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Conference on
Ultrafast Transmission Systems in Optical Fibres

13 - 17 February 1995

Some New Theoretical Results on Many Channel Soliton WDM Systems

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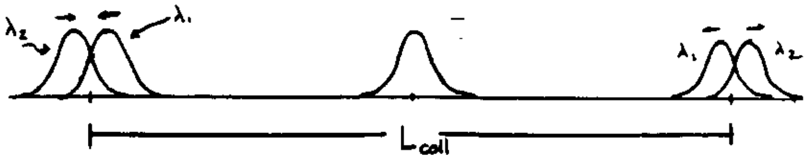
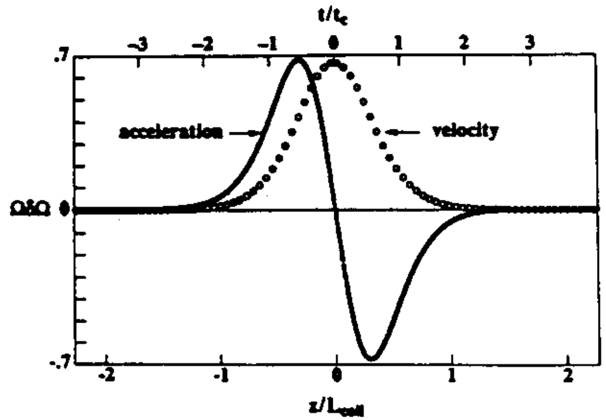
WDM With Solitons

Soliton Soliton Collision in Unperturbed Fiber (D constant and no loss)



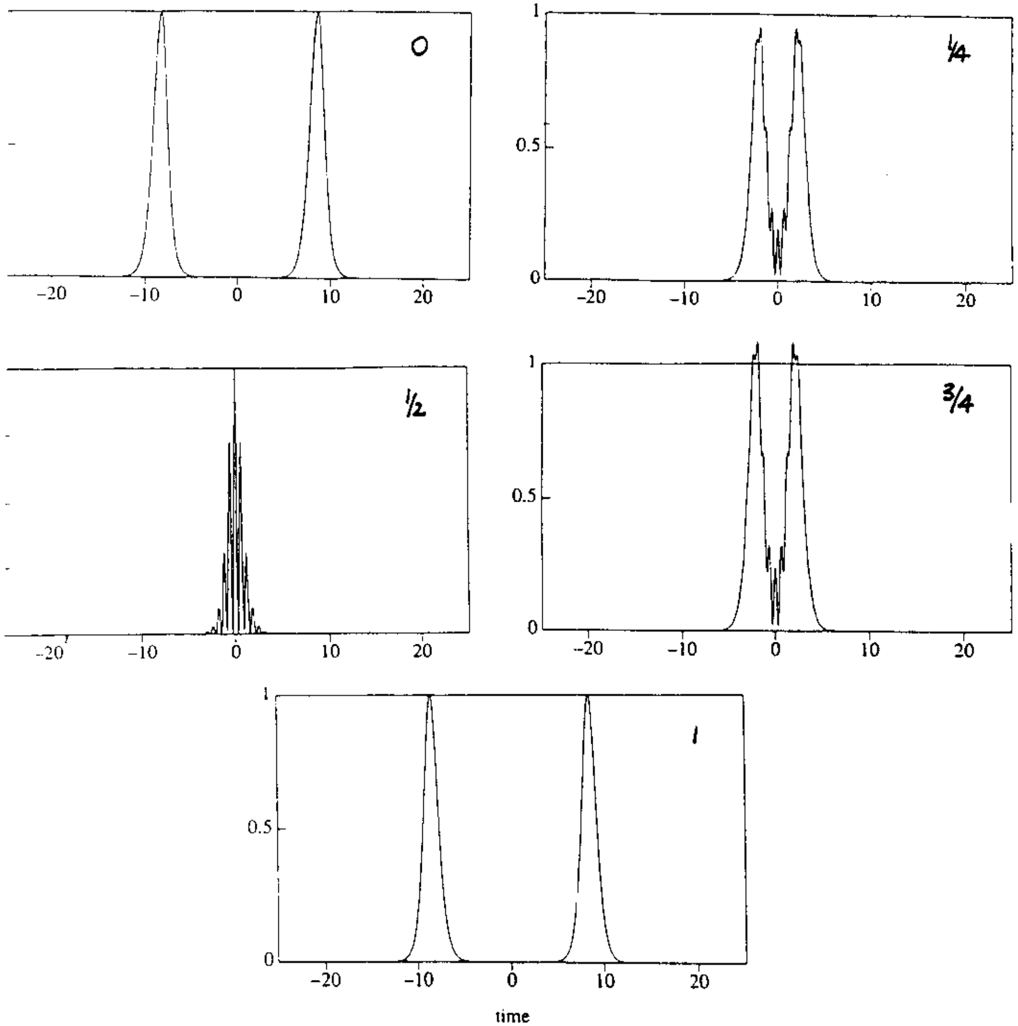
$$L_{coll} = \frac{z_c \tau}{t_c \Omega} = \frac{2\tau}{D\Delta\lambda} \qquad 2\,\Omega = \Delta f_{channel}$$



Only result of collision: $\delta t = \pm 0.1786 \, \frac{1}{\tau(\Delta f)^2}$

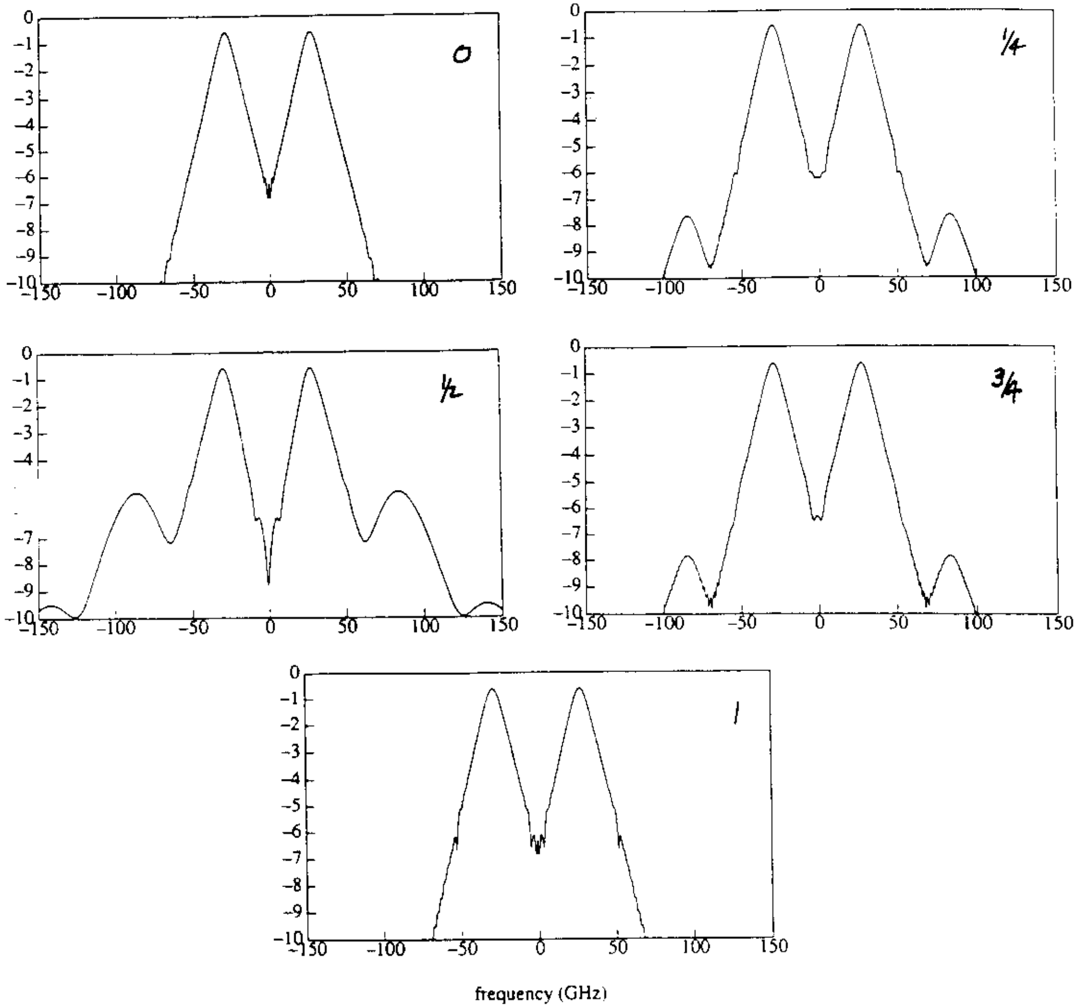
Wavelength Division Multiplexing with Solitons in Ultra Long Distance Transmission Using Lumped Amplifiers L. F. Mollenauer, S. G. Evangelides and J. P. Gordon. JLT Vol. 9 No. 3 march 1991

Soliton Collision in Lossless Fiber



Horizontal axes: time in s. u. Vertical axes: Power in s. u.

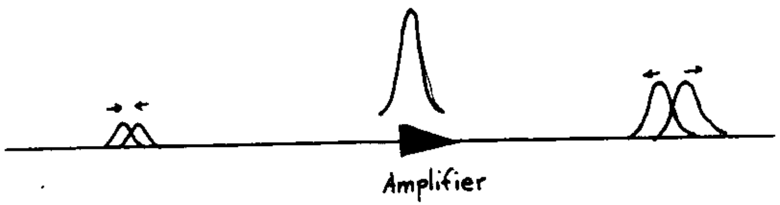
Soliton Collision in Lossless Fiber



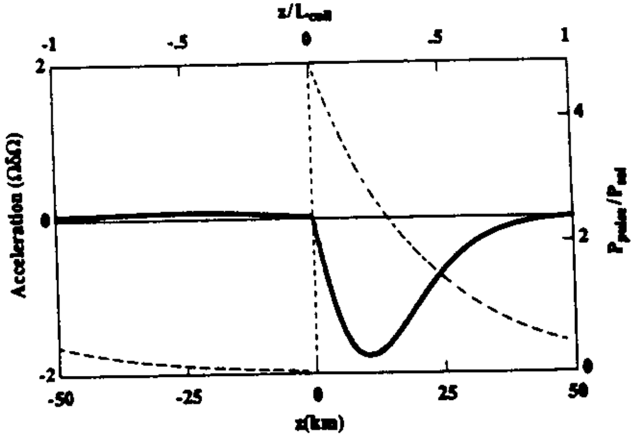
Horizontal axes: Frequency GHz, Vertical axes: Log (I) s. u.

Soliton Collisions
In the Presence of Perturbations

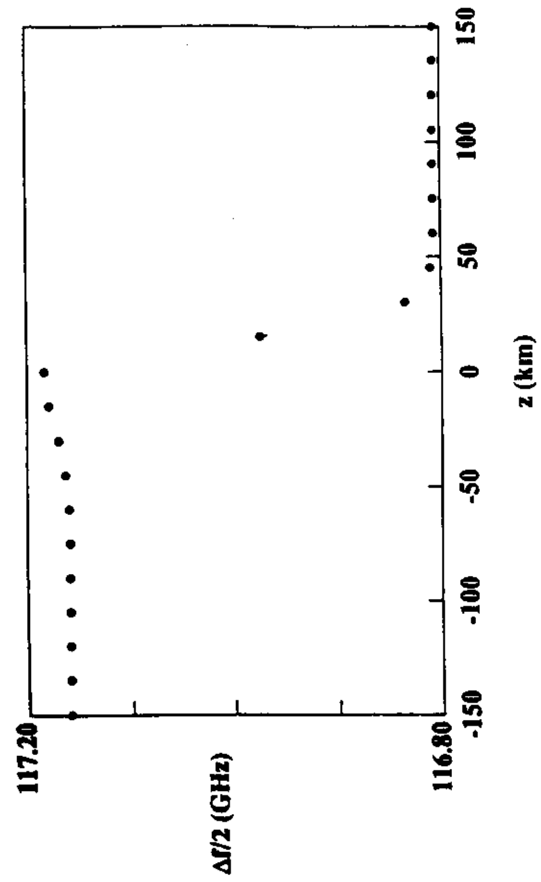
Collision centered at an amplifier



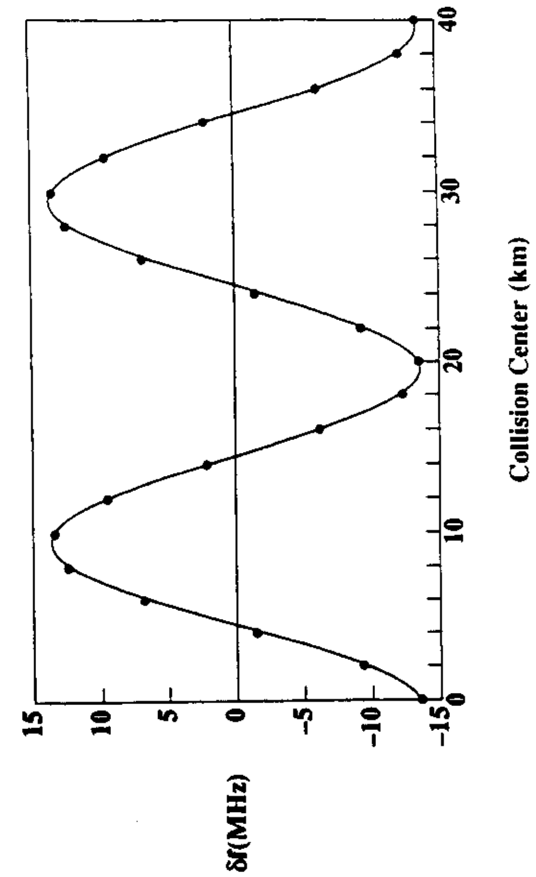
The collision is no longer symmetric resulting in a net velocity change for both solitons. This will manifest itself as extra timing jitter at the reciever.



VELOCITY (Δf/2) OF COLLIDING SOLITONS
COLLISION CENTERED AT AMPLIFIER, 1 AMP/100 km

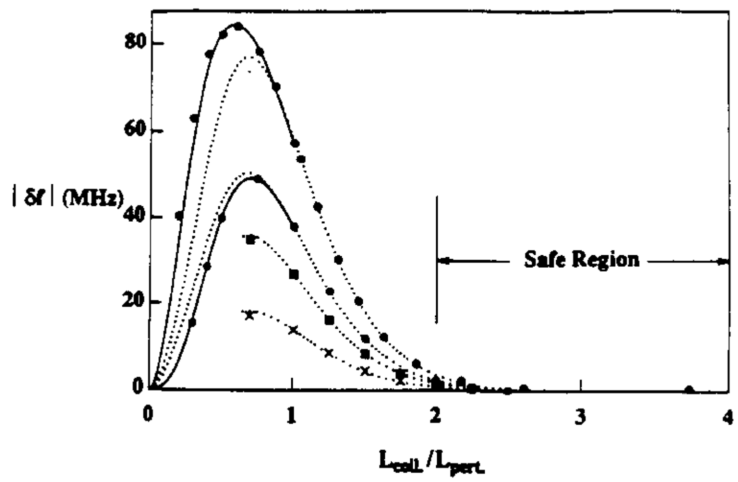


δf OF SOLITONS vs COLLISION CENTER
LUMPED AMPS AT 0,20,40, etc. km and L_{coll} = 20 km.

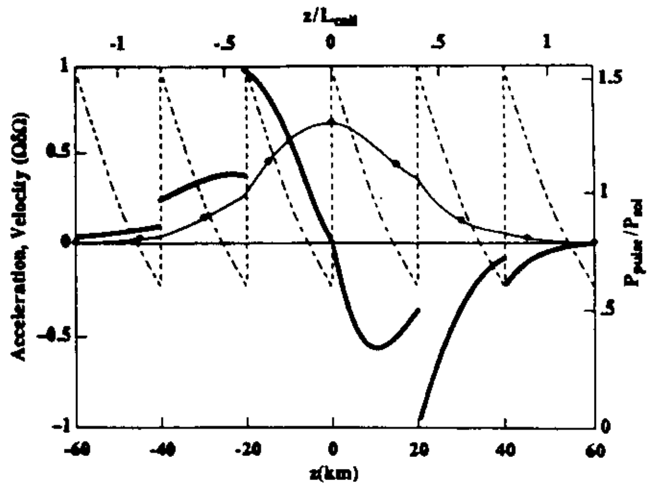


Wavelength Division Multiplexing
Using Solitons in the Presence of Perturbations

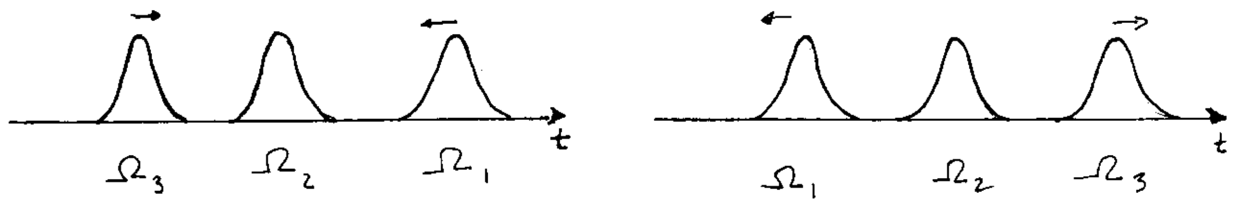
The Solution



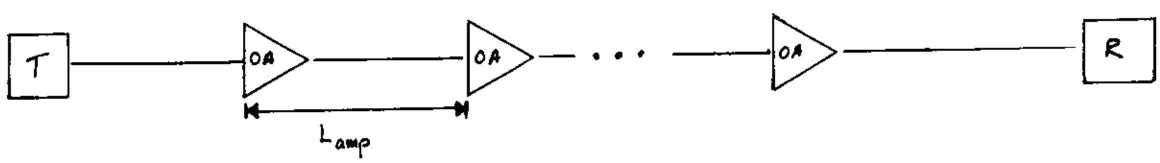
Why it works



The Three Soliton Collision



Periodic Gain and Loss in the Transmission Line as a Perturbation on the Soliton Collision



In this case the NLS is written as:

$$\frac{\partial u}{\partial z} = \frac{1}{2}\alpha(z)u + \frac{i}{2}\frac{\partial^2 u}{\partial t^2} + i|u|^2 u$$

where: $\alpha(z) = -\gamma + \gamma L_a \sum_n \delta(z - n L_a)$

Rewrite u(z,t) , the soliton wave function as: $(G(z))^{1/2}\, u(z,t)$ $\quad G(z) = 1 + 2\sum_{n>0} G_n \sin\left[\frac{2\pi n z}{L_a} + \theta_n\right]$

G(z) now represents fluctuations in the soliton intensity with distance and can replace the gain loss term in the standard NLS yielding:

$$\frac{\partial u}{\partial z} = \frac{i}{2}\frac{\partial^2 u}{\partial t^2} + iG(z)\;|u|^2 u$$

Perturbation Solution

$$\frac{\partial u}{\partial z} = \frac{i}{2}\frac{\partial^2 u}{\partial t^2} + i|u|^2 + i(G(Z) - 1)|u|^2 u$$

Define the perturbing term as: $\left[\frac{\partial u}{\partial z}\right]_p = i(G(z) - 1)$

Using the unperturbed soliton solution: $u_s = \sum_{j=1}^{3} A\,\mathrm{sech}(At - q_j)\exp(-i\Omega_j t + i\phi_j)$ in the above equation gives three coupled equations.

$$\frac{\partial u_1}{\partial z} = \frac{i}{2}\frac{\partial^2 u_1}{\partial t^2} + iG(z)\left[(|u_1|^2 + 2|u_2|^2 + 2|u_3|^2)u_1 + u_3^*(u_2)^2\right]$$

$$\frac{\partial u_2}{\partial z} = \frac{i}{2}\frac{\partial^2 u_2}{\partial t^2} + iG(z)\left[(|u_2|^2 + 2|u_1|^2 + 2|u_3|^2)u_2 + 2u_2^* u_1 u_3\right]$$

$$\frac{\partial u_3}{\partial z} = \frac{i}{2}\frac{\partial^2 u_3}{\partial t^2} + iG(z)\left[(|u_3|^2 + 2|u_2|^2 + 2|u_1|^2)u_3 + u_1^*(u_2)^2\right]$$

Perturbation Term for u_1

$$(\partial u_1/\partial z)_p = iG(z)(|u_1|^2 - 1)u_1 + i2G(z)(|u_2|^2 + |u_3|^2)u_1 + iG(z)u_3^* u_2^2$$

$iG(z)(\lvert u_1\rvert^2 - 1)u_1$	Gives the soliton a periodic phase shift.
$i2G(z)(\lvert u_2\rvert^2 + \lvert u_3\rvert^2)u_1$	Gives the frequency shift for 2 soliton collisions.
$iG(z)u_3^* u_2^2$	Four wave mixing term, new effects.

Calculation of Frequency Shifts

Using the following equation:

$$d\Omega/dz = -\mathrm{Im}\int dt \tanh(At-q)u_s^*(\partial u/\partial z)_p$$

After **much work** we get:

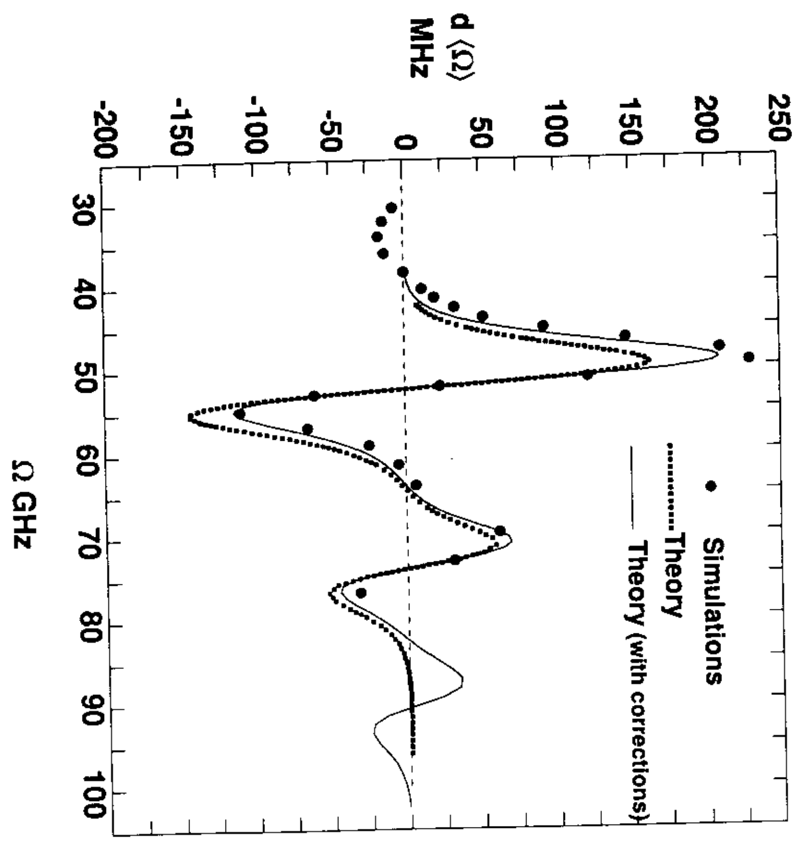
$$\Delta\Omega_1 = -\sum_n \frac{\zeta_n G_n}{2\Omega_s}\cos(\Phi-\theta_n)\int dx \cos(\zeta_n x)K(x,-x)$$

Where $\zeta_n = \Omega_s - 2\pi n/(L_a\Omega_s)$ $\quad x = \Omega_s z \quad \Phi = 2\phi_{20} + \phi_{10} + \phi_{30} \quad \theta_n = \tan^{-1}\left[\frac{\gamma L_a}{2\pi n}\right] \quad ; \quad G_n = \sin(\theta_n)$

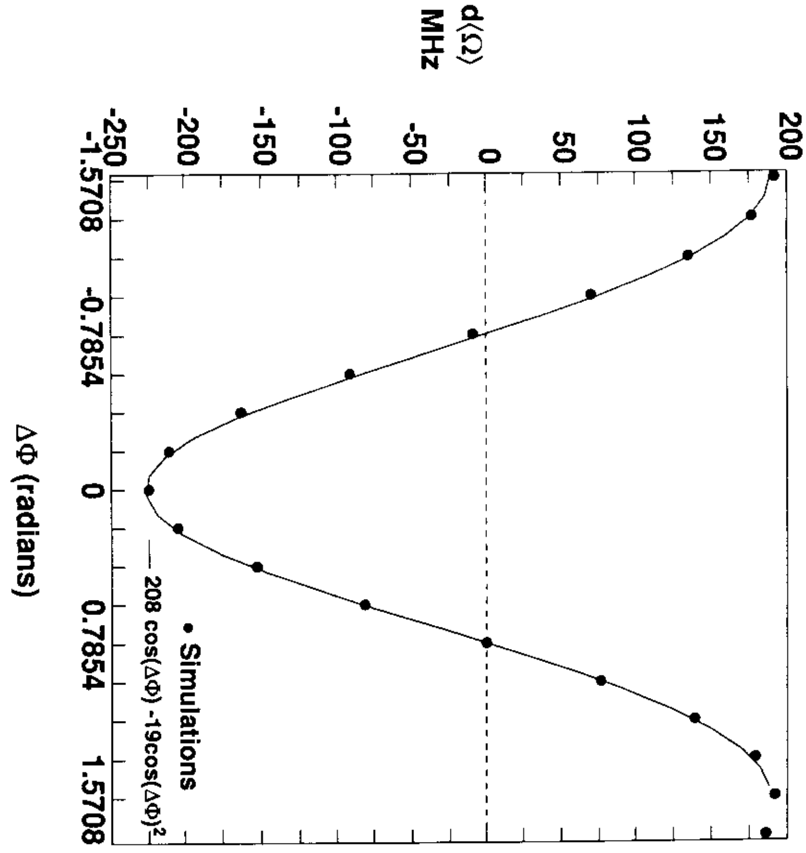
NOTE: $\Delta\Omega_1 = -\Delta\Omega_3$ and $\Delta\Omega_2 = 0$

At resonance: $(\Omega_s = \sqrt{2\pi n/L_a})$

Three Channel WDM
$d\langle\Omega\rangle$ vs. Ω
L_{amp} = 45 km



Three Channel Soliton Collision
d $\langle\Omega\rangle$ vs. total phase Φ
Ω = 49.57GHz
L_{amp} = 45 km



Calculation of Energy Shifts

Using the following equation:

$$dA/dz = \int dt\, u_s^* \,(\partial u/\partial z)_p$$

After **much work** we get:

$$\Delta A_1 = -\sum_n \frac{G_n}{\Omega_s}\cos(\Phi-\theta_n)\int dx\,\cos(\zeta_n x)K(x,-x)$$

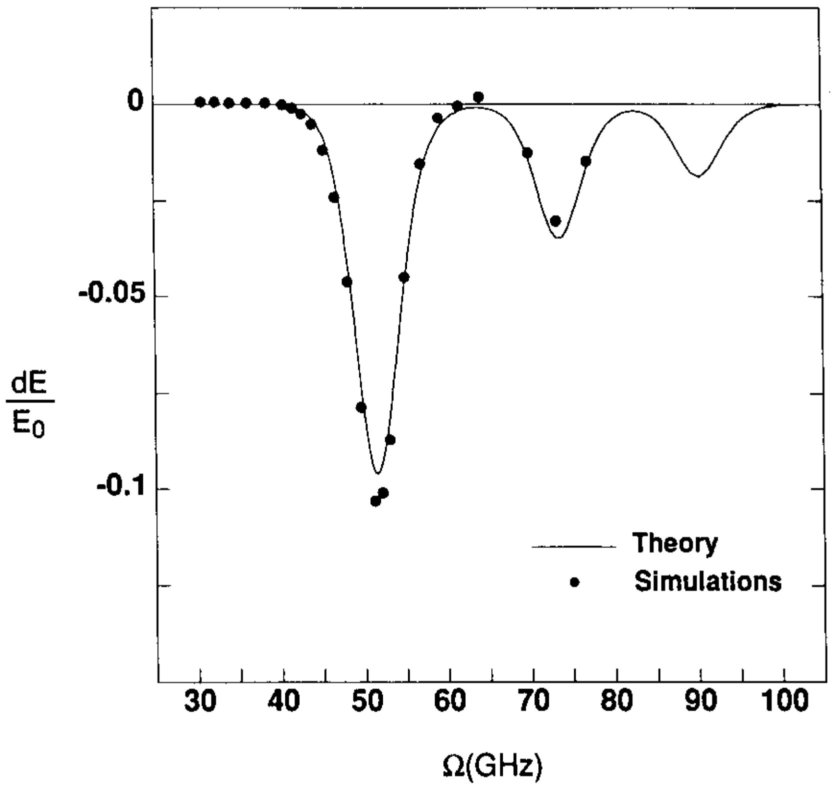
Where $\zeta_n=\Omega_s-2\pi n/(L_a\Omega_s)$ $\quad x = \Omega_s z \quad \Phi = 2\phi_{20} + \phi_{10} + \phi_{30} \quad \theta_n = \tan^{-1}\left[\frac{\gamma L_a}{2\pi n}\right]$; $G_n = \sin(\theta_n)$

NOTE: $\Delta A_1 = \Delta A_3$ and $\Delta A_2 = -\Delta A_1$

Three Channel WDM

$\frac{dE}{E}$ **vs.** Ω

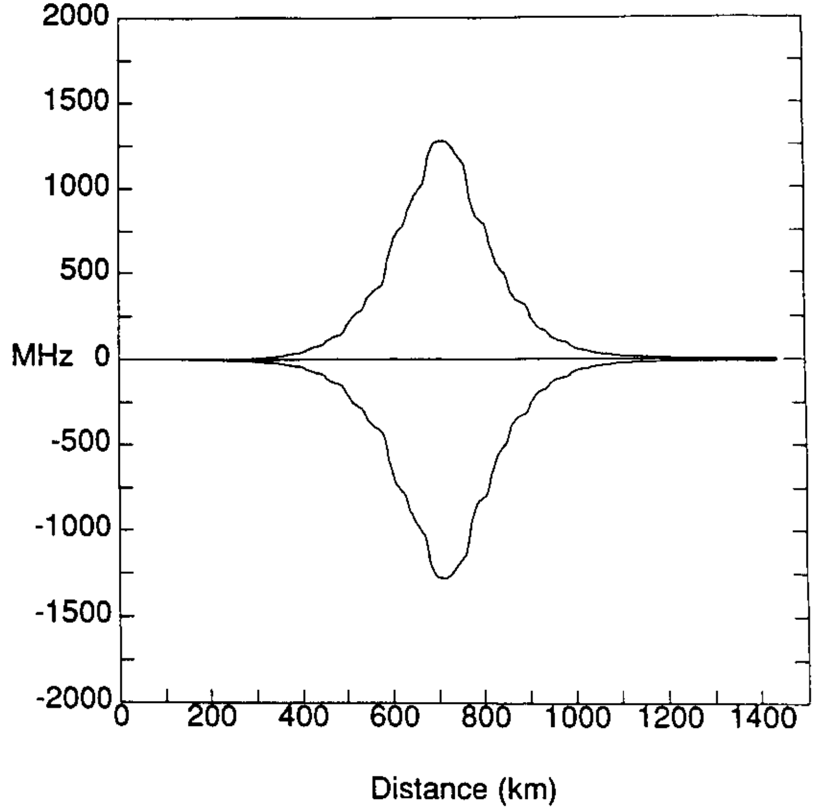
Lamp = **45 km**



Three Channel WDM

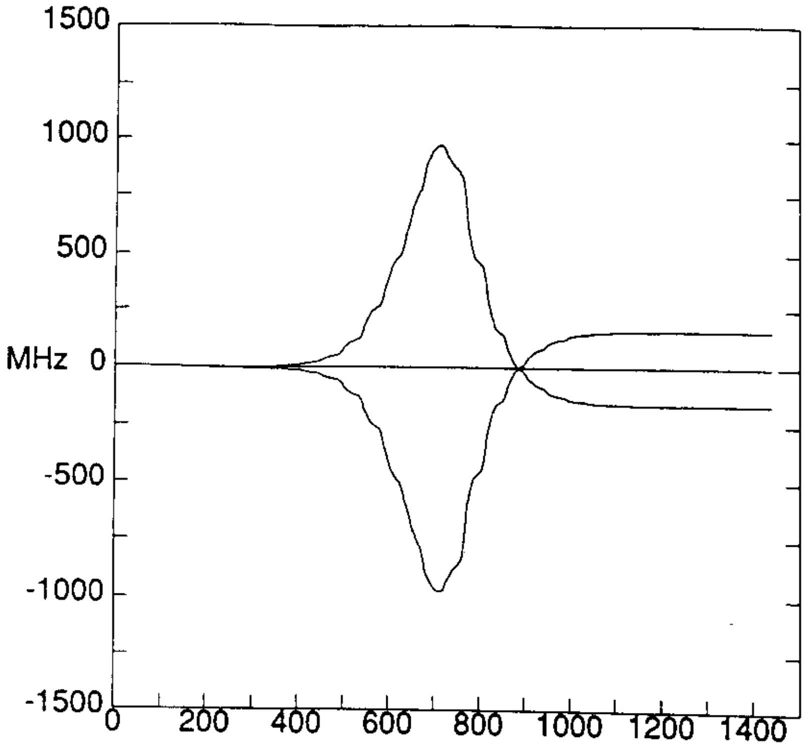
d ⟨ ω ⟩ During a Collision

$\Delta\omega_{chan}$ = **38.267 GHz**

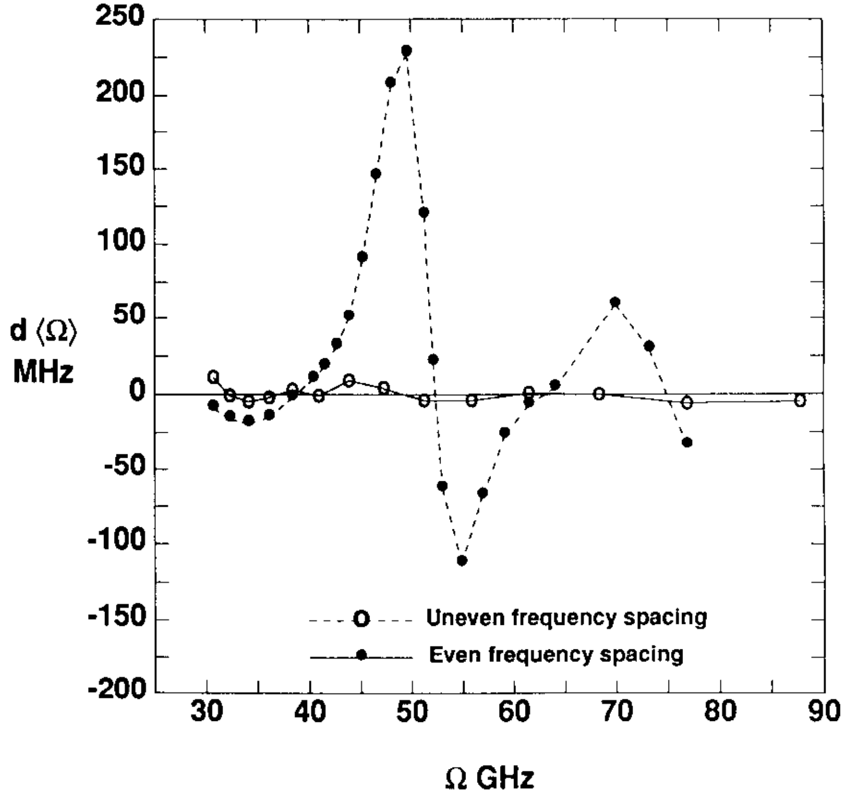


Three Channel WDM
d ⟨ ν ⟩ During a Collision

$\Delta\omega_{chan}$ = **47.097 GHz**



Three Channel WDM
d ⟨Ω⟩ vs. Ω
L_{amp} = **45 km**



CONCLUSIONS

- In the presence of periodic fluctuations in power due to fiber loss and amplifier gain a solitons at 3 equally spaced frequencies can exchange energy and effect frequency shifts on one another during a collision.

- This is essentially a four wave mixing process where the periodicity of the transmission line phase matches the waves.

- The analytic theory explains the simulation theory very well.

- Fo a given amplifier spacing there are some forbidden channel spacings near the resonances of $\Delta\Omega$ and ΔA.

$$\Omega_{resonance} = \sqrt{2\pi n/L_{amp}} \qquad |\Delta A|_{max} = 1.7 G_1 \sqrt{L_{amp}/n}$$