INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

SMR.858 - 11

V

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

12 June - 28 July 1995

BASICS OF CFT, STRING COMPACTIFICATIONS, MIRROR SYMMETRY AND SPACETIME TOPOLOGY CHANGE

B. GREENE
Newman Lab. of Nuclear Studies
Cornell University
Ithaca, NY
USA

Please note: These are preliminary notes intended for internal distribution only.

MAIN BUILDING STRADA COSTIERA, 11 TEL. 2240111 TELEFAX 224163 TELEX 460392 ADRIATICO GUEST HOUSE VIA GRIGNANO, 9 TEL. 224241 TELEFAX 224531 TELEX 460449
MICROPROCESSOR LAB. VIA BEIRUT, 31 TEL. 224471 TELEFAX 224600 TELEX 460392 GALILEO GUEST HOUSE VIA BEIRUT, 7 TEL. 22401 TELEFAX 2240310 TELEX 460392

LECTURE 4

SUMMARY OF LECTURES 0,1,2,3:

I) INTRODUCTION/MOTIVATION/BASICS OF CFT

II) THE $N=2$ SUPERCONFORMAL ALGEBRA

- GENERAL PROPERTIES (REP THEORY; (C,C), (a,c) RINGS, etc.)
- EXAMPLES (FREE, CY σ-MODEL, L-G, MIN MODELS)

III) FAMILIES OF $N=2$ THEORIES

- TRULY MARGINAL OPERATORS (BASED ON (C,C), (a,c) FIELDS)
- MODULI SPACES I: CFT/NLσM

IV) INTERRELATIONS BETWEEN REALIZATIONS OF $N=2$ SCA

- LG/MIN MODELS ($W_{LG} = \Psi^{P+2} \longleftrightarrow$ LEVEL P (A-INV) MM)
- LG/CALABI-YAU ($W_{LG}(\Psi_1, \dots, \Psi_5) \underset{\text{RELATED}}{\overset{\text{"PHASE"}}{\longleftrightarrow}}$ CY: $W_{LG}(z_1, \dots, z_5) = 0$)
- MODULI SPACE II

V) MIRROR MANIFOLDS

- MOTIVATION, • CONSTRUCTION
- MODULI SPACE III
- IMPLICATIONS

* VI) SPACETIME TOPOLOGY CHANGE: MILD/DRASTIC

IV.) SPACETIME TOPOLOGY CHANGE

(ASPINWALL, BRG, MORRISON; WITTEN)

- BASIC IDEAS: GEOMETRY OF UNIVERSE IS DYNAMICAL. MIGHT TOPOLOGY BE AS WELL?

- ONLY POSSIBLE IN A THEORY DIFFERING FROM CLASSICAL GENERAL RELATIVITY ON SMALL SCALES (cf: SINGULARITY RESULTS)

  - "QUANTUM" GRAVITY:

CURVATURE FLUCTUATIONS: $\left(\hbar G/c^3\right)^{1/2}/L^3 \simeq \frac{L_{pl}}{L^3}$

$$L \longrightarrow L_{pl} \Rightarrow \text{HUGE}$$

  - STRING THEORY:
    - JUST NEED CLASSICAL STRING THEORY FOR "MILD" TOPOLOGY CHANGE (Aspinwall, BRG, Morrison; Witten)
    - NEED QUANTUM EFFECTS FOR "DRASTIC" TOPOLOGY CHANGE (BRG, Morrison, Strominger)

IV A) A PUZZLE FOR MIRROR SYMMETRY

M		W
COMPLEX STRUCTURE DEFORMATIONS	$\longleftrightarrow$	KÄHLER STRUCTURE DEFORMATIONS
KÄHLER STRUCTURE DEFORMATIONS	$\longleftrightarrow$	COMPLEX STRUCTURE DEFORMATIONS

PROBLEM: THE PARAMETER SPACES OF COMPLEX STRUCTURE AND KÄHLER STRUCTURE DEFORMATIONS ARE QUITE DIFFERENT: DO NOT APPEAR TO BE ISOMORPHIC.

KÄHLER STRUCTURE

- J = KÄHLER FORM = $i g_{i\bar{j}}\, dX^i \wedge dX^{\bar{j}}$

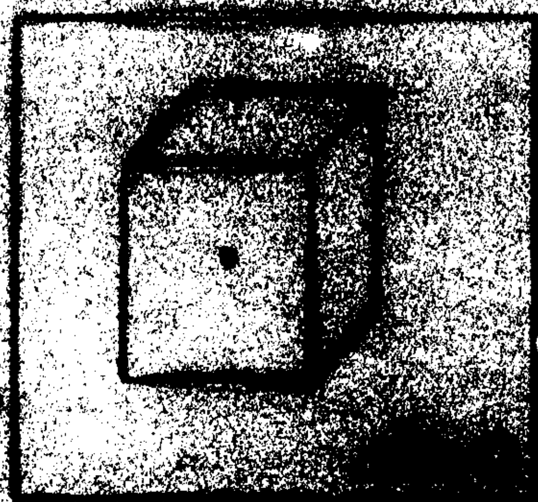
- POSITIVITY CONDITIONS:

$$\int_M J \wedge J \wedge J > 0$$

$$\int_{S \subset M} J \wedge J > 0$$

$$\int_{C \subset M} J > 0$$

J SATISFIES ⇒ sJ $s > 0 \in \mathbb{R}$



- COMPLEXIFICATION:

STRING THEORY:

ANTISYM

$$J \rightarrow K = B + iJ$$

INVARIANCE: $B \rightarrow B + \Lambda$, $\Lambda \in H^2$

$$W_\ell = e^{2\pi i (B_\ell + i J_\ell)}$$

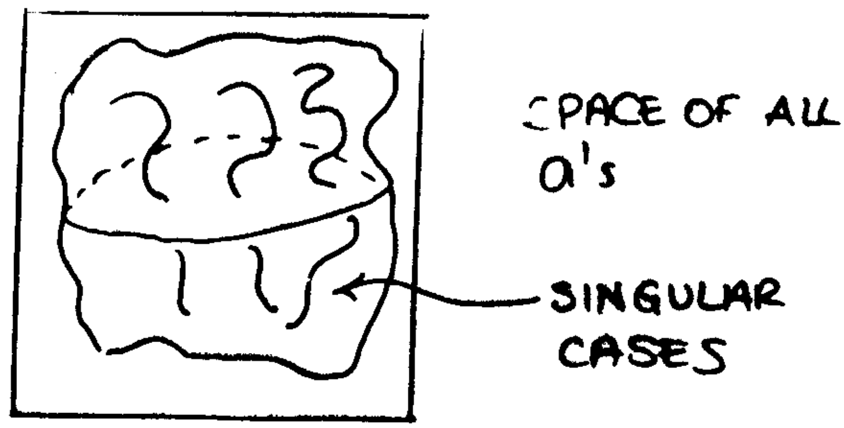
COMPLEX STRUCTURE

- TYPICAL CY's:

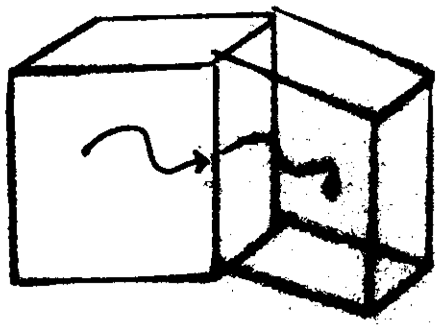
$$M =: z_1^{p_1}+z_2^{p_2}+z_3^{p_3}+z_4^{p_4}+z_5^{p_5}+a_1 z_1 z_2 z_3 z_4 z_5 + a_2 z_1^{q_1} z_2^{q_2} + \cdots + a_{\text{dim of C.S.}}\, z_1^{r_1}\cdots z_5^{r_5} = 0$$

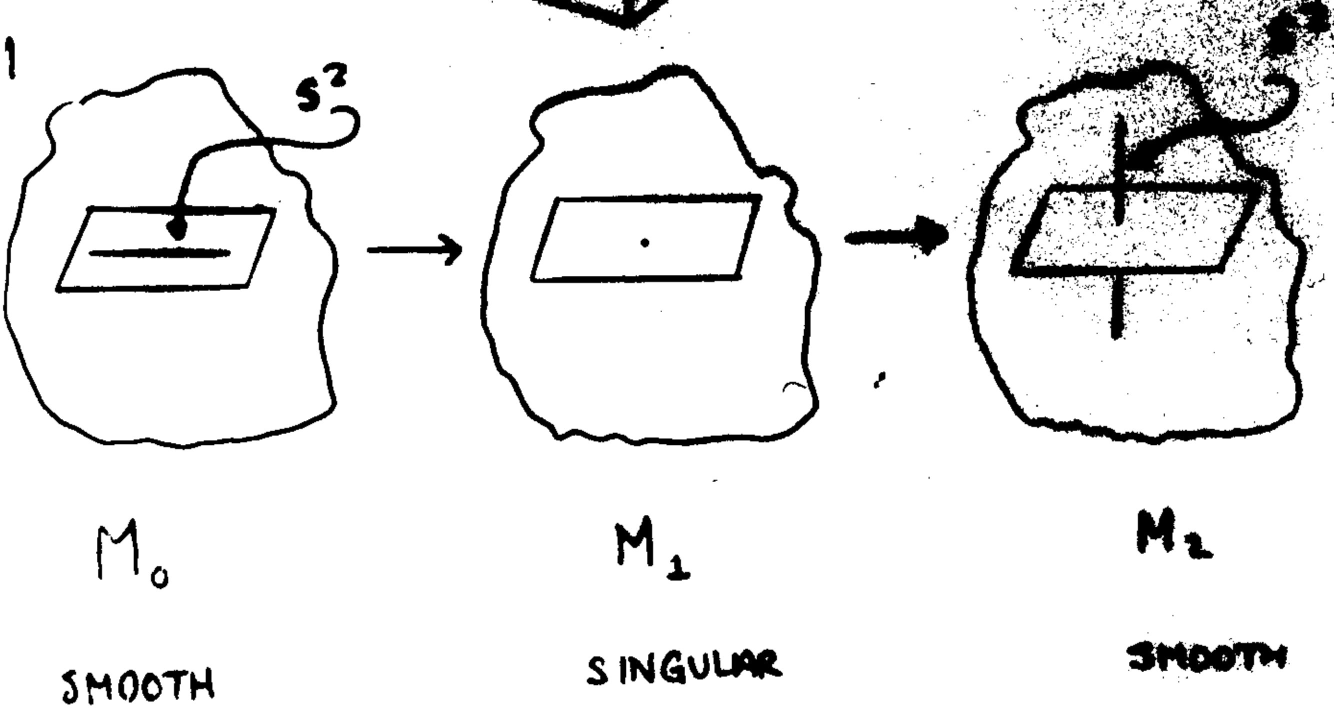
- CHANGING THE VALUES OF THE a's CHANGES THE COMPLEX STRUCTURE

- RESTRICTION ON a's: $P=0$ & $\partial P/\partial z_i = 0 \;\forall i$ MUST NOT HAVE A COMMON SOL'N. THIS IS ONE EQN IN a's.



IV. B) A Possible Resolution





- Mathematical operation: "Flop"
- Topology of $M_2 \neq$ Topology of M_1
- Can repeat → Adjoin numerous regions each being Kähler parameter space for "flopped" manifolds

1.2

• MATHEMATICALLY:

- ADJOINING ALL OF THESE KÄHLER CONES YIELDS • "ENLARGED" KÄHLER MODULI SPACE
  - TORIC VARIETY
- THIS OBJECT IS ISOMORPHIC MATHEMATICALLY TO COMPLEX STRUCTURE MODULI SPACE

• PHYSICALLY: STILL A TROUBLING DISCREPANCY:

KÄHLER $\mathcal{M}$ COMPLEX ST. $\tilde{\mathcal{M}}$

SINGULAR CY SPACE

REAL CODIMENSION 1   REAL CODIMENSION 2

¿RESOLUTION? ¿DO WE TRUST THAT CFT'S ON LHS WALLS ARE SINGULAR? THESE ARE STRONGLY COUPLED THEORIES: α'/R^2 NOT SMALL.

• ON RHS: $\alpha'/\tilde{R}^2$ IS (CAN BE TAKEN) SMALL! STRONG COUPLING $\longmapsto$ WEAK COUPLING. $\Rightarrow$ TRUST RHS - [illegible].

EXAMPLE (CHOSEN FOR CALCULATIONAL EASE)

- $M =: z_0^3 + z_1^3 + z_2^6 + z_3^9 + z_4^{18} = 0$ IN $W\mathbb{P}^4_{(6,6,3,2,1)}$
- MIRROR : $\tilde{M} = M/G \qquad G = (\mathbb{Z}_3)^3$

$$(z_0, \dots, z_4) \mapsto (\alpha\beta z_0, \alpha^{-1} z_1, \beta^{-1} z_2, \gamma^{-1} z_3, z_4)$$

$$\alpha^3 = \beta^3 = \gamma^3 = 1$$

- KÄHLER MODULI SPACE OF M : 5-DIMENSIONAL WITH $\underline{\underline{5}}$ CELLS (CORRESPONDING TO 5 TOPOLOGICALLY DISTINCT SPACES).

- COMPLEX STRUCTURE OF $\tilde{M}$: 5-DIMENSIONAL

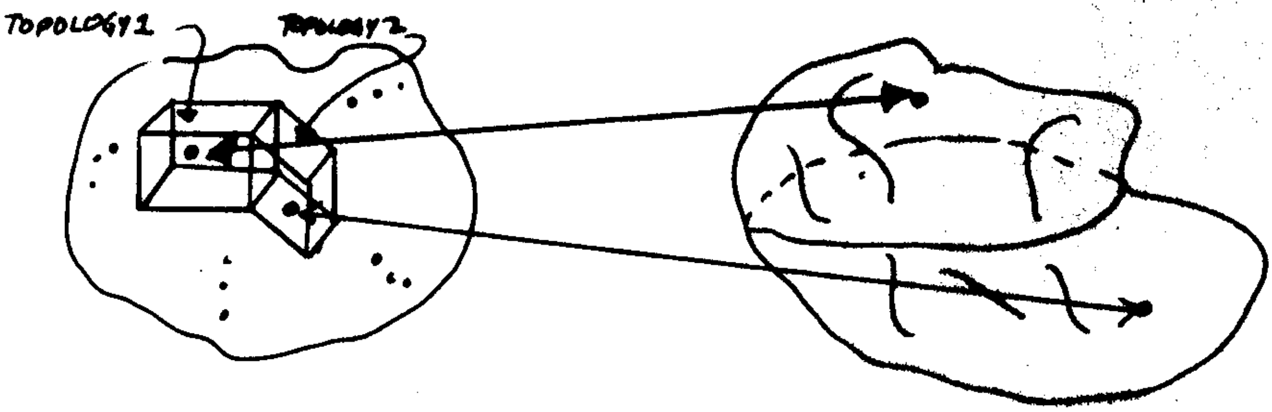
$$z_0^3 + z_1^3 + z_2^6 + z_3^9 + z_4^{18} + a_1 z_0 \cdots z_4 + a_2 z_2^3 z_4^9 + a_3 z_3^6 z_4^6 + a_4 z_3^3 z_4^{12} + a_5 z_2^3 z_3^3 z_4^3 = 0$$

- WRITE : $a_i = s^{l_i}$

- SEEK 5 CHOICES OF $\vec{l}$ SO THAT AS $s \to \infty$ GET EQUALITY.
- TORIC GEOMETRY

SENSITIVE TEST:

- IF CFT REQUIRES ADDITIONAL REGIONS, I.E., IF FLOP TRANSITION IS PHYSICAL, IT MUST BE THAT ANY POINT ON KÄHLER SIDE HAS A CORRESPONDING POINT ON COMPLEX STRUCTURE SIDE YIELDING IDENTICAL PHYSICS.



$M_{(\text{KÄHLER})}$ PARAMETER SPACE

$\mathcal{M}_{(\text{COMPLEX ST.})}$ PARAMETER SPACE

$$\langle A_{(\alpha)} A_{(\beta)} A_{(\gamma)} \rangle$$

$$\int_{M_p} b_{(\alpha)} \wedge b_{(\beta)} \wedge b_{(\gamma)} + O(e^{-nR}) \stackrel{?}{=} \int_{\mathcal{M}} \Omega(t) \wedge h_{(\alpha)} \wedge h_{(\beta)} \wedge h_{(\gamma)} \Omega + 0$$

TOPOLOGICAL INVARIANT

FIVE DISTINCT TOPOLOGIES :

Resolution	1	2	3	4	5
$\frac{(E_1^3)(E_4^3)}{(E_1^2 E_4)(E_1 E_4^2)}$	-7	$0/0$	$0/0$	∞	9
$\frac{(E_2^2 E_4)(E_3^2 E_4)}{(E_2 E_3 E_4)(E_2 E_3 E_4)}$	2	4	0	$0/0$	$0/0$
$\frac{(E_2 E_3 E_4)(H E_2^2)}{(E_2^2 E_4)(H E_2 E_3)}$	1	1	1	0	$0/0$
$\frac{(E_2 E_3 E_4)(H E_1^2)}{(E_1^2 E_4)(H E_2 E_3)}$	2	1	∞	$0/0$	0

Table 1: Ratios of intersection numbers
(3-POINT FUNCTIONS)

FIVE DIRECTIONS IN COMPLEX STRUCTURE MODULI SPACE OF $\tilde{M}$:

Resolution	1	2	3	4	5
Exponents	$(\frac{11}{9}, 1, \frac{4}{3}, \frac{5}{3}, \frac{5}{3})$	$(\frac{7}{6}, \frac{1}{2}, 1, 1, \frac{5}{2})$	$(\frac{3}{2}, \frac{1}{2}, 2, 3, \frac{3}{2})$	$(\frac{13}{9}, 2, \frac{5}{3}, \frac{4}{3}, \frac{4}{3})$	$(\frac{11}{6}, \frac{7}{2}, 1, 1, \frac{3}{2})$
$\frac{\langle\varphi_1^3\rangle\langle\varphi_4^3\rangle}{\langle\varphi_1^2\varphi_4\rangle\langle\varphi_1\varphi_4^2\rangle}$	$-7-181s^{-1}+\ldots$			$-\frac{2}{5}s^2-\frac{129}{250}s+\ldots$	$9+289s^{-1}+\ldots$
$\frac{\langle\varphi_2^2\varphi_4\rangle\langle\varphi_3^2\varphi_4\rangle}{\langle\varphi_2\varphi_3\varphi_4\rangle\langle\varphi_2\varphi_3\varphi_4\rangle}$	$2-5s^{-1}+\ldots$	$4-22s^{-1}+\ldots$	$0+2s^{-1}+\ldots$		
$\frac{\langle\varphi_2\varphi_3\varphi_4\rangle\langle\varphi_0\varphi_2^2\rangle}{\langle\varphi_2^2\varphi_4\rangle\langle\varphi_0\varphi_2\varphi_3\rangle}$	$1+\frac{1}{2}s^{-1}+\ldots$	$1+\frac{3}{2}s^{-2}+\ldots$	$1+4s^{-1}+\ldots$	$0-2s^{-1}+\ldots$	
$\frac{\langle\varphi_2\varphi_3\varphi_4\rangle\langle\varphi_0\varphi_1^2\rangle}{\langle\varphi_1^2\varphi_4\rangle\langle\varphi_0\varphi_2\varphi_3\rangle}$	$2+27s^{-1}+\ldots$	$1-\frac{1}{2}s^{-1}+\ldots$	$-2s-33+\ldots$		$0+4s^{-2}+\ldots$

Table 4: Asymptotic ratios of 3-point functions

REMARKS

1) GENERIC:

- ANY CY / N=2 SCFT
- GENERIC PATH IN MODULI SPACE RESULTS IN TOPOLOGY CHANGE

2) MICROSCOPIC ANALYSIS:

- $\langle \mathcal{O}_i \mathcal{O}_j \mathcal{O}_k \rangle = \int_M b_i \wedge b_j \wedge b_k$ + STRING CORRECTIONS
- THROUGH WALL: DISCONTINUITY ($\int_M b_i \wedge b_j \wedge b_k$) =

  − DISCONTINUITY (STRING CORRECTIONS)

3) OTHER REGIONS:

- IN THE EXAMPLE DISUSSED:

5 TOPOLOGICALLY DISTINCT CY REGIONS
27 ORBIFOLD REGIONS *
67 "HYBRID" REGIONS
1 LANAU-GINSBURG REGION
100 TOTAL

4) SUPERSYMMETRY BREAKING

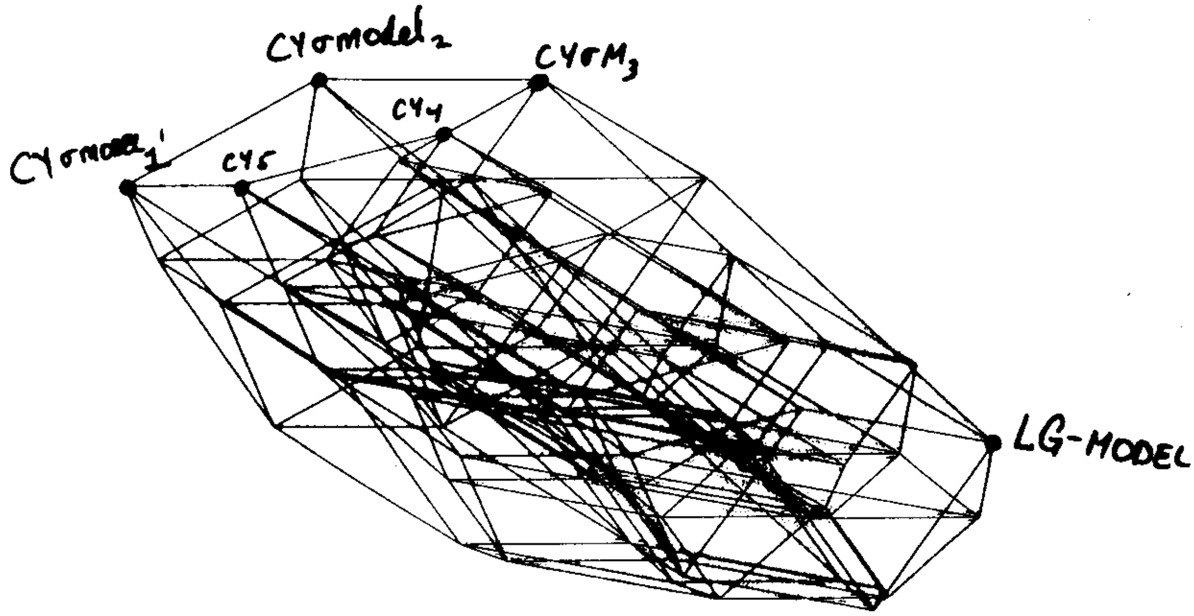


Figure 2: The web formed by rational curves connecting limit points.

$$M: \quad x_1^3 + x_2^3 + x_3^6 + x_4^9 + x_5^{18} + \dots = 0 \quad \text{in } W\mathbb{P}^4(6,6,3,2,1)$$

$$h^{1,1}_{\text{toric}} = 5$$

$$h^{2,1} = 79$$

$\mathbb{CP}^3$

# 3	2	1	1	0
# 2	1	0	0	2
# 1	0	1	1	0
# 1	0	1	1	0

h11 = 4 **NT = 15** **NCY = 3**

# 2	2	1	0	0
# 2	1	1	1	0
# 1	0	0	2	0
# 1	0	0	0	2
# 1	1	0	0	1

h11 = 5 **NT = 68** **NCY = 5**

# 3	1	1	1	1	0	0	0
# 2	1	0	0	0	0	1	1
# 2	0	1	1	0	1	0	0
# 2	0	0	0	1	1	1	0
# 1	0	0	0	1	0	0	1

h11 = 5 **NT = 1302** **NCY = 246**

4)

# 2	1	1	1	0	0	0	0	0	0
# 2	1	0	0	0	1	1	0	0	0
# 2	0	1	1	1	0	0	0	0	0
# 2	0	0	0	1	0	0	1	1	0
# 2	0	0	0	0	1	1	0	0	1
# 2	0	0	0	0	0	0	1	1	1

h11 = 6 **NT = 850** **NCY = 850**

# 2	1	1	1	0	0	0	0	0	0
# 2	1	0	0	1	1	0	0	0	0
# 2	0	1	0	0	0	1	1	0	0
# 2	0	0	1	0	0	0	0	1	1
# 2	0	0	0	1	0	1	0	1	0
# 2	0	0	0	0	1	0	1	0	1

h11 = 6 **NT = 33415** **NCY = 33415**

# 2	1	1	1	0	0	0	0
# 2	1	0	0	1	1	0	0
# 2	0	1	0	1	0	1	0
# 2	0	0	1	1	0	0	1
# 2	0	0	0	0	1	1	1

h11 = 5 NT = 22784 NCY = 302

# 3	2	1	1	0	0	0
# 1	1	0	0	1	0	0
# 1	0	1	0	0	1	0
# 1	0	0	1	0	0	1
# 1	0	0	0	1	1	0
# 1	0	0	0	1	0	1
# 1	0	0	0	0	1	1

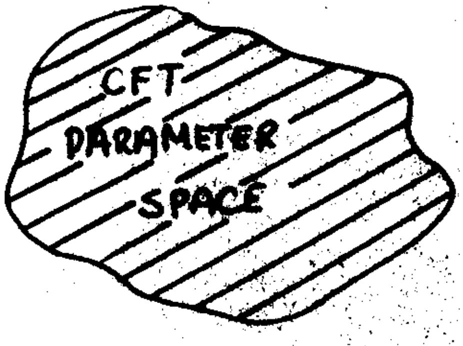
h11 = 7 NT = 29229 NCY = 1022

# 3	1	1	1	1	0	0
# 1	1	0	0	0	1	0
# 1	1	0	0	0	0	1
# 1	0	1	0	0	1	0
# 1	0	0	1	0	0	1
# 1	0	0	0	1	0	1
# 1	0	0	0	0	1	1

h11 = 7 NT = 21278 NCY = 5220

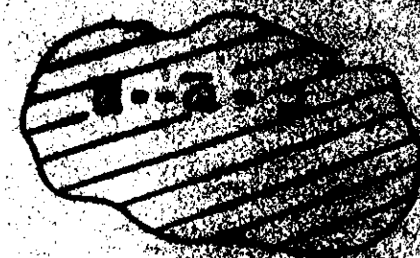
MODULI SPACE OF $N=2$ SCFT's

CFT

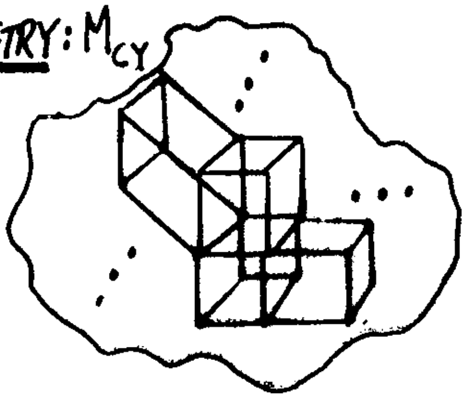


$\cong$

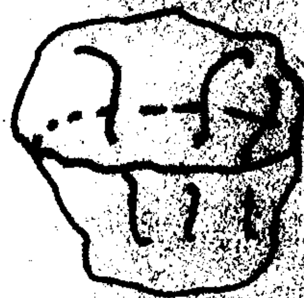
$\times$



GEOMETRY: M_{CY}



$\times$



GEOMETRY: $\tilde{M}_{CY}$

$\times$