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REVIEW OF 2D SUSY SIGMA MODELS AND RELATION TO 4D GAUGE THEORIES

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Please note: These are preliminary notes intended for internal distribution only.

$\underline{\underline{N=2}}$ Witten. Journal of Math. Phys

$(A_\mu, \lambda_\alpha, \bar\lambda_{\dot\alpha})$ ← real $N=1$

$(\varphi, \psi_\alpha, \ldots)$ chiral in adj

$(\bar\varphi, \bar\psi_{\dot\alpha}, \ldots)$ antichiral.

Twisting on R^4

$H = SU(2)_L \times SU(2)_R \times SU(2)_I$

$SU(2)_I$ rotates $Q_{\alpha i}$, $\bar Q_{\dot\alpha i}$ (acts on i).

$SU(2)_{R'} \xrightarrow{\text{diag}} SU(2)_R \times SU(2)_I$

Right rotations should be accompanied by ~~a~~ isotopic rotation.

$K = SU(2)_L \times SU(2)_{R'}$

$Q_{\alpha i}$: $(1/2, 0, 1/2) \to (1/2, 1/2)$

$\bar Q_{\dot\alpha i}$: $(0, 1/2, 1/2) \to (0, 1) \oplus (0, 0)$

$Q = \epsilon^{\dot\alpha i} \bar Q_{\dot\alpha i}$ ← scalar (!)

BRST charge

Different spin assingment

λ_{ij} ← self dual 2-form

$\bar{\lambda}$ ← scalar

λ_i ← vector.

BRST multiplet
$$Q A_j = \lambda_j$$
$$Q \lambda_j = D_j \phi$$
$$Q \phi = 0$$

$\mathcal{O}^{(0)} = \operatorname{Tr} \phi^2$ ← BRST inv. observables.

Descent Eq. $[Q, \mathcal{O}^{(n)}] = d\, \mathcal{O}^{(n-1)}$

$I(\Sigma) = \int_{\Sigma_n} \mathcal{O}^{(n)}$ ← integrated over n-cycles BRST inv.

$\mathcal{O}^{(0)} = \operatorname{Tr} \phi^2 ; \qquad \mathcal{O}^{(1)} = \operatorname{Tr} \phi \lambda_j \; dx^j$

$\mathcal{O}^{(2)} = \operatorname{Tr} (\lambda_i \lambda_j + \ldots)\, dx^i \wedge dx^j , \; \ldots$

BRST inv. corr. functions

$$\langle \mathcal{O}(x_1) \ldots \mathcal{O}(x_n)\; I(\Sigma_1) \ldots I(\Sigma_m) \rangle$$

Charge conservation $4n + 2m = \dim \mathcal{M}_K = 8K - \frac{3}{2}(\chi + \sigma)$

Need

Ground state structure is the same on any 4-manifold

This may follow from the assumption of

1) mass gap (which is not present for $N=2$)

But one can arrange a mass gap by softly breaking $N=2 \to N=1$

~~unbroken, unconfined gauge symmetries~~

2) $N=1$ theory is confined (no unbroken, unconfined gauge symmetries).

$\Rightarrow$ Dynamically generated mass scale, mass gap.

Mild metric dependence

$$g \to t g$$

$$\langle 1 \rangle \simeq \exp - \mathcal{L}_{eff}$$

$$\mathcal{L}_{eff} = \int \sqrt{g}\, d^4x \left[u + v R + w R^2 + \dots \right]$$

$[u] = n$ $\qquad \int d^4x \sqrt{g}\, u \sim t^{4-n}$

For $n > 4$ $\quad t \to \infty$, u dep. decouple

$\underline{t \to \infty}$ Manifold is almost flat.

Different $\mathcal{O}(x_i)$ does not interact

$$\langle \mathcal{O}(x_1) \dots \mathcal{O}(x_n) \rangle \simeq (\langle \mathcal{O}(x) \rangle)^n \cdot \langle 1 \rangle$$

or $$\langle \exp \lambda \mathcal{O}(x) \rangle = \exp \lambda \langle \mathcal{O} \rangle$$

Similarly $$\langle I(\Sigma_1)\, I(\Sigma_2) \rangle \simeq \zeta \cdot \#(\Sigma_1 \cap \Sigma_2) \cdot \langle 1 \rangle$$

or in general: $$\left\{ \begin{array}{l} \langle I(\Sigma) \rangle = \int_\Sigma d^2x \langle (\lambda_i \lambda_j + \dots) \rangle \\ \langle (\lambda_i \lambda_j + \dots) \rangle = 0 \quad \text{by Lorentz inv.} \end{array} \right.$$

$$\left\langle \exp \sum \alpha_i\, I(\Sigma_i) \right\rangle = \exp \frac{1}{2} \sum \alpha_i \alpha_j \; \#(\Sigma_i \cap \Sigma_j)$$

or we now can collect diff. contributions

$$\left\langle \exp \sum \alpha_i I(\Sigma_i) + \lambda \mathcal{O} \right\rangle =$$

$$= \sum_{\rho \in \text{Vacua}} C_\rho \exp\left(\frac{\eta_\rho}{2} \sum \alpha_i \alpha_j (\Sigma_i \# \Sigma_j) + \lambda \langle \mathcal{O} \rangle_\rho \right)$$

Clarification

On an arbitrary 4-manifold one actually can not justify the assumptions.
But if manifold is Kahler one actually get 2 BRST operators

$$Q = Q_1 + Q_2$$

For our purpose it is enough to keep only one of them while the second (together with $N=2$) can be broken.

In $\mathcal{N}=1$ formalism.

Mass perturbation

$$\Delta\mathcal{L} = -m \int d^4x\, d^2\theta \; \mathrm{Tr}\, \Phi^2 + h.c.$$

or

$$\Delta\mathcal{L} = -\int \omega \wedge d^2\bar{z}\, d^2\theta \; \mathrm{Tr}\, \phi^2 + h.c.$$

on a generic Kahler manifold with $H^{2,0} \neq 0$.

$$\boxed{\omega \subset H^{2,0}}$$

We just introduce <u>mass</u> perturbation which

1) BRST trivial w/r to Q_1
   (breaks Q_2 - inv)

2) BReaks $\mathcal{N}=2 \longrightarrow \mathcal{N}=1$

Corr. functions are ~~[illegible]~~ independent on m or ω.

Example $SU(2)$.

$N=1$ $SU(2)$ SUSY YM.

In the bulk $m \neq 0$

$U(1)_R \to \mathbb{Z}_4$ (in general $\mathbb{Z}_{2h}$)

$h \leftarrow$ rank

and $\mathbb{Z}_4 \to \mathbb{Z}_2$ by the choice of vacuum.

$\exists\, 2!$ vacua

Let normalize onepators $\mathcal{O}$; $I(\Sigma)$

such that $\langle \mathcal{O} \rangle_{\pm} = \pm 2$

$\eta_{\pm} = \pm 1$

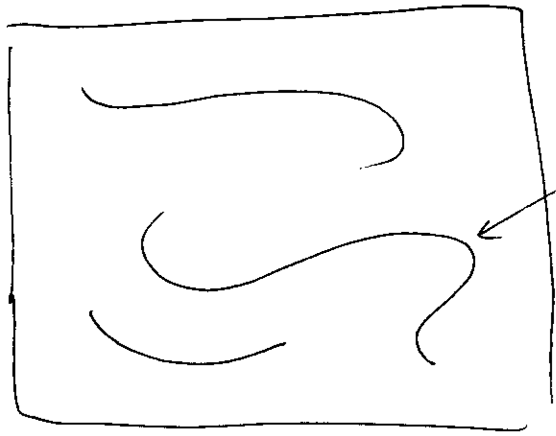
A little bit more involved analisis produce

$$\boxed{C_- = \exp\left(-\frac{3i\pi}{8}(\chi + \sigma)\right) C_+}$$

NB !

Subtlety : $\omega \in H^{2,0}$ might has zeroes.

In this region one actually hase the jump $\mathbb{Z}_2 \to \mathbb{Z}_4$

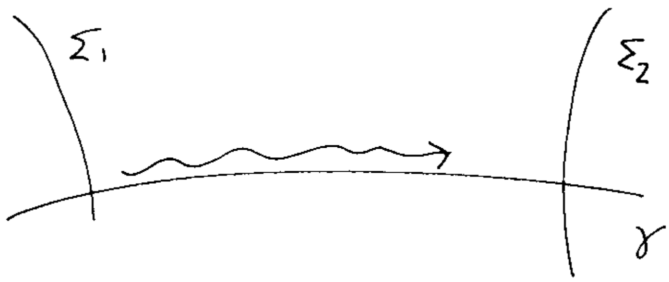


Kanonical divisor $\gamma = \sum_{\psi} \gamma_\psi$

$$\omega|_\gamma = 0$$

In ~~the~~ the neighbourhood of γ one has additional fermionic zero modes ~~localized~~ (additinal degrees of freedom) localized around γ.

Almost like the cosmic string.



Correlations may propagate along the cosmic string.

$$\langle I(\Sigma) \rangle = \sum_{\psi} (\Sigma \# \gamma_\psi)\, v_\psi$$

$v_\psi = \langle V_\psi \rangle$ local operators, inserted on cosmic string.

$\mathbb{Z}_2 \to \mathbb{Z}_4$

There is a vacum splittig

$$|+\rangle \to |+\pm\rangle$$

$$|-\rangle \to |-\pm\rangle$$

$$\langle \ldots \rangle = c_+ \exp\left[\frac{\zeta_+}{2}\sum \alpha_i \alpha_j (\Sigma_i \# \Sigma_j) + \lambda \langle 0 \rangle_+\right] \prod_\gamma \left(e^{\phi_\gamma^{++}} + t_\gamma e^{\phi_\gamma^{+-}}\right)$$

$$+ c_- \exp\left[\frac{\zeta_-}{2}\sum \alpha_i \alpha_j (\Sigma_i \# \Sigma_j) + \lambda \langle 0 \rangle_-\right] \prod_\gamma \left(e^{\phi_\gamma^{-+}} + t_\gamma e^{\phi_\gamma^{--}}\right)$$

$$\phi_\gamma^{\sigma\tilde\sigma} = \sum_i \alpha_i (\Sigma_i \# \gamma_\gamma)\, v^{\sigma\tilde\sigma}$$

$(\sigma\tilde\sigma) \leftarrow$ labels different vacua.

$\langle 0 \rangle_\pm = \pm 2$;

$\zeta_\pm = 1$

$v^{++} = v$

$v^{+-} = -v$

$v^{-+} = -iv$

$v^{--} = iv$

$\alpha: \quad \lambda \to i\lambda \qquad \phi \to \phi$

$\beta: \quad \lambda \to \lambda \qquad \phi \to -\phi$

$\langle I(\Sigma) \rangle = \sum (\Sigma \# \gamma_\gamma) V_\gamma$

$\alpha V = iV$ | $\alpha|+\rangle = |-\rangle$

$\beta V = -V$ | $\alpha|-\rangle = |+\rangle$

$\alpha|++\rangle = |-+\rangle$

$\alpha|-+\rangle = |+-\rangle$

$\alpha|+-\rangle = |--\rangle$

$\alpha|--\rangle = |++\rangle$

Action $\alpha^2\beta$ is unbroke

$\Rightarrow \quad \alpha^2 \equiv \beta$

$\langle V \rangle \simeq \langle \lambda \psi \rangle \quad \Rightarrow$

$V^{++} = v$

$V^{-+} = iv$

$V^{+-} = -v$

$V^{--} = -iv$

Mass scale

$N=2 \quad E = \mu e^{-\frac{1}{2b g^2}}$

$N=1 \quad E = \mu e^{-\frac{1}{2\tilde{b} g^2}}$

where $b = 2\tilde{b}/3$

Let m be the scale of

$N=2 \to N=1$

$N=2 \quad \frac{1}{g^2} = \frac{1}{g_0^2} - 2b \ln \frac{\mu}{E}$

$N=1 \quad E = e^{-1/2b' g^2} m^{1-b/b'} \mu^{b/b'} =$

$= e^{-(\cdot)} m^{1/3} \mu^{2/3}$

$N=1 \quad \frac{1}{g^2} = \frac{1}{g_0^2} - 2b \ln \frac{\mu}{m} - 2b' \ln \frac{m}{E}$

$\langle \lambda\lambda \rangle_{N=1} \simeq m \mu^2 e^{-(\cdot)}$

$\langle \mathrm{Tr}\, \phi^2 \rangle \simeq \mu^2 e^{-(\cdot)}$