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College on Computational Physics

15 May - 9 June 1995

Computational Studies of High Energy Physics

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Computational Studies of High Energy Physics

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at Boston University

using TM's CM-5

Outline

- Symetry (anomalous)
- Vacuum structure of gauge theory
- The sphaleron and topological transitions
- The computational Problem (anomalous baryon number violation)
- The Computational Strategy
- Computer animation of results
- More on the computation

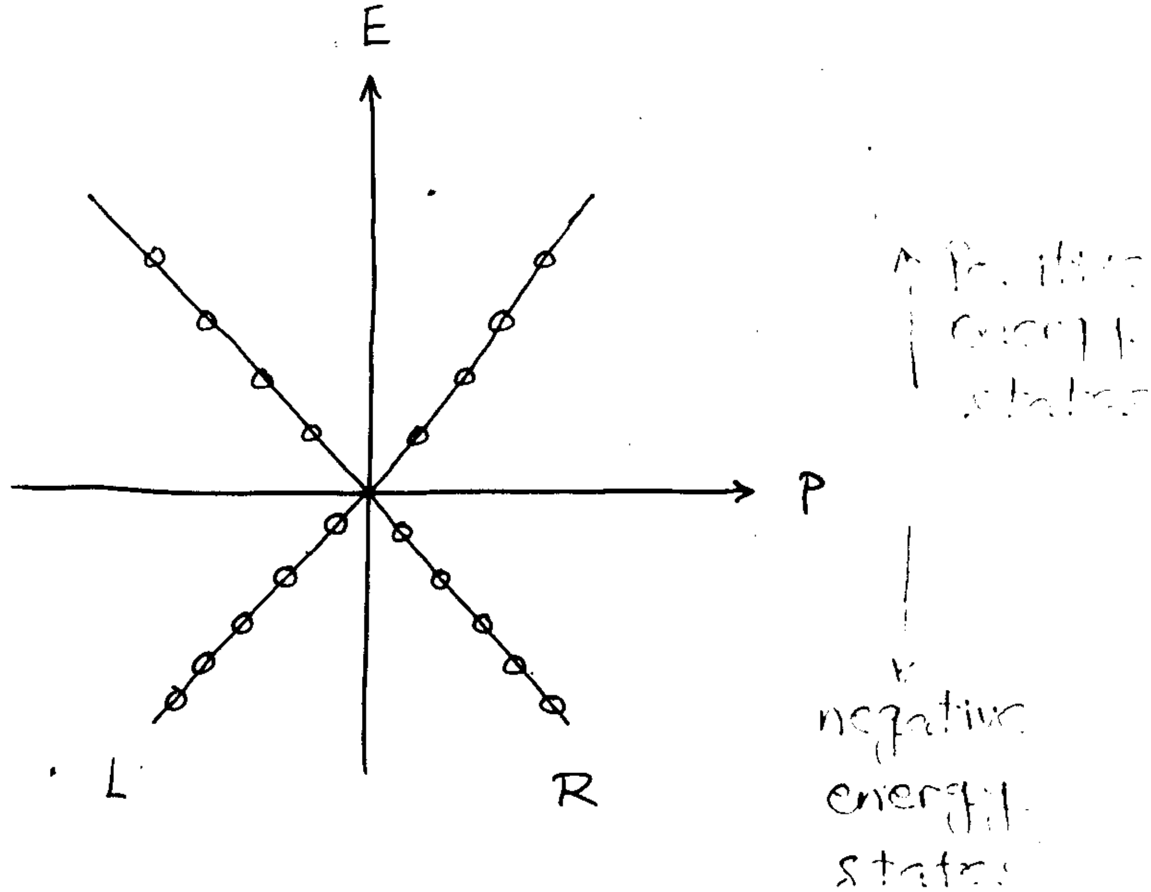
Symmetry

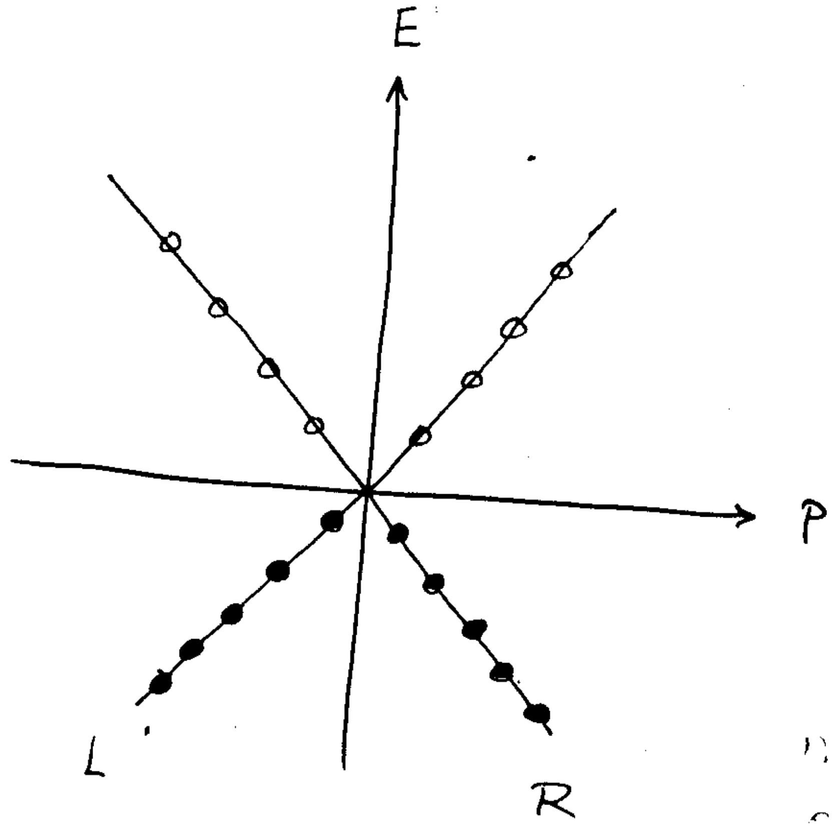
	global	local
[illegible]	chiral symm.	weak
[illegible]	electric charge	electromag.

There is yet more: Anomalous Symm.
(purely quantum)

- a symmetry that is broken by quantum effects.

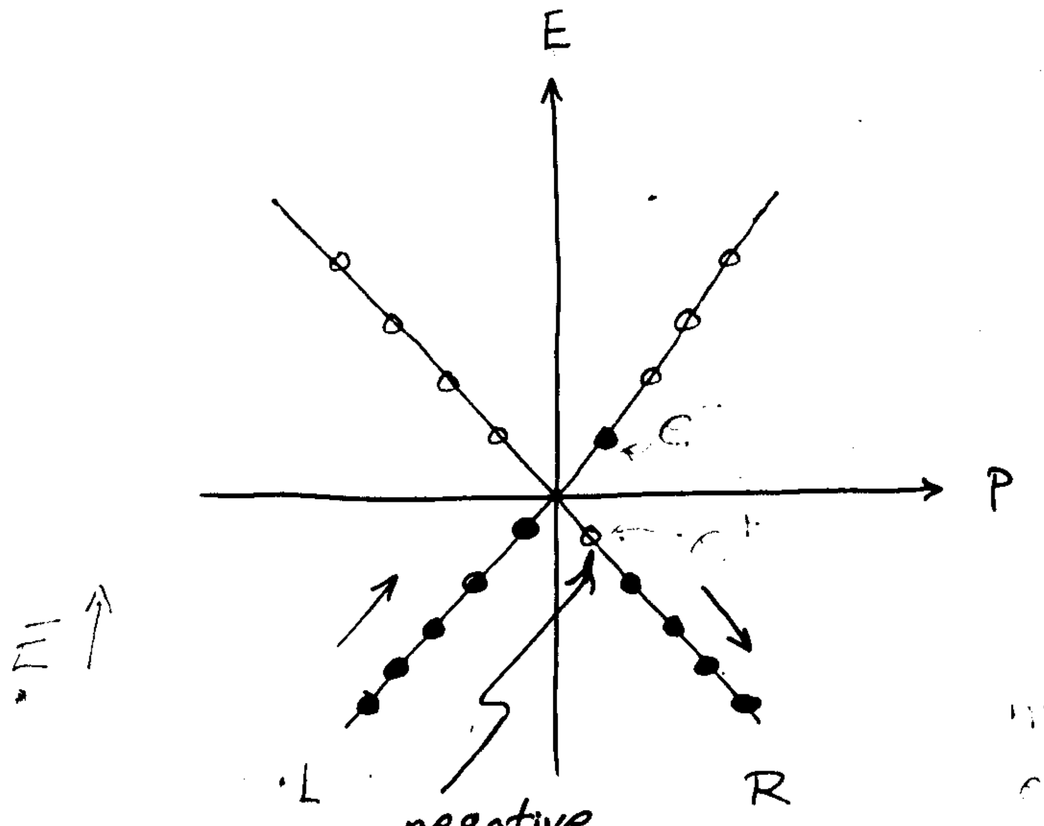
— Chiral gauge symmetries (weak interactions) suffer from this.


E
P
L
R
negative energy states



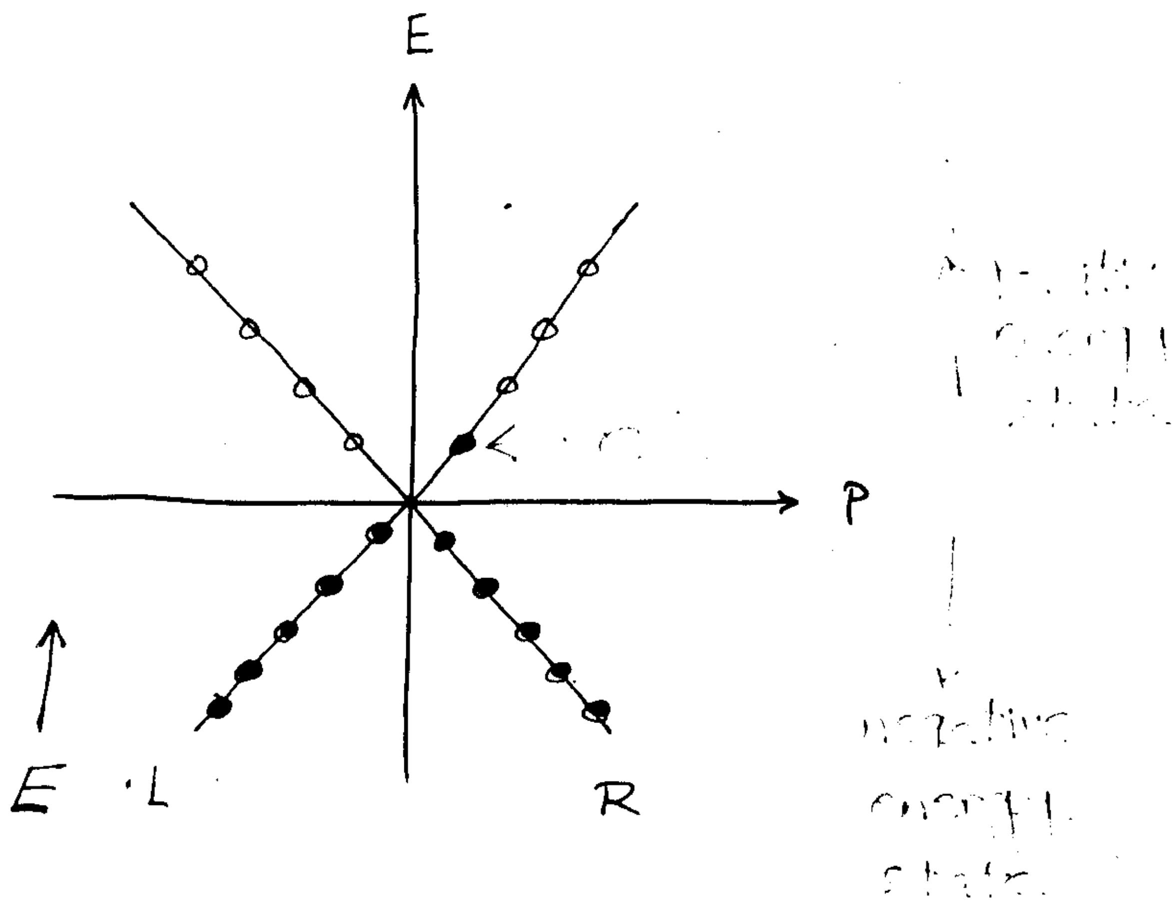
The vacuum

(all negative energy states are filled)



negative
energy hole
= anti-particle

Anomalous production



Chiral theory in which L handed particles are charged and R handed particles are neutral.

Thus, even though Q = electric charge is classically conserved, it is not quantum mechanically.

Vacuum Structure

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + |\underbrace{D_\mu \varphi}|^2 - \frac{\lambda}{2}(|\varphi|^2 - v^2)^2$$

$$D_\mu \varphi \equiv \partial_\mu \varphi - ieA_\mu \varphi$$

(one space + one time dim) : 1 + 1 $(\mu = 0, 1)$

This is called the Abelian Higgs model.

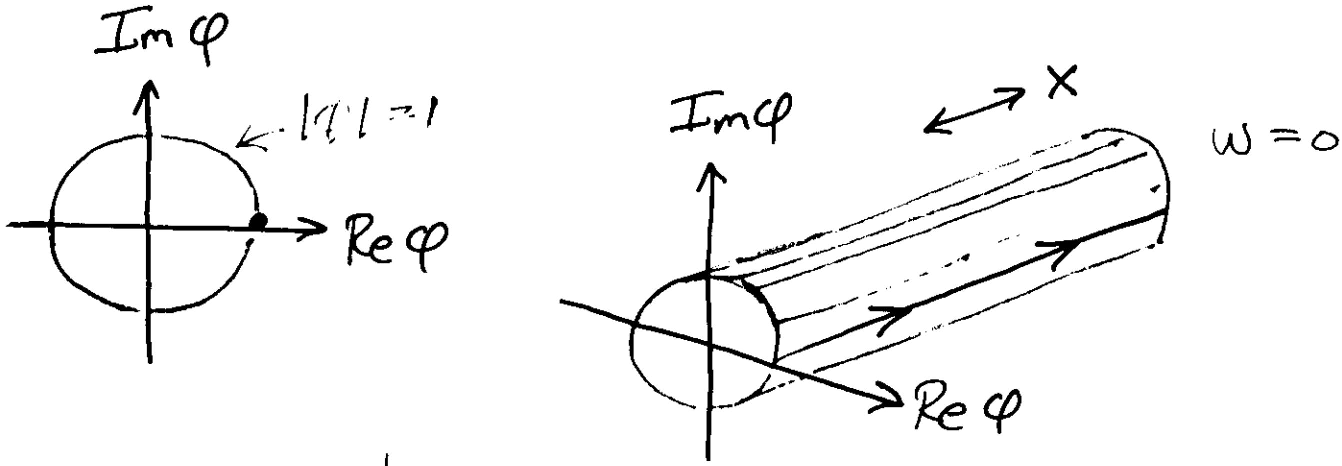
The vacuum has $|\varphi| = v \; (\neq 0)$

$\varphi(x = +\infty) = \varphi(x = -\infty)$: Periodic B.C.

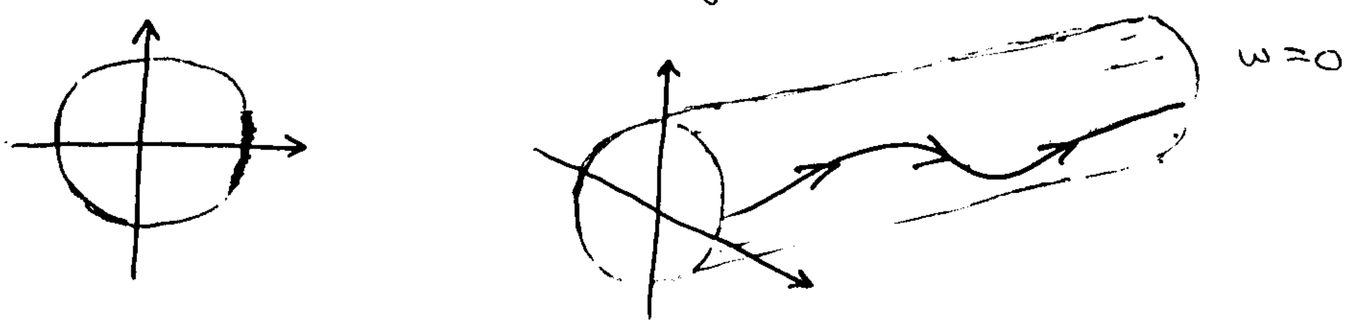
$$\varphi_{vac} : S^1 \longrightarrow U(1) \sim S^1 \Longrightarrow$$

$$\underbrace{W[vac]}_{\text{winding } \#.} \in \mathbb{Z}$$

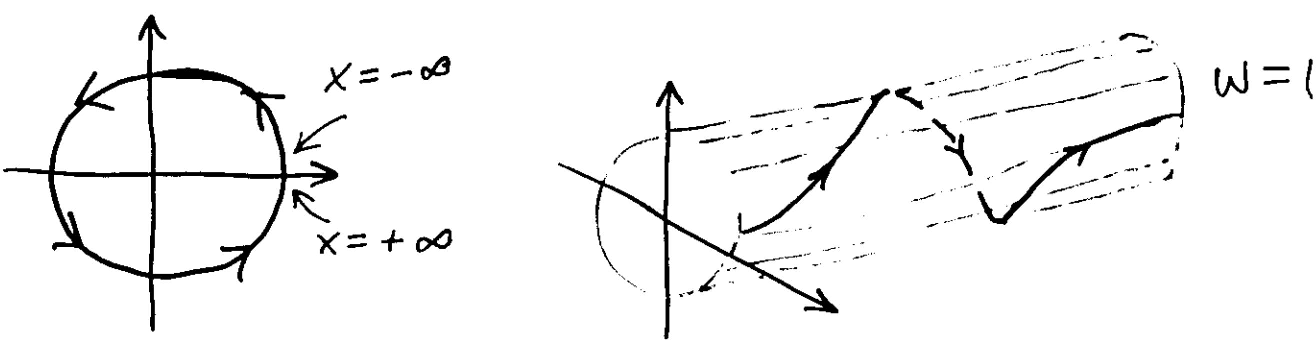
Vacuum Structure of 1+1 Abelian Higgs (Jackiw + Rebbi)



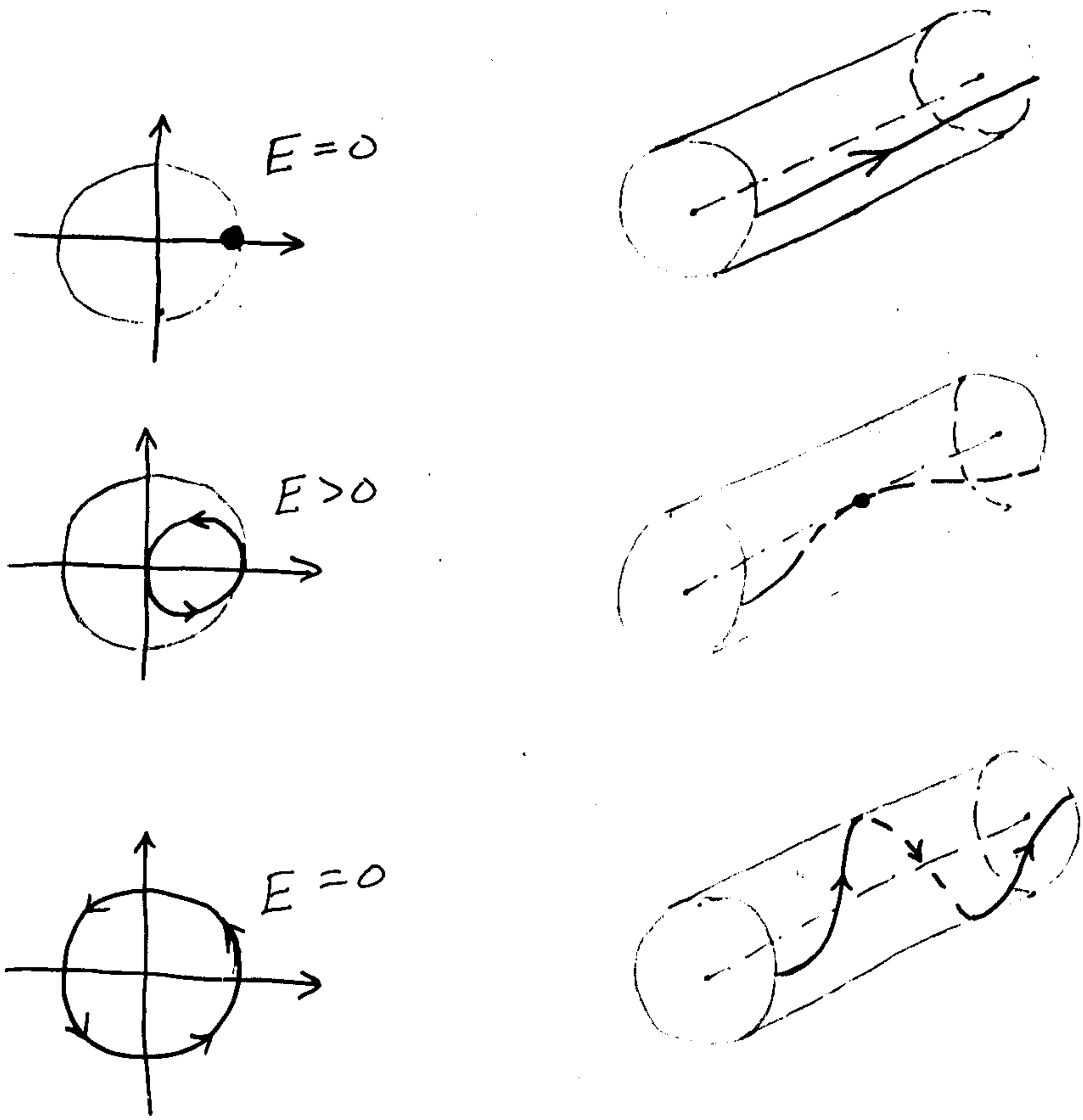
↓ topologically equivallent



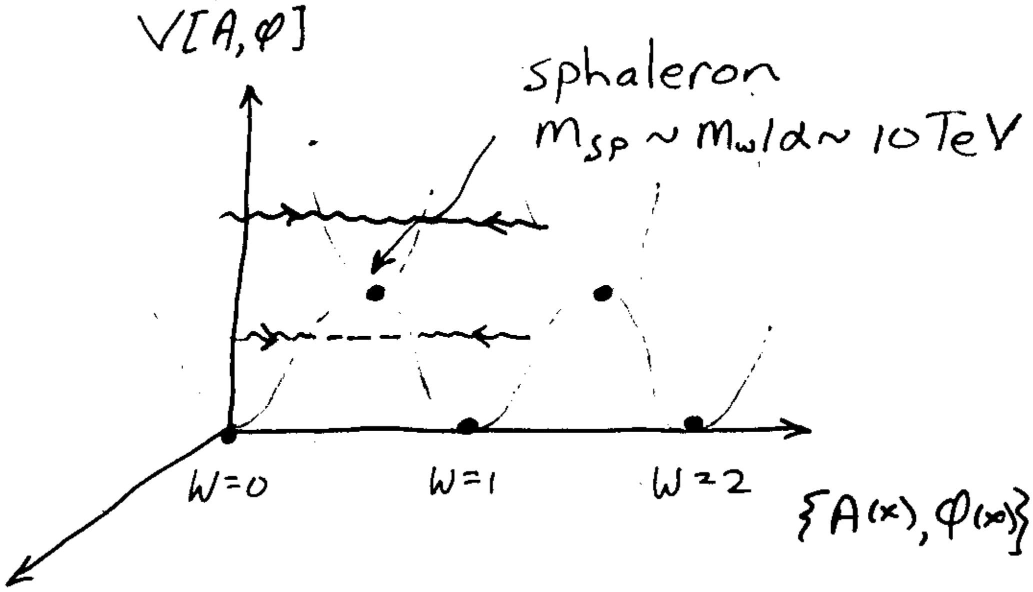
↓ topologically non-equiv.



In transitions between inequiv. vacua, you must leave the manifold of vacuum configurations (go over the sphaleron)



The sphaleron

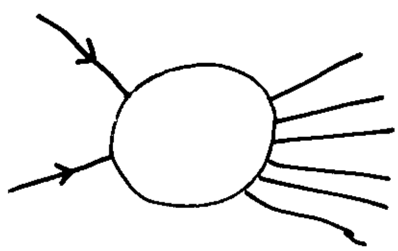


Field space: $A(x) = \int d^3p \, \tilde{A}(p) \underbrace{e^{i\underline{p}\cdot\underline{x}}}_{e_p(\underline{x}) = \text{basis vectors}}$

Axial anomaly $\Rightarrow$ $\Delta B = \Delta W$ (non perturbative!!)

low energy: $\sigma \sim e^{-4\pi/\alpha}$ (tunneling)

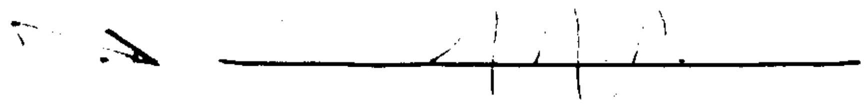
High Energy: σ ?   High T: $\sigma \sim \mathcal{O}(1)$



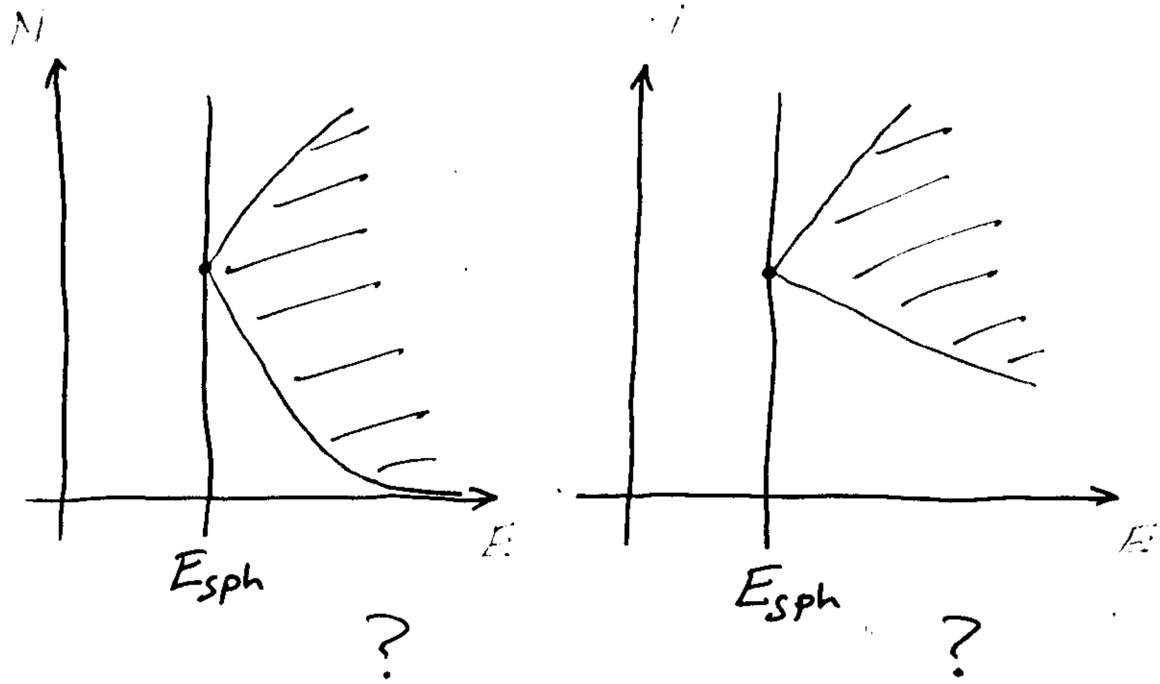
The Computational Problem

Linear: Identify N

passes over the sphaleron



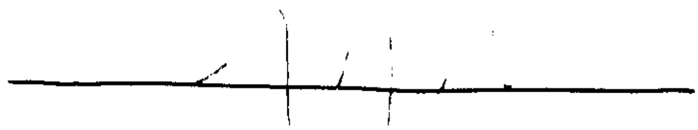
Topology Change ($\Delta W \neq 0$)



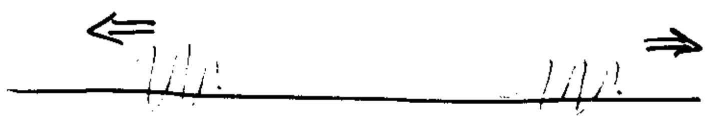
Find the boundary in the E–N plane for solutions with $\Delta W \neq 0$ (pass over the sphaleron).

Computational Strategy

Implement a time reversed evolution:



start w/ init. config on top of the barrie (i.e. $\exists\, x \ni \phi(x) = 0$) parametarized by $\{c_i\}$.



Evolve until linear regime and pull out N: $N = \sum_n |a_n|^2$.

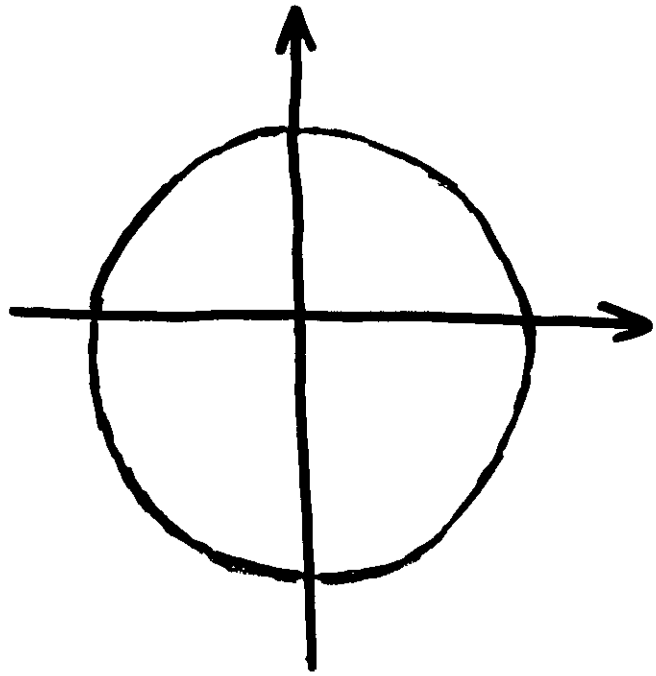
where $\varphi_{asym} = \sum_n a_n \varphi_{(n)}$

normal modes

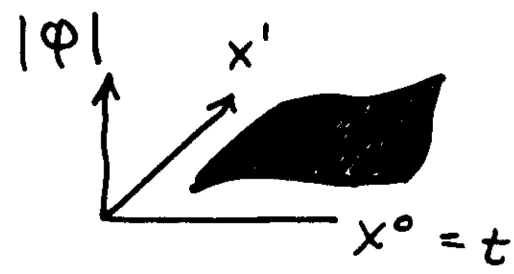
Computational Details

- gauge invariant (space) discretization of equations of motion.
- time evolution implemented with the leap-frog algorithm
  - 2^{nd} order accurate
  - Exact time reversal
  - Conserves E exactly in the linear regime
- gauge invariant subltety.
  When pulling out the ∂_n, must gauge transform to "small" perturbations about trivial vacuum.

phase is color coded :



$\varphi(x^1, x^0) \leftarrow$ 3-D colored surface



color code may not be unique.
Map will be chosen to give pretty pictures.

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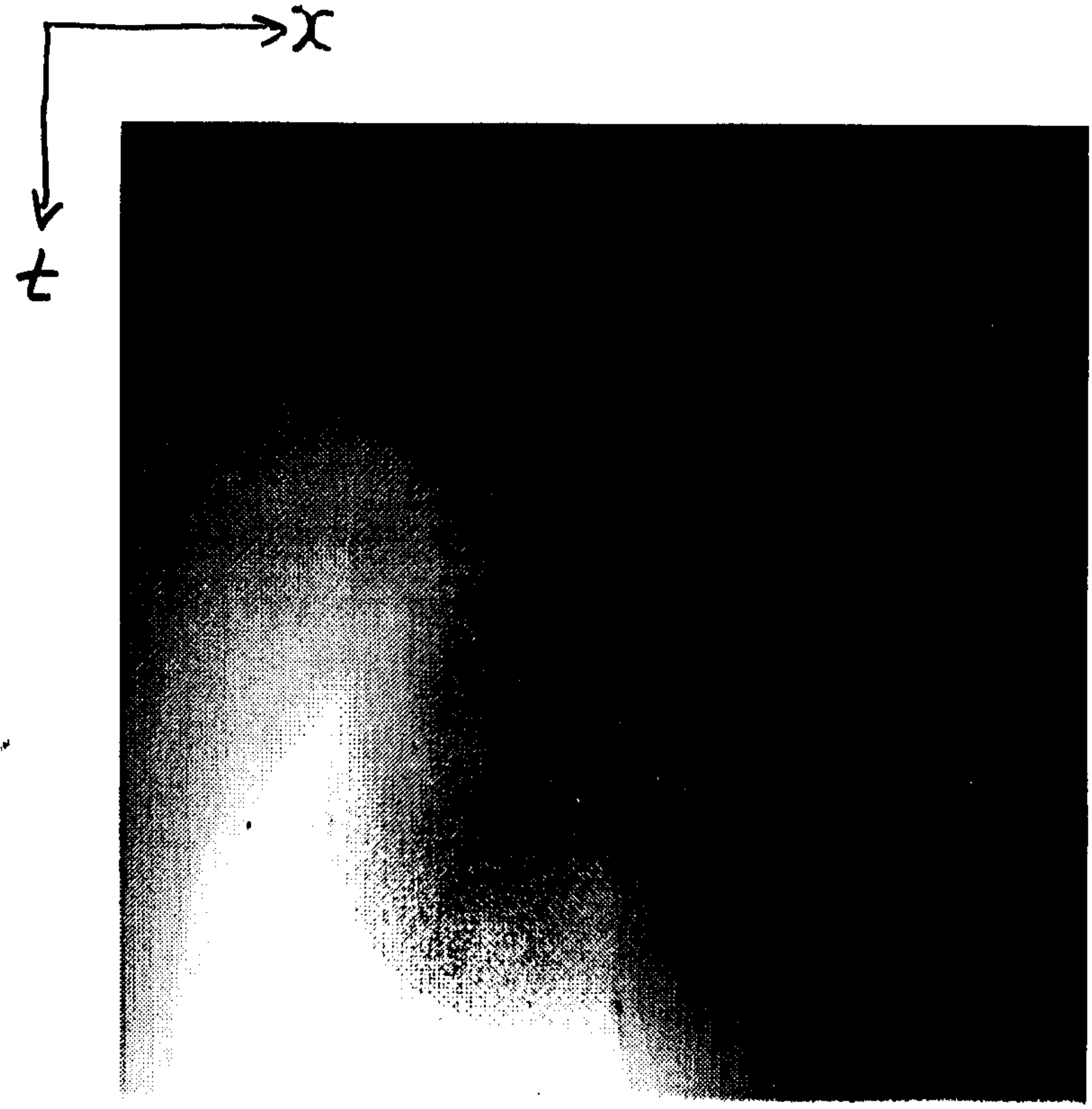
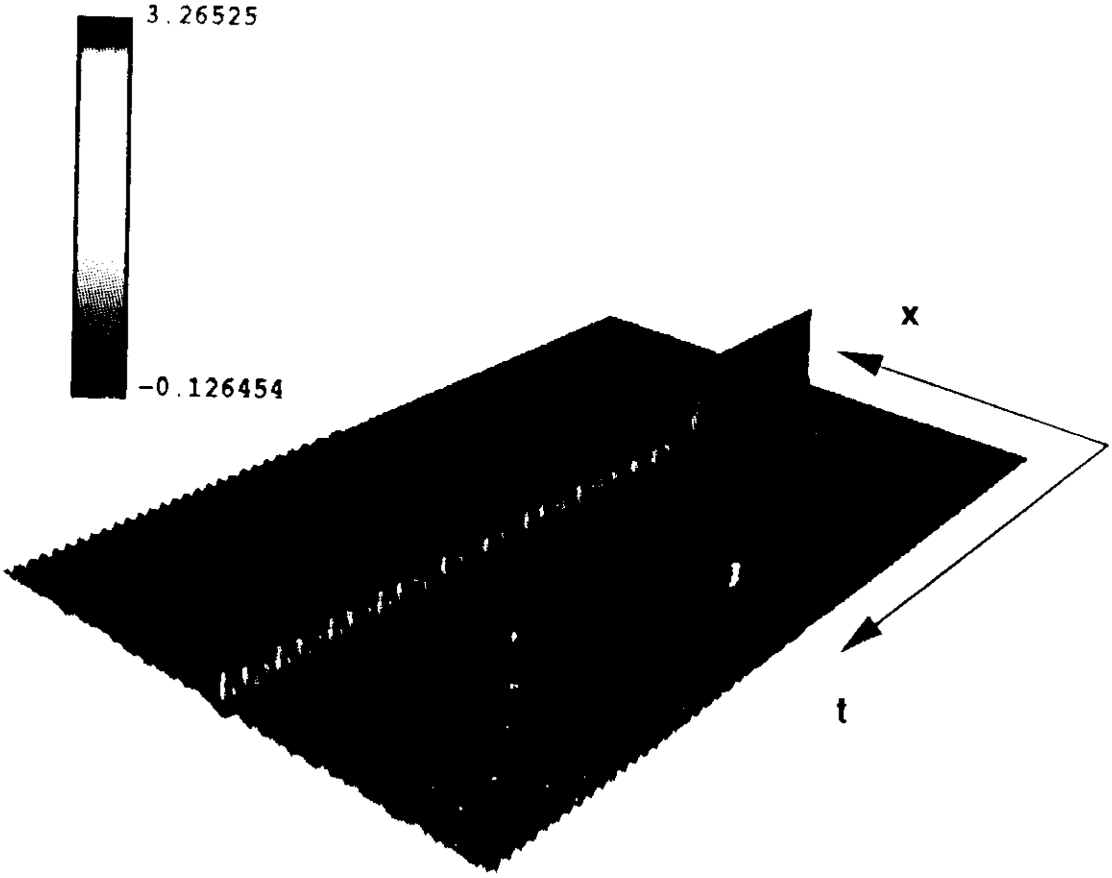


Instanton transition in the 2-d Abelian Higgs model



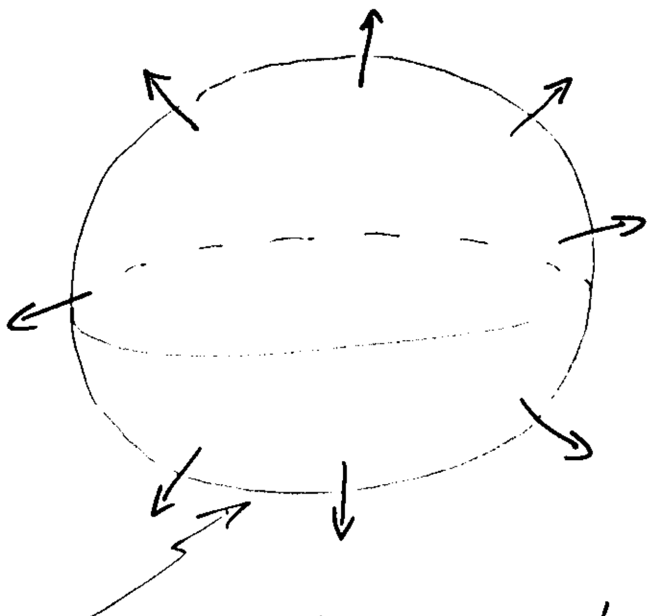
Sphaleron decay in 2D Abelian
Higgs model: the Higgs field

The Standard model in 3+1 dim

$$\left.\begin{matrix} A_\mu(\underline{x},t) \\ \Phi(\underline{x},t) \end{matrix}\right\} \xrightarrow[\text{reduction}]{\text{radial}} \begin{matrix} a_\mu(r,t) \\ \chi(r,t) \\ \phi(r,t) \end{matrix}$$

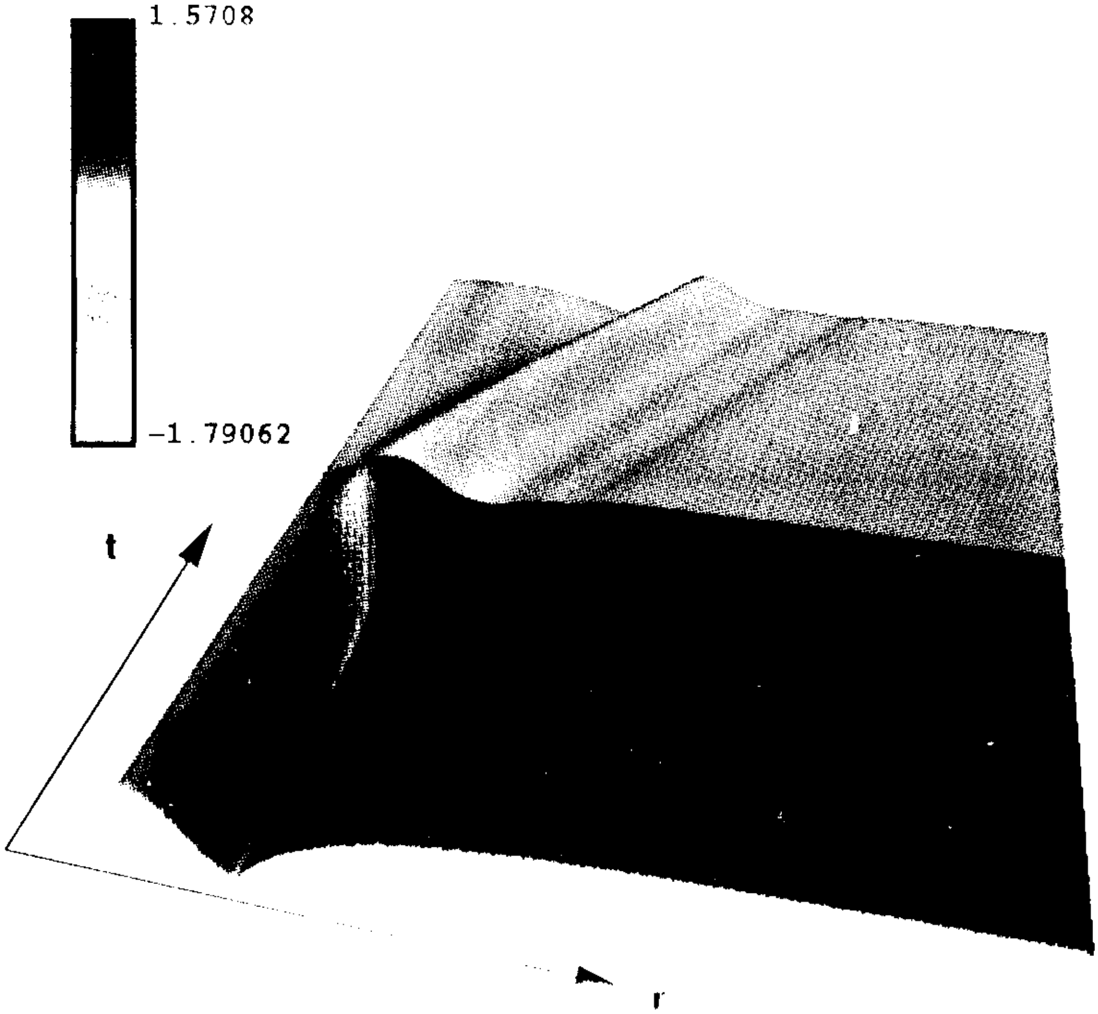
$\mu = 0,1,2,3$ $\qquad\qquad\qquad\qquad$ $\mu = 0,1$

Radial symm. gives an effective theory in 1+1 dim (r,t).

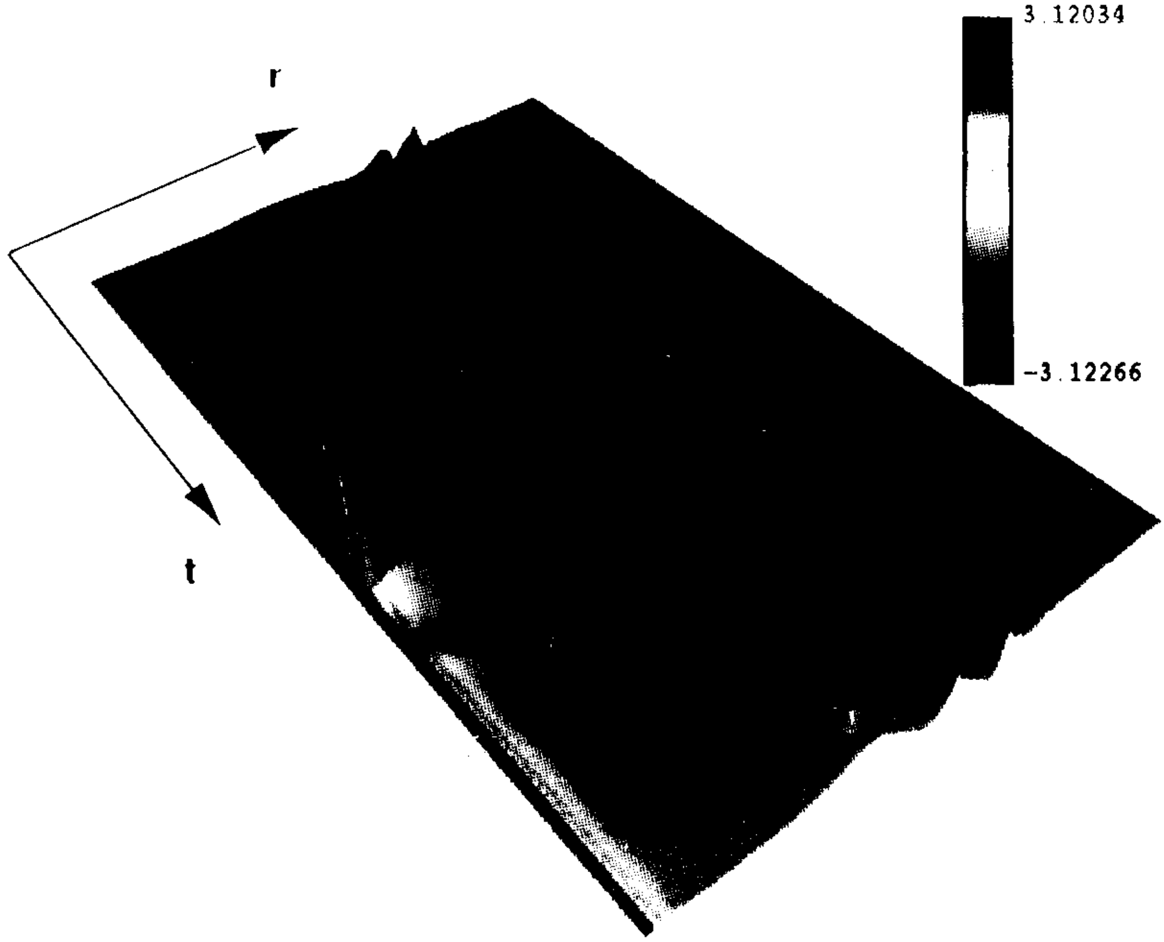


Energy density $\rho \sim \frac{1}{r^2} \sim \frac{1}{t^2}$

Much better linearization properties.



Sphaleron decay in the 4D SU(2) Higgs model:
the gauge field



Topology changing process in the
4D SU(2) Higgs model:
the gauge field

More on the Computation

1. $\chi = \chi_{sph} + \delta\chi$
   $\varphi = \varphi_{sph} + \delta\varphi$
   $\pi_\chi = \delta\pi_\chi$
   $\cdots$

   $\Big\}$ parametrize by c_i

   note: $\delta\chi(r=0) = \delta\chi(r=\infty) = 0$ etc.

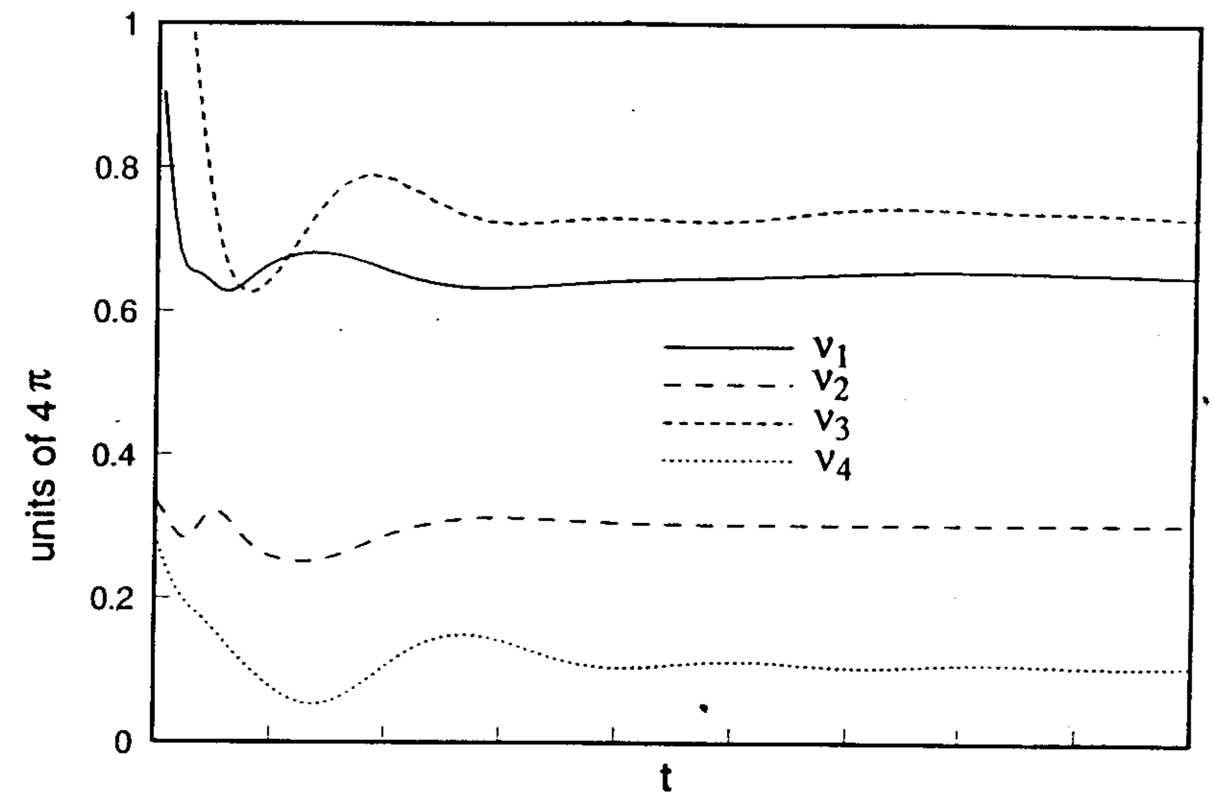
   $\Rightarrow \ \exists\, r_i > 0 \ni \chi(r_i) = 0.$

2. Evolve for a short time
3. Gauge transform to $\operatorname{Arg}\chi = -\frac{\pi}{2}$
4. Evolve until system linearizes $\{ \epsilon \cong \epsilon_{lin} \cong \sum \omega_n |a_n|$
5. Extract normal mode amplitudes a_n and calculate $\nu = \sum_n |a_n|^2 \equiv \nu(c_i)$
   - discretized normal modes numerically
   - continuous normal modes analytically

4 types of modes

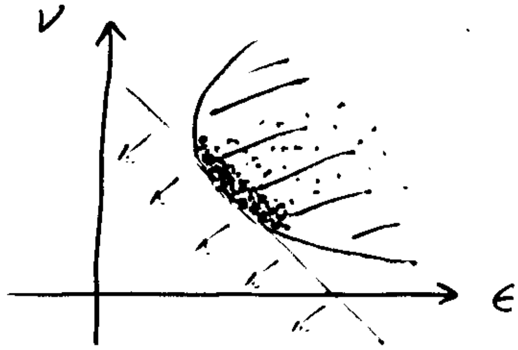
1 of mass m_h (Higgs)

3 of mass m_w (gauge bosons)



$N = 511$ 50 modes
$\Delta r = 0.1$
$\lambda = 0.1$

Sample $\{C_i\}$ stochastically with a weight $e^{-\beta\epsilon-\mu\nu}$ favoring small ν.



$\lambda = 0.1$

$N = 256$

$\Delta r = 0.2$

dim of $\{C_i\} = 20 \times 8$

normal modes $= 4 \times 50$

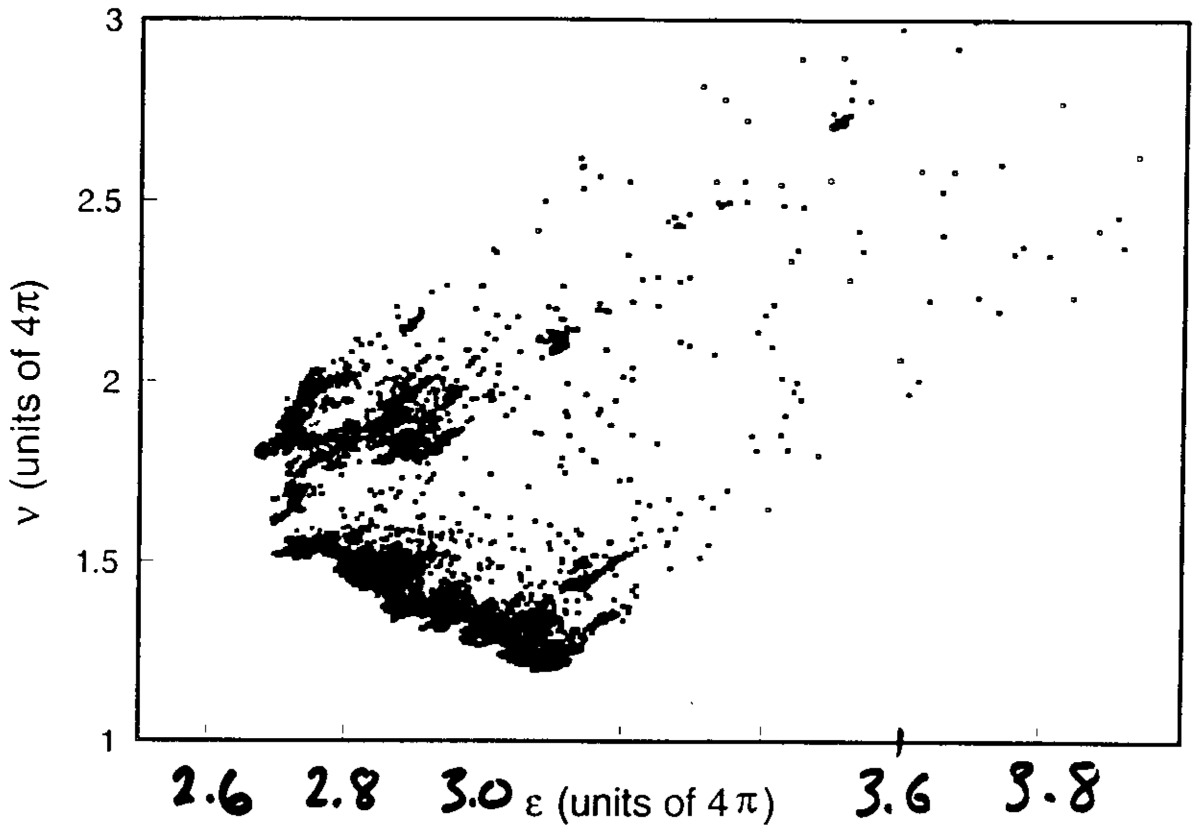
Approximately 50,000 data points

(60 hrs CPU)

note: U.V. cutoff gives $\omega_{max} \sim \frac{1}{\Delta r}$

In practice $\omega_{max} = \frac{c}{\Delta r}$ $\quad c < 1$

$\therefore \quad \min \frac{\nu}{\epsilon} \sim \frac{1}{\omega_{max}} \sim \frac{\Delta r}{c}$


3
2.5
2
1.5
1
ν (units of 4π)
2.6
2.8
3.0
ε (units of 4π)
3.6
3.8

III. Plans for the future

- Our MC results are at their limit for our grid size
  - Larger lattice + bigger comp.
- Implement the full complex RST evolution. (below barrier)
  - initial testing w/ 1+1 Dim. Abelian Higgs complete. (formidable numerical problem)
  - adapt to 3+1 SU(2) Higgs and see what happens.