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Lecture II

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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EXACT RESULTS IN SUSY GAUGE THEORIES

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Please note: These are preliminary notes intended for internal distribution only.

Last time:

$$V^{\text{classical}}_{\text{squarks}} = \sum_a \Big(\sum_r \phi_r^+ T_r^a \phi_r \Big)^2$$

(all matter fields)

Classical susy gauge theories can have "moduli spaces" of degenerate but <u>physically</u> inequiv. vacua labeled by $\langle \phi_r \rangle$, such that $\sum_r \phi_r^+ T_r^a \phi_r = 0 \quad \forall a$, modulo gauge transformations.

"D-flat directions."

[Classical D flat directions / (gauge equiv)] $\longleftrightarrow$ $\langle X^r \rangle$ gauge inv composites subject to classical rel'ns

Next example:

$\mathcal{N}=1$ $SU(N_c)$ w/ N_f flds Q_f in N_c & N_f $\tilde{Q}_{\tilde{f}}$ in $\overline{N_c}$. = "SQCD w/ N_c colors & N_f flavors".

Q_{fc} & $\tilde{Q}_{\tilde{f}\tilde{c}}$ are chiral superfields.

D-flat directions (up to gauge & flavor rotations)

$N_f < N_c$:

$$Q_{fc} = \tilde{Q}_{\tilde{f}\tilde{c}} = \begin{array}{c} \xleftrightarrow{\text{colors}} \\ \begin{pmatrix} a_1 & & & & \\ & \ddots & & & \\ & & a_{N_f} & 0 & \cdots \end{pmatrix} \end{array} \Big\updownarrow N_f$$

$N_f \geq N_c$:

$$Q = \begin{pmatrix} a_1 & & \\ & \ddots & \\ & & a_{N_c} \\ & & \end{pmatrix} \qquad \tilde{Q} = \begin{pmatrix} \tilde{a}_1 & & \\ & \ddots & \\ & & \tilde{a}_{N_c} \\ & & \end{pmatrix}$$

with $|a_i|^2 - |\tilde{a}_i|^2 =$ const. indep. of i

For generic a_i $\langle Q \rangle$ & $\langle \hat{Q} \rangle$

break by the Higgs mechanism

$$SU(N_c) \longrightarrow SU(N_c - N_f) \qquad N_f < N_c - 1$$

$$\emptyset \qquad N_f \geq N_c - 1$$

$$\rightarrow \underbrace{2 N_c N_f}_{Q \,\&\, \hat{Q}} - \big(\underbrace{N_c^2 - 1 - (N_c - N_f)^2 + 1}_{\text{eaten à la Higgs}} \big)$$

$= N_f^2$ light matter fields for $N_f \leq N_c - 1$

or $2 N_c N_f - (N_c^2 - 1)$ light matter fields for $N_f \geq N_c - 1$

Gauge invariant description:

$$M_{f\tilde{g}} = Q_f \tilde{Q}_{\tilde{g}}$$ "mesons"

$= N_f^2$ light fields for $N_f \leq N_c - 1$.

For $N_f \geq N_c$

$$M_{f\tilde{g}} = \begin{pmatrix} a_1\tilde{a}_1 & & & \\ & \ddots & & \\ & & a_{N_c}\tilde{a}_{N_c} & \\ & & & 0 \\ & & & & 0 \end{pmatrix} = \text{rank } N_f \text{ at most.}$$

& baryons $B = Q_{f_1 c_1} \cdots Q_{f_{N_c} c_{N_c}} \varepsilon^{c_1 \cdots c_{N_c}}$

& antibaryons $\tilde{B}_{\tilde{f}_1 \cdots \tilde{f}_{N_c}}$

subject to constraints.

E.g. $\underline{N_f = N_c}$: $M_{f,g}$, B, $\tilde{B}$

subject to $\det M = B\tilde{B}$.

E.g. $N_f = N_c + 1$

$M_{f\tilde{g}}$, $\quad B^f \equiv \epsilon^{f_1 \cdots f_{N_c} f} B_{f_1 \cdots f_{N_c}}$,

and $\tilde{B}^{\tilde{f}}$ satisfy

$$M_{f\tilde{g}} B^f = 0$$

$$M_{f\tilde{g}} \tilde{B}^{\tilde{g}} = 0$$

$$\text{"} \det M \, (M^{-1})^{f\tilde{g}} \text{"} = B^f \tilde{B}^{\tilde{g}}$$

$\uparrow$ cofactor matrix

We will first focus on $N_f < N_c$

$W_{classical}(M) = 0$. Use holomorphy & symmetries to find the form of $W_{exact}(M)$

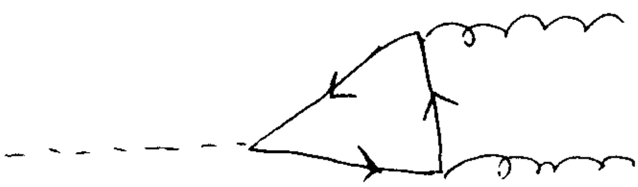
Classical Symmetries: Global flavor symm:

$$\underbrace{SU(N_f) \times SU(N_f) \times U(1)_Q \times U(1)_{\tilde{Q}}}_{\text{act on superfields i.e. commute w/ susy}} \times \underset{\substack{\uparrow \\ \text{does not} \\ \text{commute w/ susy}}}{U(1)_R}$$

$U(1)_R$ counts # gluinos − # quarks

Ex Check that these are symmetries of $\mathcal{L}_{classical}$

At the quantum level the $U(1)$s are violated by the standard ABJ anomaly

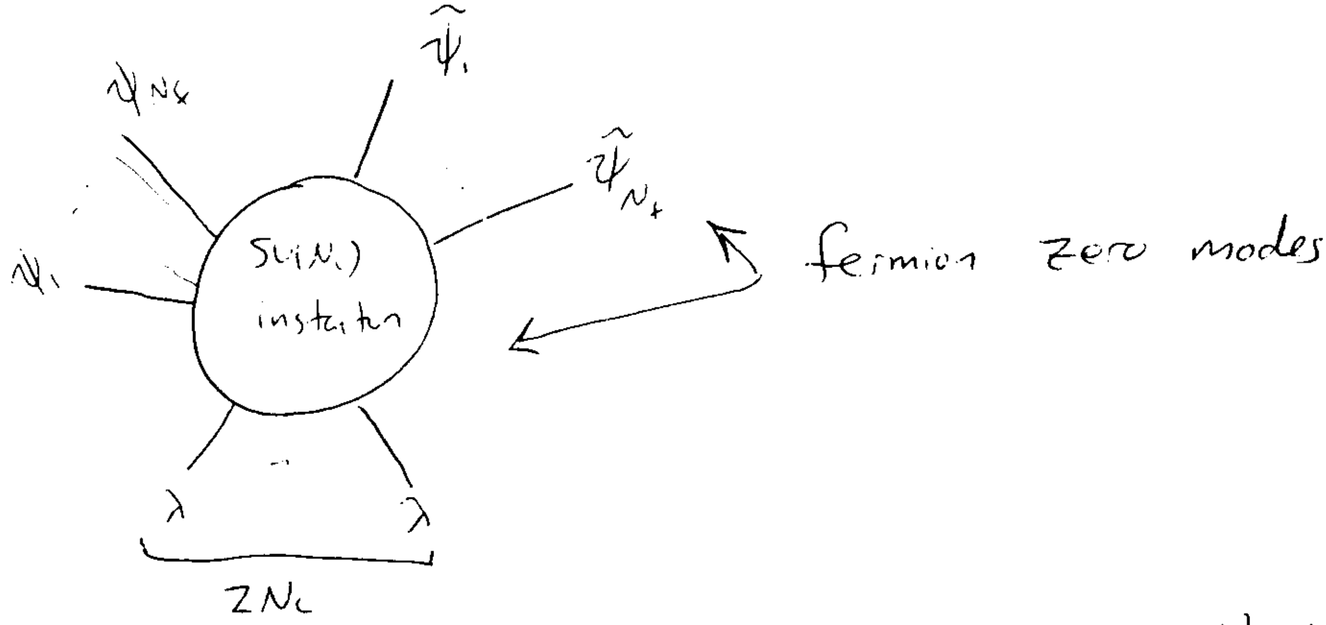


$j_{U(1)}$

$$\Rightarrow \partial_\mu j^\mu_{U(1)} \sim F\tilde{F}$$

$$\Rightarrow \Delta(U(1)\text{ charge}) \sim \int F\tilde{F}$$

charge violation in an instanton background.



$$\sim e^{-S_{INST}} = e^{-\frac{8\pi^2}{g^2(\mu)} + i\theta} = \left(\frac{\Lambda}{\mu}\right)^{3N_c - N_f}$$

integrate the β function for SQCD.

Rather than thinking of the U(1)s as being anomalous, think of Λ as a field's expectation value (like the dilaton in string thy) which "spontaneously" breaks the U(1)s. Then Λ carries a charge under the U(1)s and $\mathcal{L}_{eff}$ then respects the symmetries $\longleftrightarrow$ selection rules.

$M_{f\tilde{g}}$ is in the (N_f, N_f) of $SU(N_f)\times SU(N_f)$

$\det M$ = $SU(N_f)\times SU(N_f)$ flavor singlet

W_{eff} can only depend on M as

$W_{eff}(\det M)$.

gluinos - quarks

	$U(1)_Q$	$U(1)_{\tilde{Q}}$	$U(1)_R$
$\det M$	N_f	N_f	0
$\Lambda^{3N_c-N_f}$	N_f	N_f	$2(N_c-N_f)$
$W_{eff}(\Lambda, \det M)$	0	0	2

for $\mathcal{L}_{eff} = \int d^2\theta\, W_{eff} + \cdots$ to be invariant.

$$\boxed{\Rightarrow \quad W_{eff} = a_{N_c,N_f}\left(\frac{\Lambda_{N_c,N_f}^{3N_c-N_f}}{\det M}\right)^{1/(N_c-N_f)}}$$

constants.

Note:

- For $N_f = N_c - 1$ $W_{eff} \sim e^{-S_{INST}}$
  & ∴ it can be computed exactly simply by doing a 1-instanton calculation to determine the constant $a_{N_c, N_f = N_c - 1}$.

- W_{eff} is non perturbative.

Perturbative non renormalization thm
$\Rightarrow$ $W_{eff} = 0$ to all orders in pert. thy

Here this is a simple consequence of holomorphy & the need for having definite charge violation
$\int F\tilde{F} \neq 0$.

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Consider the limit $\langle Q_{N_f} \rangle = \langle \tilde{Q}_{N_f} \rangle$ = large i.e. $M_{N_f N_f}$ large.

$$\begin{array}{c} SU(N_c) \text{ w/} \\ N_f \text{ flavors} \end{array} \xrightarrow[\langle Q_{N_f} \rangle]{\text{Higgs}} \begin{array}{c} SU(N_c-1) \text{ w/} \\ N_f - 1 \text{ flavors.} \end{array}$$

$E \uparrow$

(N_c, N_f)
$$e^{-\frac{8\pi^2}{g^2(\mu)} + i\theta} = \left(\frac{\Lambda_{N_c, N_f}}{\mu} \right)^{3N_c - N_f}$$

———————— $\langle Q_{N_f} \rangle$ ————————

(N_c-1, N_f-1)
$$e^{-\frac{8\pi^2}{g^2(\mu)} + i\theta} = \left(\frac{\Lambda_{N_c-1, N_f-1}}{\mu} \right)^{3(N_c-1)-(N_f-1)}$$

matching $g^2(\mu)$ at $\mu = \langle Q_{N_f} \rangle$

$$\Rightarrow \quad \frac{\Lambda_{N_c, N_f}^{3N_c - N_f}}{M_{N_f N_f}} = \Lambda_{N_c-1, N_f-1}^{3(N_c-1)-(N_f-1)}$$

In this limit

$$W_{eff} = a_{N_c,N_f} \left(\frac{\Lambda_{N_c,N_f}^{3N_c-N_f}}{M_{N_cN_f} \det \hat{M}} \right)^{1/(N_c-N_f)}$$

$\hat{M}$: N_f-1 light flavors

$$= a_{N_c,N_f} \left(\frac{\Lambda_{N_c-1,N_f-1}^{3(N_c-1)-(N_f-1)}}{\det \hat{M}} \right)^{1/(N_c-1-(N_f-1))}$$

$$\Longrightarrow \quad a_{N_c,N_f} = a_{N_c-1,N_f-1} = a_{N_c-N_f,0}.$$

Another limit: give Q_{N_f} a mass instead of a vev.

by adding $W_{tree} = m\, Q_{N_f} \tilde{Q}_{N_f}$

$$(N_c, N_f) \xrightarrow[m]{} (N_c, N_f-1)$$

(M)

$$W = a_{N_c,N_f}\left(\frac{\Lambda_{N_c,N_f}^{3N_c-N_f}}{\det M}\right)^{1/(N_c-N_f)} + m\,M_{N_fN_f}$$

Integrate out $M_{N_fN_f}$: $M_{N_fN_f} \to \langle M_{N_fN_f}\rangle$

$\langle M_{N_fN_f}\rangle$ solves $\dfrac{\partial W}{\partial M_{N_fN_f}} = 0$

$$W_{low} = W\Big|_{\langle M_{N_fN_f}\rangle}$$

$$= \left(\frac{a_{N_c,N_f}}{N_c-N_f}\right)^{\frac{N_c-N_f}{N_c-N_f+1}} (N_c-N_f+1)\,\cdot$$

$$\cdot\left(\frac{m\,\Lambda_{N_c,N_f}^{3N_c-N_f}}{\det \hat{M}}\right)^{1/(N_c-N_f+1)}$$

light fields

$$E \uparrow \quad \frac{(N_c, N_f)}{(N_c, N_f - 1)} \quad m \quad \frac{\left(\frac{\Lambda_{N_c,N_f}}{\mu}\right)^{3N_c - N_f}}{\left(\frac{\Lambda_{N_c,N_f-1}}{\mu}\right)^{3N_c-(N_f-1)}}$$

$$\Longrightarrow \quad m\,\Lambda_{N_c,N_f}^{3N_c-N_f} = \Lambda_{N_c,N_f-1}^{3N_c-(N_f-1)}$$

$$\text{so} \quad a_{N_c,\,N_f-1} = \left(\frac{a_{N_c,N_f}}{N_c-N_f}\right)^{\frac{N_c-N_f}{N_c-N_f+1}} (N_c - N_f +$$

$$\Rightarrow \left(\frac{a_r}{r}\right)^r = a_1 = a_{N_c,\,N_f=N_c-1}$$

$$\Rightarrow W = (N_c - N_f)\left(\frac{a_{2,1}\,\Lambda_{N_c,N_f}^{3N_c-N_f}}{\det M}\right)^{1/(N_c-}$$

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We only need to find $a_{\tau,1}$!

Affleck, Dine, & Seiberg (84)

showed by an instanton calculation that $a_{\tau,1} \neq 0$.

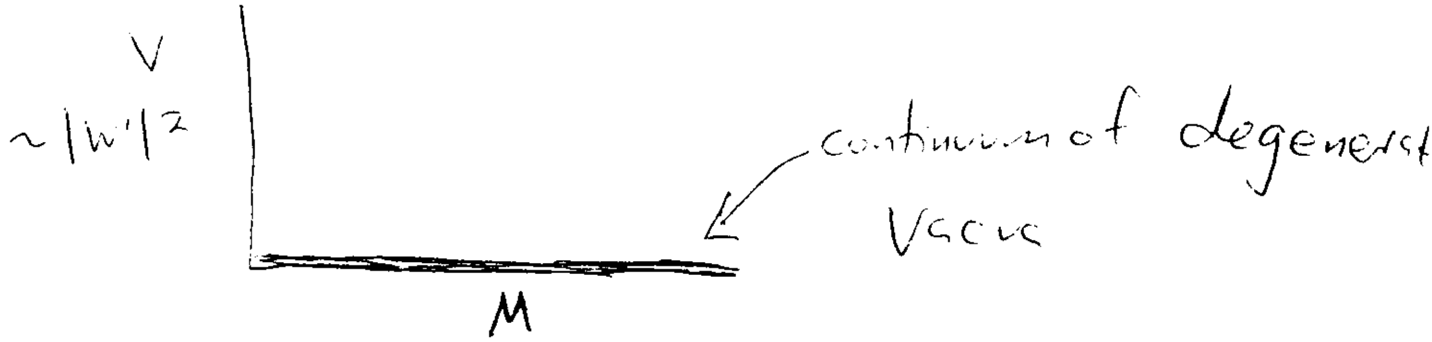
Normalize Λ s.t. $a_{\tau,1} = 1$

($\overline{DR}$ scheme).

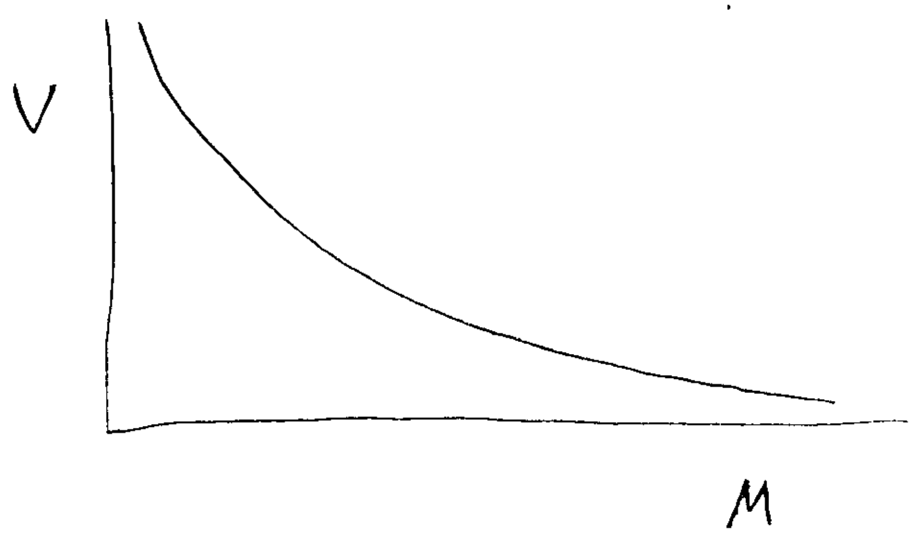
$$W_{eff} = (N_c - N_f) \left(\frac{\Lambda^{3N_c - N_f}}{\det M} \right)^{1/(N_c - N_f)}$$

exact effective superpotential

Classically



Quantum mechanically



No vacuum - they runs to

$\langle M \rangle = \infty$ = weak coupling.

Consider giving all fields a mass by adding $W_{tree} = \mathrm{Tr}\, m\, M$

mass matrix.

$$\frac{\partial}{\partial M_{f\tilde{g}}} \left(W_{eff} + \mathrm{Tr}\, m M \right) = 0$$

for all f & $\tilde{g}$ for a susy vacuum

Find: N_c sol'ns:

$$\langle M_{f\tilde{g}} \rangle = \left(\Lambda^{3N_c - N_f} \det m \right)^{1/N_c} \left(\frac{1}{m} \right)^{f\tilde{g}}$$

$$W_L(m) = \left(W_{eff} + \mathrm{Tr}\, m M \right)_{\langle M \rangle}$$

$$W_L(m) = N_c \left(\Lambda_{N_c, N_f}^{3N_c - N_f} \det m \right)^{1/N_c}$$

$$W_{eff}(M) \longleftrightarrow W_L(m)$$

Legendre' transform.

$$\frac{\partial W_L}{\partial m} = \langle M \rangle .$$

Take m large

SUSY QCD

———— m ————

SUSY YM (No Matter)

E

N_c vacua $\langle M \rangle \longrightarrow N_c$ vacua

for SUSY $SU(N_c)$ YM

$\Rightarrow \operatorname{Tr}(-1)^F = N_c$ ✓

(Witten).

Note that $\Lambda_{N_c,N_f}^{3N_c-N_f} \det m$

$= \Lambda_{N_c,0}^{3N_c}$

so $W_L = W_{SU(N_c)\ YM}$

$W_{N_c,0} = N_c\ \varepsilon_{N_c}\ \Lambda_{N_c,0}^{3}$

$\uparrow$ N_c^{th} root of unity.

labeling the vacuum choice.

This is associated with gaugino

condensation: $\langle \lambda\lambda \rangle_{SU(N_c)\ YM} = \langle W_\alpha^2 \rangle$

$= \varepsilon_{N_c}\ \Lambda_{N_c,0}^{3}$

To see that, since $\mathcal{L} = \cdot + \log \Lambda_{N_c,0}^{3N_c} \cdot$

$\cdot \int W_\alpha^2 d^2\theta$,

$$\langle W_\alpha^2 \rangle = \frac{\partial W_{N_c,0}}{\partial \log \Lambda^{3N_c}} = \varepsilon_{N_c}\ \Lambda_{N_c,0}^{3}$$

Note also that for $N_f < N_c - 1$

$$W_{N_c, N_f} = (N_c - N_f) \left(\frac{\Lambda_{N_c, N_f}^{3N_c - N_f}}{\det M} \right)^{1/(N_c - N_f)}$$

$$= (N_c - N_f)\, \varepsilon_{(N_c - N_f)}\, \Lambda^3_{N_c - N_f, 0}$$

(for M large)

$$= W_{N_c - N_f, 0}$$

i.e. for $N_f < N_c - 1$

W_{N_c, N_f} is associated with gaugino condensation in the unbroken $SU(N_c - N_f)$. For $N_f = N_c - 1$, $W_{N_c, N_f = N_c - 1}$ as we noted before, is due to an instanton in the broken $SU(N_c)$.

The Legendre transform can be used to obtain new & interesting results - "integrating in".

Simple ex $SU(2)_1 \times SU(2)_2$ with matter G in the $(2,2)$. Light field $= X \equiv GG$

$$\epsilon^{\lambda} \quad W = W_{eff}(X) + mX \qquad \left(\frac{\Lambda_1}{\mu}\right)^5 \quad \left(\frac{\Lambda_2}{\mu}\right)^5$$

$$\xrightarrow{m}$$

$$W_L = (W_{eff} + mX)_{\langle X \rangle} \qquad \left(\frac{\Lambda_{1,L}}{\mu}\right)^6 \quad \left(\frac{\Lambda_{2,L}}{\mu}\right)^6$$

$$= 2\epsilon_1 (\Lambda_{1,L}^6)^{1/2} + 2\epsilon_2 (\Lambda_{2,L}^6)^{1/2}$$

$$= 2\epsilon_1 (m\Lambda_1^5)^{1/2} + 2\epsilon_2 (m\Lambda_2^5)^{1/2}$$

$\epsilon_1, \epsilon_2 = \pm 1 \longleftrightarrow$ 4 vac of $SU(2)_1 \times SU(2)_2$

Inverse Legendre transf $\Rightarrow$

$$W_{eff} = \frac{\Lambda_1^5}{X} + \frac{\Lambda_2^5}{X} + 2\varepsilon_1\varepsilon_2 \frac{\Lambda_1^{5/2}\Lambda_2^{5/2}}{X}$$

$$W_{eff} = W_{eff}(\varepsilon = \pm 1, X)$$

discrete quantum parameter

$\frac{\Lambda_1^5}{X} \leftarrow$ instanton in broken $SU(2)_1$

$\frac{\Lambda_2^5}{X} \leftarrow$ inst in broken $SU(2)_2$

$\pm 2\frac{\Lambda_1^{5/2}\Lambda_2^{5/2}}{X} \longleftarrow$ gaugino condensation in unbroken $SU(2)_D$

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Another Ex: $SU(2)_1 \times SU(2)_2$ with Q in the $(2,2)$ and $L \in (1,2)$; $\tilde{L} \in (1,2)$

Light matter: $X = Q^2 \qquad Y = L\tilde{L}$

E Λ

$$W = W_{eff}(X,Y) + mY \qquad e^{-8\pi^2/g_1^2} = \left(\frac{\Lambda_1}{\mu}\right)^5$$

m

$$e^{-8\pi^2/g_2^2} = (\Lambda_2/\mu)^4$$

$$W_L = \Big(W_{eff}(X,Y) + mY\Big)_{\langle Y \rangle}$$

$$= \frac{\Lambda_1^5}{X} + \frac{m\Lambda_2^4}{X} + \frac{2\varepsilon\, m^{1/2}\,\Lambda_1^{5/2}\Lambda_2^2}{X}$$

inverting $\Rightarrow$ $W_{eff} = \dfrac{\Lambda_1^5 Y}{XY - \Lambda_2^4}$

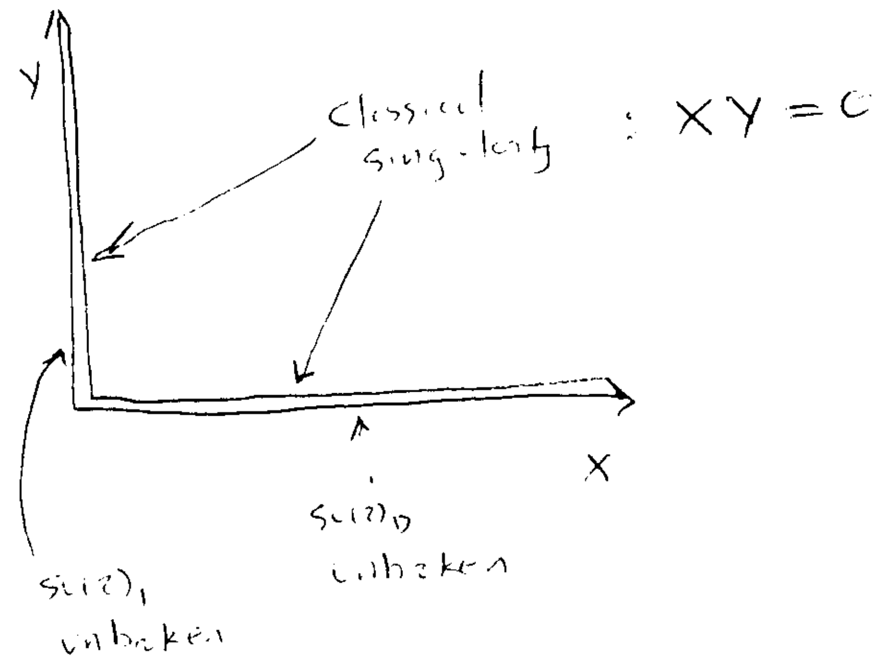
$$= \sum_{(1,n)} \frac{\Lambda_1^5}{X}\left(\frac{\Lambda_2^4}{XY}\right)^n$$

∞ instanton sum

exactly computes & sums up all ∞ # of instanton contributions!

$$W_{eff} = \frac{\Lambda_1^5 Y}{XY - \Lambda_2^4}$$

classical



quantum