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SMR.858 - 19

Lecture III

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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EXACT RESULTS IN SUSY GAUGE THEORIES

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Please note: These are preliminary notes intended for internal distribution only.

Lecture 3:

Last time SQCD with $N_f < N_c$

$$W_{eff} = (N_c - N_f)\left(\frac{\Lambda_{N_c,N_f}^{3N_c - N_f}}{\det M}\right)^{1/(N_c - N_f)}$$

Now consider $N_f = N_c$:

Recall: light fields = N_f^2 mesons $M_{f\tilde{g}}$, baryon $B = \det Q$, $\tilde{B} = \det \tilde{Q}$ with

$$\det M = B\tilde{B} \quad \underline{\text{classically}}.$$

Classical moduli space of vacua

$$= \{ M_{f,\tilde{g}},\ B,\ \tilde{B} \mid \det M = B\tilde{B} \}$$

Note: 1) This space has a singular submanifold : $B = \tilde{B} = 0$, M_{fg} with $\text{rank}(M) \leq N_c - 2$. since

$$d\left(\det M - B\tilde{B}\right) = 0$$ on this submanifold $\Longrightarrow$

singular submanifold.

2) In addition, the classical metric on this space, obtained from the classical Kahler potential, has orbifold singularities on submanifolds :

$\mathbb{Z}_2$ orbifold sing. in M for $\det M = 0$

$\mathbb{Z}_{N_c}$ orbifold sing in B at $B = 0$

$\mathbb{Z}_{N_c}$ orbifold sing. in $\tilde{B}$ @ $\tilde{B} = 0$.

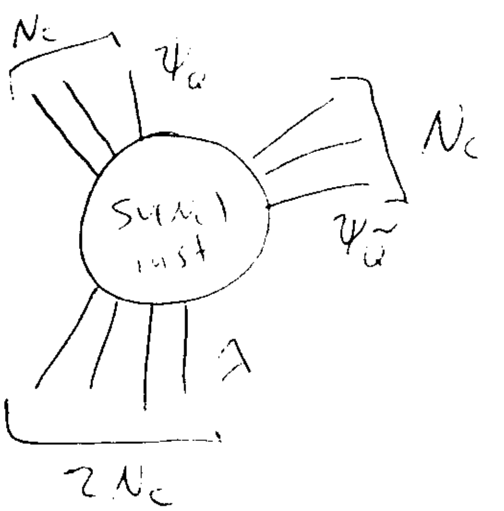
As with $N_f < N_c$ we might expect the classical accidental degeneracy of this classical moduli space of vacua to be lifted by quantum effects with some W_{eff}...

<u>But it isn't!</u>

Symmetries $SU(N_f)^2 \Rightarrow$

$$W_{eff} = W_{eff}(\det M, B, \tilde{B}).$$

$U(1)_Q$, $U(1)_{\tilde{Q}}$, $U(1)_R$ - consider instanton



$$\sim e^{-S_{inst}} = \left(\frac{\Lambda}{\mu}\right)^{2N_c}$$

	$U(1)_Q$	$U(1)_{\tilde{Q}}$	$U(1)_R$
$\det M$	N_f	N_f	0
B	N_f	0	0
$\tilde{B}$	0	N_f	0
Λ^{2N_c}	N_f	N_f	0
W	0	0	2

gluinos − quarks

It is impossible to make a W of R-charge 2 out of charge zero ingredients!

$$\therefore \; W_{\underline{exact}} = 0 .$$

The degeneracy is exact!

There is a quantum moduli space of degenerate but physically inequiv. Vacua.

<u>Question</u>: "What happens to the singular submanifolds of the classical moduli space?"

In the quantum theory either the singularity is <u>smoothed</u> out by quantum effects or it reflects the presence of some <u>additional</u> <u>massless</u> degrees of freedom, eg: monopoles, gluons, exotic composites, ...

(5) <u>Note</u> singularites are at strong coupling. Quantum effects importan

Consider giving all N_c flavors masses by adding $W_{tree} = \mathrm{Tr}\, m M$.

Using the results from last time,

$$\langle M \rangle = \frac{\partial W_L(m)}{\partial m} \leftarrow \text{1PI superpotential}$$

$$W_L = N_c \left(\det m \, \Lambda^{3N_c - N_f} \right)^{1/N_c}$$

$$\text{so } \langle M_{f\tilde{g}} \rangle = \left(\Lambda^{3N_c - N_f} \det m \right)^{1/N_c} \left(\frac{1}{m} \right)_{f\tilde{g}}$$

$$\therefore \langle \det M \rangle = \left(\Lambda^{3N_c - N_f} \det m \right)^{N_f/N_c} \frac{1}{\det m}$$

For $N_f = N_c \Rightarrow$

$$\langle \det M \rangle = \Lambda^{2N_c} \quad \text{indep. of } m!$$

☺

Suggests $\langle \det M \rangle = \Lambda^{2N_c}$ even

for $m = 0$. We need to be careful -

we could have $\langle B \rangle$ & $\langle \tilde{B} \rangle$ non zero

by adding sources for them,

$W_{tree} = \mathrm{Tr}\, mM + bB + \tilde{b}\tilde{B}$

$$\Rightarrow \quad \boxed{\det M - B\tilde{B} = \Lambda^{2N_c}}$$

(Seiberg '94).

<u>Note</u>

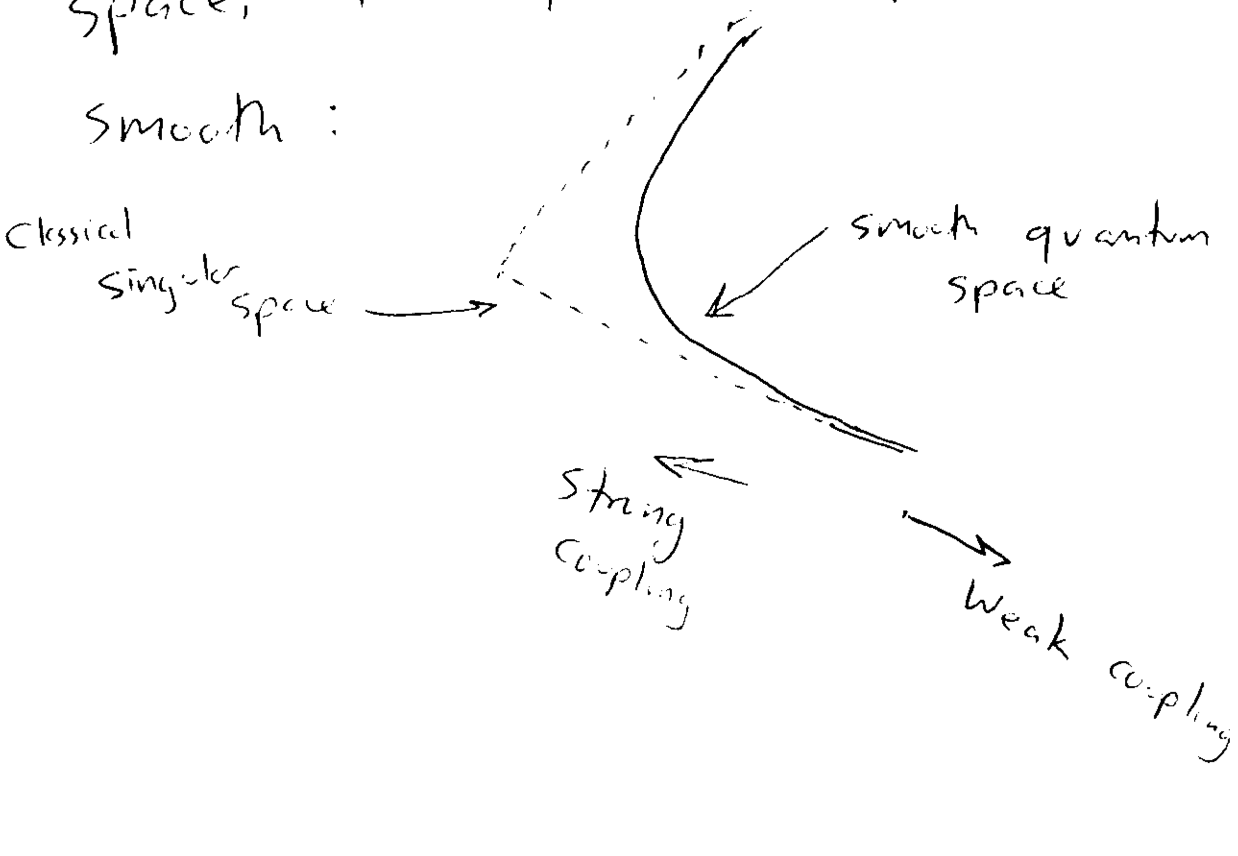
1) This is, again, <u>exact</u>. The quantum correction is due to an instanton.

2) The quantum moduli space

$$\{ M_{fg}, B, \tilde{B} \mid \det M - B\tilde{B} = \Lambda^{2N_c} \}$$

is a quantum deformation of the classical space

For large $M, B, \tilde{B}$ the quantum space doesn't differ much from the classical one. This is physically reasonable since large $M, B, \tilde{B}$ $\rightarrow$ weak coupling (asymp. freedom). However, at strong coupling – i.e. small $M, B, \tilde{B}$, – the difference is dramatic. Unlike the classical space, the quantum space is smooth:



We can describe $N_F = N_c$ by introducing a Lagrange multiplier to implement the constraint:

$$W_{N_c, N_c} = A \left(\det M - B \tilde{B} - \Lambda^{2N_c} \right)$$

$$\Rightarrow \quad W = 0 \quad \text{with} \quad \det M - B\tilde{B} = \Lambda^{2N_c}.$$

<u>Exercise</u>: Add $W_{tree} = m M_{N_c N_c}$ to give a mass to the N_c th flavor.

Using $\Lambda^{2N_c}_{N_c, N_c} m = \Lambda^{2N_c+1}_{N_c, N_c-1}$,

show that

$$\left(W_{N_c, N_c} + m M_{N_c, N_c} \right)_{\langle M_{N_c, N_c} \rangle}$$

$$= W_{N_c, N_c-1} = \frac{\Lambda^{2N_c+1}_{N_c, N_c-1}}{\det \hat{M}}$$

$N_c - 1$ light fields

$N_f = N_c + 1$:

Gauge invs $\quad M_{f\tilde{g}}, \quad B^f, \quad \tilde{B}^{\tilde{g}}$

where $B^f = \epsilon^{f f_1 \cdots f_{N_c}} \epsilon^{c_1 \cdots c_{N_c}} Q_{f_1 c_1} \cdots Q_{f_{N_c} c_{N_c}}$

$\tilde{B}^{\tilde{g}} = \epsilon \epsilon \tilde{Q} \cdots \tilde{Q}.$

Classical moduli space of vacua

$$\{ M_{f\tilde{g}}, B^f, \tilde{B}^{\tilde{g}} \mid \text{classical relations} \}$$

classical relations : $\quad M_{f\tilde{g}} B^f = 0$

$M_{f\tilde{g}} \tilde{B}^{\tilde{g}} = 0 , \quad$ " $\underset{\uparrow \text{ cofactor matrix.}}{\det M \, (M^{-1})^{f\tilde{g}}}$ " $= B^f \tilde{B}^{\tilde{g}}$

E.g. $\quad Q = \begin{pmatrix} a_1 & & \\ & \ddots & \\ & & a_{N_c} \\ & & \end{pmatrix} \quad \tilde{Q} = \begin{pmatrix} \tilde{a}_1 & & \\ & \ddots & \\ & & \tilde{a}_{N_c} \\ & & \end{pmatrix}$

$M = \begin{pmatrix} a_1\tilde{a}_1 & & & \\ & \ddots & & \\ & & a_{N_c}\tilde{a}_{N_c} & \\ & & & 0 \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ \vdots \\ a_1 \cdots a_{N_c} \end{pmatrix} \quad \tilde{B} = \begin{pmatrix} 0 \\ \vdots \\ \tilde{a}_1 \cdots \tilde{a}_{N_c} \end{pmatrix}$

As before, this space is singular near the origin - i.e. at strong coupling in the quantum theory.

"Is it smoothed out?" No.

Consider giving masses to all N_c+1 flavors via $W_{tree} = \mathrm{Tr}\, m M$.

$$\langle M_{f\tilde{g}} \rangle = \left(\Lambda^{3N_c - N_f} \det m \right)^{1/N_c} \left(\frac{1}{m} \right)_{f\tilde{g}}$$

Taking $m_{f\tilde{g}} \to 0$ in different ratios, find $\langle M_{f\tilde{g}} \rangle$ can be anywhere on the classical moduli space.

1) "How do we get the above $\langle M_{fg} \rangle$?"

2) "What fields become massless near the origin?"

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Sol'n (Seiberg '94)

$$W_{\substack{eff \\ N_f = N_c+1}} = \frac{1}{\Lambda^{2N_c-1}} \left(M_{f\tilde{g}} B^f \tilde{B}^{\tilde{g}} - \det M \right)$$

with $M, B, \tilde{B}$ all indep.

① Eqns of motion $\Rightarrow$ classical constraints. Away from the origin - i.e. at weak coupling - the components which are classically constrained are very massive. At the origin these quantum compts of $M, B, \tilde{B}$ are all massless - quantum massless mesons and baryons at strong coupling!

② Adding mass terms $W_{eff} + \mathrm{Tr}\, m M$ $\Rightarrow$ correct $\langle M \rangle$.

Ex: Add a mass for just the N_c+1 th flavor. Integrate out M_{N_c+1, N_c+1} & recover the quantum moduli space constraint for the low energy $N_f = N_c$ thy

Another check: 't Hooft anomaly matching:

There is a large, unbroken, anomaly free, global symmetry at the origin.

'tHooft Anomalies computed with the classical massless spectrum must match those computed with the quantum massless spectrum. When quantum massless spectrum $\neq$ classical massless spectrum, this is a highly non-trivial check.

't Hooft anomaly matching argument: add a spectator sector which cancels the classical obstructions to gauging the global symmetries. In the quantum theory they are also gauged without obstruction. Subtract the spectator sector on both sides.

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At $M = B = \tilde{B} = 0$ there is an unbroken $SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_R$ global symm.

<u>Classical spectrum</u>. <u>$N_f = N_c + 1$</u>

		$SU(N_f)_L$	$SU(N_f)_R$	$U(1)_B$	$U(1)_R$
$N_c \cdot$	Q	$\square$		1	$1/N_f$
N_c	$\tilde{Q}$		$\square$	-1	$1/N_f$
N_c^2-1				0	1

<u>Quantum spectrum</u>.

M	$\square$	$\square$	0	$2/N_f$
B	$\bar{\square}$		N_f	N_c/N_f
$\tilde{B}$		$\bar{\square}$	$-N_f$	N_c/N

<u>examples</u>:

$SU(N_f)_L^3$: $N_c \overset{?}{=} N_f - 1$ ✓

$$U(1)_R^3: \quad 2N_cN_f\left(\frac{1}{N_f}-1\right)^3 + N_c^2 - 1 \overset{?}{=} \left(\frac{2}{N_f}-1\right)^3 N_f^2 + 2N_f\left(\frac{N_c}{N_f}-1\right)^3 \checkmark$$

etc...