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REVIEW OF 2D SUSY SIGMA MODELS AND RELATION TO 4D GAUGE THEORIES

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Please note: These are preliminary notes intended for internal distribution only.

Review Supersymmetric QCD

① Non renormalization theorems

$$\mathcal{L} = \int d^4\theta\, K(..) + \int d^2\theta\, W(..) + \int d^2\bar{\theta}\, \overline{W}(...)$$

All renormalizations can be reabsorbed in D-terms.

② Chiral superfield

$$\Phi(y) = \varphi(y) + \theta^\alpha \psi_\alpha(y) + \theta^\alpha \theta_\alpha F(y)$$

$$y^\mu = x^\mu + \bar{\theta}\sigma^\mu\theta$$

Supersymmetry Transform

$$Q_\alpha \varphi = \psi_\alpha \qquad \bar{Q}^{\dot{\alpha}} \varphi = 0$$

$$Q_\alpha \psi_\beta = \epsilon_{\alpha\beta} F \qquad \bar{Q}^{\dot{\alpha}} \psi^\beta = (\sigma^\mu)^{\dot{\alpha}\beta} \partial_\mu \varphi$$

$$Q_\alpha F = 0$$

Consider

$$\langle \varphi_1(x_1)\, \varphi_2(x_2) \ldots \varphi_N(x_N) \rangle = F(x_1 \ldots x_N)$$

$\varphi_i \leftarrow$ Lowest Components of Chiral superfields

$F(x_1, \ldots x_N) \leftarrow x_i$ independent.

Proof is Almost obvious $\quad \partial_\mu \varphi = (\sigma_\mu)_{\alpha\dot\alpha} [\bar{Q}^{\dot\alpha} \psi^\alpha]_-$

3) Condensates

— Pure SQCD

Supergeneralization of $F_{\mu\nu}$

$$W_\alpha = \lambda_\alpha + \theta_\alpha D + \theta^\beta (\sigma^{\mu\nu})_{\alpha\beta} F_{\mu\nu} + \ldots$$

$S = \mathrm{Tr}\, W^\alpha W_\alpha \cong W^2 \leftarrow$ chiral superfield.

$$\langle \lambda^\alpha \lambda_\alpha (x_1) \ldots \lambda^\alpha \lambda_\alpha (x_N) \rangle$$

$\uparrow$ independent of the positions of $x_1, \ldots x_N$.

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In PERTURBATION Theory

$\langle \lambda\lambda \dots \lambda\lambda \rangle \equiv 0$ by charge conservation.

1) For SQCD with matter

$$S = S_{SQCD} + \int \bar{\Phi} e^{V} \Phi + \int P(\Phi) + \int \bar{P}(\bar{\Phi})$$

$$\langle \lambda\lambda \dots \lambda\lambda \quad \varphi^{i_1}\varphi_{j_1} \dots \varphi^{i_n}\varphi_{j_n} \rangle$$

1) again, lowest components of chiral superfields

2) Position independent.

5) Non Perturbative Results.

$$\langle (\lambda\lambda)^2 \rangle \simeq c_2 \Lambda^6$$

$$\langle (\lambda\lambda)^4 \rangle \simeq c_4 \Lambda^{12}$$

$$\vdots$$

$$\langle (\lambda\lambda)^{2n} \rangle \simeq c_{2n} \Lambda^{6n}$$

For $SU(2)$

Remarkable result

$$\boxed{C_{2n} = (C_2)^n}$$

Each correlation function get saturated by instanton contribution

$$\langle \underbrace{(\lambda\lambda)^2 \cdots (\lambda\lambda)^2}_{n} \rangle_{n\text{-instanton}}.$$

Instanton 1) $F = \tilde{F}$ $\Rightarrow$ SD connections.

2) $\frac{1}{8\pi^2}\int F \wedge \tilde{F} = n.$

In the presence of Instanton

$\Rightarrow$ precisely $4n$ gluino zero modes.

For $SU(2)$

$\mathbb{Z}_4$ symmetry get broken to $\mathbb{Z}_2$.

There are 2 vacua $\langle +|$, $\langle -|$

or $\langle 1|$ & $\langle 2|$

by going to the limit $|x_1 - x_2| \to \infty$

$$\langle \lambda\lambda(x_1)\, \lambda\lambda(x_2) \rangle_1 = \langle + | \lambda\lambda | - \rangle \langle - | \lambda\lambda | + \rangle$$

6) Clustering For $SU(N)$ (general case)

$\langle (\lambda\lambda)^N \rangle_1 \neq 0$ $\leftarrow$ get saturated by $n = 1$ instanton.

Non anomalous $\mathbb{Z}_{2N}$ get broken to $\mathbb{Z}_2$ (by instanton).

There ARE N different Vacua

e_i $\quad i = 1, \dots N$

f^i $\quad i = 1 \dots N$

Two different basis.

$$\boxed{f^i = \sum_{k=1}^{N} \omega^{-ik} e_k}$$

By clustering

Either $\langle e_{i+1} | \lambda\lambda | e_i \rangle = c \cdot \Lambda^3$

or $\langle f^i | \lambda\lambda | f^i \rangle = \omega^i c \Lambda^3$

$\Rightarrow$ Then $\langle (\lambda\lambda)^k \rangle = \frac{1}{N} \sum_{i=1}^{N} \left[\langle f^i | \lambda\lambda | f^i \rangle \right]^k \simeq$

$$\simeq c \Lambda^{3k} \delta_{k,N}.$$

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Simple Example of 1-instanton Calculations for $SU(2)$

~~[illegible]~~ $W_\alpha(x,\theta) = \lambda_\alpha + \theta^\beta (\sigma^{\mu\nu})_{\alpha\beta} F_{\mu\nu} + \dots$

superfield.

Instanton $(n=1)$.

$x_0 \leftarrow 4$ (position) $\quad \rho \leftarrow 1$ (scale)

$3 \leftarrow SU(2)$ Rotation (Fix the gauge)

+ Additional 4 gluino zero modes, parametrized by α and $\bar{\beta}$

Superformalizm

$$W^2(x,\theta) \simeq \frac{\rho^4}{\left[(x-x_0)^2 + \rho^2\right]^4}\left[(\theta-\alpha) - (x_L - x_0)_\mu \sigma^\mu \bar{\beta}\right]^2$$

Supersymmetry transformations

$$\delta x_0^\mu = \bar{\varepsilon}(\sigma^\mu)\alpha \qquad \delta\rho^2 = (\bar{\varepsilon}\cdot\bar{\beta})\rho^2$$

$$\delta\alpha = \varepsilon \qquad \delta\bar{\beta} = \bar{\beta}(\bar{\varepsilon}\cdot\bar{\beta})$$

Dimensions $[\alpha] = -1/2$ $\quad [\rho] = -1$

$[\beta] = 1/2$. $\quad [\mu] = 1$

Instanton Measure $\sim C\,\mu^6\, d\rho^2\, d^4x_0\, d^2\theta\, d^2\bar{\beta}$

1) Measure is supersymmetric invariant
2) Power of μ is determined on the dimensional grounds.
3) $\mu \leftarrow$ is Renormalization scale

$$\langle W^2 \, W^2 \rangle = C \mu^6 \int d^2\rho^2 d^4x_0 \, d^2\theta \, d^2\bar{\beta} \times e^{-8\pi^2/g^2}$$

$$\times \frac{\rho^8 \left[(\theta-\alpha) - (x_L - x_0)^\mu \sigma_\mu \bar{\beta}\right]^2 \left[(\tilde{\theta}-\alpha) - (\tilde{x}_L - x_0)^\mu \sigma_\mu \bar{\beta}\right]^2}{\left[(x-x_0)^2+\rho^2\right]^4 \left[(\tilde{x}-x_0)^2+\rho^2\right]^4}$$

Simple Exercise

Important

1) The result of integration is $x, \tilde{x}$ independent !!

2) $\langle W^2 \, W^2 \rangle \simeq C \cdot \mu^6 e^{-8\pi^2/g^2(\mu)} = C \Lambda^6$

3) $\Lambda \leftarrow$ Dynamically generated scale

$$\Lambda = \mu \, e^{-4\pi^2/3g^2(\mu)}$$

8) SQCD (Pure YM!!).

Condition that Λ is μ-independent implies That 1-loop beta function is Exact

$$\frac{d\Lambda}{d\mu} = 0 \implies \boxed{\beta(g) = -\frac{3}{8\pi^2} g^3}$$

9 Comments on ~~[illegible]~~ beta-function } For $\boxed{N=1}$

(i) For SQCD (Super YM).

1-loop beta function is ~~Exact~~ COMPUTABLE

(ii) For SQCD ⊕ matter.

Full Beta function is written in terms of 1-loop beta function (β_1) and anomalous dimensions

For $SU(N)$

$$\beta(g) \simeq -\frac{g^3}{16\pi^2} \frac{3C_2 - \sum_j T(R_j)(1-\gamma_j)}{1 - C_2 g^2/8\pi}$$

This is an <u>Exact</u> Result.

10 For $N=2$ $\boxed{\text{1-loop Beta function is Exact}}$

11 For $N=4$ $\beta \equiv 0$.

Supersymmetric QCD coupled to matter

$$S_{inter} = \int \bar{\Phi} e^{V} \Phi + \bar{\tilde{\Phi}} e^{-V} \tilde{\Phi} + \int P(\Phi, \tilde{\Phi}) + \int \bar{P}(\bar{\Phi}, \bar{\tilde{\Phi}})$$

$\Phi, \tilde{\Phi}$ transforms as $R, \tilde{R}$
↑ both chiral

For simplicity $P(\Phi, \tilde{\Phi}) = m \tilde{\Phi}\Phi$

) R-symmetry

$$U_R(1) : \begin{cases} \lambda \to e^{i\beta}\lambda \\ \varphi \to e^{i\beta}\varphi \\ \tilde{\varphi} \to e^{i\beta}\tilde{\varphi} \end{cases}$$

Broken by anomaly to $\mathbb{Z}_{2N} \longrightarrow \mathbb{Z}_2$ (by condensate)

Another Important Symmetry (Broken by j-th mass)

$$\lambda \to e^{-i\alpha/2N}\lambda$$

$$(\tilde{\varphi}^{\ell}\, \psi_{\ell}) \to e^{i\alpha\delta_{\ell j}} (\tilde{\varphi}_{\ell}\, \psi_{\ell})$$

$$(\tilde{\varphi}^{\ell}\, \varphi_{\ell}) \to e^{i\alpha(\frac{1}{2}\delta_{\ell j} - 1/2N)} (\tilde{\varphi}^{\ell}, \varphi_{\ell})$$

Rotation of phase of m
$m = e^{i\alpha}|m|$

This symmetry is Explicitly broken:

$$\partial^\mu J_\mu^{5\,(j)} = m_j \tilde{\psi}^j \psi_j$$

Also Usual anomaly in Axial current $\sim U_A(1)$.

$U_R(1)$ & $U_A(1)$ can be combined in non anomalous ~~symmetry~~ symmetry

$$U_X(1): \quad \begin{aligned} \lambda &\to \lambda e^{-iM\beta} \\ (\varphi, \tilde{\varphi}) &\to e^{i(N-M)\beta} (\varphi, \tilde{\varphi}) \\ (\psi, \tilde{\psi}) &\to e^{iN\beta} (\psi, \tilde{\psi}) \end{aligned}$$

Condensates

$$\langle \underbrace{\lambda\lambda \cdots \lambda\lambda}_{p} \, \phi^{i_1} \tilde{\phi}_{j_1} \cdots \phi^{i_q} \phi_{j_q} \rangle$$

m - dependence can be Uniquely fixed

For $SU(N)$ this Correlation function get Saturated by n- instanton config If $\boxed{p+q = nN}$

Comments on Konishi anomaly

SUSY YM coupled to $N=1$ matter

$$S_{tot} = S_{YM} + S_{Matter}$$

$$S_{matter} = \int \left(\bar{\Phi} e^{gV} \Phi + \bar{\Phi}' e^{g\tilde{V}} \Phi' \right) d^4\theta\, d^4x\, +$$

$$+ \int W(\Phi, \Phi')\, d^2\theta\, d^4x \; + \; \text{c.c.}$$

In principle one may consider an arbitrary ~~...~~ potential $W(\Phi, \Phi')$

Important coupling

$$\bar{\varphi}^i \lambda^a (T^a)_{ij} \psi^j$$

$$\Phi = \varphi + \theta\psi + \theta\theta F \qquad (\text{fundamental})$$

$$\Phi' = \;\ldots\ldots \qquad (\text{antifundamental})$$

Supersymmetry transformations.

$$[\bar{Q}^{\dot\alpha}\ \varphi] = 0$$

$$\{\bar{Q}^{\dot\alpha}\ \psi^{\alpha}] = \sqrt{2}\,\sigma_{\mu}^{\dot\alpha\alpha}\,\partial_{\mu}\varphi$$

$$[Q_{\alpha}\ F] = 0$$

$$[Q_{\alpha}\ \psi_{\beta}] = \epsilon_{\alpha\beta}F$$

Equation of motion for F

$$F = \partial_{\varphi}W$$

flavour indeces

Comes from regularization

$$\bar{\Psi}^{\dot\alpha i}(x+\epsilon)\,e^{\int_{x}^{x+\epsilon} A\cdot dx}\,\varphi_j(x)$$

Konishi anomaly:

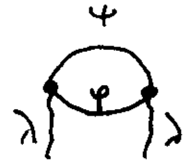
$$[\bar{Q}_{\dot\alpha}\ \bar{\Psi}^{\dot\alpha i}\varphi_j] = \bar{F}^{i}\varphi_j + ig\,\epsilon_{\mu}\,\bar{\Psi}^{\dot\alpha i}(\sigma^{\mu})_{\dot\alpha\beta}\,\lambda^{\beta}\varphi_j$$

$$\frac{\partial}{\partial\varphi^{i}}W\,\varphi_j + ig\,\epsilon_{\mu}\bar{\Psi}^{i}\sigma^{\mu}\lambda\,\varphi_j = [\bar{Q}_{\dot\alpha}, *]$$

At the level of Corr. functions

$$\left\langle \frac{\partial}{\partial\varphi^i} W \varphi_j \right\rangle = -ig\,\epsilon_\mu \left\langle \bar{\varphi}^i \sigma^\mu \lambda \varphi_j \, e^{\int \psi T^a \lambda^a \bar{\varphi}} \right\rangle =$$

interaction from the action

$=$ 

$$\Rightarrow \left\langle \frac{\partial}{\partial\varphi^i} W \varphi_j \right\rangle = \frac{g^2}{32\pi^2} \langle \lambda\lambda \rangle$$

Relation between Condensates.

Konishi Anomaly in SUPERINVARIANT formulation

$$\frac{1}{4}\bar{D}^2 \bar{\Phi}_i e^V \Phi_j = \frac{g^2}{32\pi^2} \delta^i_j C_2 W^2 + \phi^i \frac{\partial W}{\partial \phi^j}$$

Bound

$$S \geq \frac{8\pi^2}{g^2} K$$

where K is Pontriagin number.

Solution of SD or ASD Eq. is called INSTANTON.

These solutions are parameterized by MODULI space

(only for compact manifolds) $\dim \mathcal{M}_K = 8K - \frac{3}{2}(\chi + \sigma)$ | $S^4 : 8K-3$, $R^4 : 8K$

For Example ~~for~~ For $SU(2)$ S^4 and $K=1$ (or R^4)

there are 5 parameters (8 parameters)

x_0^μ $\# = 4$ (center of the mass)

ρ $\# = 1$ (scale)

U $\# 3$ ($3 - SU(2)$ angles)

Plan 1) Solution for $K=1$

2) ADM construction

3) Instanton measure

4) Instantons in Supersymmetric theories

... is given by

$$A_\mu^a = \frac{2}{g}\, \eta_{a\mu\nu}\, \frac{(x-x_0)^\nu}{(x-x_0)^2+\rho^2}$$

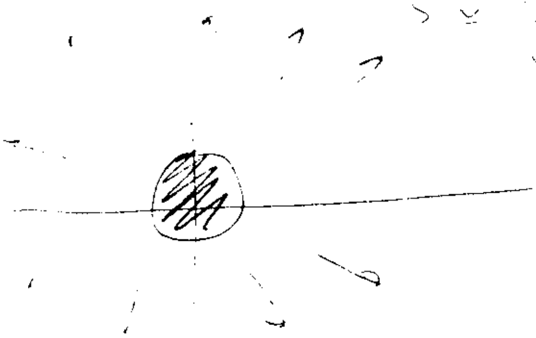
$$F_{\mu\nu}^a = -\frac{4}{g}\, \eta_{a\mu\nu}\, \frac{\rho^2}{\left[(x-x_0)^2+\rho^2\right]^2}$$

$x_0 \leftarrow$ position of ~~the~~ center of the mass

$\rho \leftarrow$ Instanton size.

As $x \to \infty$

$S \simeq \dfrac{x_\mu \tau_\mu^+}{\sqrt{x^2}}$



One can make a singular gauge transformation with $U = \dfrac{i\,\tau_\mu^+ (x-x_0)_\mu}{\sqrt{(x-x_0)^2}}$

$U(x)$ is ill deffined at $x = x_0$

under singular g.t. $S \to \tilde{S}$ s.that $\tilde{S}(x\to\infty) \to 1$

In this gauge

$$\bar{A}_\mu^a = -\frac{1}{g}\, \bar{\eta}_{a\mu\nu}\, \partial_\nu \ln\left(1+\frac{\rho^2}{(x-x_0)^2}\right)$$

Zero modes

SD equation $F = *F$

For $k=1$ $\quad A_\mu^a = \frac{2}{g}\, \eta^a_{\mu\nu} \frac{(x-z)^\nu}{(x-z)^2+\rho^2}$

⊕ SD satisfies the gauge condition $\partial^\mu A_\mu = 0$
(as well as $D^\mu A_\mu \doteq 0$)

e are going to use a background method.
he interaction simplifies, if we choose
background dependent gauge

$$\boxed{D_A^\mu \, \delta A_\mu = 0}$$

Zero modes: $\quad A + \omega \quad ; \quad \omega \leftarrow$ zero mode

$$\begin{cases} d_A\omega = * \, d_A \omega & : \text{variation of SD} \\ D_A^\mu \, \omega_\mu = 0 & : \text{gauge} \end{cases}$$

zero modes span the tangent
space to the instanton moduli space.

Different zero modes

1) Translations

2) Dilatations

3) $SU(2)$ rotations

) Translations

$$\omega^a_\mu(\nu) = \frac{\partial}{\partial z^\nu} A^a_\mu + D_\mu \Lambda^a(\nu) \,,$$

where $\Lambda^a(\nu)$ generates a gauge transformation

$$\Lambda^a = \frac{2}{g} \eta^a{}_{\nu\lambda} \frac{(x-z)^\lambda}{(x-z)^2+\rho^2}$$

For simplicity we put $\rho = 1$

$$\Rightarrow \quad \omega^a_\mu(\nu) = -\frac{4}{g} \eta^a{}_{\mu\nu} \frac{1}{\rho^2 + (x-z)^2}$$

$$\langle \omega(\nu) | \omega(\epsilon) \rangle \simeq \delta_{\nu\epsilon} \frac{\mathrm{const}}{g^2}$$

It is easy to check $D^\mu_A \omega_\mu(\nu) \equiv 0$

1B (Put $\rho = 1$) During the class.

Dilatations

$$\omega^a_\mu = \frac{\partial}{\partial \rho} A^a_\mu = -\frac{4}{g}\,\eta^a_{\mu\nu}\,\frac{x^\nu \rho}{(x^2+\rho^2)^2}$$

we ignore z-dependence.

$$\langle \omega | \omega \rangle = \frac{\text{const}}{g^2}$$

Again, it is easy to check that this solution satisfy g.c.

Similar, the su(2) rotations produce the zero modes

$$\omega^a_\mu(b) = \frac{2}{g}\,\eta_{a\rho\mu}\,\bar{\eta}_{b\rho\lambda}\,\frac{2\rho\, x^\lambda}{(\rho^2+x^2)^2}$$

$[A] \sim \frac{1}{\rho}$ $\quad [\delta_z A] = [\delta_\rho A] = \frac{1}{\rho^2}$

but $\delta_{su(2)} A \approx \frac{1}{\rho}$

Zero mode Normalization

$$\langle \omega_b | \omega_c \rangle \simeq \delta_{bc}\,\frac{\text{const}}{g^2}\,\rho^2$$

$\Rightarrow$ Contribution to the measure $\left(\frac{1}{g^2}\right)^{n_B/2} \rho^3 \simeq \left(\sqrt{S_0}\right)^{n_B} \rho^{\dim G}$

[redacted]

As the result we end up with

$$\int d^4z\, d\rho\; M^8 \left(\frac{8\pi^2}{g^2}\right)^4 \rho^3\, e^{-\frac{8\pi}{g^2}+\phi_1} \simeq$$

$$\simeq \int \frac{d^4z\, d\rho}{\rho^5} (M\rho)^8 \left(\frac{8\pi^2}{g^2}\right)^4 e^{-\frac{8\pi}{g^2}+\phi_1}$$

ϕ_1 ← contribution of non zero modes.

For SUSY YM $\phi_1 = 0$.

For bosonic configuration

$$\phi_1 \simeq -\frac{2}{3}\ln M\rho + o(g^2)$$

$\phi_1 \simeq$  + h.o. terms.

Once again M is UV cut-off.

g ← coupling const at a scale M

$$g = g(M)$$

$$\int \frac{d^4z\, d\rho}{\rho^5} \left(\frac{8\pi}{g^2}\right)^4 e^{-\frac{8\pi}{g^2} + 8\ln M\rho - \frac{2}{3}\ln M\rho + \dots}$$

$$\frac{8\pi}{g^2(\rho)} = \frac{8\pi}{g^2(\mu)} - 2\cdot\frac{11}{3}\ln M\rho$$

↑ 1-loop β-function.

Supersymmetric extensions

$$\int \frac{d^4z\, d\rho\, d\xi_i}{\rho^5} \left(\frac{8\pi^2}{g^2}\right)^{(n_B-n_F)/2} (M\rho)^{n_B - \frac{1}{2}n_F} \cdot e^{-\frac{8\pi^2}{g^2}}$$

zero mode normalization.

zero mode det

N = 1.

Instanton configuration does not invariant under SUSY transformation. Acting by half of SUSY generators (chiral) one produces a fermionic zero modes

Fermionic Zero modes.

$N=1$ $\qquad (A_\mu, \lambda_\alpha, \bar\lambda_{\dot\alpha}, \dots)$.

$$\begin{cases} F = * F & \text{SD condition} \\ D_A \lambda = 0 & \leftarrow \text{zero mode eq.} \end{cases}$$

check with zumino ?

An instanton configuration gives rise to a CHIRAL Superfield.

Namely $F^{\text{ins}}_{\mu\nu}$ can be rearranged together with fermionic zero modes ~~[illegible]~~ into chiral superfield.

For $K=1$ there are $2N$ zero modes for $SU(N)$ gauge group.

In general there are $2N \cdot K$ zero modes. (?).

For $K=1$

2-zero modes correspond to supertranslations

$$\theta \to \theta + \epsilon$$

$$\bar\theta \to \bar\theta$$

$$x \to x + i\,\epsilon\,\bar\sigma\,\bar\theta$$

+ there 2 - zero modes, that correspond to superconformal transformations.

The instanton moduli space is ~~a~~ 8|4 dimensional supermanifold.

Instanton Superfield

$$W^a_\alpha = \underset{\uparrow \text{zero mode}}{\lambda^a_\alpha} + (\sigma^{\mu\nu})_{\alpha\beta}\,\theta^\beta \underset{\nearrow \text{bosonic configuration.}}{F^a_{\mu\nu}} - (\theta\cdot\theta)(D_\mu \bar\lambda_{\dot\alpha})^a\,\bar\sigma^\mu_{\alpha\dot\alpha}$$

$$\mathrm{Tr}\,W^2 = \frac{\cancel{\ }}{g^2}\,\frac{\rho^4}{\left[(x_L - z)^2 + \rho^2\right]^2} \times \left[(\theta - \chi) - (x_L - z)_\mu\,\sigma^\mu\,\bar\beta\right]$$

Parameters : z, ρ, g

$\chi, \bar\beta$ ← Grassman parameters

SUSY transform:

$$\delta \cancel{z} = 2i\,\chi\,\sigma\,\bar\epsilon \qquad \delta\chi = 0$$

$$\delta\rho = \rho\left(-2i\,\bar\epsilon\cdot\bar\beta + 3\,\bar\epsilon\cdot\bar\epsilon\;\bar\beta\cdot\bar\beta\right) \qquad \delta\bar\beta = -4i\,\bar\beta\,(\bar\epsilon\cdot\bar\beta)$$

Explicit computation of instanton measure:

$$\int \frac{d^4 z \, d\rho \, d^2\chi \, d^2\bar\beta}{\rho^5} \left(\frac{8\pi^2}{g^2}\right)^{\frac{1}{2}(n_B - n_F)} (\rho M)^{n_B - \frac{1}{2}n_F} e^{-8\pi^2/g^2}$$

<u>Special correlation functions</u> (Insertion).

To compute corr. function one just need to insert the classical configuration *.

$$\langle W^2(1)\, W^2(2) \rangle = M^6 e^{-8\pi^2/g^2} \frac{1}{g^4} \times$$

$$\int d^2\chi \, d^2\bar\beta \, d^4 z \, d\rho \cdot \rho \quad W^2(x_{1L}, \theta_1)\, W^2(x_{2L}, \theta_2) =$$

$$= \dots = c \left(\frac{8\pi^2}{g^2}\right)^4 M^6 e^{-8\pi^2/g^2} \leftarrow x_1, x_2 \text{ independent}$$

$\Rightarrow$ Condensates ; Special class of corr. functions that are point indep.

In perturbation theory this corr. function is identically equal to zero, due to fermion number conservation.

Bla-Bla-Bla on chiral symmetries