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Amplifier Noise and Kerr Nonlinearity in Long-Haul Tansmission Systems Operating Close to Zero Dispersion

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Noise and nonlinearity in long-haul transmission systems (operating close to zero dispersion)

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OUTLINE

- Long distance communications: soliton vs. "linear"
- ASE noise + Kerr nonlinearity
- ASK, PSK
- Dispersion, polarization
- Rare earth vs. semiconductor amplifiers
- Semiconductor amplifier nonlinearity
- Solitons or zero dispersion?

Solitons vs. “linear” transmission

Historically:

Solitons: **nonlinearity** used to compensate **dispersion**

Now: dispersion shifted fibers are available

Why do we need solitons?

Solitons: **dispersion** used to compensate **nonlinearity**

Self phase modulation (Kerr nonlinearity)

$$E(0,t) \xrightarrow{z} E(0,t)\exp(i\Delta\phi)$$

$$\Delta\phi = \delta |E(0,t)|^2 z$$

$$\delta = \frac{2\pi n_2}{\lambda A_{\text{eff}}}$$

n_2 nonlinear index

A_{eff} effective area of the fiber

λ free space wavelength

(c)

Fiber loss

0.2 dB/km @ 1.5 μm 3 dB/15 km

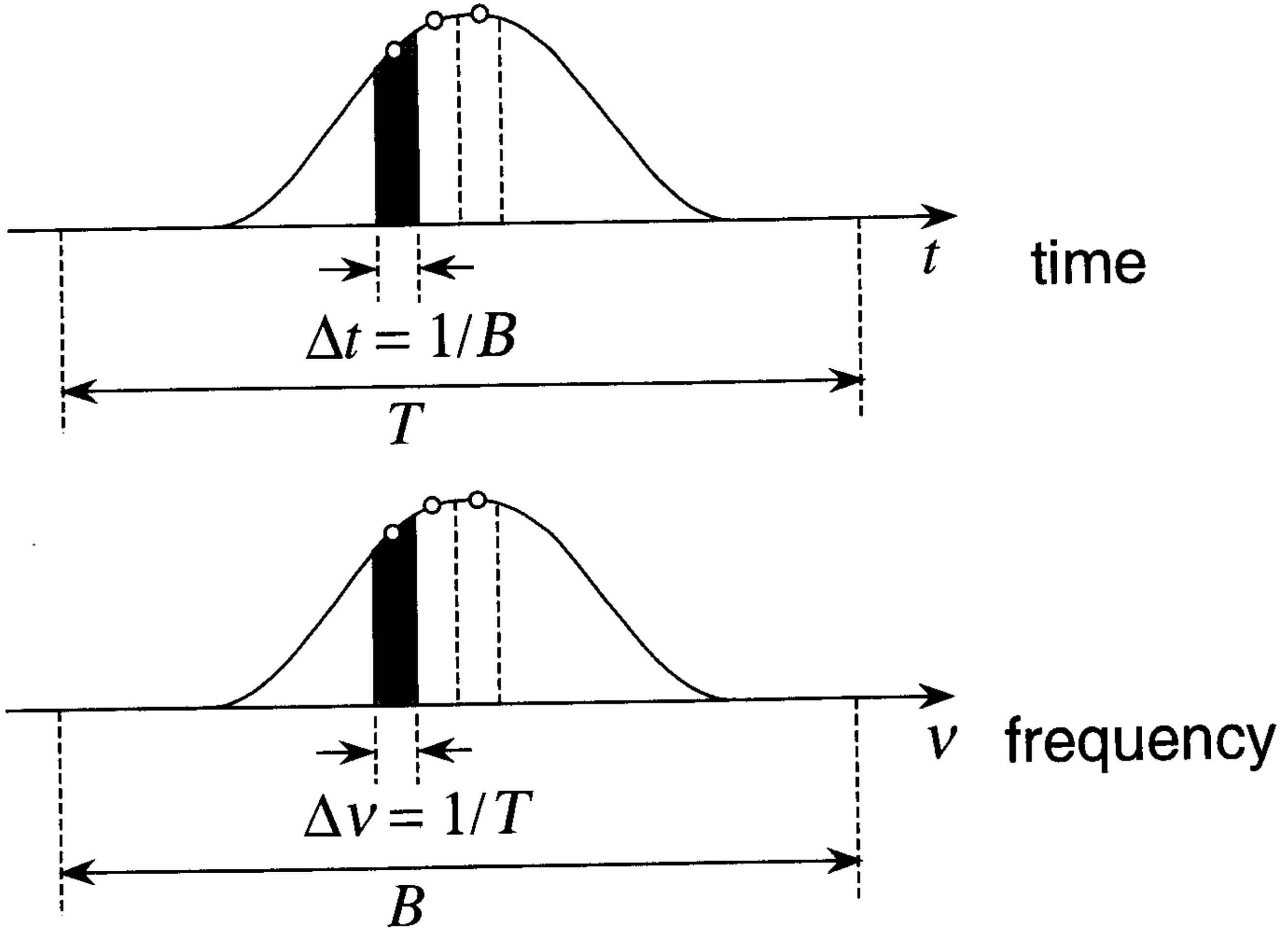
0.4 dB/km @ 1.3 μm 3 dB/7.5 km

Amplifiers are required

$\Downarrow$

Ase noise

Degrees of freedom of a pulse - time and frequency



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Degrees of freedom of a pulse 1

Pulse of bandwidth B and duration T

Defined by the value of the (complex) field at $m = B\,T$ points

Number of degrees of freedom: $2\,m = 2\,B\,T$

Each degree of freedom is an independent mode of the field

Degrees of freedom of a pulse 2

$E(t)$ is normalized such that $|E(t)|^2$ is the power in Watts

Energy contained in each mode $\varepsilon = \Delta t\,|E(t_i)|^2 = \dfrac{1}{B}\,|E(t_i)|^2$

Number of photons in each mode $\dfrac{\varepsilon}{\hbar\omega_0} = \dfrac{1}{B\hbar\omega_0}\,|E(t_i)|^2$

$a_i = \dfrac{\sqrt{\Delta t}\,E(t_i)}{\sqrt{\hbar\omega_0}} = \dfrac{E(t_i)}{\sqrt{B\hbar\omega_0}}$ $a_i^\dagger a_i$ number of photons

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ASE noise

$$b_i = G^{1/2} a_i + (G-1)^{1/2} c_i^\dagger$$ Haus and Mullen, '63

$$\langle c^\dagger c \rangle = 1$$

The noise power added, scaled to the field $E(t)$ is

$$\left\langle |n(t)|^2 \right\rangle = B\,\hbar\omega_0\,(G-1)$$

at each of the m independent points of the field

THE ASE NOISE CANNOT BE AVOIDED

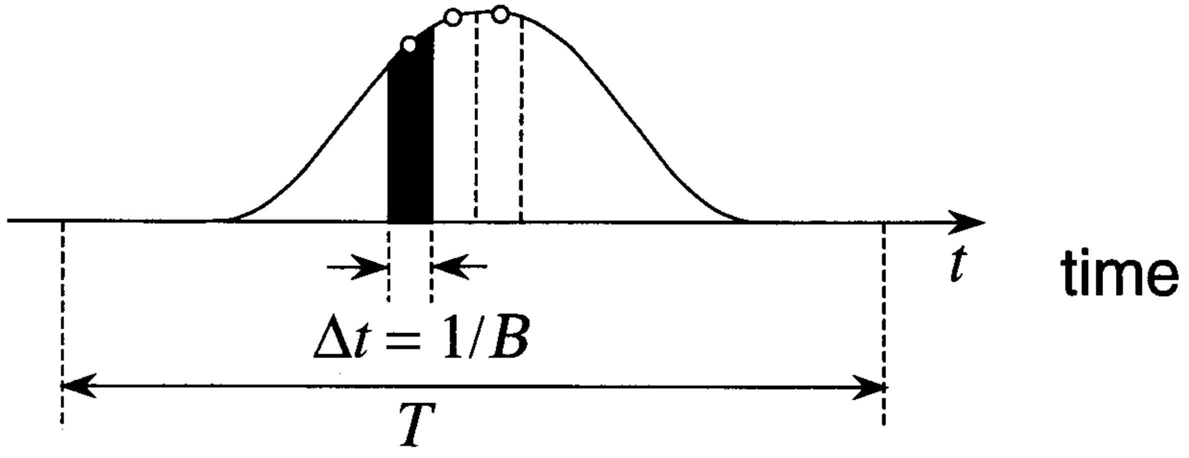
It is the consequence of:

a) the linearity and phase independence of the amplification

b) the preservation of the commutation rules, i.e. of the bosonic character of the field

H. A. Haus and J. R. Mullen, Phys. Rev. **128**, 2407 (1963)

Zero dispersion



The independent “slots” of the field travel independently

Intensity noise

Kerr nonlinearity does not affect the intensity

$$\left\langle |E(z,t)|^2 \right\rangle = |E(0,t)|^2 + K_{\text{noise}} z$$

$$K_{\text{noise}} = \hbar \omega_0 \, \theta_a \frac{G-1}{z_{\text{amp}}} B$$ ASE power per unit length

θ_a inversion factor of the amplifier

Self-phase modulation induced phase noise

The signal-ASE beating induces intensity noise

$$|E(t)+n(t)|^2 \cong |E(t)|^2 + E^*(t)n(t) + E(t)n^*(t)$$

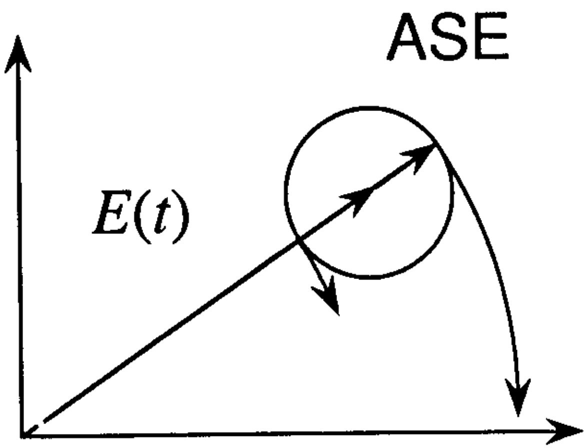
that couples with self-phase modulation to give phase noise

$$\Delta\phi(z) \cong \delta z\left[E^*(t)n(t) + E(t)n^*(t)\right]$$

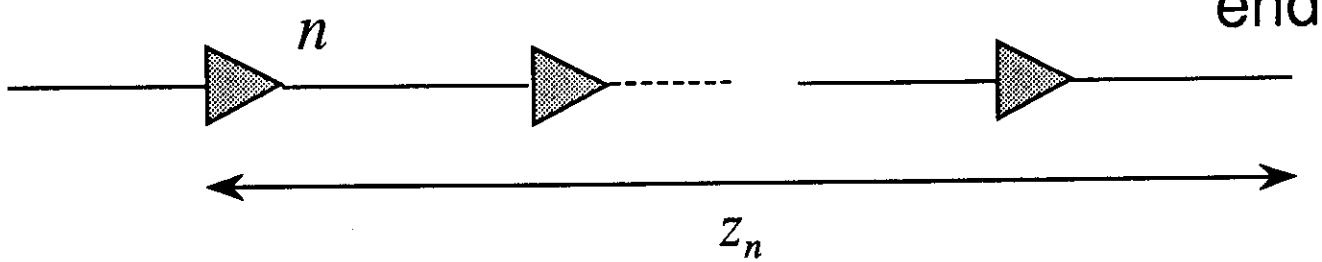
(Gordon and Mollenauer, Optics Letters <u>15</u>, 1351, (1990)

Phase noise 1

The signal-ASE beating increases phase noise



$$\langle E(t,z)\rangle \approx E(t,0)\underbrace{\left\langle \exp\left[i\sum_n \Delta\phi_n(z_n)\right]\right\rangle}_{\exp\left[-\frac{1}{2}\left\langle\left[\sum_n \Delta\phi_n(z_n)\right]^2\right\rangle\right]}$$



Phase noise 2

$$\langle E(t,z)\rangle = E(t,0)\exp\left(-\frac{z^3}{z_{\text{noise}}^3}\right)$$

$$\frac{1}{z_{\text{noise}}^3} = \frac{(\delta r)^2}{3} K_{\text{noise}} |E(z,t)|^2$$ length of "dephasing"

$$K_{\text{noise}} = \hbar\omega_0\,\theta_a \frac{G-1}{z_{\text{amp}}} B$$ ASE power per unit length

$$r = \frac{1-\exp(-2\gamma z_{\text{amp}})}{2\gamma z_{\text{amp}}}$$ path average coefficient

γ loss per unit length

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Signal spectrum 1

$$\left\langle \left| \tilde{E}(z,\omega) \right|^2 \right\rangle = \frac{1}{T_0} \int dt \int dt' \left\langle E(z,t) E^*(z,t') \right\rangle \exp[i(t-t')\omega]$$

$$dt \longrightarrow \Delta t = \frac{1}{B}$$

$$\left\langle \left| \tilde{E}(z,\omega) \right|^2 \right\rangle = \frac{\Delta t}{T_0} \int dt \left\langle \left| E(z,t) \right|^2 \right\rangle$$

$$+ \frac{1}{T_0} \int dt \int dt' \left\langle E(z,t) \right\rangle \left\langle E^*(z,t') \right\rangle \exp[i(t-t')\omega] \Big|_{t \neq t'}$$

Signal spectrum 2

$$\left\langle \left|\tilde{E}(z,\omega)\right|^2 \right\rangle = \frac{1}{BT_0}\int dt\left[\left\langle \left|E(z,t)\right|^2 \right\rangle - \left|\left\langle E(z,t)\right\rangle\right|^2\right]$$

$$+\frac{1}{T_0}\int dt\int dt' \left\langle E(z,t)\right\rangle \left\langle E^*(z,t')\right\rangle \exp[i(t-t')\omega]$$

$$\left\langle \left|E(z,t)\right|^2 \right\rangle = \left|E(0,t)\right|^2 + K_{\text{noise}} z$$

$$\left\langle \left|\tilde{E}(z,\omega)\right|^2 \right\rangle = \frac{1}{BT_0}\int dt\left[\left|E(0,t)\right|^2 + K_{\text{noise}} z - \left|E(0,t)\right|^2 \exp\left(-2\frac{z^3}{z_{\text{noise}}^3}\right)\right]$$

$$+\frac{1}{T_0}\exp\left(-2\frac{z^3}{z_{\text{noise}}^3}\right)\int dt\int dt' E(0,t)E^*(0,t')\exp[i(t-t')\omega]$$

A. M., JOSA B 11, 462 (1994); EL. LETT. 29, 2136 (1993)

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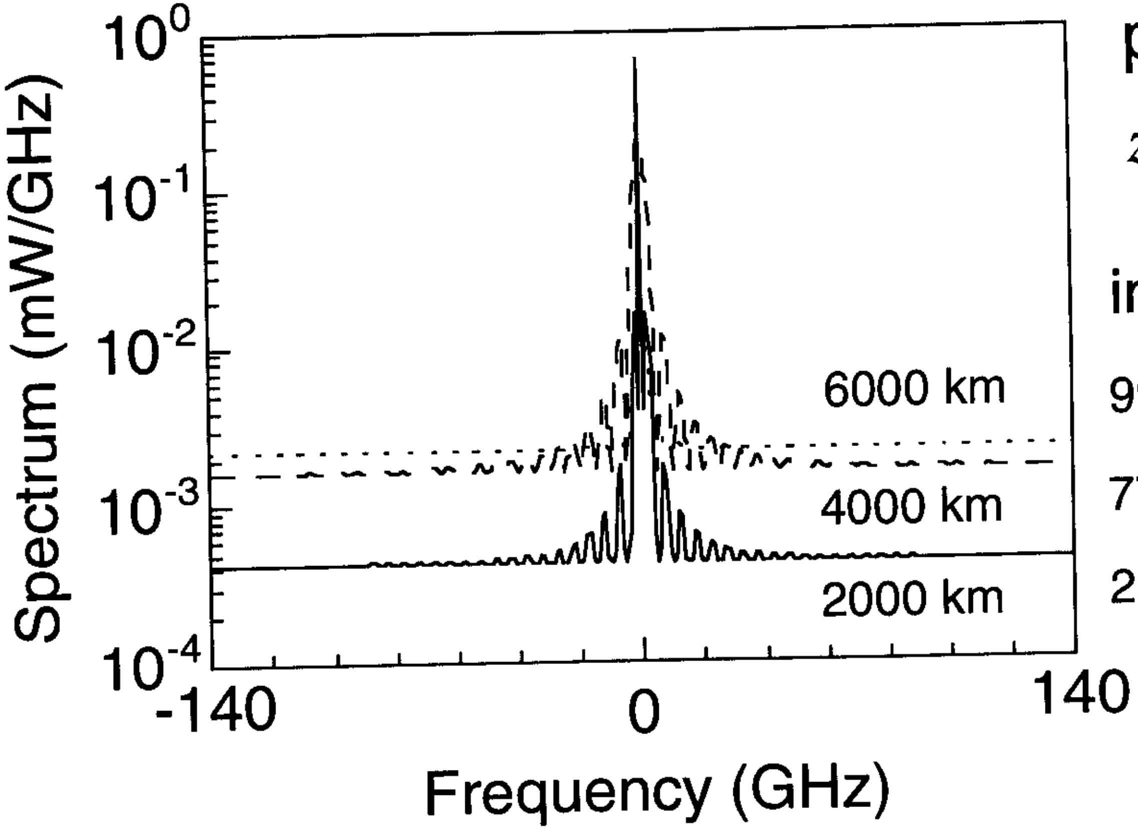
Rise of the phase-noise background



power 0.5 mW

z_{noise} = 4600 km

in the background of power

99 %

77 %

22 %

$$\exp\left(-2\frac{z^3}{z_{\text{noise}}^3}\right)$$

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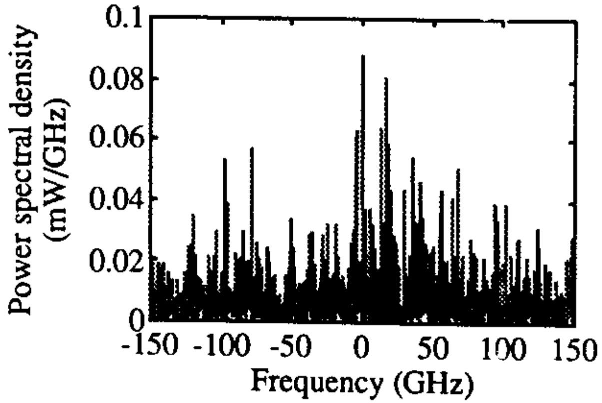
1 mW signal, $\beta'' = 0$, R = 5 GBit/s, amp. spacing 60 km

9000 km



(simulation by Francesco Matera)

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Noise Background 1

It is phase noise: the performance of amplitude modulated system is not affected if a square law detector is used.

$$\left\langle |E(z,t)|^2 \right\rangle = |E(0,t)|^2 + K_{\text{noise}} z$$

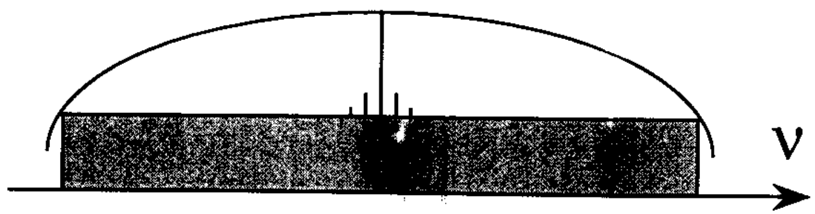
The intensity noise is the same of a linear system, with ASE noise and without Kerr nonlinearity

Noise background 2

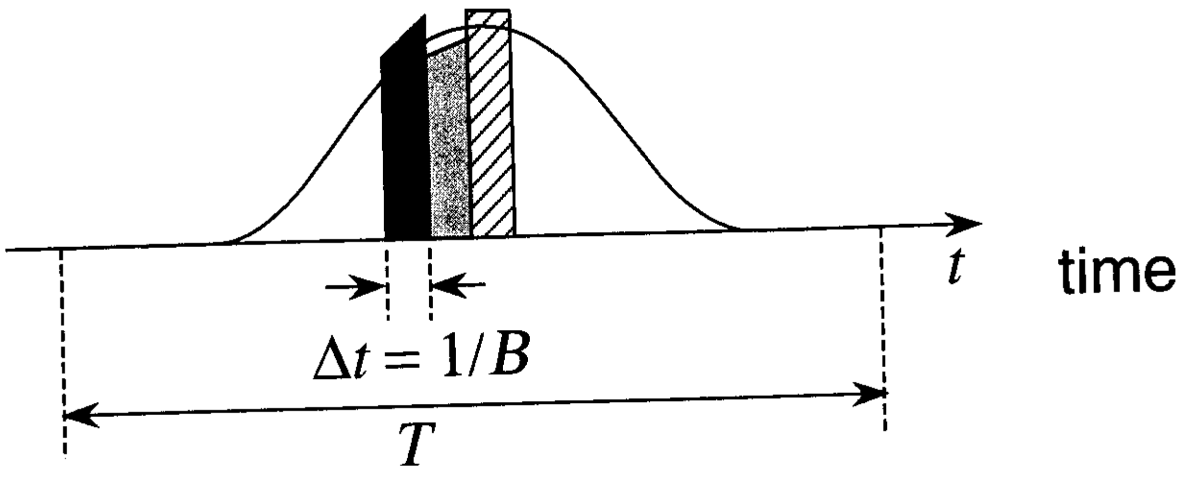
Indirect effects of the noise background

a) If narrow optical filters are used before detection, the received power decreases and the phase noise induces amplitude noise. Larger (linear) ASE noise at receiver.

b) The limited bandwidth of the in line elements (Erbium gain, for instance) produces loss of signal power in the wings of the broadened spectrum.

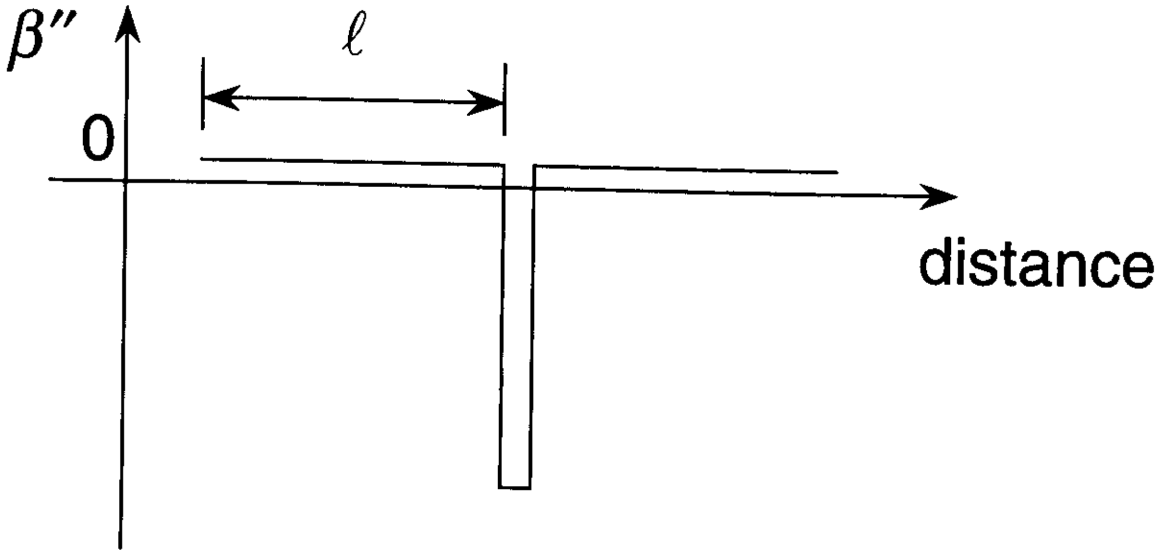


Zero dispersion



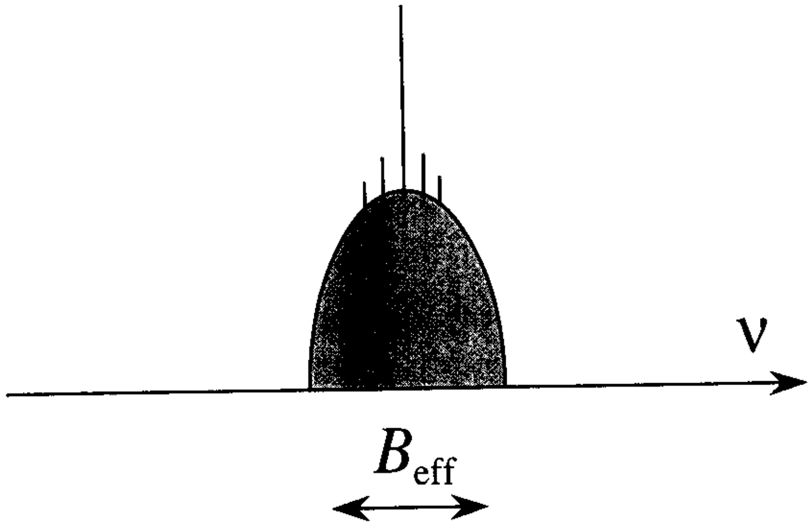
Intensity fluctuations do not “average” during propagation

Dispersion management



E. Lichtman and S.G. Evangelides, El. Lett. 30, 346 (1994)

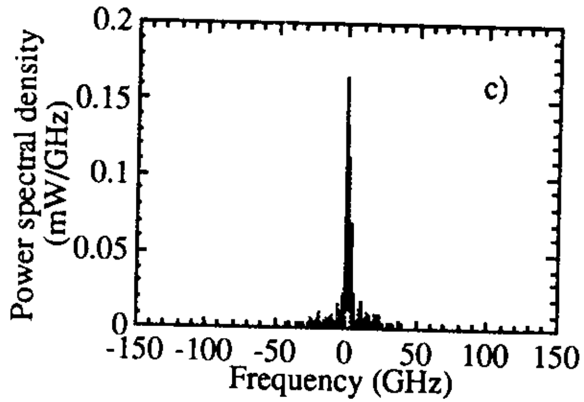
Dispersion 6



The link supports intensity fluctuations with bandwidth less than B_{eff}. Fluctuations with wider bandwidth are averaged out

1 mW signal, $\beta'' = -20\ ps^2/km$ or $+1\ ps^2/km$,

R = 5 GBit/s, amp. spacing 60 km, 9000 km

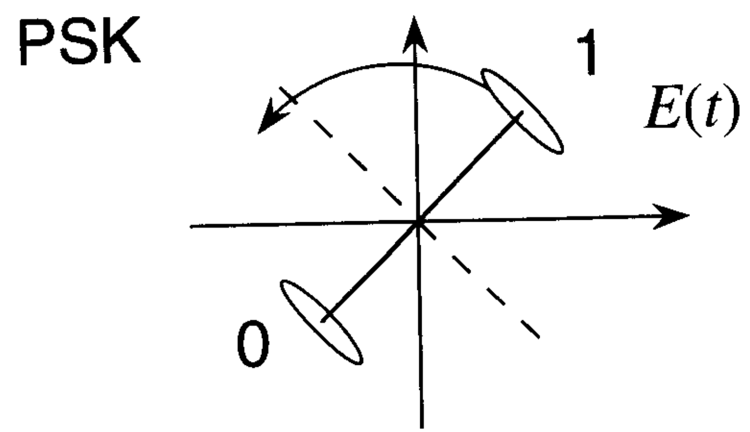


(simulation by Francesco Matera)

Phase-shift keying modulation format

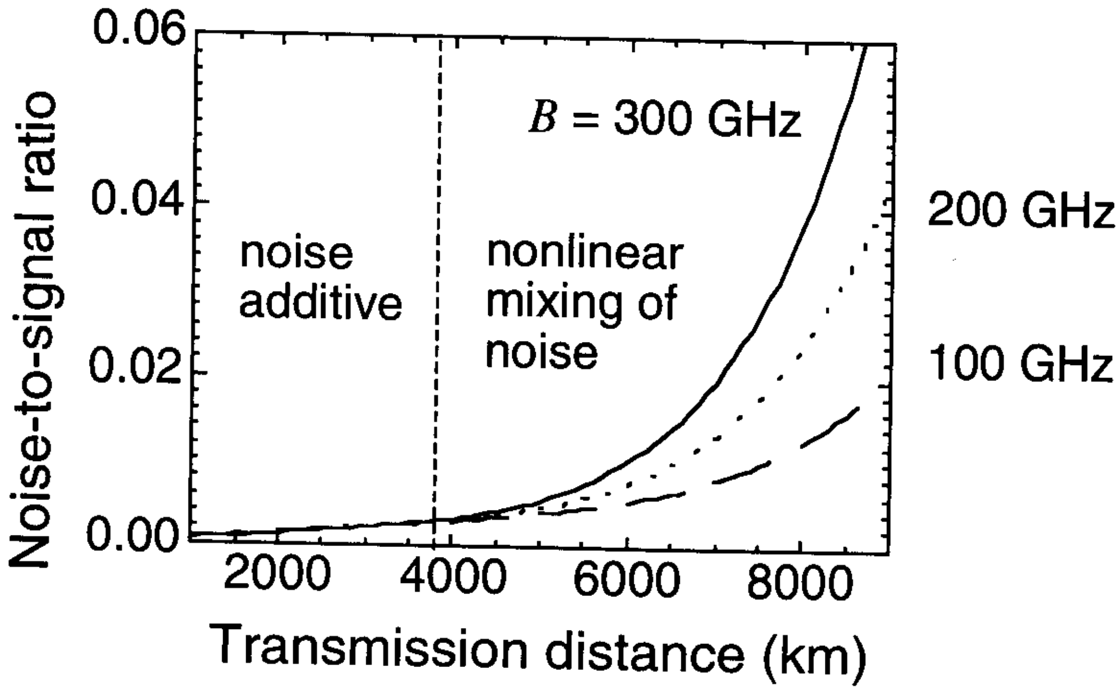
Intensity modulation is (relatively) unaffected by the broadening of the spectrum (phase noise)

Phase modulation is affected



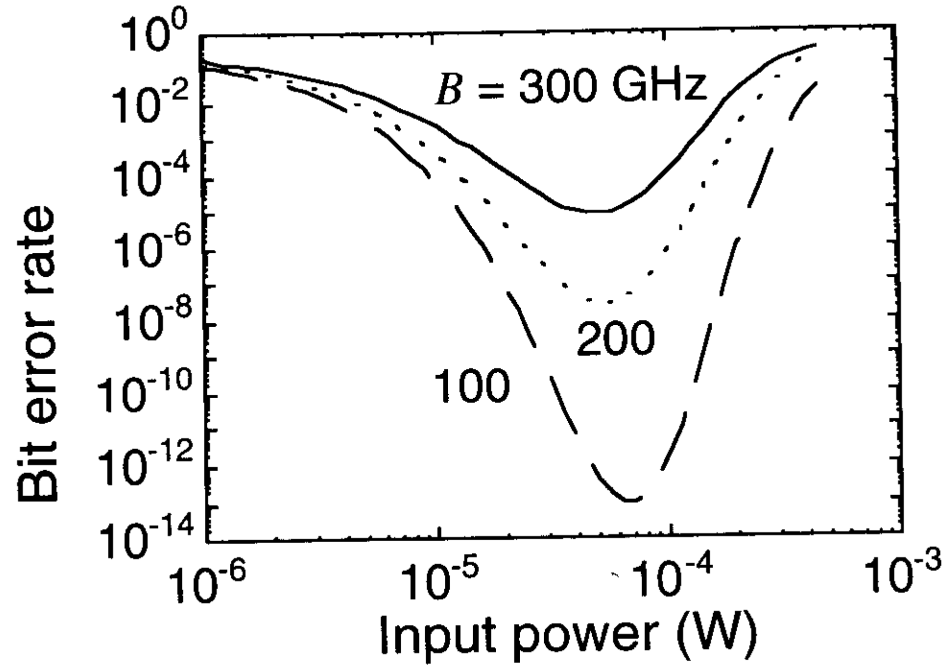
Noise-to-signal ratio for a PSK signal

Input power = 0.1 mW, 5 GBit/s



Bit error rate with PSK transmission

9,000 km, 5 GBit/s



A.M., Journal of Lightwave Technol. 11, 1993 (1994)

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Semiconductor amplifiers

Cheaper than fiber amplifiers

Pumped electrically

Cover wavelength range from 0.7 to 1.6 μm

Problems:

Lifetime ~ 200 ps $\Rightarrow$ Highly nonlinear, chirp, pattern effects

High coupling loss

Upgrading problem

Technological problem:

10 GBit/s over 1000 km of step-index fibers:

λ_0 = 1300 nm

loss = 0.4 dB/km

Possible solution: semiconductor amplifiers @ 1300 nm

Total ASE noise

1000 km, 0.4 dB/km, 40 km amplifier spacing

$B_{\text{detection}} = 20$ GHz

Bit rate = 10 GHz

$\theta_a = 10$

$\lambda = 1300\ \mu$m

Total noise power at detection = 30 μW

Noise background

1000 km, 0.4 dB/km, 40 km amplifier spacing

Bit rate = 10 GHz $\theta_a = 10$ $\delta = 2.7\ 10^{-3}$ 1/(mW km)

$\lambda = 1300\ \mu$m B = 300 GHz

power at the amplifier output = 10 mW

z_{noise} = 1100 km

To avoid a large noise background: work detuned from λ_0

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Rate equation for carriers in semiconductor amplifiers

$$\frac{dn}{dt} = -\frac{n}{\tau_s} - a(n - n_0)\frac{|E(z,t)|^2}{A_{\text{eff}}\,\hbar\omega_0} + \frac{J}{ed}$$

n	carrier density (cm^{-3})	e	electron charge
n_0	transparency carrier density	τ_s	spontaneous lifetime
d	stripe height	A_{eff}	area of the optical mode
a	differential gain (cm^2)	$|E(z,t)|^2$	optical power (W)
J	current density (Ampere/cm^2)		

Gain compression and phase shift (to first order)

$$\Delta g(t) = -\left(\frac{\bar{g}}{\bar{g}-\gamma_{sc}}\right)\left(1-\frac{1}{G}\right)\int_{-\infty}^{t}\frac{dt'}{\tau_s}e^{-\frac{t-t'}{\tau_s}}\times\frac{I^{(0)}(L,t')}{P_s}$$

$$\Delta\phi(t) = \frac{\alpha}{2}\left(\frac{\bar{g}}{\bar{g}-\gamma_{sc}}\right)\left(1-\frac{1}{G}\right)\int_{-\infty}^{t}\frac{dt'}{\tau_s}e^{-\frac{t-t'}{\tau_s}}\times\frac{I^{(0)}(L,t')}{P_s}$$

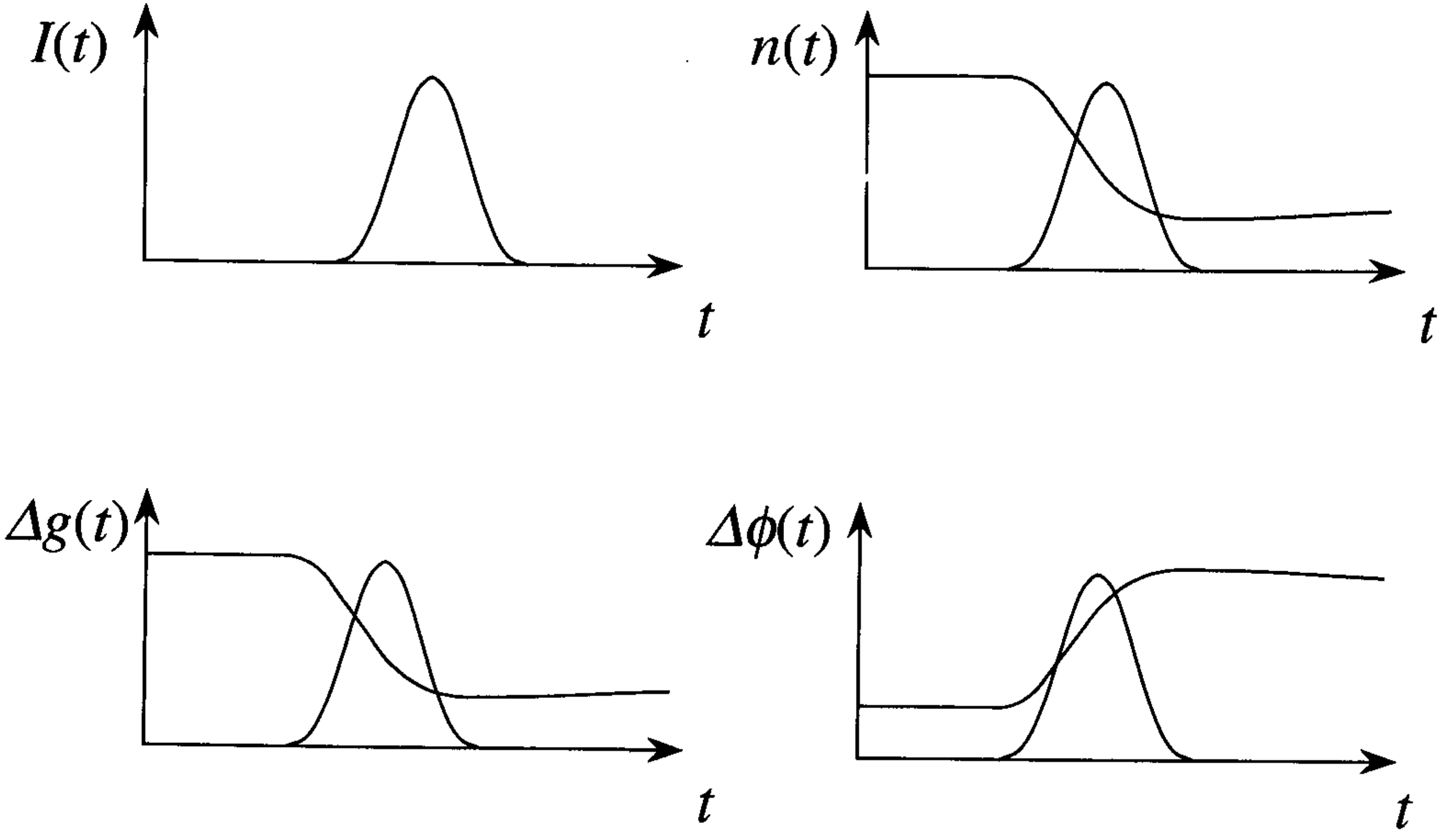
$G = e^{(-\gamma_{sc}+\bar{g})L}$ small signal gain α Henry's factor

$P_s = \frac{A_{eff}\hbar\omega_0}{a\tau_s}$ Saturation power γ_{sc} waveguide loss

$\bar{g}$ small signal gain coefficient

Gain compression and phase shift

pulsewidth << τ_s



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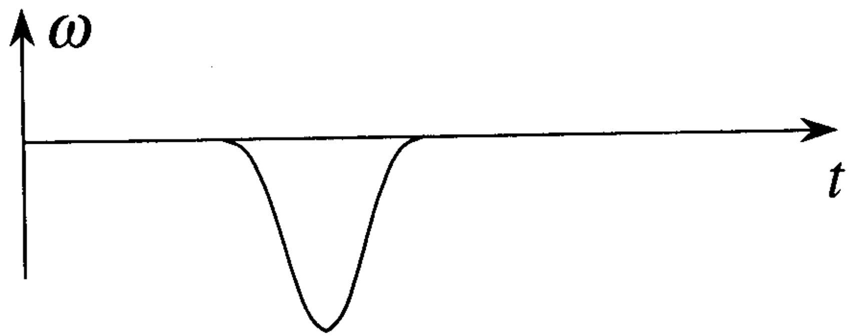
Saturation induced chirp

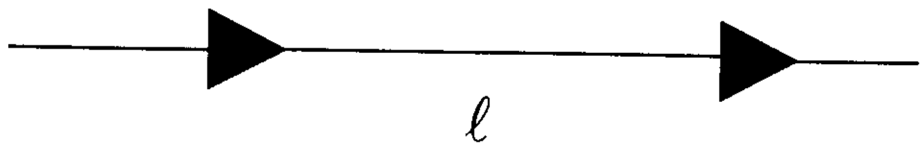
$$\Delta\phi(L,t) = \frac{\alpha}{2}\left(\frac{\bar{g}}{\bar{g}-\gamma_{sc}}\right)\left(1-\frac{1}{G}\right)\int_{-\infty}^{t}\frac{dt'}{\tau_s}e^{-\frac{t-t'}{\tau_s}} \times \frac{I^{(0)}(L,t')}{P_s}$$

pulsewidth << τ_s

$$\Delta\omega(L,t) = -\frac{d}{dt}\Delta\phi(L,t) \approx -\frac{\alpha}{2}\left(\frac{\bar{g}}{\bar{g}-\gamma_{sc}}\right)\left(1-\frac{1}{G}\right)\frac{I^{(0)}(L,t)}{\tau_s P_s}$$



Can solitons be used with semiconductor amplifiers?



$$\Delta \nu_{\mathrm{amp}}(L,t) \approx -\frac{\alpha}{4\pi\,\tau_s} \times \frac{I^{(0)}(L,t)}{P_s}$$

$$\Delta \nu_{\mathrm{fiber}}(L,t) \approx \frac{\delta\ r\, z_{\mathrm{amp}}}{2\pi} \frac{d}{dt} I^{(0)}(L,t) \qquad r = \frac{1-\exp(-\gamma z_{\mathrm{amp}})}{\gamma z_{\mathrm{amp}}}$$

is $\left|\Delta \nu_{\mathrm{fiber}}\right| \quad >> \quad \left|\Delta \nu_{\mathrm{amp}}\right| \quad ?$

Numerical example

$$I_s(L,t) = I_0 \,\mathrm{sech}^2\left(1.763\, t / \tau_{\mathrm{FWHM}}\right) \qquad \left.\frac{d}{dt} I_s(L,t)\right|_{\max} = \frac{1.161}{\tau_{\mathrm{FWHM}}} I_0$$

$$\delta = 2.7 \times 10^{-3}\,\mathrm{mW\,/\,km} \qquad z_{\mathrm{amp}} = 40\,\mathrm{km} \qquad \tau_{\mathrm{FWHM}} = 20\,\mathrm{ps}$$

amplifier $P_s = 10\,\mathrm{mW}$ $\tau_s = 200\,\mathrm{ps}$

$r = 0.265$ $0.4\,\mathrm{dB\,/\,km}$, $@1.3\,\mu\mathrm{m}$

$$\left|\frac{\Delta \nu_{\mathrm{fiber}}}{I_0}\right|_{\max} \approx 0.264\,\mathrm{GHz\,/\,mW} \qquad \left|\frac{\Delta \nu_{\mathrm{amp}}}{I_0}\right|_{\max} \approx 0.040\,\alpha\,\mathrm{GHz\,/\,mW}$$

Frequency shift induced by the amplifiers

1. For large α, the frequency shift induced by the amplifiers can be of the same order of the frequency shift induced by the Kerr nonlinearity of the fiber: the usual NLS solitons may be difficult to achieve.

2. The red shift induced by each amplifier can be of the order of some GHz for output power of the order of tens of mW.

3. The frequency shift is not compensated by the dispersion, it introduces spectral broadening.

Averaged propagation equation of a transmission line with semiconductor amplifiers

$$\frac{\partial}{\partial z}E(z,t) = -\,i\frac{\beta''}{2}\frac{\partial^2}{\partial t^2}E(z,t) + i\delta r\left|E(z,t)\right|^2 E(z,t)$$

$$-\frac{(1-i\alpha)}{2z_{\mathrm{amp}}}\frac{\overline{g}}{\overline{g}-\gamma_{\mathrm{sc}}}\left(1-\frac{1}{G}\right)\left[\int_{-\infty}^{t}\frac{dt'}{\tau_s}e^{-\frac{t-t'}{\tau_s}}\frac{\left|E(z,t')\right|^2}{P_s}\right]E(z,t)$$

Averaged propagation equation of a transmission line with semiconductor amplifiers and in-line filters

$$\frac{\partial}{\partial z}E(z,t)=$$

$$\left\{\left(\frac{\Delta g}{z_{\mathrm{amp}}}+i\Delta\Psi\right)+(\Delta t-i\Delta\omega g'')\frac{\partial}{\partial t}+\left(-i\frac{\beta''}{2}+\frac{g''}{2}\right)\frac{\partial^2}{\partial t^2}+i\delta n|E(z,t)|^2+\right.$$

$$\left.-\frac{(1-i\alpha)}{2z_{\mathrm{amp}}}\frac{\overline{g}}{\overline{g}-\gamma_{\mathrm{sc}}}\left(1-\frac{1}{G}\right)\left[\int_{-\infty}^{t}\frac{dt'}{\tau_s}e^{-\frac{t-t'}{\tau_s}}\frac{|E(z,t')|^2}{P_s}\right]\right\}E(z,t)$$

Δg excess gain of the amplifiers

Δt time delay per unit distance

$$g''=\frac{1}{z_{\mathrm{amp}}}\frac{\partial^2 g}{\partial\omega^2}L$$

"filter" curvature

Solution for τ_s >> pulsewidth

$$E(z,t) = A_0\left[\operatorname{sech}\left(\frac{t}{\tau}\right)\right]^{1+i\beta}$$

Pulse area $$\tau^2 A_0^2 = -\frac{\beta''}{\delta r}\left[\left(1-\frac{\beta^2}{2}\right) \mp \frac{3}{\underset{2}{\cancel{4}}}\frac{g''}{\beta''}\beta\right] = \xi \quad \Rightarrow \quad \beta'' < 0$$

Chirp parameter $$\beta = \frac{3}{2}\frac{\beta''}{g''} - \left[\left(\frac{3}{2}\frac{\beta''}{g''}\right)^2 + 2\right]^{1/2}$$

Energy balance

$$\frac{\Delta g}{z_{\mathrm{amp}}} = -\frac{1}{4\xi^2}\left[(1-\beta^2)\frac{g''}{2} + \beta\beta''\right]W^2 + \frac{1}{4z_{\mathrm{amp}}}\frac{\overline{g}}{\overline{g}-\gamma_{\mathrm{sc}}}\left(1-\frac{1}{G}\right)\frac{W}{\tau_s P_s}$$

$$W = 2A_0^2\tau$$ pulse energy

$$\Delta\omega = -\frac{1}{g''}\left(\frac{\alpha+\beta}{\beta^2+1}\right)\left(\frac{1}{2z_{\mathrm{amp}}}\right)\left(\frac{\overline{g}}{\overline{g}-\gamma_{\mathrm{sc}}}\right)\left(1-\frac{1}{G}\right)\frac{A_0^2\tau^2}{\tau_s P_s}$$

is independent of the pulse energy!

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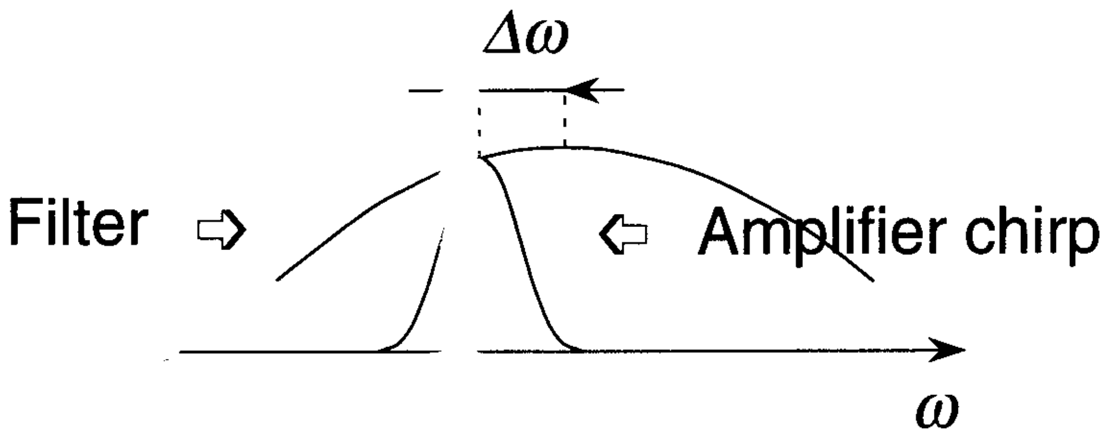
Pulse filtering



Benefits of filters with semiconductor amplifiers

- It reduces pattern effects, as it automatically stabilizes the gain. Due to pattern effects Δg is different from pulse to pulse. This produced different W for different pulses, but not the death of pulses.

- Without filters, different W implies a different frequency shift $\Rightarrow$ velocity shift if dispersion is non zero. With filters, the frequency shift $\Delta\omega$ is independent of the piulse energy W.

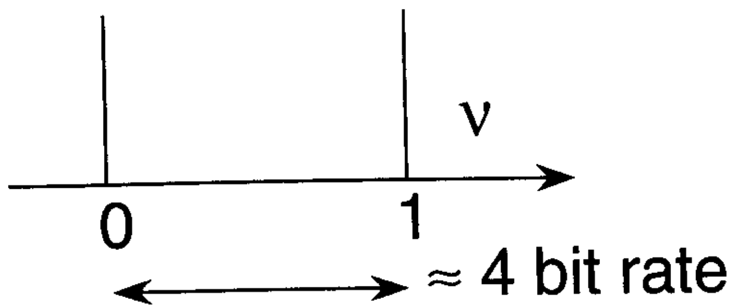
Avoiding spectral broadening

The spectrum of IMDD signals broadens too. $\Rightarrow$ no problem if dispersion is zero.

If the power is kept constant, no spectral broadening.

1. DPSK. It requires coherent detection.

2. Wide deviation frequency shift keying (FSK). Instead of zeros, ones are transmitted at another frequency. Ones are distinguished by optical filtering at detection



3. Possible also FDM, with unequal channel spacing to avoid FWM cross talk.

SUMMARY

1. Large effect of ASE noise + Kerr nonlinearity over transoceanic distances.

2. Large effects are mainly phase noise.

3. Semiconductor amplifiers: introduce a nonlinearity which, unlike Kerr nonlinearity, is not compensated by dispersion (solitons). It can be compensated with filters. Its effect can be minimized if transmission formats in which the transmitted optical power is constant are used (PSK or FSK).