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SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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EXACT RESULTS IN SUSY GAUGE THEORIES

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Exact results in SUSY gauge theories.

K. Intriligator.

References.

- N. Seiberg, "The Power of Holomorphy – Exact Results in 4D SUSY Field Theories" hep th/9408013
- N. Seiberg, "The Power of Duality – Exact Results in 4D SUSY Field Theories" hep-th/9506077
- K. Intriligator and N Seiberg "Phases of N=1 Supersymmetric Gauge Theories and Electric-Magnetic Triality" hepth/9506084

AND REFERENCES THEREIN.

Lecture Plan

1) Duality & phases of gauge theories.
   Holomorphy & symmetries
   $N=1$ SUSY QCD

2) Finish SUSY QCD for $N_f < N_c$.
   Integrating out & in, etc.
   Related examples

3) Quantum moduli spaces of vacua

4) Duality in $N=1$ SUSY QCD.

5) Other examples of duality.
   An example with triality.

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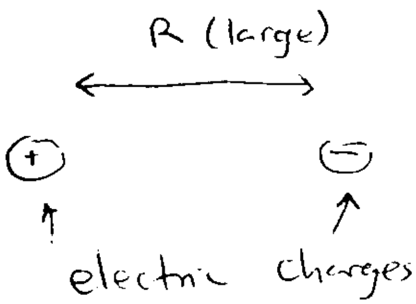
Aspects of 4d $N=1$ susy gauge theories are exactly solvable. Susy gauge theories can be used as toy models for gaining general insight into the dynamics of gauge theories. It is possible to study strong coupling phenomena such as confinement, chiral symmetry breaking, ...
SUSY makes the theories more tractable but the dynamics involved are standard to gauge theories - eg instantons, monopoles, condensates..

The exact results suggest that strong coupling phenomena can be given a dual description in terms of weakly coupled degrees of freedom

Phases of gauge theories and duality



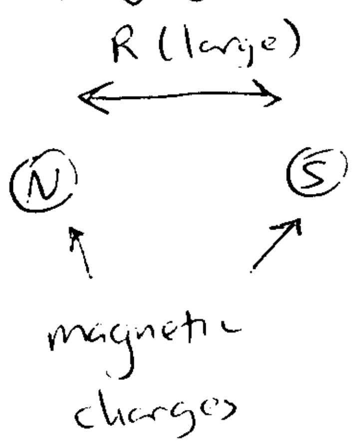
Coulomb: $V(R) \sim \frac{1}{R}$

free electric: $V(R) \sim \frac{1}{R \log(R\Lambda)}$

free magnetic: $V(R) \sim \frac{\log(R\Lambda)}{R}$

Higgs: $V(R) \sim \text{const.}$

confining: $V(R) \sim \sigma R$



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confining: $V(R) \sim \text{const.}$

All of the above for either Abelian or Non Abelian.

<u>Note</u>: free electric $\leftrightarrow$ free magnetic.

upon exchanging electric $\leftrightarrow$ magnetic charges.

Also Higgs $\leftrightarrow$ confinement under

electric $\leftrightarrow$ magnetic. Confinement can

thus be understood as dual Meissner

effect associated with a condensate

of monopoles (Mandelstam & 't Hooft).

Electric $\leftrightarrow$ magnetic takes Coulomb $\leftrightarrow$ Coulomb

For Abelian Coulomb phase this is standard

Not obvious for non-Abelian Coulomb

phase. It was suggested by

Montonen & Olive. The simplest

version is true only for $N=4$

and finite $N=2$ theories.

Extended to $N=1$ by Seiberg.

Holomorphy in the effective action

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massive vector bosons, etc...

$\mathcal{L}_{eff}$ (light fields) has a linearly realized susy

possible susy breaking scale

Light fields

<u>matter</u>

chiral superfields: $X_r = \phi_r + \Theta_\alpha \psi_r^\alpha + \ldots$ (scalar: ϕ_r; Weyl fermion: ψ_r^α, $\alpha = 1,2$)

conjugate anti-chiral superfields: $X_r^+ = \phi_r^+ + \bar{\Theta}_{\dot\alpha} \psi_r^{\dot\alpha}$

<u>gauge</u>

(light in the Coulomb phase) supermultiplet involves $A_\mu, \lambda_\alpha, \lambda^+_{\dot\alpha}$

gauge boson: A_μ; gauginos: $\lambda_\alpha, \lambda^+_{\dot\alpha}$

Focus on two types of contributions to $\mathcal{L}_{eff}$

- $\int d^2\theta \, W_{eff}(X_r, \underset{\text{coupling constants}}{g_I})$

  $\uparrow$ superpotential

  $\longrightarrow$ scalar potential & Yukawa couplings

- $\int d^2\theta \, \mathrm{Im}\left[\tau(X_r, g_I) W_\alpha^2 \right]$

  $\longrightarrow$ gauge kinetic terms

W_α^2 gives the susy completion of $F^2 + iF\tilde{F}$

so $$\tau \sim \frac{\theta_{eff}}{2\pi} + \frac{4\pi i}{g_{eff}^2} .$$

The <u>key fact</u>.

$\boxed{N=1}$ susy $\Rightarrow$ W_{eff} & τ are holomorphic in the chiral superfields X_r - indep. of the X_r^{+}.

A new ingredient (Seiberg '93)

coupling constants g_I $\longleftrightarrow$ expectation values of additional chiral superfields

$\therefore$ W_{eff} & τ must be holomorphic in the g_I - indep. of $g_I^\dagger$.

The requirements of holomorphy in field + coupling constant space, symmetries, selection rules, and reproducing known behavior in understood limits often suffice to <u>exactly</u> determine W_{eff} and τ. The results can be highly non-trivial, revealing interesting properties of strongly coupled gauge theories.

Classical SUSY gauge theories & D-flat directions:

Gauge group G with matter Φ_r ← rep of G

$$\mathcal{L} = \mathcal{L}_0 + \sum_{r,a} \phi_r^\dagger T_r^a \lambda^a \psi_r + h.c.$$

← obvious kinetic terms

$$+ \sum_a \left(\sum_r \phi_r^\dagger T_r^a \phi_r \right)^2.$$

D flat directions: $\sum_r \phi_r^\dagger T_r^a \phi_r = 0 \quad \forall a.$

E.g. $U(1)$ gauge theory with matter

Q charge 1

$\tilde{Q}$ charge -1

} = chiral superfields

$$V_{scalar} = \left(Q^\dagger Q - \tilde{Q}^\dagger \tilde{Q} \right)^2$$

$= 0$ for $\langle Q \rangle = \langle \tilde{Q} \rangle = v$

$\forall$ complex v. Continuum of degenerate vacua. $v \neq 0 \Rightarrow U(1)$ Higgsed

light field $= X = Q\tilde{Q}$.

$\langle X \rangle = v^2$ = unconstrained. X is a classical moduli, $V_{classical}(X) = 0$.

X is not a Goldstone boson – quantum effects could give it a potential $W_{eff}(X)$. The vacuum degeneracy is not protected by any symmetry – in fact, vacua with different $\langle X \rangle$ are physically inequivalent e.g. $M^2_{photon} \sim |X|$.

Classical $\mathbb{Z}_2$ orbifold singularity at $X = 0$, since $K_{cl} = QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger \sim (XX^\dagger)^{1/2}$, reflects the fact that the photon is massless for $\langle X \rangle = 0$.

A general statement about D-Flat directions.

Classical D flat directions $\longleftrightarrow$ $\langle X^r \rangle$

$\uparrow$ gauge inv. composites subject to classical relations

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$N=1$, $SU(N_c)$ with N_f fields Q_f in the $\underset{\sim}{N_c}$ & N_f fields $\tilde{Q}_{\tilde{f}}$ in the $\overline{\underset{\sim}{N_c}}$ of $SU(N_c)$. $=$ "SUSY QCD with N_c colors & N_f flavors."

$$Q_{fc} = (\text{squark})_{fc} + \Theta^\alpha (\text{quark})_{fc,\alpha} + \cdots$$

$$\tilde{Q}_{\tilde{f}\tilde{c}} = (\widetilde{\text{squark}})_{\tilde{f}\tilde{c}} + \Theta^\alpha (\widetilde{\text{quark}})_{\tilde{f}\tilde{c}\alpha} + \cdots$$

$=$ chiral superfields.

D-flat directions (up to gauge and flavor rotations)

$\underline{N_f < N_c}$

$$\begin{array}{c} \longleftarrow \text{colors} \longrightarrow \\ Q_{fc} = \tilde{Q}_{fc} = \begin{pmatrix} a_1 & & & & \\ & \ddots & & & \\ & & a_{N_f} & 0 & \cdots \end{pmatrix} \end{array} \quad \updownarrow \text{flavors}$$

$\underline{N_f \geq N_c}$

$$Q_{fc} = \begin{pmatrix} a_1 & & \\ & \ddots & \\ & & a_{N_c} \\ & & \\ & & \end{pmatrix} \qquad \tilde{Q}_{fc} = \begin{pmatrix} \hat{a}_1 & & \\ & \ddots & \\ & & \hat{a}_{N_c} \\ & & \\ & & \end{pmatrix}$$

with $|a_i|^2 - |\hat{a}_i|^2 = \text{const.}$ indep. of i.

<u>Exercise</u> check that these are D-flat.

For generic a_i $\langle Q \rangle$ & $\langle \tilde{Q} \rangle$

break by the Higgs mechanism

$$SU(N_c) \longrightarrow SU(N_c - N_f) \qquad N_f < N_c - 1$$

$$\emptyset \qquad N_f \geq N_c - 1$$

$$\rightarrow \underbrace{2 N_c N_f}_{Q \,\&\, \tilde{Q}} - \underbrace{\left(N_c^2 - 1 - (N_c - N_f)^2 + 1 \right)}_{\text{eaten à'la Higgs}}$$

$= \quad N_f^2$ light matter fields for $N_f \leq N_c - 1$

or $2 N_c N_f - (N_c^2 - 1)$ light matter fields

for $N_f \geq N_c - 1$

Gauge invariant description:

$$M_{f\tilde{g}} = Q_f \tilde{Q}_{\tilde{g}}$$ "mesons"

$= N_f^2$ light fields for $N_f \leq N_c - 1$.

For $N_f \geq N_c$

$$M_{f\tilde{g}} = \begin{pmatrix} a_1 \tilde{a}_1 & & & \\ & \ddots & & \\ & & a_{N_f}\tilde{a}_{N_f} & 0 \\ & & & 0 \end{pmatrix}$$ = rank N_f at most.

& baryons $B = Q_{f_1 c_1} \cdots Q_{f_{N_c} c_{N_c}} \epsilon^{c_1 \cdots c_{N_c}}$

& antibaryons $\tilde{B}_{\tilde{f}_1 \cdots \tilde{f}_{N_c}}$

subject to constraints.

E.g. $\underline{N_f = N_c}$: $M_{f,g}$, B, $\tilde{B}$

subject to $\det M = B\tilde{B}$.

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E.g. $N_f = N_c + 1$

$M_{f\tilde{g}}$, $B^f \equiv \epsilon^{f_1 \cdots f_{N_c} f} B_{f_1 \cdots f_{N_c}}$,

and $\tilde{B}^{\tilde{f}}$ satisfy

$$M_{f\tilde{g}} B^f = 0$$

$$M_{f\tilde{g}} \tilde{B}^{\tilde{g}} = 0$$

$$\text{“} \det M \, (M^{-1})^{f\tilde{g}} \text{”} = B^f \tilde{B}^{\tilde{g}}$$

↑ cofactor matrix

We will first focus on $N_f < N_c$

$W_{classical}(M) = 0$. Use holomorphy & symmetries to find the form of $W_{exact}(M)$

Classical Symmetries: Global flavor symm=

$$\underbrace{SU(N_f)\times SU(N_f)\times U(1)_Q\times U(1)_{\tilde{Q}}}_{\text{act on superfields i.e. commute w/ SUSY}}\times \underset{\substack{\uparrow\\ \text{does not}\\ \text{commute w/ SUSY}}}{U(1)_R}$$

$U(1)_R$ counts # gluinos − # quarks

Ex. Check that these are symmetries of $\mathcal{L}_{\text{classical}}$

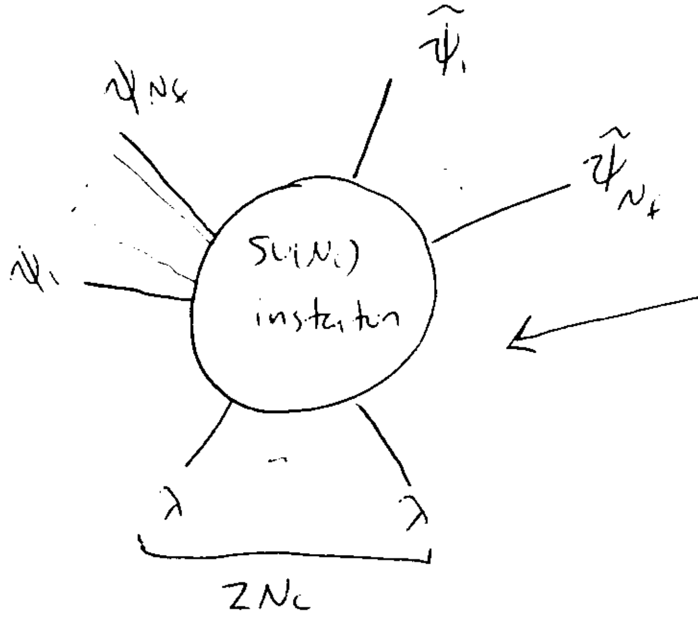
At the quantum level the $U(1)$s are violated by the standard ABJ anomaly

$j_{U(1)}$

$$\Rightarrow \partial_\mu j^\mu_{U(1)} \sim F\tilde{F}$$

$$\Rightarrow \Delta\left(U(1)\text{ charge}\right) \sim \int F\tilde{F}$$

charge violation in an instanton background



fermion zero modes

$$\sim e^{-S_{INST}} = e^{-\frac{8\pi^2}{g^2(\mu)} + i\theta} = \left(\frac{\Lambda}{\mu}\right)^{3N_c - N_f}$$

integrate the β function for SQCD.

Rather than thinking of the U(1)s as being anomalous, think of Λ as a field's expectation value (like the dilaton in string thy) which "spontaneously" breaks the U(1)s. Then Λ carries a charge under the U(1)s and $\mathcal{L}_{eff}$ then respects the symmetries $\longleftrightarrow$ selection rules.

$M_{f\tilde{g}}$ is in the (N_f, N_f) of $SU(N_f) \times SU(N_f)$

$\det M =$ $SU(N_f) \times SU(N_f)$ flavor singlet

W_{eff} can only depend on M as

$W_{eff}(\det M)$.

gluinos - quarks

	$U(1)_Q$	$U(1)_{\tilde{Q}}$	$U(1)_R$
$\det M$	N_f	N_f	0
$\Lambda^{3N_c - N_f}$	N_f	N_f	$2(N_c - N_f)$
$W_{eff}(\Lambda, \det M)$	0	0	2

for $\mathcal{L}_{eff} = \int d^2\theta \, W_{eff} + \ldots$ to be invariant.

$$\boxed{\Rightarrow \quad W_{eff} = a_{N_c, N_f} \left(\frac{\Lambda^{3N_c - N_f}_{N_c, N_f}}{\det M} \right)^{1/(N_c - N_f)}}$$

constants.

Note:

- For $N_f = N_c - 1$ $\quad W_{eff} \sim e^{-S_{INST}}$ & ∴ it can be computed exactly simply by doing a 1-instanton calculation to determine the constant $a_{N_c, N_f = N_c - 1}$.

- W_{eff} is non perturbative.

Perturbative non renormalization thm
$\Rightarrow W_{eff} = 0$ to all orders in pert. thy

Here this is a simple consequence of holomorphy & the need for having definite charge violation

$\int F\tilde{F} \neq 0$.