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VI

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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BASICS OF CFT, STRING COMPACTIFICATIONS, MIRROR SYMMETRY AND SPACETIME TOPOLOGY CHANGE

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Please note: These are preliminary notes intended for internal distribution only.

LECTURE V

- SUMMARY OF LECTURE IV:
  - Complex STRUCTURE PARAMETER SPACE of M DOES NOT APPEAR TO BE ISOMORPHIC TO KÄHLER PARAMETER SPACE of MIRROR W
  - RESOLUTION:
    * KÄHLER SPACE $\longrightarrow$ ENLARGED KÄHLER SPACE
    * SOME CLASSICAL MATH. SINGS ARE STILL PHYSICALLY SMOOTH IN CFT

$\Rightarrow$ * "MILD" SPACETIME TOPOLOGY CHANGE

- LECTURE V
  - CONSIDER SINGULARITIES WHICH ARE TRULY BADLY BEHAVED AS CFT's
  - USE STRUCTURE of TYPE II STRINGS TO SHOW PHYSICALLY SMOOTH

$\Rightarrow$ * "DRASTIC" SPACETIME TOPOLOGY CHANGE

I) INTRODUCTION / REVIEW

• BASIC SET-UP:

- STRING PROPAGATION : $M_4 \times \{(2,2)\ c=9\ \text{SCFT}\}$
- PARTICULAR REALIZATION : $M_4 \times \{\text{Calabi-Yau 3-fold}\}$

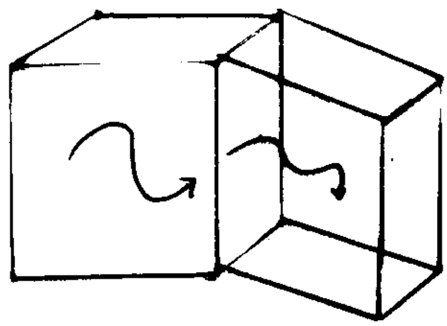
• PARAMETERS:

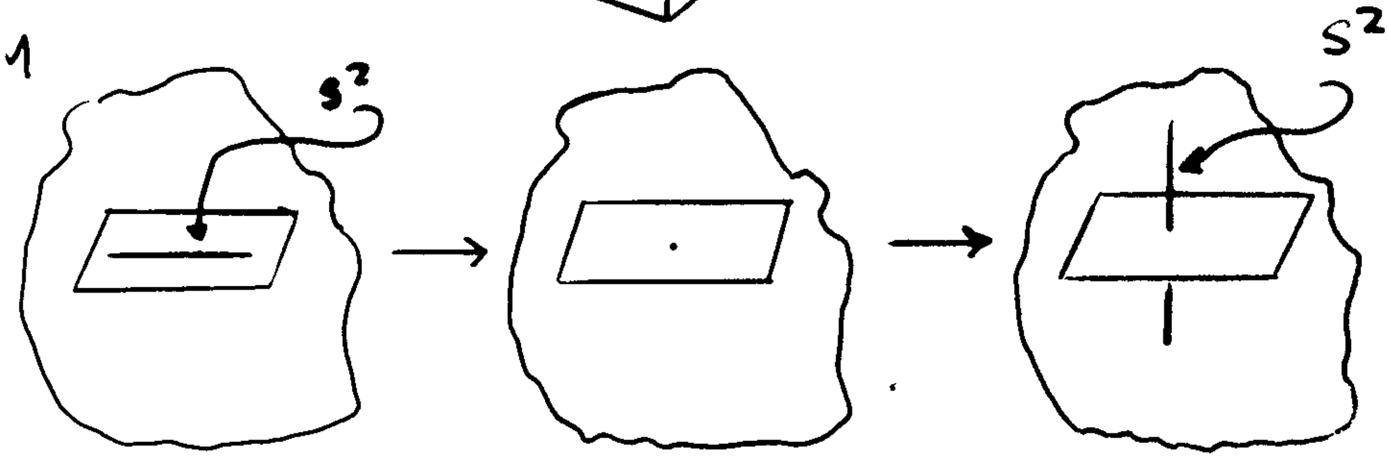
- MARGINAL OPERATORS : $\begin{cases} Q = \bar{Q} = 1 \\ Q = -\bar{Q} = 1 \end{cases}$

- GEOMETRICAL DEFORMATIONS : $\begin{cases} \text{Kähler Structure} \\ \text{Complex Structure} \end{cases}$

- MIRROR SYMMETRY : $\begin{cases} KS_M \longleftrightarrow CS_W \\ CS_M \longleftrightarrow KS_W \end{cases}$

• UNDERSTANDING THE PHASES





M_0	M_1	M_2
SMOOTH	SINGULAR	SMOOTH

- MATHEMATICAL OPERATION: "FLOP"
- TOPOLOGY OF M_2 ≠ TOPOLOGY OF M_1
- CAN REPEAT → ADJOIN NUMEROUS REGIONS EACH BEING KÄHLER PARAMETER SPACE FOR "FLOPPED" MANIFOLDS

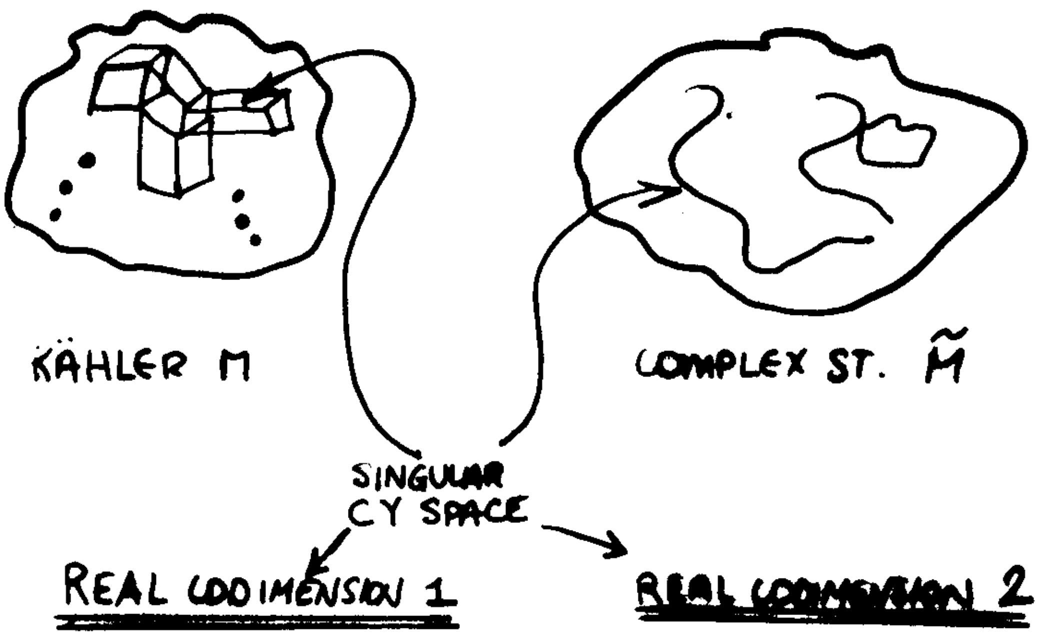
2.2

• MATHEMATICALLY: 23.5 23.6 23.7

• ADJOINING ALL OF THESE KÄHLER CONES YIELDS • "ENLARGED" KÄHLER MODULI SPACE
  • TORIC VARIETY

• THIS OBJECT IS ISOMORPHIC MATHEMATICALLY TO COMPLEX STRUCTURE MODULI SPACE

• PHYSICALLY: STILL A TROUBLING DISCREPENCY:



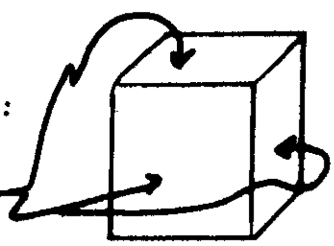
¿RESOLUTION? ¿DO WE TRUST THAT CFT's ON LHS WALLS ARE SINGULAR? THESE ARE STRONGLY COUPLED THEORIES : α'/R^2 NOT SMALL.

• ON RHS: $\alpha'/\tilde{R}^2$ IS (CAN BE TAKEN) SMALL!
STRONG COUPLING $\longmapsto$ WEAK COUPLING. $\Rightarrow$ TRUST RHS - NONSINGULAR.

• THUS, BY MIRROR SYMMETRY HAVE LEARNED:

– MATHEMATICAL KÄHLER PARAMETER SPACE:

MATH. SINGULAR:



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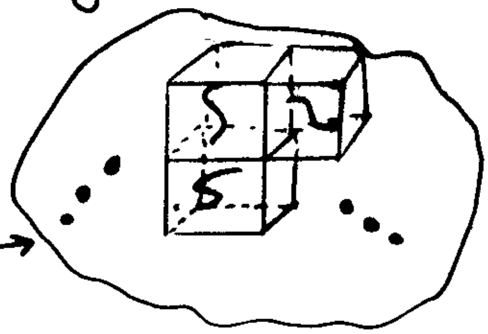
– PHYSICALLY: a) MUST ADJOIN ALONG COMMON WALLS:

• (AT MOST) PHYSICALLY SINGULAR ON CODIMENSION 1 SUBSET OF FACES →



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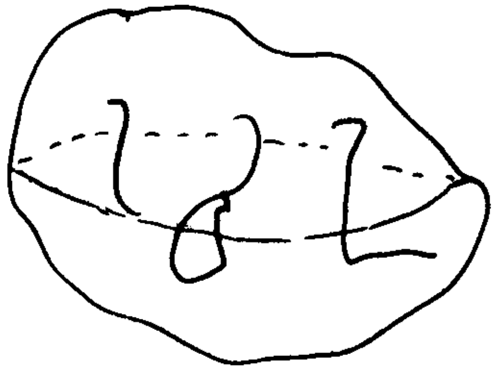
b) MIRROR TO:



W

• (AT MOST) SINGULAR ON REAL CODIMENSION 2 (TRUST PERT. TH*)

• ¿ IS CFT REALLY SINGULAR AT THESE POINTS ?
IS STRING THEORY SINGULAR " " " ?

N=2 SUSY AND SPECIAL GEOMETRY

- TYPE II STRING ON $M_4 \times CY \Rightarrow N=2$ SUSY IN SPACETIME

- FOR IIB STRING: • $h^{2,1} \leftrightarrow$ SPACETIME VECTOR MULTIPLETS

$$\left(A_\mu \ \ {}^{\psi}_{\lambda} \ \ \phi \right) \leftrightarrow U(1)^{h^{2,1}}$$

• $h^{1,1} \leftrightarrow$ HYPERMULTIPLETS

(FERMIONS; 2 complex SCALARS)

- MODULI SPACE:

complex st.

LOCAL GEOMETRY

$\updownarrow$

"SPECIAL GEOMETRY"

CONIFOLDS AND STROMINGER'S RESOLUTION

- COORDINATES ON CS MODULI SPACE:
  - (A_I, B^J) $\quad I,J = 0,\dots,h^{2,1}$ SYMPLECTIC HOMOLOGY BASIS OF $H^3(M,\mathbb{Z})$.
    $(A_I \cap B^J = \delta_I^J ;\ A_I \cap A_J = B_I \cap B_J = 0)$
  - LET $z^J = \int_{B^J} \Omega$, $\quad G_I = \int_{A_I} \Omega$; Ω = HOL 3-FORM
    $\{z^J\}$ = GOOD LOCAL (PROJECTIVE) COORDS ON $\mathcal{M}_{c.s.}$,
    $G_I = G_I(z)$.
- CONIFOLD POINT IN MODULI SPACE (ROUGHLY): SOME $z^J \to 0$
  - <u>MATHEMATICAL IMPLICATION</u>: CURVATURE SINGULARITY:
    - MONODROMY AROUND $z^J = 0$: $\quad z^J \to z^J$
      $G_J \to G_J + z^J$
      $\Rightarrow$ NEAR $z^J = 0$, $G_J(z^J) = \frac{1}{2\pi i} z^J \ln z^J$ + SINGLE VALUED
    - SPECIAL GEOMETRY : $K = -\ln(i\bar{z}^I G_I - i z^I \bar{G}_I)$
      METRIC: $g^{\mathcal{M}}_{I\bar{J}} = \partial_I \partial_{\bar{J}} K \Rightarrow g^{\mathcal{M}}_{J\bar{J}} \sim \ln(z^J \bar{z}^J)$
  - <u>PHYSICAL IMPLICATION</u>:
    - 4D LOW-ENERGY EFFECTIVE FIELD THEORY $\to$ SINGULAR NLσM ON $\mathcal{M}_{cs}$ $\sim \int d^4x\, g^{\mathcal{M}}_{I\bar{J}}\, \partial\phi^I \partial\phi^{\bar{J}}$

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• RESOLUTION:

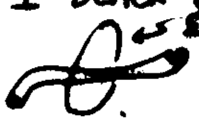
• SINGULARITY IN $\mathcal{L}^{eff}$ $\longleftrightarrow$ PREVIOUSLY MASSIVE STATES BECOME MASSLESS

• CURE SINGULARITY: INCLUDE THESE DEGREES OF FREEDOM IN $\mathcal{L}^{eff}$

• WHICH STATES?

· TYPE IIB: EXTREMAL CHARGED R-R BLACK HOLES

– 10-D: $\exists$ 3+1 BLACK-BRANE OF CHARGE $Q_{\Sigma_5} = \int_{\Sigma_5} F^{(5)}$



; ASSUME $Q = g_s \mathbb{Z}$.

– 4-D: WRAP 3-BRANE AROUND 3 CYCLES A_I, B^J ON CY.
$\Rightarrow$ 4-D CHARGED BLACK HOLES

4D: 

· 4-D EM CHARGES: $\int_{A_I \times S^2} F^{(5)} = g_s \cdot n_I$; $\int_{B^J \times S^2} F^{(5)} = g_s \cdot m^J$

· 4-D EXTREMAL (BPS) MASS: (CERESOLE, D'AURIA, FERRARA, VAN PROEYEN)

$$M = g_s e^{K/2} |m^I G_I - n_I z^I|$$

· CONSIDER: $n_I = \delta_{IJ}$ $m^I = 0$ FOR J FIXED WITH $z^J \to 0$.

$\Rightarrow M \to 0$ AT CONIFOLD POINT.

• WILSONIAN $\mathcal{L}^{eff}$ CONTAINING MASSLESS N=2 B. HOLE HYP.: SMOOTH

• INTEGRATE OUT B. HOLE $\to$ SHOULD RECOVER SINGULAR THEORY:

RUNNING COUPLING $\to$ $\tau_{JJ} \sim \frac{1}{2\pi i} \cdot \ln z^J$ + SINGLE VALUED

• $N=2 \Rightarrow \tau_{JJ} = \partial_J G_J \Rightarrow G_J = \frac{1}{2\pi i} z^J \ln z^J$ + SING. VALUED

$\Rightarrow$ REPRODUCED SINGULARITY $\Rightarrow$ INCLUDING BH's RESOLVES CONIFOLD

II. Conifold Transitions & Topology Change

(BRG, D. Morrison, A. Strominger)

• **Basic Idea:** Consider a less "generic" conifold point in $\mathcal{M}^{c.s.}$ for which

- More than 1, say P, 3-cycles degenerate
- These P - 3cycles are not homologically independent but satisfy R homology relations.

• **Claim:** • Bosonic potential for scalars in black hole hypermultiplets H^a $a=1,\dots,P$ has R flat directions.

• Moving along such flat directions takes us to another branch of type II string moduli space corresponding to propagation on a topologically distinct CY space.

• **Clarifying Remark:**

• In previous results yielding topology change (Witten; Aspinwall, BRG, Morrison) Hodge #'s stay fixed — only more subtle top. invariants change

• In present context: Hodge #'s jump

$$(h^{2,1}, h^{1,1}) \longmapsto (h^{2,1}-P+R,\ h^{1,1}+R)$$

even though physics is smooth.

• Useful Background:

A) Conifold Points in $\mathcal{M}^{CS}$: (Lefschetz, Candelas, de la Ossa, Candelas, Green, Hübsch)

• Discriminant Locus ($P=0=dP$ for CY $P=0$)
$\rightarrow$ Singular CY's

• Conifold Locus: singularities on CY are nodes:

• Locally Node: $\sum_{i=1}^{4} (\omega_i)^2 = 0$; Singular at apex as $C = dC = 0$.

• Homogeneous Eqn $\Rightarrow$ $(\lambda\omega_1, \dots, \lambda\omega_4)$ satisfy if $(\omega_1, \dots, \omega_4)$ do.
$\Rightarrow$ CONE

• Base of cone: Int. cone with sphere in $\mathbb{R}^8$

$$\sum_{i=1}^{4} |\omega_i|^2 = r^2$$

• Write: $\vec{w} = (\omega_1, \omega_2, \omega_3, \omega_4) = \vec{x} + i\,\vec{y}$

Then Eqns: $\vec{x}\cdot\vec{x} = r^2/2$; $\underbrace{\vec{y}\cdot\vec{y} = r^2/2 \,;\, \vec{x}\cdot\vec{y} = 0}$

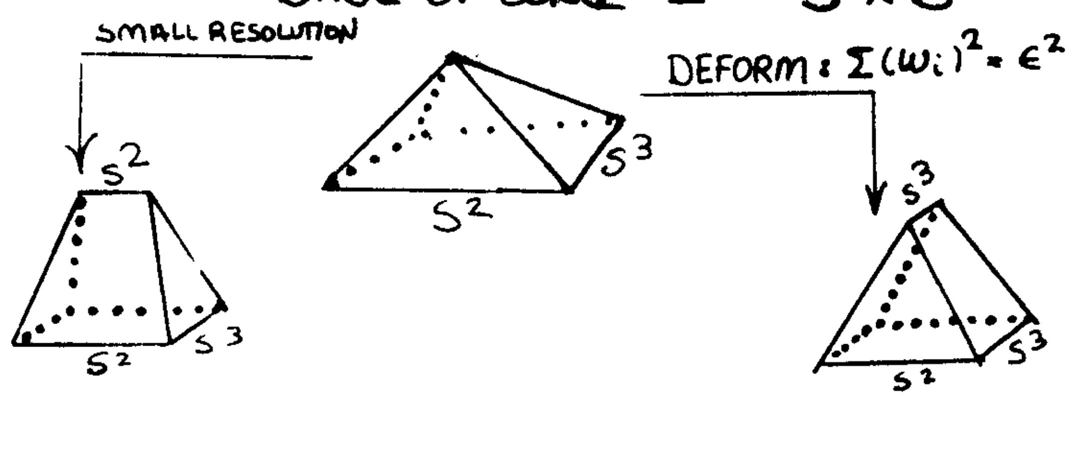
Locus: $S^3_{r/\sqrt{2}}$; $S^2_{r/\sqrt{2}}$ over S^3

$\rightarrow$ Base of cone $=$ $S^2 \times S^3$

$\epsilon = 0 \rightarrow XY - UV = 0$

$$\downarrow$$

$$\begin{cases} XY - UV = 0 \\ [X, \dots, V] \times [\lambda_1, \lambda_2] \\ \begin{pmatrix} X & U \\ V & Y \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = 0 \end{cases}$$



- **MONODROMY:**

IF γ^a $a=1,\dots,k$ ARE k-VANISHING 3 CYCLES AT A CONIFOLD POINT, THEN A 3-cycle δ UNDERGOES MONODROMY $\delta \mapsto \delta + \sum_{a=1}^{k} (\delta \cap \gamma^a)\, \gamma^a$ ABOUT THIS PT IN $\mathcal{M}^{cs}$.

B) TYPE IIB STRING THEORY ON A CY 3-FOLD

$\mathcal{N}=2$ SUSY:
- $h^{2,1}$ $U(1)$ Vector Multiplets (+ GRAVIPHOTON)
  $h^{1,1}+1$ Hypermultiplets (INCLUDING AXION/DILATON)

EXAMPLE:

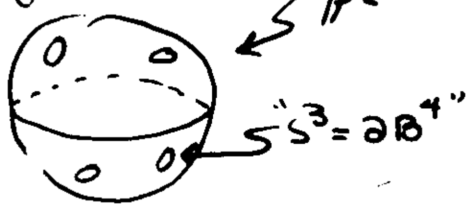
- BEGIN WITH QUINTIC IN $\mathbb{P}^4$ $\left(\begin{matrix} h^{1,1}=1 \\ h^{2,1}=101 \end{matrix}\right)$
- DEFORM COMPLEX STRUCTURE TO 86 DIM LOCUS

$$P(x) = x_1\, g(x) + x_2\, h(x) = 0$$

(g, h QUARTICS)

- NOTE:
  - THIS IS SINGULAR: P & dP VANISH AT
    $\{x_1 = x_2 = g(x) = h(x) = 0\}$ i.e. 16 POINTS
    (LYING ON $\mathbb{P}^2$ $x_1 = x_2 = 0$ IN $\mathbb{P}^4$)
  - THESE ARE NODES (MATRIX OF 2nd derivatives NONDEG.
  - THE 16 S^3's WHICH HAVE DEGENERATED, γ^a $a=1,\dots,16$
    ARE NOT ALL HOMOLOGICALLY INDEPENDENT. RATHER

$$\sum_{a=1}^{16} \gamma^a = 0.$$



$$\partial\left[\mathbb{P}^2 - \sum_{a=1}^{16} B^a\right] = \sum_{a=1}^{16} \gamma^a$$

BLACK HOLE RESOLUTION

• SINGULARITY:

- BASIS (A_I, B^J) : $I, J = 1, \dots, 204$
- VANISHING CYCLES $\gamma^1, \dots, \gamma^{16}$ WITH $\sum \gamma^a = 0 \Rightarrow$ CAN TAKE
  $\gamma^1, \dots, \gamma^{15} = B^1, \dots, B^{15}$; $\gamma^{16} = -\sum_{I=1}^{15} B^I$
- AS USUAL:
  $$Z^I = \int_{B^I} \Omega \quad ; \quad G_J = \int_{A_J} \Omega$$
- MONODROMY AROUND $\{ Z^1 = Z^2 = \cdots = Z^{15} = 0 \}$
  $$\delta \to \delta + (\delta \cap \gamma^a)\gamma^a \qquad a = 1, \dots, 16$$
  $$\int_\delta \Omega \sim \frac{1}{2\pi i} \sum_{a=1}^{16} (\delta \cap \gamma^a)\left(\int_{\gamma^a} \Omega\right)\left(\ln \int_{\gamma^a} \Omega\right) + \text{single valued}$$

$$\Rightarrow G_J = \int_{A_J} \Omega \sim \frac{1}{2\pi i} Z^J \ln Z^J + (1)\left(-\sum_{I=1}^{15} Z^I\right)\left(\ln\left(\sum_{I=1}^{15} Z^I\right)\right)$$

+SINGLE VALUED

(AS BEFORE THIS DETERMINES SINGULARITY VIA SPECIAL GEOMETRY)

• MASSLESS STATES : 16 DEGENERATING 3-cycles
$\Rightarrow$ 16 MASSLESS BH hypermultiplets
$H^a \quad a = 1, \ldots, 16$

• WILSONIAN THEORY SMOOTH.

¿ DO WE RECOVER KNOWN SINGULARITY IF WE INTEGRATE OUT BH multiplets?

• NEED BH CHARGES UNDER $U(1)^{15}$

• H^a ARISES FROM γ^a : $U(1)_J$ CHARGE $= Q^a_J = A_J \cap \gamma^a$
AS TAKE $F^{(5)} = [\sum \alpha^I F_I^{(2)}]_{S.D.}$ w/ α^I = dual of A_I

• $\Rightarrow Q^a_J = \delta^a_J \quad 1 \leq a \leq 15 ; \quad Q^{16}_J = -1 \quad \forall J$.

• RUNNING COUPLES FROM INTEGRATING OUT THE H^a:

$$\tau_{IJ} \sim \frac{1}{2\pi i} \sum_{a=1}^{16} Q^a_I Q^a_J \ln(m^a) + \text{s.v.}, \quad m^a \sim e^{K/2} |Q^a_I Z^I|$$

$$\Rightarrow \quad \tau_{IJ} \sim 1/2\pi i \, \delta_{IJ} \ln Z^J + 1/2\pi i \ln\left(\sum_{K=1}^{15} Z^K\right) + \text{S. VALUED}$$

Now: $\tau_{IJ} = \partial_I G_J$ ($N=2$ SPECIAL GEOM) $\Rightarrow$

$$G_J \sim 1/2\pi i \, Z^J \ln Z^J + 1/2\pi i \left(\sum_{I=1}^{15} Z^I\right)\left(\ln\left(\sum_{I=1}^{15} Z^I\right)\right) + \text{S.V.}$$

LOW ENERGY EFFECTIVE ACTION WITH BLACK HOLES

• **MAIN QUESTION**: ¿WHAT IS PHYSICAL SIGNIFICANCE OF NONTRIVIAL HOMOLOGY RELATIONS BETWEEN VANISHING CYCLES?

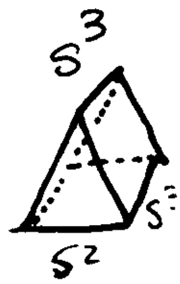
• SCALAR BH POTENTIAL:

• $E^{I}_{\alpha\beta} = \left(\sum_{a=1}^{16} Q^{I}_{a} \, \varepsilon_{\alpha\gamma} h^{\dagger (a)}_{\gamma} h^{(a)}_{\beta} - (\alpha \leftrightarrow \beta) \right)$; $I = 1, \dots, 15$; $\alpha, \beta = 1, 2$

WITH $h^{(a)}_{1}, h^{(a)}_{2} \subset H^{(a)}$ $a = 1, \dots, 16$.

• $V \sim \sum E^{I}_{\alpha\beta} E^{\alpha\beta}_{I}$

• FLAT DIRECTIONS?

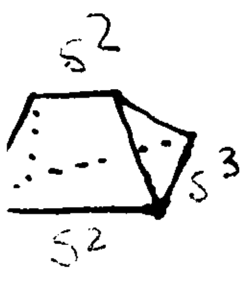


• $\langle h^{(a)}_{i} \rangle = 0$; $\langle$ SCALARS IN 15 VECTORS $\rangle \neq 0$

• BLACK HOLES GET MASS; MOVE BACK TO SMOOTH QUINTIC

• $Q^{I}_{a} \sim A_I \cap \gamma^a$ w/ $\sum_{a=1}^{16} \gamma^a = 0 \Rightarrow \sum_a Q^{I}_{a} = 0 \;\; \forall I$

$\Rightarrow \langle h^{(a)}_{\alpha} \rangle = v^{\alpha} \;\; \forall a$ IS FLAT (UNIQUE UP TO GAUGE T.)



• 15 VECTORS / 15 HYPERS GET MASS ; 1 HYPER MASSLESS

$h^{2,1}_{OLD} = 101 \rightarrow 101 - 15 = 86$; $h^{1,1}_{OLD} = 1+1 \rightarrow 2+1$

PRECISELY HODGE NUMBERS GET FROM SMALL RESOLUTION!
(CANDELAS, GREEN, HUBSCH)

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- $\mathcal{C}$: HOMOLOGY RELATION AMONG γ^a

  $\Rightarrow$ FLAT DIRECTION IN BH POTENTIAL

- MOVING ALONG SUCH FLAT DIRECTION TAKES US SMOOTHLY TO NEW BRANCH OF TYPE II STRING MODULI SPACE.

- APPARENTLY PHYSICALLY REALIZED CY TRANSITIONS DISCUSSED SOME YEARS AGO IN INSIGHTFUL PAPERS OF CANDELAS, GREEN, HUBSCH.

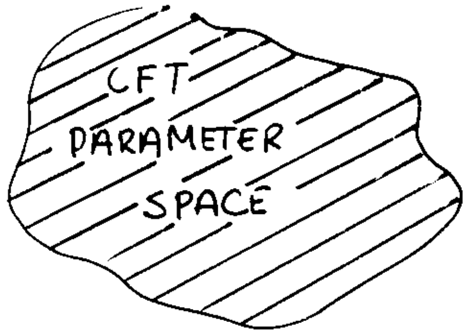
- GENERAL CASE:
  - P VANISHING CYCLES
  - R HOMOLOGY RELATIONS $\Rightarrow$ • $P-R$ $U(1)$'s
    - P BLACK HOLE HYPERMULTIPLETS
    - R FLAT DIRECTIONS IN BH POTENTIAL

  - GENERIC DEFORMATION: • LOSE $P-R$ VECTORS
    - GAIN R HYPERMULTIPLETS

$$\Rightarrow (h^{21}, h^{11}) \longmapsto (h^{21}-(P-R),\ h^{11}+R) \quad (\Rightarrow |\Delta\chi| = 2P)$$

MODULI SPACE OF $N=2$ SCFT's

CFT



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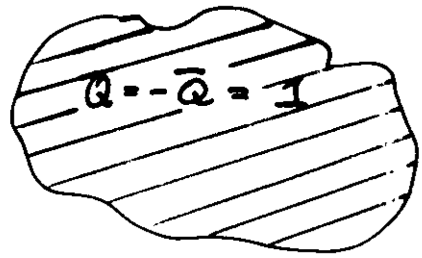
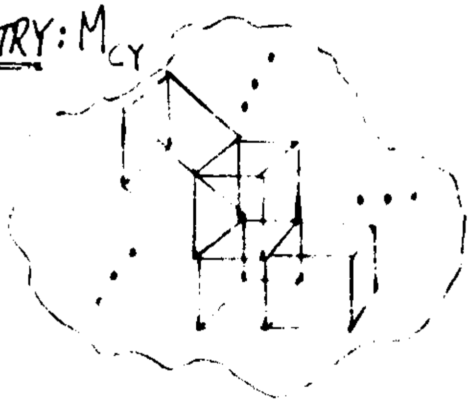


GEOMETRY: M_{CY}



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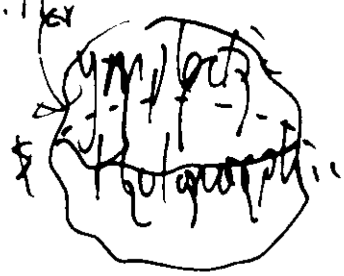
GEOMETRY: $\tilde{M}_{CY}$



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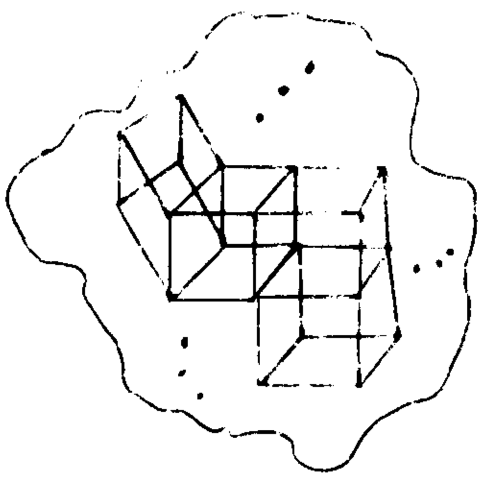

MODULI SPACE OF TYPE II STRING

III) IMPLICATIONS CONCLUSIONS

* 1) TOWARDS UNIFICATION OF STRING VACUA

* 2) PHYSICALLY SMOOTH TOPOLOGY CHANGE OF MORE "DUALITIC" SORT.

* 3) MIRROR SYMMETRY

* 4) BLACK HOLES AS ELEMENTARY PARTICLES