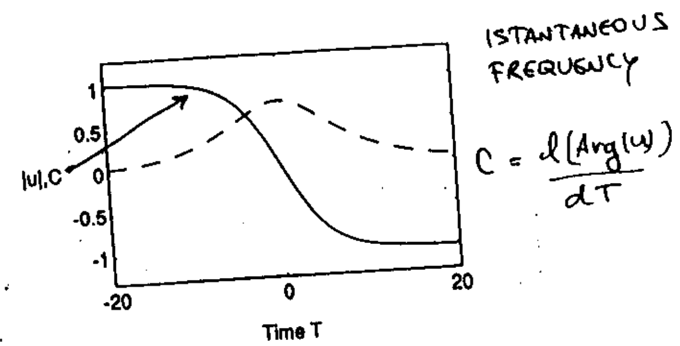
Normal dispersion SFS laser: Theory-experiment comparison

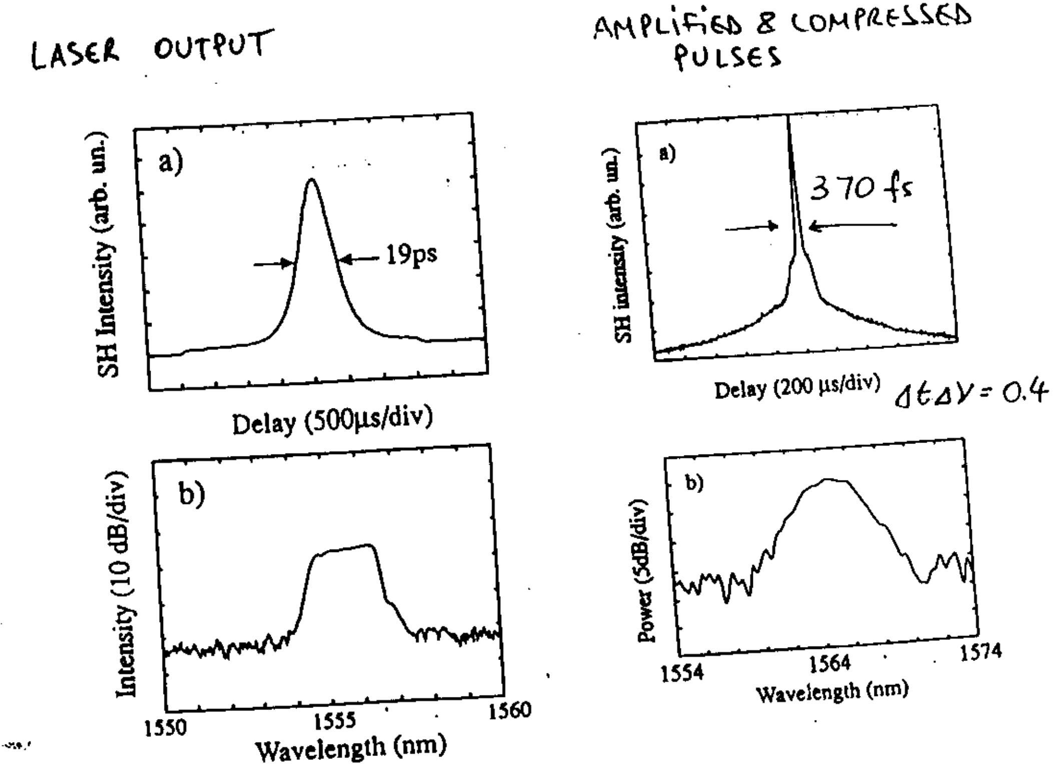
T_{fwhm} = 12 ps (experimental), 14 ps (numerical)
$\Delta\nu_{fwhm}$ = 2.2 nm (experimental), 2.2 nm (numerical)

NOTE: filter bandwidth = 2.3 nm



COMPRESSION OF CHIRPED PULSES FROM SFSL OPERATING IN THE NORMAL DISPERSION REGIME





EFFECTS OF THIRD-ORDER DISPERSION ON SOLITON PROPAGATION

One-soliton case: FREQUENCY SHIFT

$$K_1 = -2\beta_3 i \eta_1 \int_{-\infty}^{+\infty} \left\{ q_0^{(1)*} \psi_{11}^2 - q_0^{(1)} \psi_{12}^2 \right\} dT = 2\beta_3 \eta_1^2$$

TWO-SOLITON CASE: TWO DISTINCT FREQUENCY SHIFTS

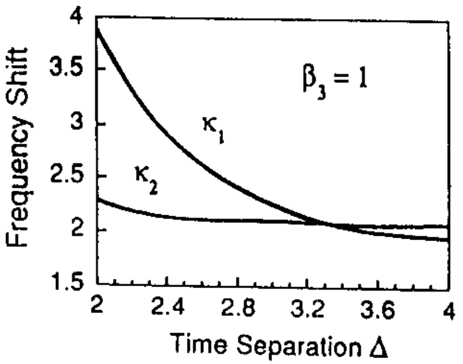


Fig. (22)

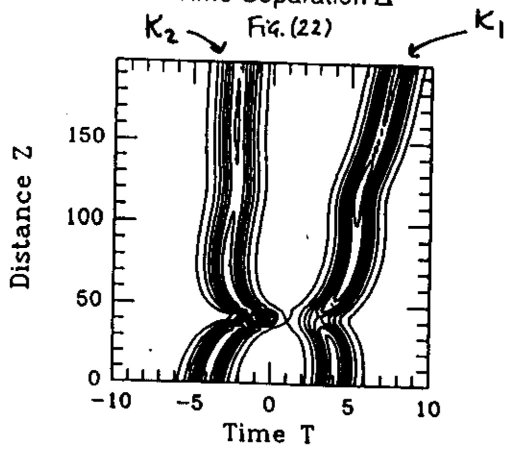


Fig. (23)

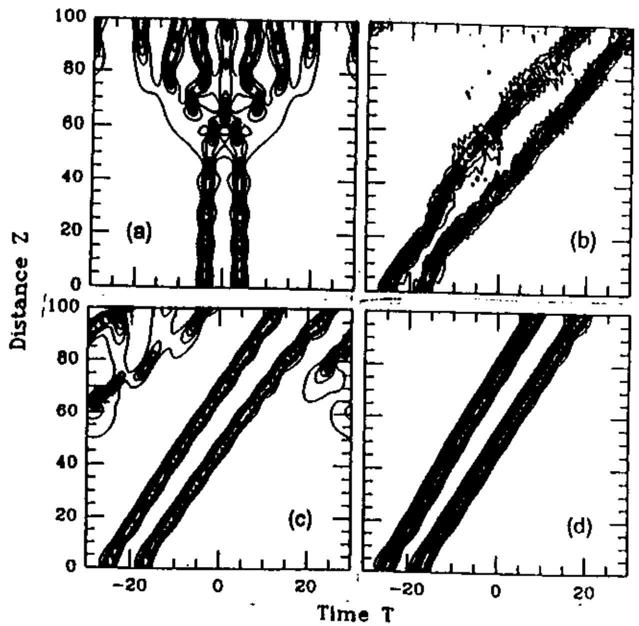
SOLITON STABILIZATION BY THIRD ORDER DISPERSION

$$P = i\delta u + i\beta u_{TT} + i\gamma |u|^2 u + i\beta_3 u_{TTT},$$

where $\beta_3 = k'''/(6|k''|t_0)$.

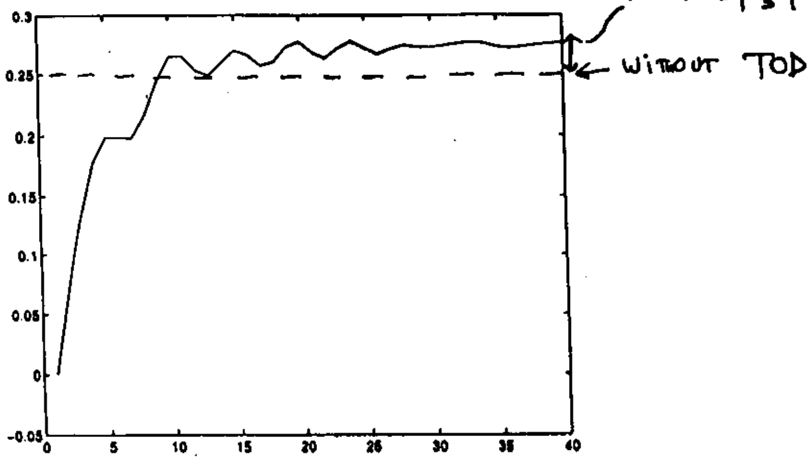
The most important consequence of TOD is an intensity dependent shift to the velocity v_s of the soliton $v_s^{-1} = \beta_3 A^2$

Radiation velocity $v_r^{-1} = 2\beta_3 A^2$ where A is the soliton amplitude.

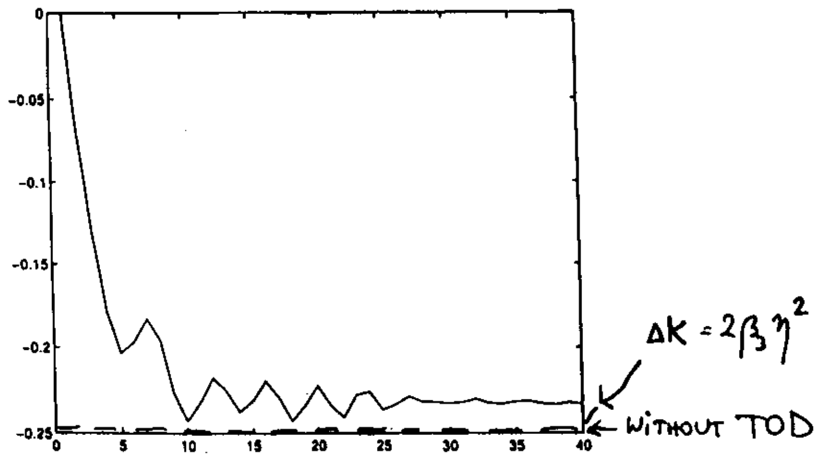


EFFECT OF THIRD ORDER DISPERSION ON SFSL OPERATING FREQUENCY (see Golovchenko et al., Opt. Lett 95)

Positive AOM frequency shift



Negative AOM frequency shift



SOLITON POLARIZATION DIVISION MULTIPLEXING

(S.G. Evangelides et al., JLT, 1992)

Othogonally Polarized Solitons $E(Z,T) =$

$$u(Z,T)\mathbf{e_1} + v(Z,T)\mathbf{e_2},$$

$$iu_Z + \frac{1}{2}u_{TT} + \left(|u|^2 + \epsilon|v|^2\right)u = P_u[u,u^*],$$
$$iv_Z + \frac{1}{2}v_{TT} + \left(|v|^2 + \epsilon|u|^2\right)v = P_v[v,v^*].$$

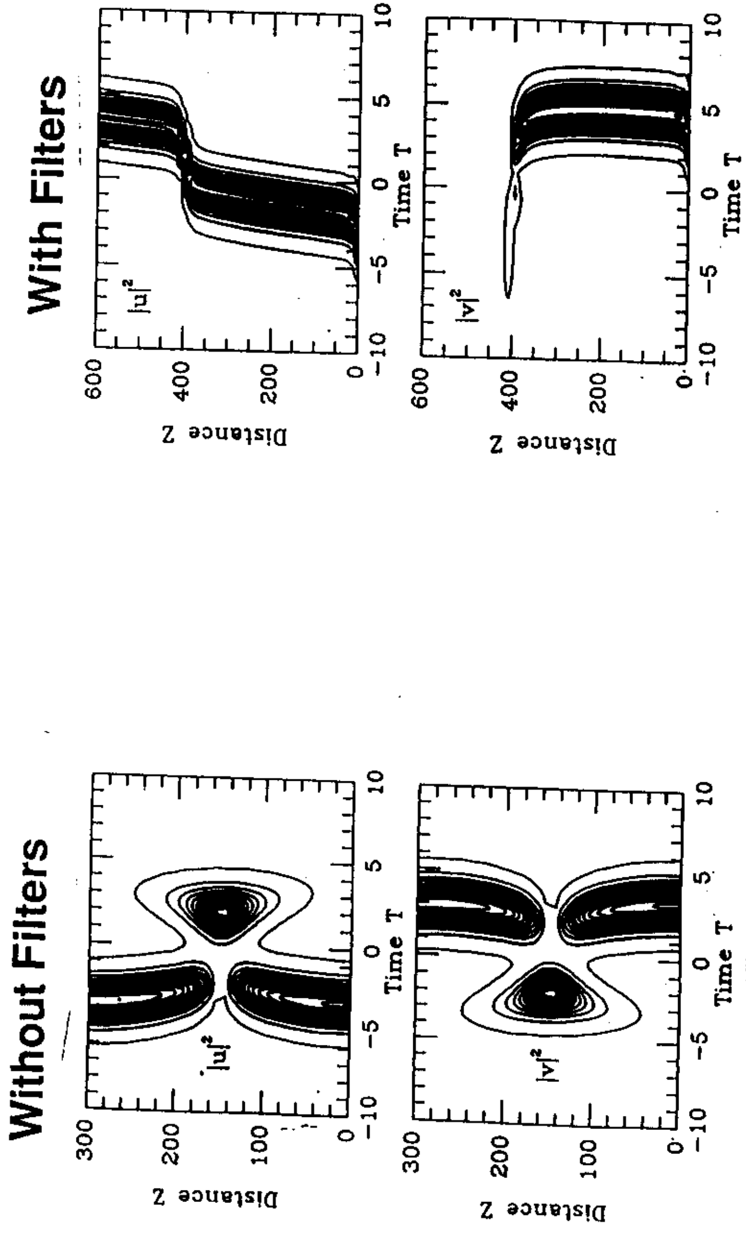
In a Sliding Frequency Filtered Transmission System

(L.F. Mollenauer et al., Opt. Lett., 1994)

$$iu_Z + \frac{1}{2}u_{TT} + \left(|u|^2 + |v|^2\right)u = i\delta u + i\beta u_{TT} - \alpha_0 T u,$$

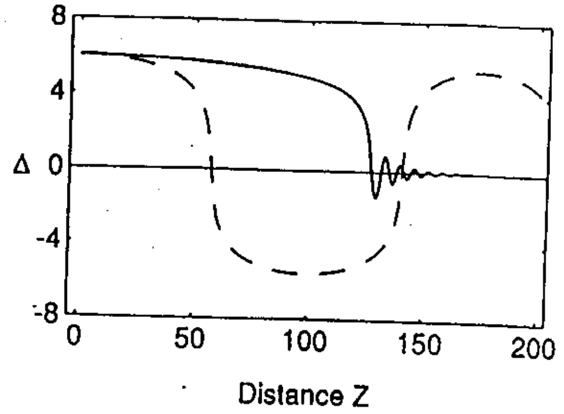
$$iv_Z + \frac{1}{2}v_{TT} + \left(|v|^2 + |u|^2\right)v = i\delta v + i\beta v_{TT} - \alpha_0 T v$$

REDUCTION OF PDM SOLITON INTERACTIONS BY SLIDING FREQUENCY FILTERING

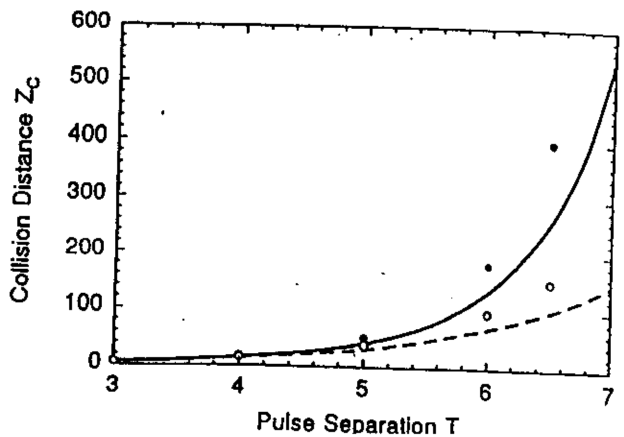


INCREASE OF PDM SOLITON COALEASCENCE DISTANCE BY SLIDING FREQUENCY FILTERING

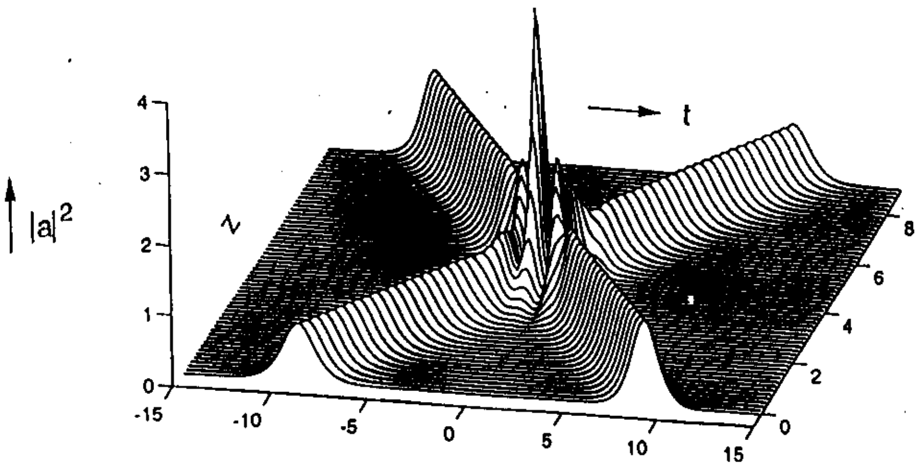
Soliton Separation Δ versus distance without --- and with — filtering



Soliton Coalescence Distance Z_C without ---- and with — filtering



SOLITON WAVELENGTH DIVISION MULTIPLEXING



$$E(Z,T) = u(Z,T)exp\{i(k_1 Z - \omega_1 T)\} + v(Z,T)exp\{i(k_2 Z - \omega_2 T)\}$$

4a

CANCELLING RESIDUAL WDM FREQUENCY SHIFTS (AND OUTPUT JITTER) BY FILTERING

$$iu_Z + \frac{1}{2}u_{TT} + a(Z)^2\left[(|u|^2 + 2|v|^2)u + u^* v^2 e^{i\Omega}\right] = i\delta_1 u + i\beta_1 u_{TT},$$

$$iv_Z + i\kappa v_T + \frac{1}{2}v_{TT} + a(Z)^2\left[(|v|^2 + 2|u|^2)v + v^* u^2 e^{-i\Omega}\right] = i\delta_2 v + i\beta_2 v_{TT}.$$

where

$$\kappa = \omega_2 - \omega_1 \qquad \Omega = 2((\omega_2 - \omega_1)T - (k_2 - k_1)Z).$$

Note: without CW suppression, collisions generate FWM sidebands at the frequencies

$$\omega_{nm} = \omega_n + 2m(\omega_2 - \omega_1) \qquad m = 0, \pm 1, \pm 2, \ldots$$

4b

collisions

one pulse at ω_1

$$u(Z,T) = \eta_1 sech(\eta_1(T-\xi_1))exp[-i\kappa_1(T-\xi_1)+\psi_1]$$

N-1 pulses at ω_2

$$v(Z,T) = \sum_{j=2}^{N} \rho_j \eta_j sech(\eta_j(T-\xi_j))exp[-i\kappa_j(T-\xi_j)+\psi_j],$$

$\{\rho_i\}$ is a pseudorandom pattern of zeroes and ones

Soliton perturbation theory assuming $\eta_i = \eta$

$$\frac{d\kappa_1}{dZ} = \alpha_0 - 4\eta^3 a(Z)^2 \sum_{j=2}^{N} \rho_j g[\eta(\xi_1-\xi_j)] - \frac{4}{3}\beta\kappa_1\eta^2$$

$$\frac{d\kappa_j}{dZ} = \alpha_0 + 4\eta^3 a(Z)^2 \rho_j g[\eta(\xi_1-\xi_j)] - \frac{4}{3}\beta\kappa_j\eta^2, \ j \geq 2$$

$$\frac{d\xi_1}{dZ} = -\kappa_1,$$

$$\frac{d\xi_j}{dZ} = -\kappa_j + \kappa, \ j \geq 2,$$

$$\frac{d\eta}{dZ} = 2\delta\eta - \frac{2}{3}\beta\eta^3.$$

$$g(x) = (3sinh(x)cosh(x) - x(1+2cosh^2(x)))/sinh^4(x)$$

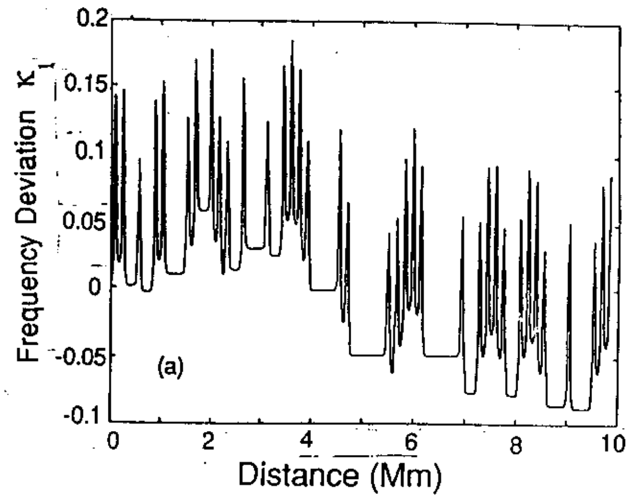
L. F. Mollenauer et al. JLT 9, 1991, p. 362

we set $\kappa_j(Z=0) = 0$, $\xi_1(Z=0) = 0$, $\xi_j(Z=0) = T_j$, $j \geq 2$, and $\eta(Z=0) = 1$.

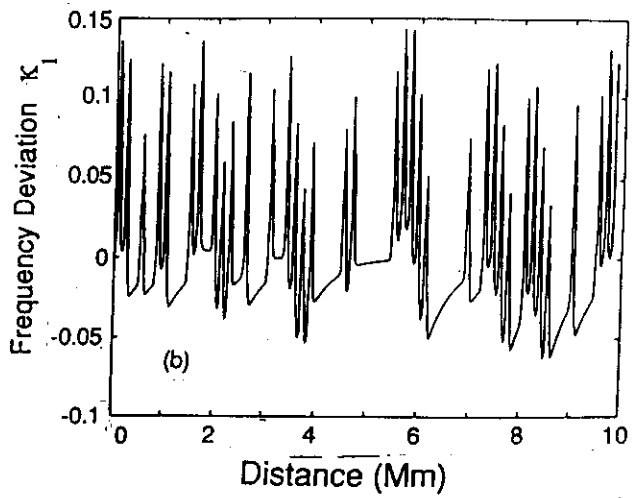
5c

FREQUENCY DEVIATIONS IN CHANNEL AT $\omega 1$ FROM THE COLLISIONS WITH A RANDOM PULSE STREAM IN CHANNEL AT $\omega 2$

Without Filtering



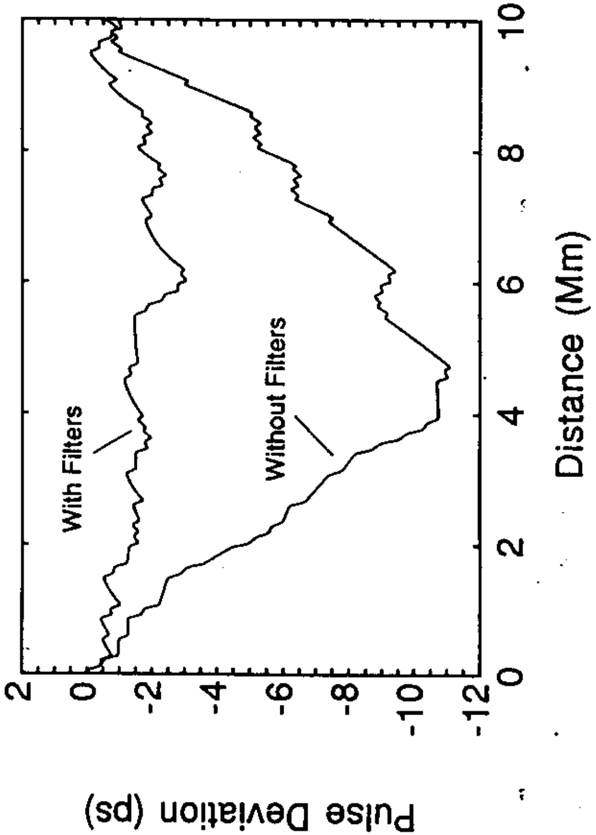
With Filtering

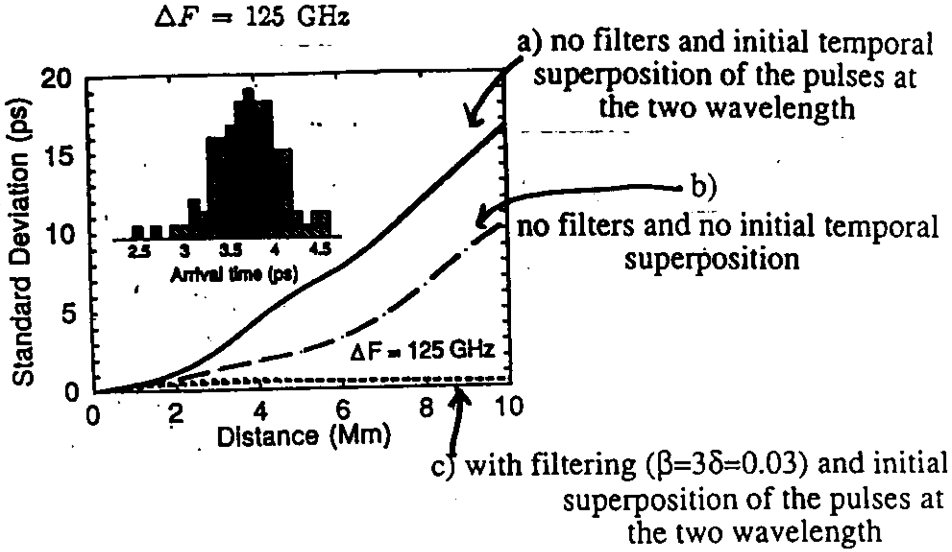


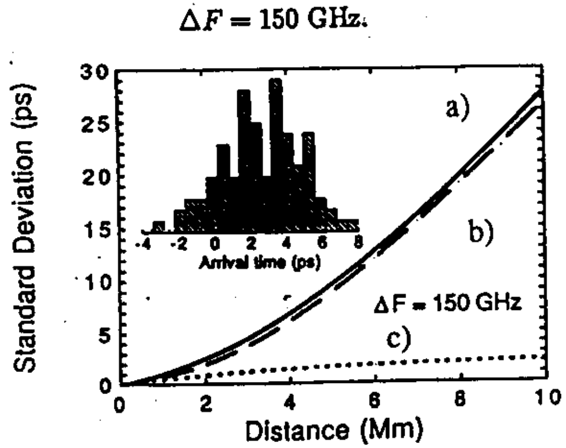
5d

FILTERING SUPPRESSES THE TEMPORAL JITTER DUE TO WDM SOLITON COLLISIONS WITH AMPLIFIER SPACING CLOSE TO THE COLLISION LENGTH

Random Walk of Soliton Position



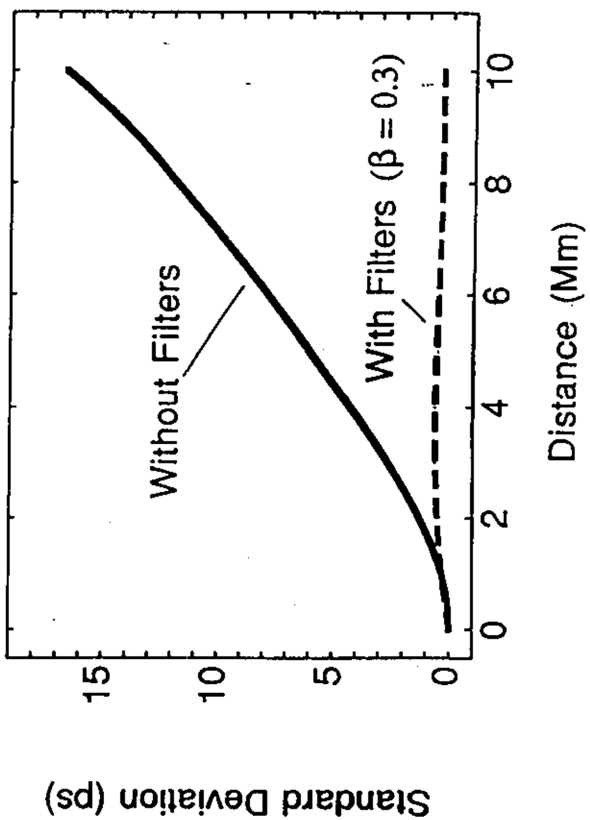




Sliding filters do not reduce the time jitter with respect the "fixed filters", but they limit the soliton instability induced by the periodical amplification and hence permit the use of narrower filters.

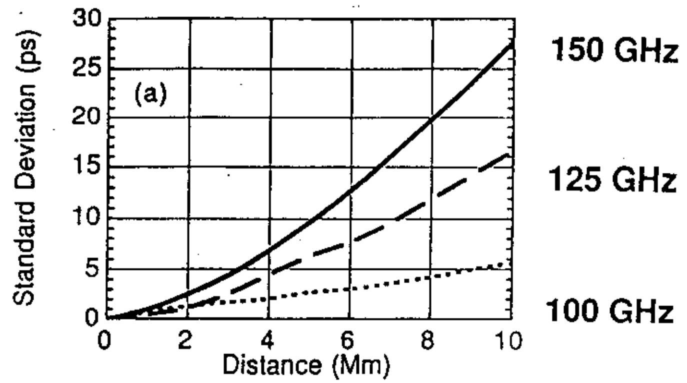
FILTERING SUPPRESSES THE TEMPORAL JITTER DUE TO WDM SOLITON COLLISIONS WITH AMPLIFIER SPACING CLOSE TO THE COLLISION LENGTH

Reduction of Output Time Jitter with Channel Separation of 125 GHz



DEPENDENCE OF WDM OUTPUT TIME JITTER ON FREQUENCY SEPARATION BETWEEN CHANNELS

Without Filtering



With Filtering