UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

H4.SMR/845-15

Second Winter College on Optics

20 February - 10 March 1995

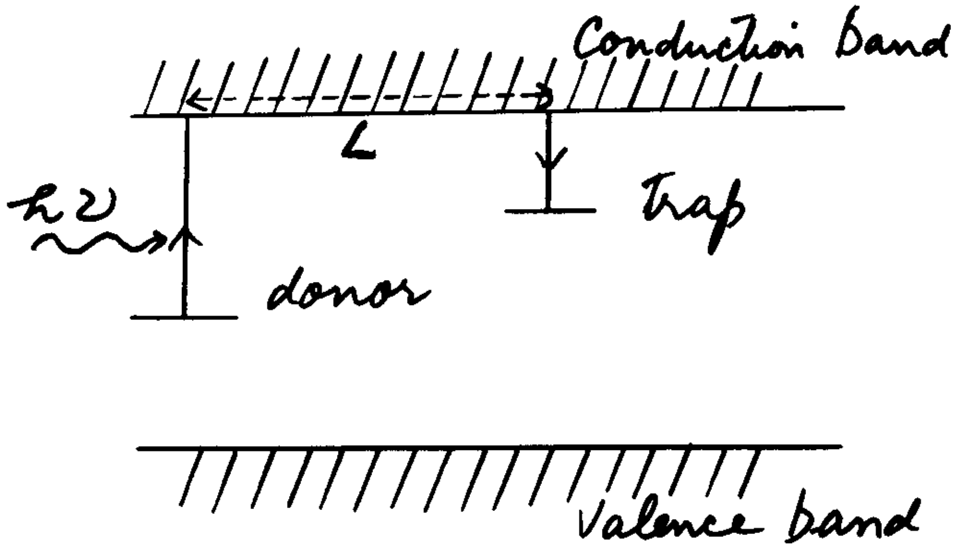
Image Correlation Using Photorefractive Materials

S. Mallick

Institut d'Optique
Orsay, France

Photorefractive effect

$LiNbO_3$, $BaTiO_3$, $Bi_{12}SiO_{20}$, polymers



Incident intensity $I(x,y)$

Energy absorbed per unit volume $I.\alpha.t.$

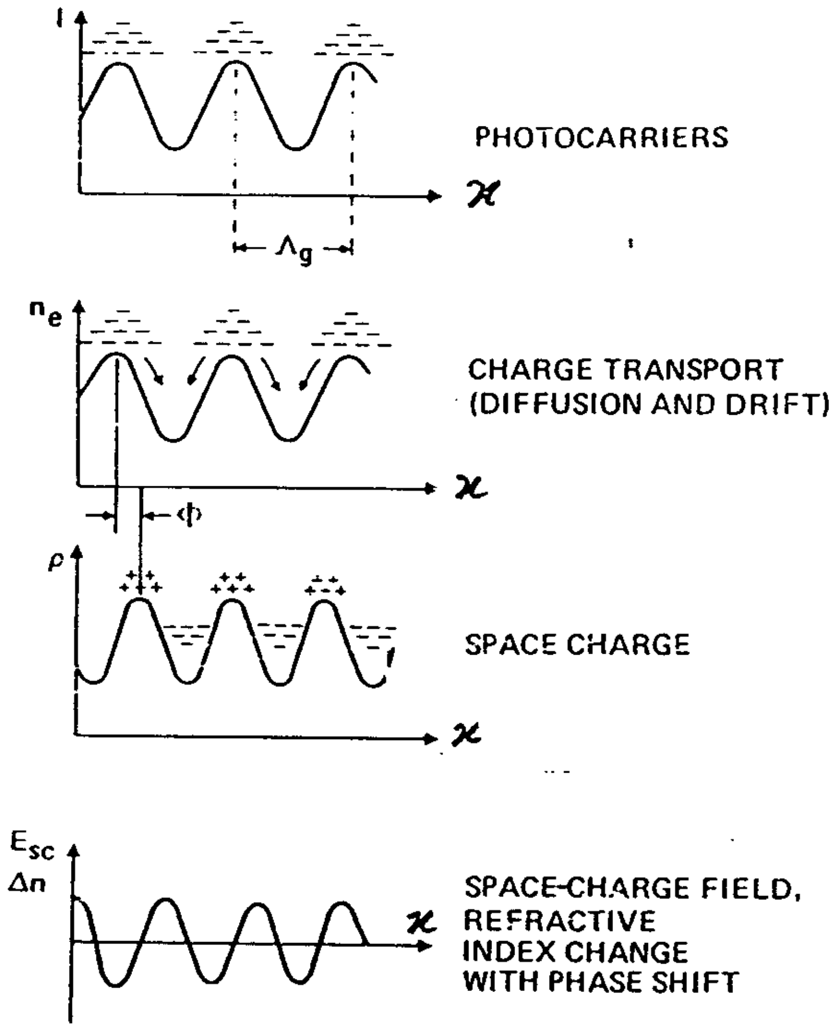
Free electron density $\frac{I(x,y)\,\alpha t}{h\nu} \cdot \phi = N$

electric polarization NeL

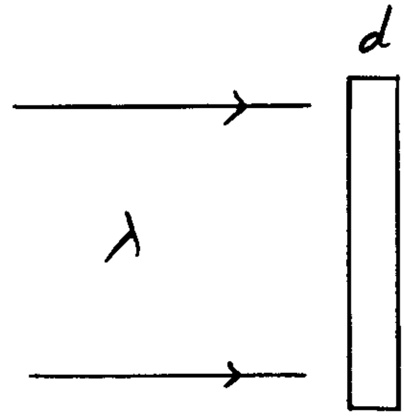
space-charge field $E_{sc} = \frac{NeL}{\varepsilon}$

refractive index change, (E.O. effect) $\Delta n \propto E_{sc}$

$\Delta n \propto I(x,y)$


I
PHOTOCARRIERS
x
Λg
ne
CHARGE TRANSPORT
(DIFFUSION AND DRIFT)
x
φ
ρ
SPACE CHARGE
x
Esc
Δn
x
SPACE-CHARGE FIELD,
REFRACTIVE
INDEX CHANGE
WITH PHASE SHIFT

Amplitude transmittance of the PR crystal is a linear function of the recorded intensity.



Amplitude at the exit plane, $t(x,y) = \exp\left[j \frac{2\pi}{\lambda} d\,(n_0 + \Delta n)\right]$

$$= e^{j \frac{2\pi}{\lambda} d n_0} \cdot e^{j \frac{2\pi}{\lambda} d\, \Delta n(x,y)}$$

$$\simeq \quad '' \quad \cdot \left[1 + j \frac{2\pi}{\lambda} d\, \Delta n(x,y)\right]$$

$$\Delta n(x,y) = \beta\, I(x,y)$$

$$t(x,y) \simeq 1 + j \frac{2\pi}{\lambda} d \cdot \beta\, I(x,y)$$

Fourier decomposition of a function $f(x,y)$

$$f(x,y) = \iint_{-\infty}^{+\infty} F(u,v)\, e^{j2\pi(ux+vy)}\, du\, dv$$

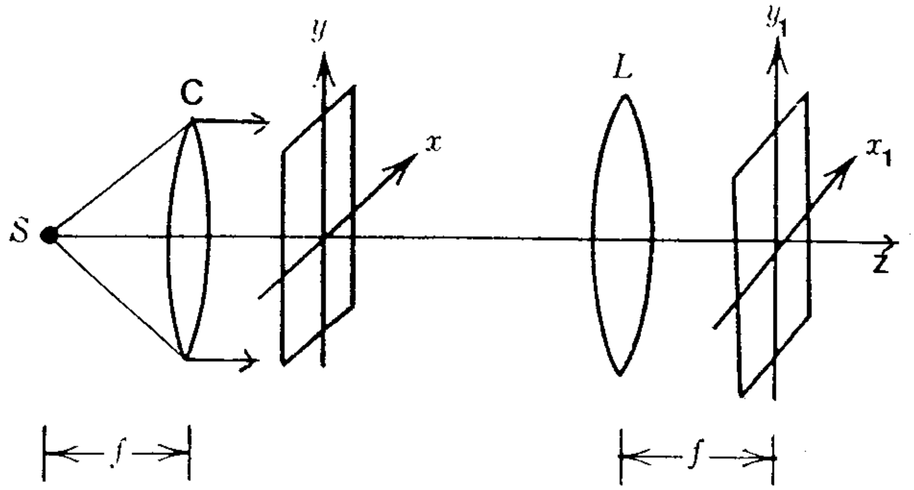
$$F(u,v) = \iint_{-\infty}^{+\infty} f(x,y)\, e^{-j2\pi(ux+vy)}\, dx\, dy$$

Intentifying the Fourier component $e^{j2\pi(ux+vy)}$ with a plane wave.

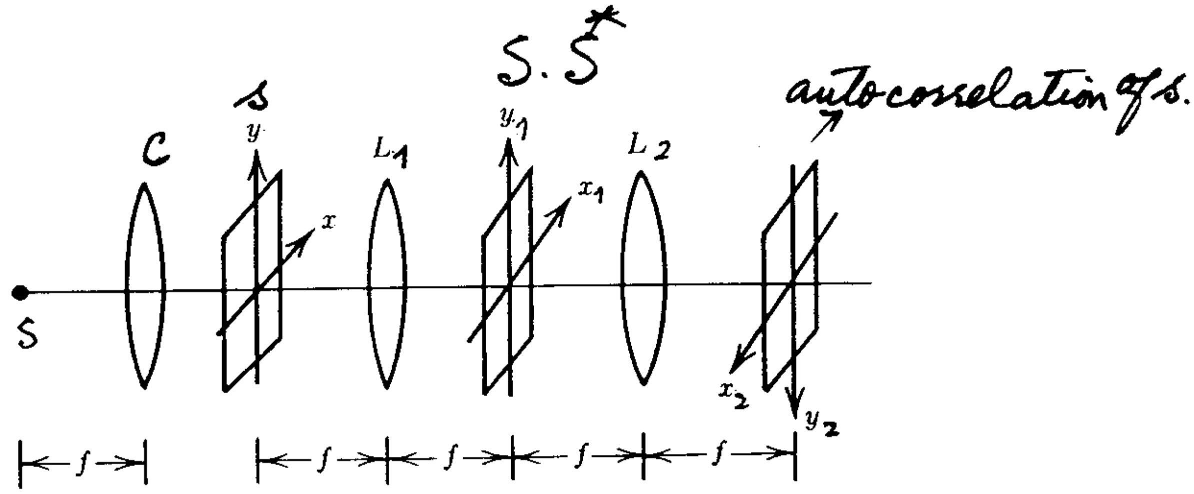
$$E(x,y,z) = \exp\left[j \left\{ \frac{2\pi}{\lambda}(\alpha x + \beta y + \gamma z) - \omega t \right\} \right]$$

$$\alpha = \lambda u, \quad \beta = \lambda v, \quad \gamma = \sqrt{1-\lambda^2(u^2+v^2)}$$

$$x_1 = u\lambda f, \quad y_1 = v\lambda f$$



Pattern recognition.



$$\text{F.T.}\left[S(u)\,S^*(u)\right] = \int S\,S^*\,e^{-j2\pi u x}\,du$$

$$= \int \left[\int s(x_1)\,e^{-j2\pi u x_1}\,dx_1\right]\left[\int s^*(x_2)\,e^{j2\pi u x_2}\,dx_2\right] e^{-j2\pi u x}\,du$$

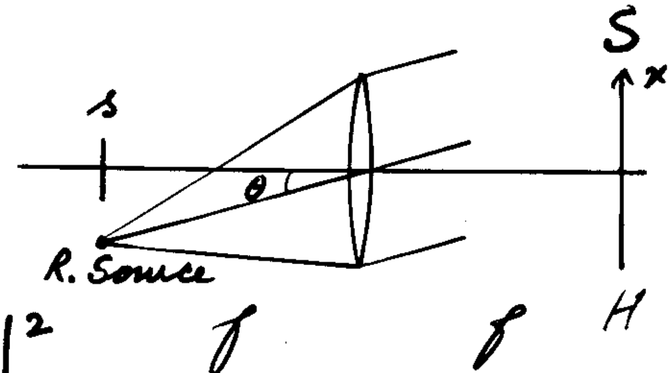
$$= \iiint s(x_1)\,s^*(x_2)\,e^{j2\pi u(x_2 - x_1 - x)}\,dx_1\,dx_2\,du$$

$$= \iint dx_1\,dx_2\,s(x_1)\,s^*(x_2)\,\delta(x_2 - x_1 - x)$$

Integrate w.r.t. x_2 $\qquad$ $x_2 = x_1 + x$

$$\int s(x_1)\,s^*(x_1 + x)\,dx_1$$

Implementing the filter of transmittance S^* holographically.



Intensity incident on the holographic plate

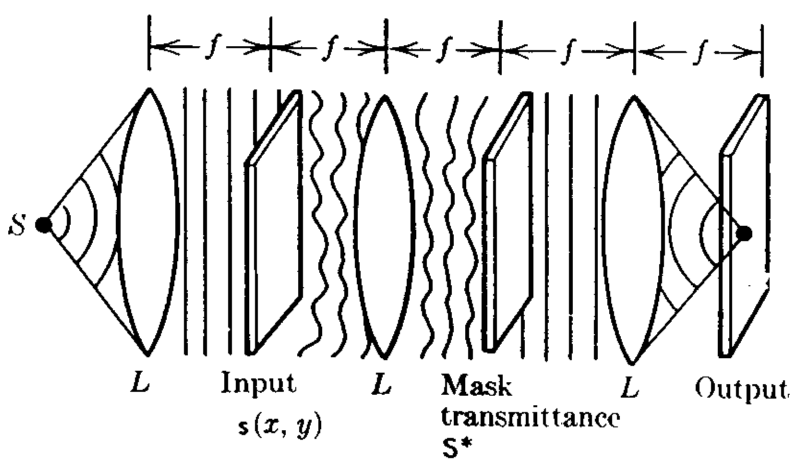
$$I = \left|S + R \exp j \frac{2\pi}{\lambda} \sin\theta\, x\right|^2$$

$$= |S|^2 + R^2 + SR\, e^{-jk\sin\theta x} + \underline{\underline{S^*}}\, R\, e^{jk\sin\theta x}$$

$$t(x,y) \propto I(x,y)$$

H acts as the Frequency plane filter.

Due to off-set θ, the correlation pattern is located off axis; $x_2 = F\sin\theta$, $y_2 = 0$



Different filter functions

amplitude incident on the filter

$$S(u, v) = |S(u, v)| \exp[j \phi(u, v)]$$

amplitude transmittance of the matched filter

$$\overset{*}{S}(u, v) = |S(u, v)| \exp[-j \varphi(u, v)]$$

amplitude transmittance of the inverse filter

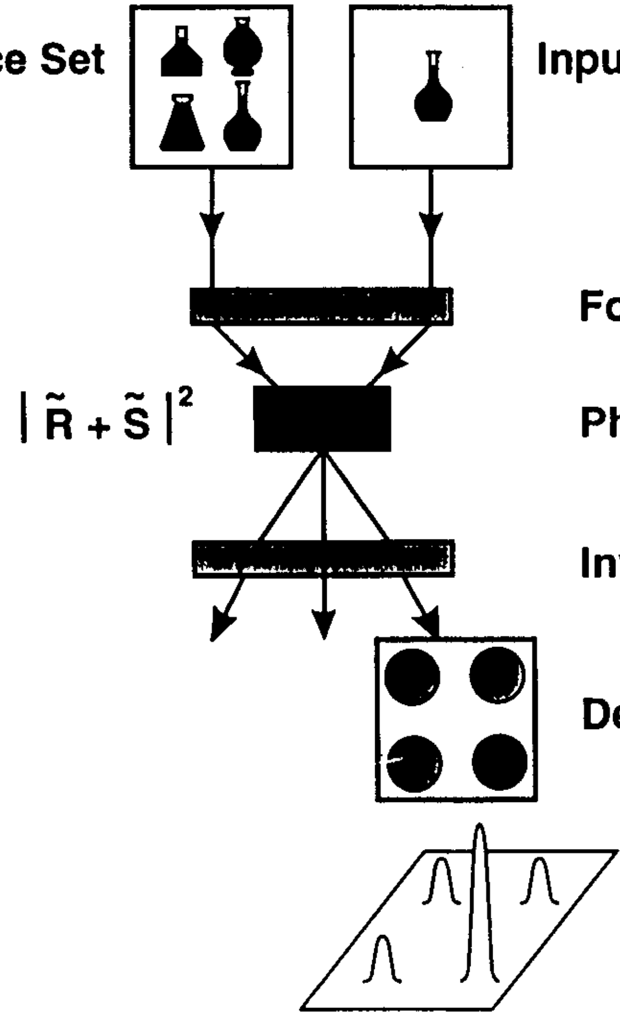
$$= \frac{\exp[-j \varphi(u, v)]}{|S(u, v)|} \cdot c.$$

phase only filter $\exp[-j \varphi(u, v)]$

matched filter in presence of noise

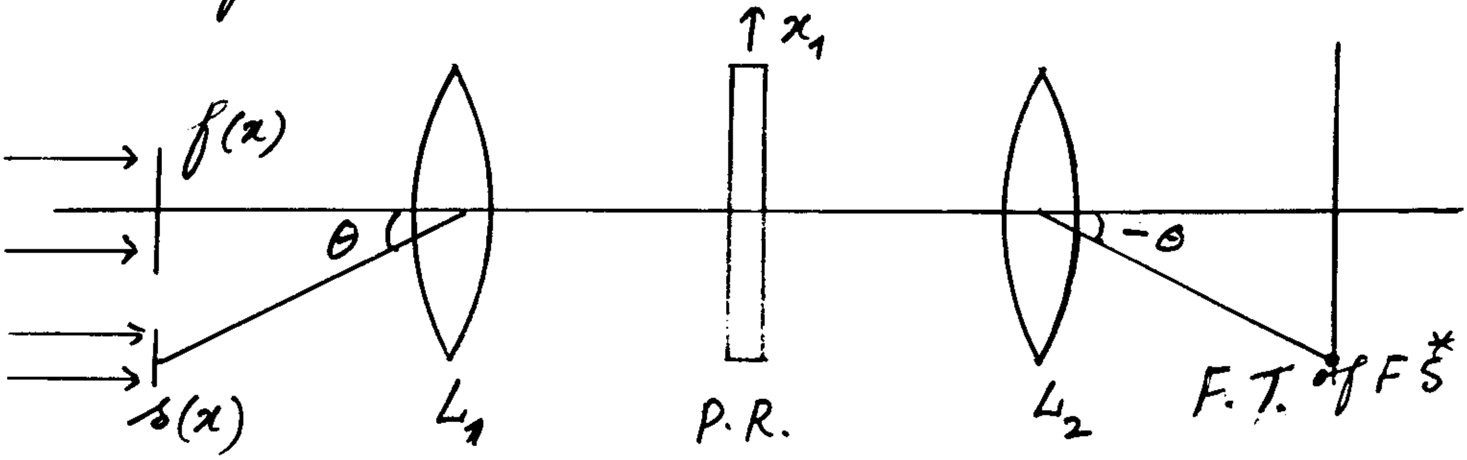
$$\frac{\overset{*}{S}(u, v)}{\text{noise spectral density}}$$

JOINT CORRELATION



DT1599/15

joint Correlator



amplitude in $x_1 y_1$ plane

$$F(u) + S(u)\, e^{j \frac{2\pi}{\lambda} \sin\theta\, x_1}$$

$$x_1 = u \lambda f_1$$

$$I(x_1, y_1) = |F(u)|^2 + |S(u)|^2 + F^* S\, e^{j\frac{2\pi}{\lambda}\sin\theta\, x_1} + F \overset{*}{S}\, e^{-j\frac{2\pi}{\lambda}\sin\theta\, x_1}$$

amplitude in output plane

$$\int_{-\infty}^{+\infty} F \overset{*}{S}\, e^{-j 2\pi u x}\, du$$

$$= \int \left[\int f(X_1)\, e^{-j2\pi u X_1}\, dX_1\right] \left[\int \overset{*}{s}(X_2)\, e^{j 2\pi u X_2}\, dX_2\right] e^{-j2\pi u x}\, dx$$

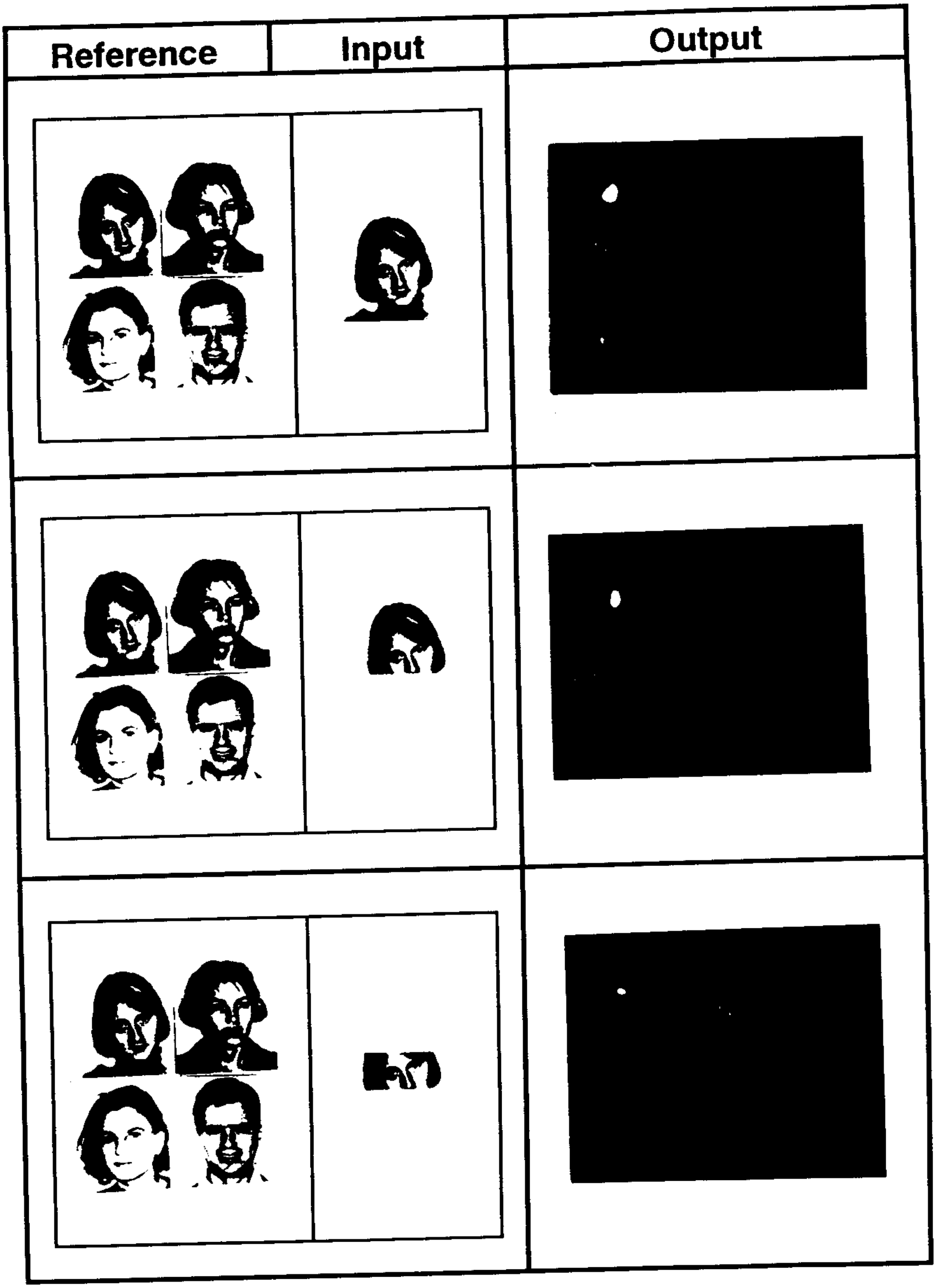
$$= \iiint dX_1\, dX_2\, du\; f(X_1)\, \overset{*}{s}(X_2)\, e^{-j2\pi u (x + X_1 - X_2)}$$

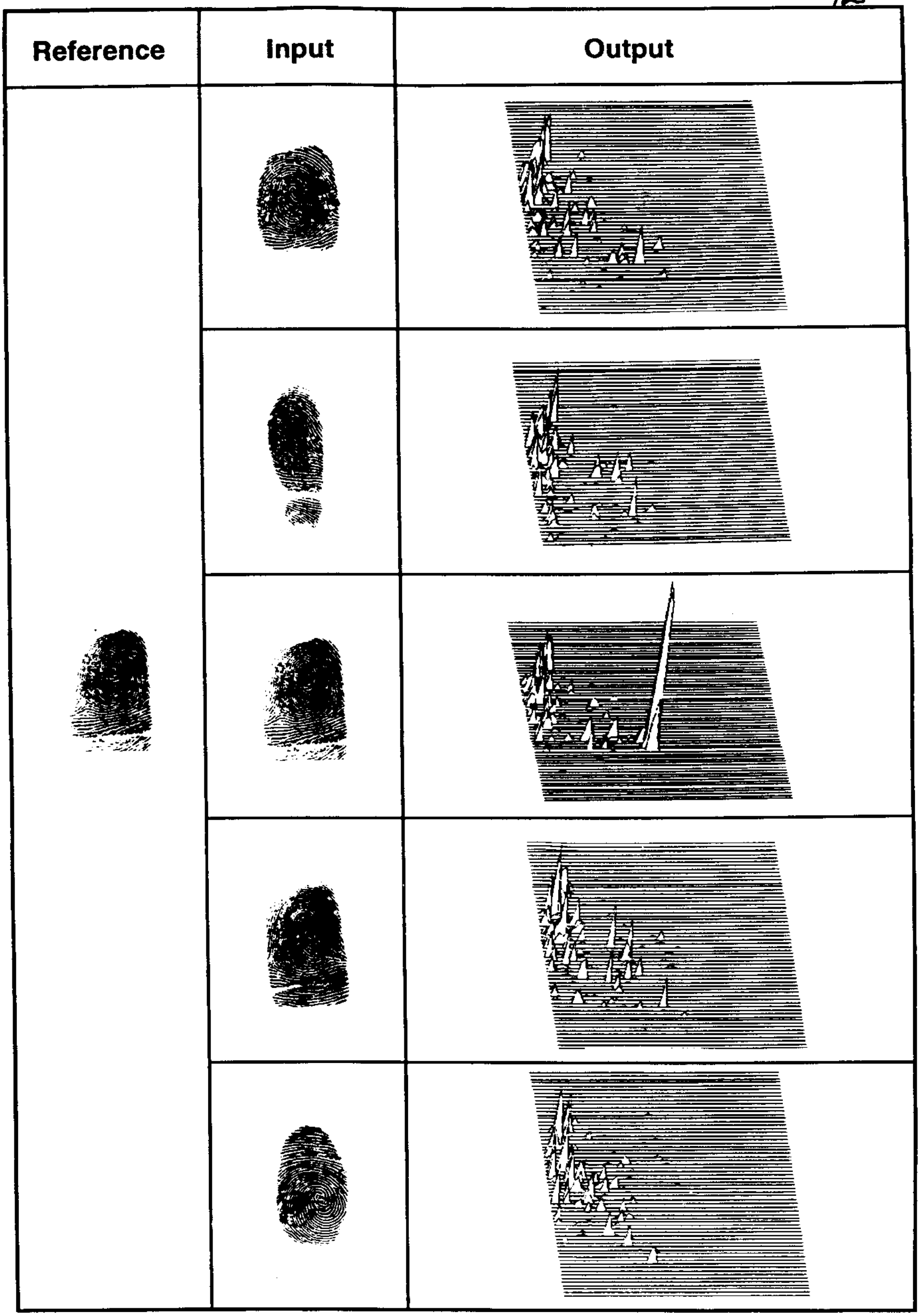
$$= \iint dX_1\, dX_2\; f(X_1)\, \overset{*}{s}(X_2)\, \delta(x + X_1 - X_2)$$

Integrate w.r.t. X_2;

$$\int f(X_1)\, \overset{*}{s}(X_1 + x)\, dX_1$$

cross correlation of functions f and s.

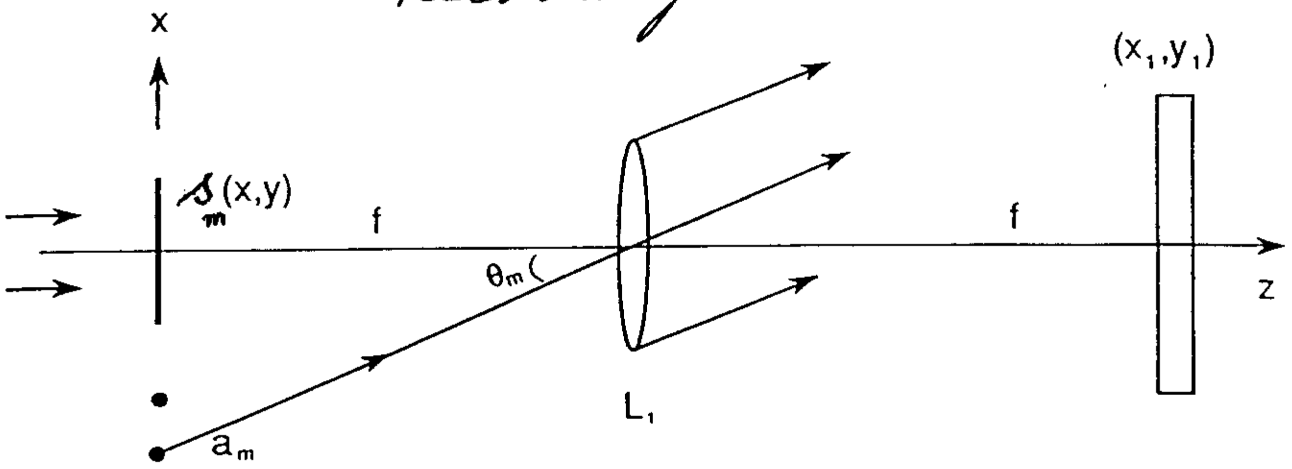

Reference
Input
Output



DT1593/03

Associative memory.

Recording.

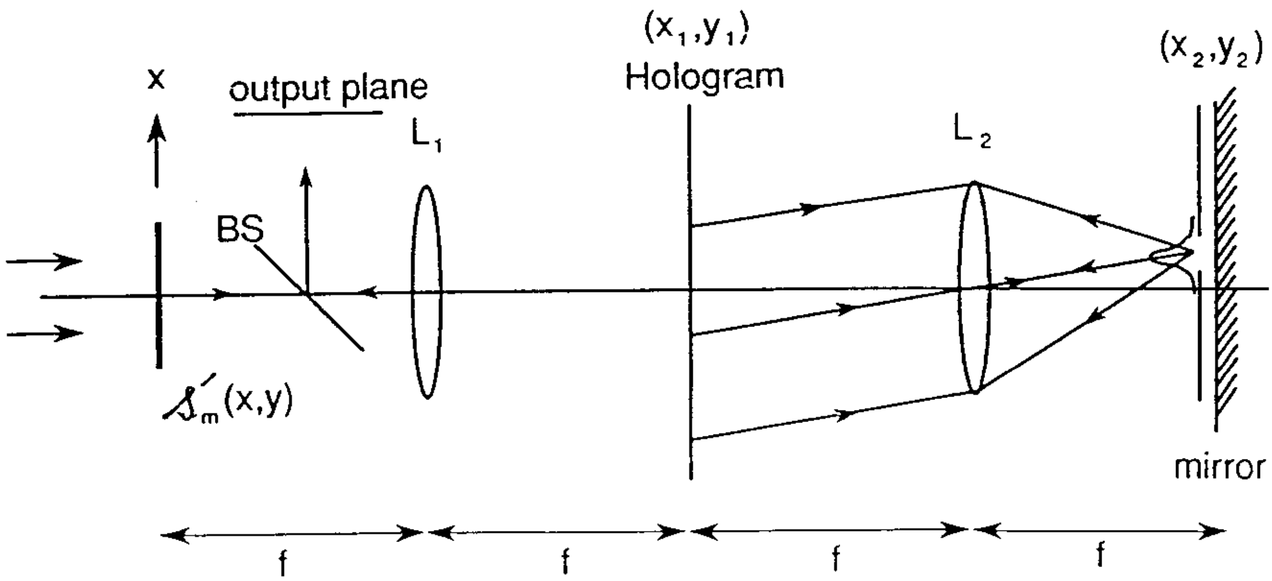


$$I = \left| S_m(x_1, y_1) + \exp jk \sin\theta_m x_1 \right|^2$$

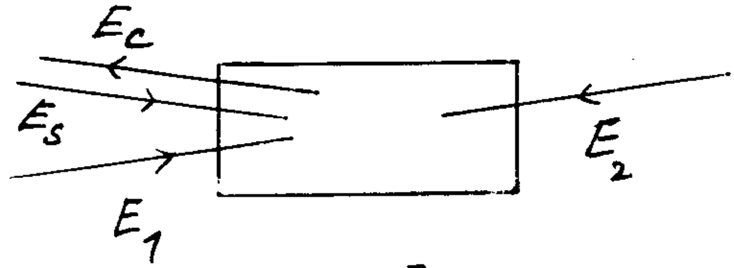
Readout.

amplitude in $x_1 y_1$ plane

$$S_m^* S_m' \exp jk \sin\theta_m x_1$$



Implementing a phase conjugate mirror



$$E_s = \underline{Re\left[A_s(\vec{r})\exp\{j(\vec{k}_s\cdot\vec{r}-\omega t)\}\right]}$$
$$= \frac{1}{2}A_s(\vec{r})\exp\{j(k_s\cdot r-\omega t)\} + c.c.$$

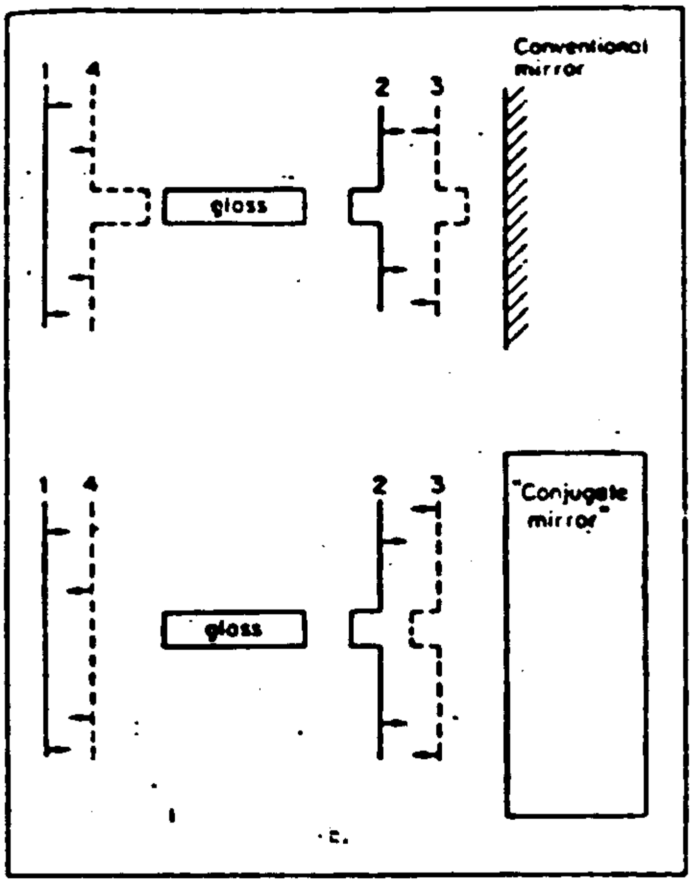
$$E_1 = \frac{1}{2}A_1 \exp\{j(k_1\cdot r-\omega t)\} + c.c.$$

$$I = \langle (E_1+E_s)^2\rangle$$
$$= \langle E_1^2\rangle + \langle E_s^2\rangle + \frac{1}{2}A_1 A_s^* \exp\{j(k_1-k_s)\cdot r\} + c.c.$$

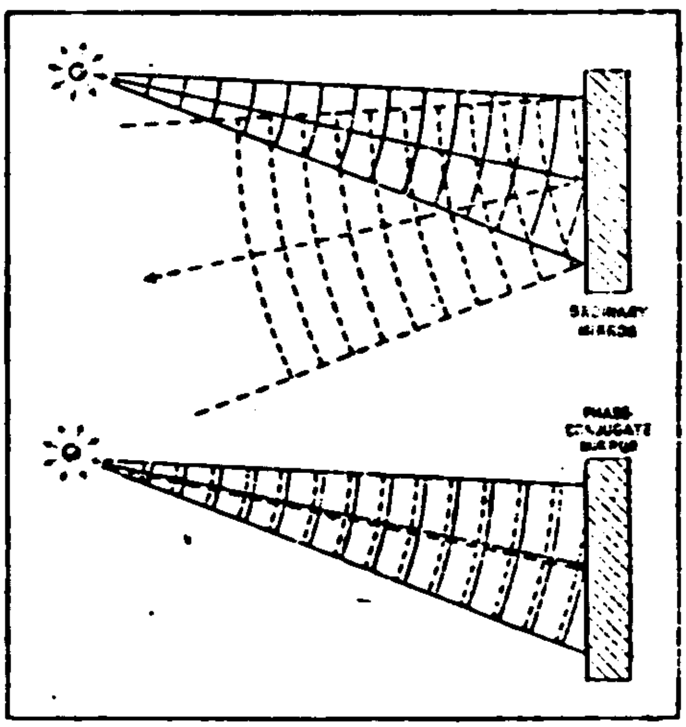
$$E_2 = \frac{1}{2}A_2 \exp\{j(-k_1\cdot r-\omega t)\} + c.c.$$

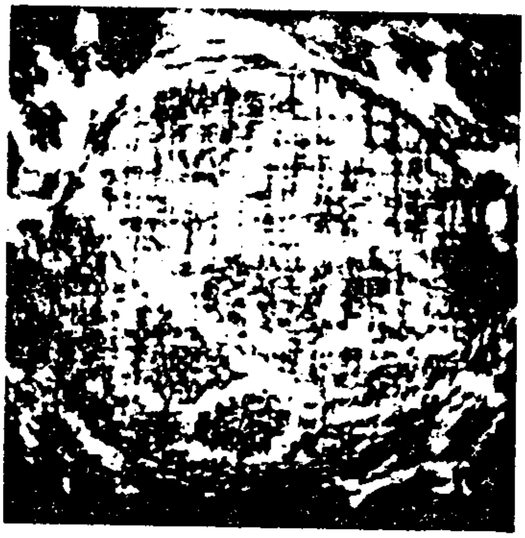
wave propagating in $-\vec{k}_s$ direction

$$E_c = \frac{1}{4}A_1 A_2 A_s^* \exp\{j(-k_s\cdot r-\omega t\} + c.c.$$
$$= \frac{1}{2}A_1 A_2\, \underline{Re\left[A_s^*\exp\{j(-k_s\cdot r-\omega t)\}\right]}$$
$$= \frac{1}{2}A_1 A_2\, \underline{Re\left[A_s\exp\{j(k_s\cdot r-\omega(-t))\}\right]}$$

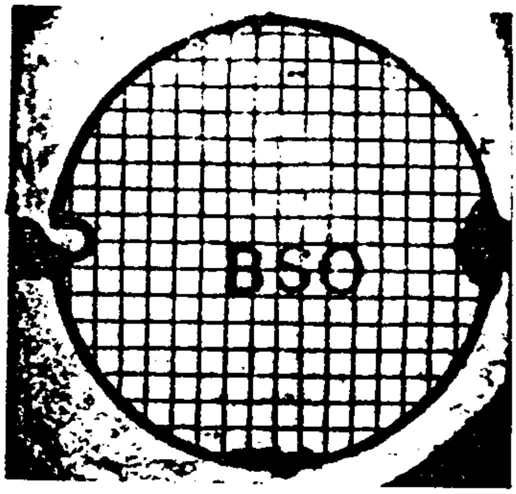


Conventional mirror
1
4
2
3
glass
1
4
2
3
"Conjugate mirror"
glass

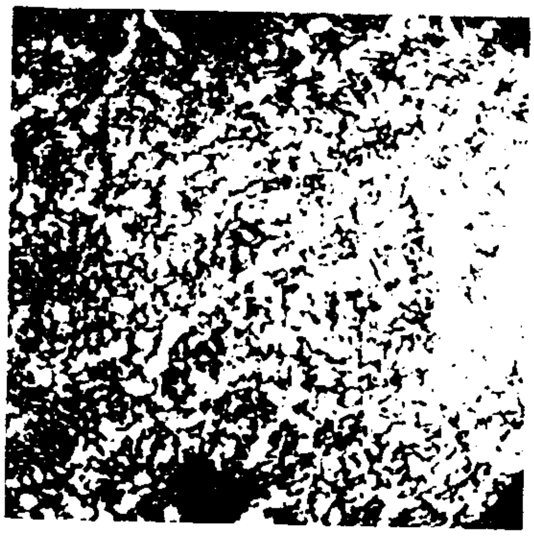




(a)



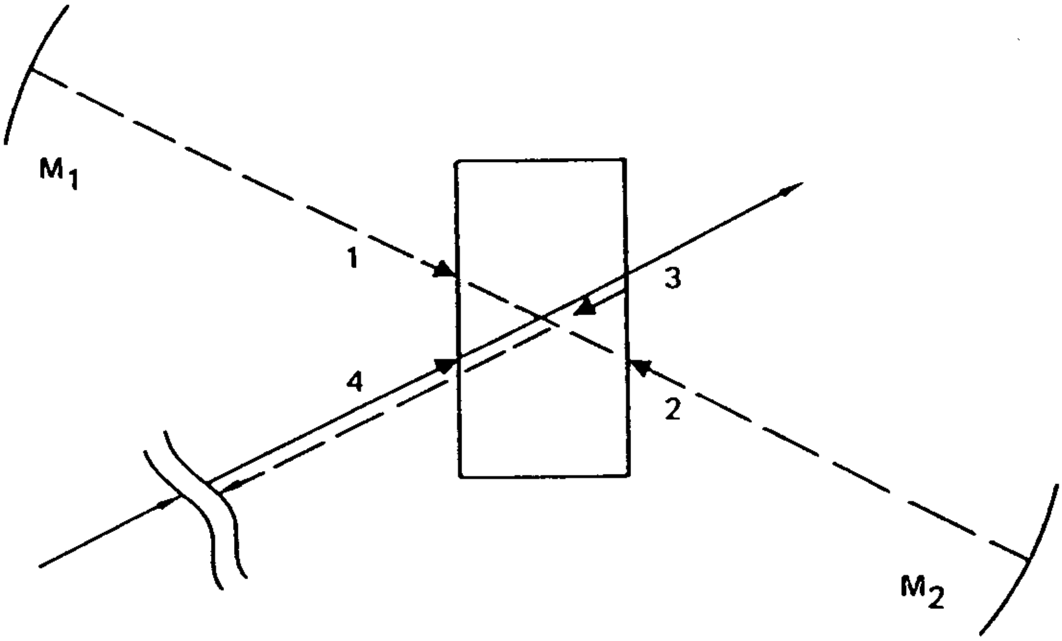
(b)



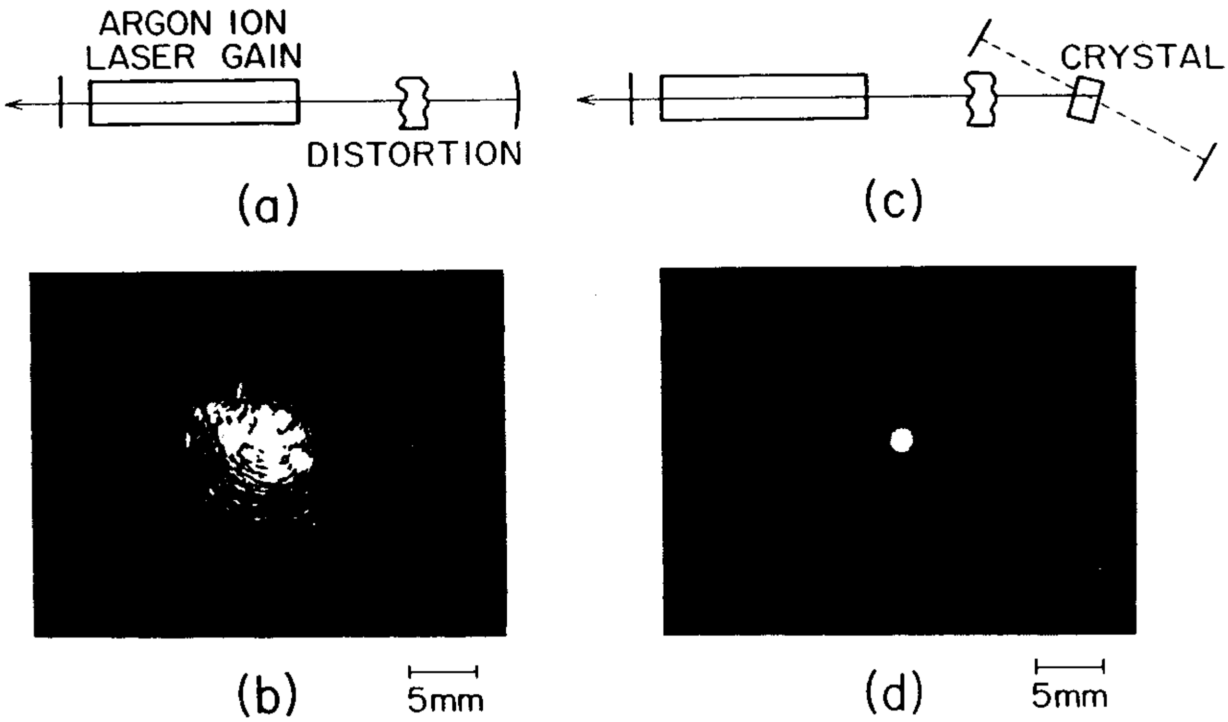
(c)

Reconstructed image with aberrating glass plate: (a) direct imaging through aberrating glass; (b) imaging via real-time phase conjugate wavefront generation; (c) imaging with a vertical misalignment of the mirror by 1·5 mrad. (After Huignard *et al.* Opt. Lett. vol 4, page 21, 1979.

Self-pumped phase conjugate mirror



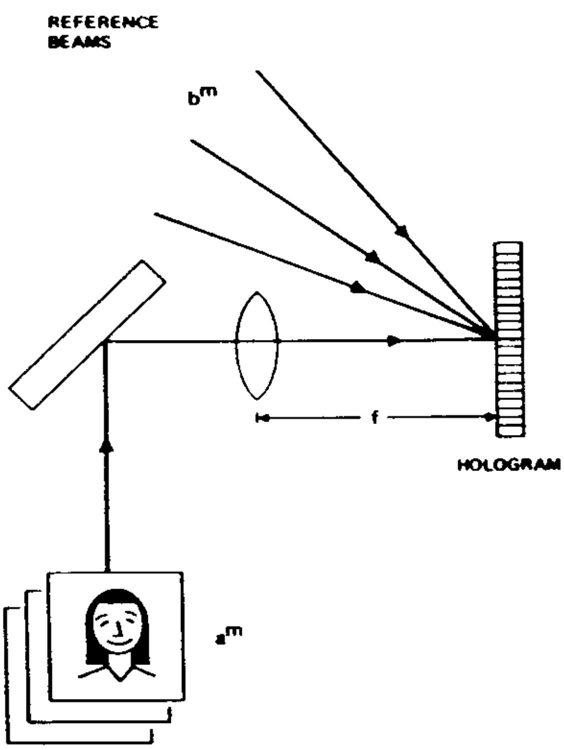
Laser performance without and with a PCM.

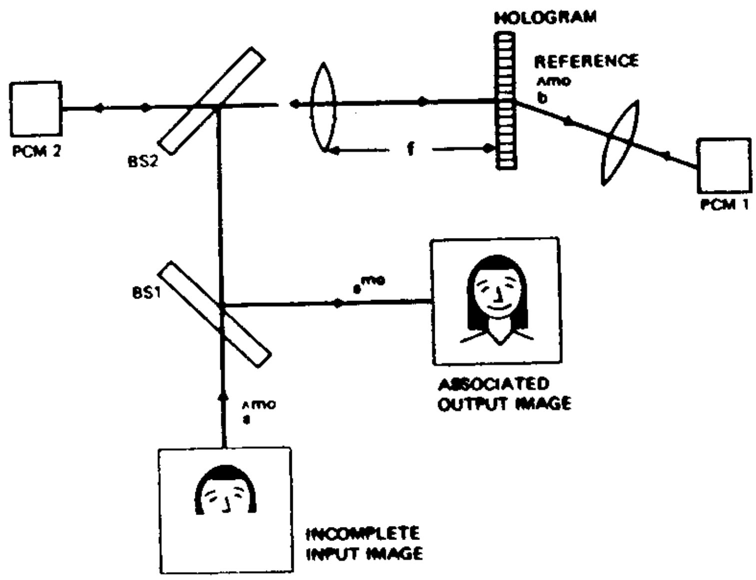


(a) (c)

(b) (d)

Associative memory using a resonator formed with two phase conjugate mirrors





(a) IMAGE STORED IN MEMORY

(b) INCOMPLETE INPUT IMAGE

(c) ASSOCIATED OUTPUT IMAGE

Reconstruction of gray-scale image fr
partial input: (a) image stored in memory;
incomplete input image; (c) associated output i
age (inversion due to mirror reflection).

image with a gray scale

(Owechko et al; App. Opt vol. 26, page 1900, (1987).