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Lecture III

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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N = 2 SUPERSYMMETRY

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Please note: These are preliminary notes intended for internal distribution only.

4.3 Symplectic transformations in $N = 2$

$\mathcal{N}_{AB}$ can be defined by

$$\partial_i F_A = \bar{\mathcal{N}}_{AB} \partial_i X^B \tag{4.19}$$

This relation is maintained if (X^A, F_A) transforms under $Sp(2n, \mathbb{R})$ in the same way as $(\mathcal{F}^A_{\mu\nu}, G_{A\mu\nu})$. Remark that the Kähler potential is also a symplectic invariant:

$$K = i(\bar{F}_A X^A - F_A \bar{X}^A) = iV^T \Omega \bar{V} \ . \tag{4.20}$$

Explicitly, such a transformation is of the form

$$\begin{aligned} \tilde{X}^A &= A^A{}_B X^B + B^{AB} F_B \ , \\ \tilde{F}_A &= C_{AB} X^B + D_A{}^B F_B \ . \end{aligned} \tag{4.21}$$

When we start from a prepotential $F(X)$, the F_A are the derivatives of F, so that the first line expresses the dependence of the new coordinates $\tilde{X}$ on the old coordinates X. If this transformation is invertible (the full symplectic matrix itself is always invertible), the $\tilde{F}_A$ are again the derivatives of an new function $\tilde{F}(\tilde{X})$ of the new coordinates,

$$\tilde{F}_A(\tilde{X}) = \frac{\partial \tilde{F}(\tilde{X})}{\partial \tilde{X}^A} \ . \tag{4.22}$$

The integrability condition for this statement is the symmetry of

$$\frac{\partial \tilde{F}_A}{\partial \tilde{X}^B} = (C + D\bar{\mathcal{N}})(A + B\bar{\mathcal{N}})^{-1} \ , \tag{4.23}$$

and is thus the same as (4.8), which was the condition that $\mathcal{S} \in Sp(2n, \mathbb{R})$. Hence we obtain a new, but equivalent, formulation of the theory, and thus of the target-space manifold, in terms of the function $\tilde{F}$. Note that the argument for the existence of $\tilde{F}$ only applies if the mapping $X^A \to \tilde{X}^A$ is invertible.

In view of the above, the field equations corresponding to two supersymmetric Yang-Mills actions characterized by different functions $F(X)$, can be identical up to a symplectic transformation. In that case the two functions describe equivalent classical field theories.

Hence we may find a variety of descriptions of the same theory in terms of different functions F.

Now it may happen that the new theory is the same as the old one: if

$$\tilde{F} = F + a + q_A X^A + c_{AB} X^A X^B \ , \tag{4.24}$$

where a and q_A can be complex, but c_{AB} should be real. E.g. the symplectic transformations with

$$\mathcal{S} = \begin{pmatrix} \mathbb{1} & 0 \\ C & \mathbb{1} \end{pmatrix} \tag{4.25}$$

are of this form. In these cases we are dealing with an invariance of the equations of motion (but not necessarily of the action as not all transformations can be implemented locally on the gauge fields). This invariance reflects itself in isometries of the target-space manifold. (Not in (4.25), as there are no transformations on the scalars.)

4.4 Interpretation of fields as moduli of a Riemann surface.

Riemann surfaces parametrized by $n(=1)$ complex moduli.

Seiberg-Witten: Landau–Ginzburg superpotential

$$0 = W(X,Y,Z;u) = -Z^2 + \frac{1}{4}\left(X^4+Y^4\right) + \frac{u}{2}X^2Y^2 \tag{4.26}$$

Identify

$$U_i = \begin{pmatrix} \int_{A^A} \omega^i \\ \int_{B^B} \omega^i \end{pmatrix} \qquad (i=1,\dots n = \text{ genus}) \tag{4.27}$$

of the n holomorphic 1-forms ω^i along a canonical homology basis:

$$A_A \cap A_B = 0 \; ; \qquad B^A \cap B^B = 0 \; ; \qquad A_A \cap B^B = -B^B \cap A_A = \delta_A^B \; . \tag{4.28}$$

Using Griffith residue theorem (C is any cycle $aA+bB$)

$$\Pi = \begin{pmatrix} \int_C \omega \\ \int_C \bar{\omega} \end{pmatrix} = \begin{pmatrix} \int_C \frac{1}{W(X,Y,Z;u)}\, dX\wedge dY \wedge dZ \\ \int_C \frac{X^2Y^2}{W^2(X,Y,Z;u)}\, dX\wedge dY\wedge dZ \end{pmatrix} \tag{4.29}$$

Leads to Picard–Fuchs equations on the periods. These are the same as the differential constraints of special Kähler spaces.

Using very general techniques of algebraic geometry, the dependence of the periods $U_i(u)$ on the moduli parameters can be determined through the solutions of the Picard–Fuchs differential system, once the manifold is explicitly described as the vanishing locus of a holomorphic superpotential $\mathcal{W}(Z,X,Y;u_i)$. In particular one can study the monodromy group Γ_M of the differential system and the symmetry group of the potential Γ_W, that are related to the full group of duality rotations Γ_D as follows:

$$\Gamma_W = \Gamma_D/\Gamma_M \tag{4.30}$$

The elements of $\Gamma_D \supset \Gamma_M$ are given by integer valued symplectic matrices $\gamma \in Sp(2n,\mathbb{Z})$ that act on the symplectic section U_i. Given the geometrical interpretation (4.27) of these sections, the elements $\gamma\in\Gamma_D \subset Sp(2n,\mathbb{Z})$ correspond to changes of the canonical homology basis respecting the intersection matrix (4.28).

E.g. for $n=1$, writing all connections explicitly, the differential equations that were mentioned in the coordinate independent treatment can be written as

$$(\partial - \hat{\Gamma})U_i + CU^i = 0 \; ; \qquad (\partial+\hat{\Gamma})U^i = 0 \; . \tag{4.31}$$

Combining these we have:

$$(\partial+\hat{\Gamma})C^{-1}(\partial-\hat{\Gamma})U_i = 0 \; . \tag{4.32}$$

Similar differential equations can be obtained from (4.29):

$$\left(\partial^2 - \frac{2u}{1-u^2}\partial - \frac{1}{4}\frac{1}{1-u^2}\right)f(u) = 0 \; , \tag{4.33}$$

where $f(u)$ is a component of (4.29).

5 Supergravity

5.1 Vector multiplets coupled to supergravity

Construction using superconformal tensor calculus

Superconformal group:

$$SU(2,2|N=2) \supset SU(2,2) \otimes U(1) \otimes SU(2)$$

(related : Kähler and quaternionic couplings of vector and hypermultiplets)

In Poincaré theory: dilatations, $U(1) \otimes SU(2)$ are fixed.

multiplets:

Weyl multiplet: (e_μ^a, ψ_μ^I) + auxiliary fields.

$n+1$ *vector multiplets*:

$$\left(X^\Lambda, \lambda^{I\Lambda}, \mathcal{A}_\mu^\Lambda\right) \qquad \text{with} \qquad \Lambda = 0, 1, ..., n.$$

Invariance under *dilatations* $\otimes\, U(1)$:

gives action for $n+1$ vectors
n spin $1/2$
n complex scalars

(hypermultiplets can also be added).

$r+1$ *hypermultiplets:*

One extra for fixing $SU(2)$ (+auxiliaries)

$\delta X^\Lambda = \epsilon_D X^\Lambda + i\epsilon_A X^\Lambda$.

Weyl weight: $\left[X^\Lambda\right] = 1$;
to form an action : $[F(X)] = 2$.

F is holomorphic function, homogeneous of 2nd degree in X^Λ

When coupling n of these so-called vector multiplets to supergravity, one again has a holomorphic prepotential $F(X)$, this time of $n+1$ complex fields, but now it must be a *homogeneous* function of degree two [6].

In action $-\frac{1}{2}i(\bar{X}^\Lambda F_\Lambda - X^\Lambda \bar{F}_\Lambda)eR$

Gauge for dilatations:

$$i(\bar{X}^\Lambda F_\Lambda - X^\Lambda \bar{F}_\Lambda) = 1 \ . \tag{5.1}$$

The physical scalar fields of this system parameterize an n-dimensional complex hypersurface, while the overall phase of the X^Λ is irrelevant in view of a local (chiral) invariance.

Only ratios $\frac{X^\Lambda}{X^0}$ occur $\rightarrow$

The embedding of this hypersurface can be described in terms of n complex coordinates z^i by letting X^Λ be proportional to some holomorphic sections $Z^\Lambda(z)$ of the projective space $P\mathbb{C}^{n+1}$ [17]. The n-dimensional space parametrized by the z^i $(i = 1, \ldots, n)$ is a Kähler space; the Kähler metric $g_{i\bar{j}} = \partial_i \partial_{\bar{j}} K(z, \bar{z})$ follows from the Kähler potential

$$K(z,\bar{z}) = -\log\left[i\bar{Z}^\Lambda(\bar{z})\,F_\Lambda(Z(z)) - iZ^\Lambda(z)\,\bar{F}_\Lambda(\bar{Z}(\bar{z}))\right], \quad \text{where} \tag{5.2}$$
$$X^\Lambda = e^{K/2}Z^\Lambda(z)\,, \qquad \bar{X}^\Lambda = e^{K/2}\bar{Z}^\Lambda(\bar{z})\,.$$

The resulting geometry is known as *special* Kähler geometry [6, 8]. The curvature tensor associated with this Kähler space satisfies the characteristic relation [18]

$$R^i_{jk}{}^\ell = \delta^i_j\delta^\ell_k + \delta^i_k\delta^\ell_j - e^{2K}\mathcal{W}_{jkm}\,\bar{\mathcal{W}}^{mi\ell}\,, \tag{5.3}$$

where

$$\mathcal{W}_{ijk} = F_{\Lambda\Sigma\Pi}(Z(z))\,\frac{\partial Z^\Lambda(z)}{\partial z^i}\frac{\partial Z^\Sigma(z)}{\partial z^j}\frac{\partial Z^\Pi(z)}{\partial z^k}\,. \tag{5.4}$$

A convenient choice of inhomogeneous coordinates z^i are the *special* coordinates, defined by

$$z^i = \frac{X^i}{X^0}\,, \qquad i = 1, \ldots, n, \tag{5.5}$$

or, equivalently,

$$Z^0(z) = 1\,, \qquad Z^i(z) = z^i\,. \tag{5.6}$$

The kinetic terms of the spin-1 gauge fields in the action are proportional to the symmetric tensor

$$\mathcal{N}_{\Lambda\Sigma}(z,\bar{z}) = \bar{F}_{\Lambda\Sigma} + 2i\frac{(\text{Im } F_{\Lambda\Gamma})(\text{Im } F_{\Sigma\Pi})X^\Gamma X^\Pi}{(\text{Im } F_{\Xi\Omega})\,X^\Xi X^\Omega}\,. \tag{5.7}$$

Positivity domain: $g_{i\bar{j}} > 0$ and $e^{-K} > 0$.

We give here some examples of functions $F(X)$ and their corresponding target spaces.

$$F = i[(X^0)^2 - (X^1)^2] \qquad \frac{SU(1,1)}{U(1)} \tag{5.8}$$

$$F = (X^1)^3/X^0 \qquad \frac{SU(1,1)}{U(1)} \tag{5.9}$$

$$F = \sqrt{X^0(X^1)^3} \qquad \frac{SU(1,1)}{U(1)} \tag{5.10}$$

$$F = iX^\Lambda\eta_{\Lambda\Sigma}X^\Sigma \qquad \frac{SU(1,n)}{SU(n)\otimes U(1)} \tag{5.11}$$

$$F = d_{ABC}X^AX^BX^C/X^0 \qquad \text{'very special Kähler'} \tag{5.12}$$

The first three functions give rise to the manifold $SU(1,1)/U(1)$. However, the first one is not equivalent to the other two as the manifolds have a different value of the curvature [22]. The latter two are, however, equivalent by means of a symplectic transformation. In the fourth example η is a constant non-degenerate real symmetric matrix. In order that the manifold has a non-empty positivity domain, the signature of this matrix should be $(+ - \cdots -)$. So not all functions $F(X)$ allow an non-empty positivity domain. The last example, defined by a real symmetric tensor d_{ABC}, defines a class of special Kähler manifolds, which we will denote as 'very special' Kähler manifolds. This class of manifolds is important in the applications discussed below.

5.2 Coordinate independent description

$$V = \begin{pmatrix} X^\Lambda \\ F_\Lambda \end{pmatrix} \in \mathbb{C}^{2n+2}$$

function of n complex variables z^i.

Constraint:

$$\langle \bar{V}, V \rangle \equiv \bar{V}^T \Omega V = -i \ .$$

Covariant derivatives have $U(1)$ connection, e.g.

$$\mathcal{D}_i X = \left(\partial_i + \frac{1}{2}(\partial_i K) \right) X \ .$$

where $K(z, \bar{z})$ will be the Kähler potential.
Symplectic sections are covariantly holomorphic:

$$V = e^{K/2} v(z) \ .$$

Constraints:

$$\begin{aligned} U_i &\equiv \mathcal{D}_i V = (\mathcal{D}_i X^\Lambda, \mathcal{D}_i L_\Lambda) \\ \mathcal{D}_i U_j &= i C_{ijk} g^{k\bar{l}} \bar{U}_{\bar{l}} \\ \mathcal{D}_i \bar{U}_{\bar{j}} &= g_{i\bar{j}} \bar{V} \\ \mathcal{D}_i \bar{V} &= 0 \ . \end{aligned}$$

leads to

$$g_{i\bar{j}} = i \langle U_i, \bar{U}_{\bar{j}} \rangle$$

5.3 Symplectic transformations

The same principles which were explained in rigid supersymmetry also work here. The duality group on the vectors is $Sp(2(n+1), \mathbb{R})$. Again the relations from supersymmetry remain valid if we simultaneously transform the vector (X^Λ, F_Λ).

Therefore

- If $\tilde{F}(\tilde{X})$ is the same function (or $\tilde{F}(X) = F(X) + c_{\Lambda\Sigma} X^\Lambda X^\Sigma$):
  proper symmetry
  (and isometry of scalar manifold)
- If $\tilde{F}(\tilde{X})$ is different function:
  pseudo symmetry
  (symplectic reparametrisation)
  (Cecotti, Ferrara, Girardello)

5.4 Examples

Example 1: $F = (X^1)^3/X^0$

$$S = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/3 \\ 0 & 0 & 1 & 0 \\ 0 & -3 & 0 & 0 \end{pmatrix}$$

leads to $F = -4\sqrt{X^0(X^1)^3}$
is a Symplectic reparametrisation.
On the other hand

$$S = \begin{pmatrix} 1+3\epsilon & \mu & 0 & 0 \\ \lambda & 1+\epsilon & 0 & 2\mu/9 \\ 0 & 0 & 1-3\epsilon & -\lambda \\ 0 & -6\lambda & -\mu & 1-\epsilon \end{pmatrix}$$

for infinitesimal ϵ, μ, λ leaves F invariant:
$SU(1,1)$ isometries.
$\delta z = \lambda - 2\epsilon z - \mu z^2/3$.
Domain: Im $z > 0$:

$$\frac{SU(1,1)}{U(1)}$$

Example 2 : $F = iX^0X^1$

$$\mathcal{N} = \begin{pmatrix} i\frac{X^1}{X^0} & 0 \\ 0 & i\frac{X^0}{X^1} \end{pmatrix} .$$

In action:

$$e^{-1}\mathcal{L}_1 = -\tfrac{1}{4}\mathrm{Re}\left[z\left(F^{+0}_{\mu\nu}\right)^2 - \tfrac{1}{4}z^{-1}\left(F^{+1}_{\mu\nu}\right)^2\right]$$

As '$SO(4)$ formulation' of pure $N = 4$ SG.

Symplectic mapping $\mathcal{S} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$

Transformation is

$$\tilde{X}^0 = X^0 \qquad \tilde{X}^1 = -F_1 = -iX^0$$
$$\tilde{F}_0 = F_0 \qquad \tilde{F}_1 = X^1 .$$

Non–invertible :
No coord. transform. between $z = X^1/X^0$ and $\tilde{X}^0$, $\tilde{X}^1$
Also $\tilde{F} = 0$. However, $A + B\mathcal{N}$ is invertible

$$\tilde{\mathcal{N}} = (C + D\mathcal{N})(A + B\mathcal{N})^{-1} = iX^1(X^0)^{-1}\mathbb{1} .$$

So now
(performing only symplectic transf., no diffeomorphism)

$$e^{-1}\mathcal{L}_1 = -\tfrac{1}{4}\mathrm{Re}\left[z\left(F^{+0}_{\mu\nu}\right)^2 - \tfrac{1}{4}z\left(F^{+1}_{\mu\nu}\right)^2\right]$$

As '$SU(4)$ formulation' of pure $N=4$ SG.

$\Longrightarrow$ there are formulations one can not directly obtain from a superspace formulation with a function F.

Similarly for matter couplings in model
(only factorised special Kähler manifold !)

$$\frac{SU(1,1)}{U(1)} \otimes \frac{SO(r,2)}{SO(r)\otimes SO(2)}$$

First formulation with $F = \frac{d_{ABC}X^A X^B X^C}{X^0}$
(very special)

New formulation with:

$$X^\Lambda \eta_{\Lambda\Sigma} X^\Sigma = 0 \ ; \qquad F_\Lambda = S\,\eta_{\Lambda\Sigma} X^\Sigma$$

has full $SO(2,r)$ as invariance of action.

6 Real and quaternionic special spaces; homogeneous and symmetric special manifolds

<u>c map</u>
special Kähler to special quaternionic
(reduction $d=4$ to $d=3$).
from n complex to $n+1$ quaternions.

<u>r map</u>
very special real to very special Kähler
reduction $d=5$ to $d=4$
from $n-1$ real to n complex

real	$\longrightarrow$	Kähler	$\longrightarrow$	quaternionic
$n-1$	r map	n	c map	$n+1$
homog.	$\longrightarrow$	homog.	$\longrightarrow$	homog.

We now consider the so-called c map [2] from a special Kähler to a quaternionic manifold. It is induced by reducing an $N=2$ supergravity action in $d=4$ space-time dimensions to an action in $d=3$ space-time dimensions, by suppressing the dependence on one of the (spatial) coordinates. The resulting $d=3$ supergravity theory can be written in terms of $d=3$ fields and this rearranges the original fields such that the number of scalar fields increases from $2n$ to $4(n+1)$.

The notion of special Kähler manifolds induces also a notion of '*special quaternionic manifolds*', which are those manifolds appearing in the image of the c map. Similarly, very special real manifolds are the manifolds defined by coupling (real) scalars to vector multiplets in 5 dimensions (characterised by a symmetric tensor d_{ABC}). Very special Kähler manifolds are induced as the image under the **r** map (dimensional reduction from 5 to 4 dimensions) and very special quaternionic manifolds as the image of the **c**∘**r** map.

These very special manifolds contains all known homogeneous non-symmetric quaternionic spaces. The homogeneous (and symmetric) special manifolds are listed in table 4 and table 5.

Table 4: Symmetric very special manifolds

	real	Kähler	quaternionic
$L(-1,0)$	$SO(1,1)$	$\left[\frac{SU(1,1)}{U(1)}\right]^2$	$\frac{SO(3,4)}{(SU(2))^3}$
$L(0,P)$	$SO(1,1)\otimes\frac{SO(P+1,1)}{SO(P+1)}$	$\frac{SU(1,1)}{U(1)}\otimes\frac{SO(P+2,2)}{SO(P+2)\otimes SO(2)}$	$\frac{SO(P+4,4)}{SO(P+4)\otimes SO(4)}$
$L(1,1)$	$\frac{S\ell(3,R)}{SO(3)}$	$\frac{Sp(6)}{U(3)}$	$\frac{F_4}{USp(6)\otimes SU(2)}$
$L(2,1)$	$\frac{S\ell(3,C)}{SU(3)}$	$\frac{SU(3,3)}{SU(3)\otimes SU(3)\otimes U(1)}$	$\frac{E_6}{SU(6)\otimes SU(2)}$
$L(4,1)$	$\frac{SU^*(6)}{Sp(3)}$	$\frac{SO^*(12)}{SU(6)\otimes U(1)}$	$\frac{E_7}{SO(12)\otimes SU(2)}$
$L(8,1)$	$\frac{E_6}{F_4}$	$\frac{E_7}{E_6\otimes U(1)}$	$\frac{E_8}{E_7\otimes SU(2)}$

Table 5: Homogeneous manifolds. In this table, q, P, $\dot{P}$ and m denote positive integers or zero, and $q \neq 4m$. SG denotes an empty space, which corresponds to supergravity models without scalars. Furthermore, $L(4m,P,\dot{P}) = L(4m,\dot{P},P)$. The horizontal lines separate spaces of different rank. The manifolds indicated by a $\star$ were not known before our classification, except for the cases $L(0,P,\dot{P})$.

	real	Kähler	quaternionic
$L(-3,P)$			$\frac{USp(2P+2,2)}{USp(2P+2)\otimes SU(2)}$
SG_4		SG	$\frac{U(1,2)}{U(1)\otimes U(2)}$
$L(-2,P)$		$\frac{U(P+1,1)}{U(P+1)\otimes U(1)}$	$\frac{SU(P+2,2)}{SU(P+2)\otimes SU(2)\otimes U(1)}$
SG_5	SG	$\frac{SU(1,1)}{U(1)}$	$\frac{G_2}{SU(2)\otimes SU(2)}$
$L(-1,P)$	$\frac{SO(P+1,1)}{SO(P+1)}$	$\star$	$\star$
$L(4m,P,\dot{P})$	$\star$	$\star$	$\star$
$L(q,P)$	$X(P,q)$	$H(P,q)$	$V(P,q)$

7 Summary

- Special Kähler manifolds are those defined by couplings of vector multiplets in $N=2$ SG.

- Have n vectors (SUSY) or $n+1$ vectors (SG) and n complex scalars.
- Usually defined by a holomorphic function; in SUSY: $F(X^A)$
  in SUGRA: $F(X^\Lambda)$ homog. of 2^{nd} degree
- Symplectic transformations lead either to isometries or symplectic reparametrisations.
- Relation with special quaternionic and very special real.
- Relation with moduli of Calabi-Yau surfaces, useful for non–perturbative results of effective string theories.

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The references are not added to these notes, as they are incomplete so far.

Here is a list of some reviews on the material presented: (all be it that they mostly concentrate on the supergravity case, rather than the rigid case as was done here)

- B. de Wit and A. Van Proeyen, "Isometries of special manifolds", Based on invited talks given at the Meeting on Quaternionic Structures in Mathematics and Physics, Trieste, September 1994; to be published in the proceedings. preprint THU-95/13, KUL-TF-95/13 ; hep-th/9505097, 25pp. *This contains a short summary of what is a special manifold. It has been written for mathematicians interested in the classification of quaternionic homogeneous spaces.*
- P. Frè and P. Soriani, "The $N=2$ Wonderland, from Calabi–Yau manifolds to topological field–theories", 476 pp. (World Scientific, 1995), to be available very soon. *This books explains formulations of $N=2$ theories, and applications as topological theories, Landau–Ginzburg models, Calabi–Yau, ... It contains a lot of geometry and topology as mathematical background, and goes to Picard–Fuchs equations and the mirror maps.*
- A. Van Proeyen, "Superconformal tensor calculus in N = 1 and N = 2 supergravity", in '*Supersymmetry and Supergravity 1983*', ed. B. Milewski, (World Scientific Publ. Co.), 93–166 (1983). *A review on tensor calculus, the multiplets, ...*
- A. Van Proeyen, "N = 2 matter couplings in d = 4 and 6 from superconformal tensor calculus", in '*Superunification and Extra Dimensions*', eds. R. D'Auria and P. Fré, (World Scientific Publ. Co.), 97–125 (1986). *Completes the previous one not only for including $d=6$, but also e.g. the construction of the general action for special Kähler was not yet included in the previous one.*