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SMR.858 - 22

Lecture IV

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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EXACT RESULTS IN SUSY GAUGE THEORIES

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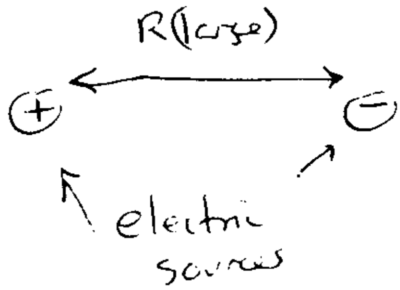
Lecture 4 $\mathcal{N}=1$ Duality.

(Seiberg '94)

Previous lectures SQCD for $N_f < N_c$, $N_f = N_c$, and $N_f = N_c + 1$. Move up to larger N_f.

$N_f > 3N_c$: Not asymptotically free. Coupling const is smaller at large distances. Spectrum = classical spectrum of quarks Q & $\tilde{Q}$ and gluons.

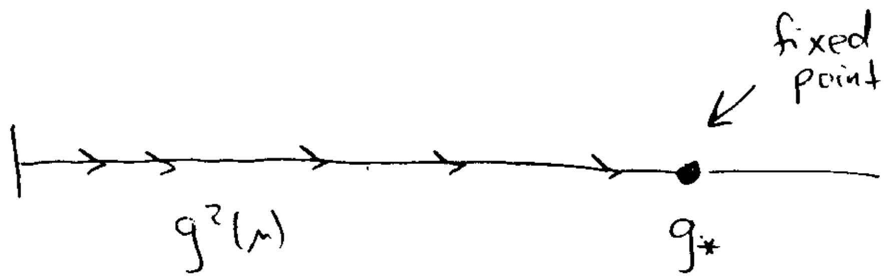


$$V(R) \sim \frac{1}{R \log(R\Lambda)} \quad \left(\begin{array}{c}\text{free}\\ \text{electric}\end{array}\right).$$

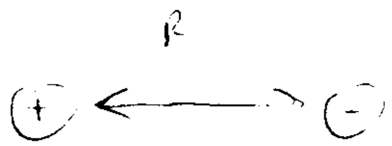
$N_f < 3N_c$: Asymptotically free coupling const.

$g^2(\mu)$ is small at large energies & gets larger at lower energies.

Claim: For $\frac{3}{2} N_c < N_f < 3N_c$ the R.G. flow hits a fixed point in the IR:



Elementary quarks & gluons are not confined but are in a non-Abelian Coulomb phase



$$V(R) \sim \frac{1}{R}$$

For large N_c & $N_f \lesssim N_c$, this can be seen in perturbation theory. Use

$$\beta(g) = -\frac{1}{16\pi^2} g^3 \frac{3N_c - N_f + N_f \gamma}{1 - \frac{N_c g^2}{8\pi^2}}$$

$$\gamma = -\frac{g^2}{8\pi^2} \frac{N_c^2 - 1}{N_c} + O(g^4).$$

E.g. consider $N_f = 3N_c - 1$ & N_c large

$$\beta(g) \simeq -\frac{1}{16\pi^2} g^3 \left(1 - \frac{3 N_c^2 g^2}{8\pi^2} + \cdots\right)$$

$= 0$ for $g_*^2 \sim 1/N_c^2$ - perturbative fixed point.

For smaller N_f, the fixed point is at larger and larger coupling g_*.

The fixed point is a 4 dimensional $N=1$ superconformal field theory.

The gauge invariant operators should be in unitary representations. As in 2d, these have been classified and they satisfy some inequalities. For spinless fields we have

$D \geq \frac{3}{2}|R|$ ← R charge

↑ dimension

$D = \frac{3}{2}R$ chiral fiel

$D = -\frac{3}{2}R$ anti chiral

$\left(\text{like} \quad |G^{\pm}_{-1/2}|\phi\rangle|^2 \geq 0 \right)$

$D(D-1) \geq 0$

$D = 0$ for $\mathbb{1}$

$D = 1$ for free field

$\partial^2 \Phi = 0.$

$\left(\text{like} \quad |L^2_{-1}|\phi\rangle|^2 \geq 0 \right)$

As in 2d $N=2$ phys., the chiral fields with $D = \frac{3}{2}R$ form a ring.

The R symmetry of the IR SCFT is inherited from the anomaly free R symm. of the UV theory:

$$R(Q) = R(\tilde{Q}) = \frac{N_f - N_c}{N_f}.$$

$$R(M = Q\tilde{Q}) = 2\left(\frac{N_f - N_c}{N_f}\right)$$

$$\text{so } D(M) = 3\left(\frac{N_f - N_c}{N_f}\right)$$

(note $D \equiv 2 + \gamma$ so $\gamma = 1 - \frac{3N_c}{N_f}$ as expected for a fixed point from our $\beta(g)$ expression $\beta(g) = f(g)\left(3N_c - N_f + N_f\gamma\right)$.)

$$D(M) \geq 1 \Rightarrow N_f \geq \frac{3}{2}N_c.$$

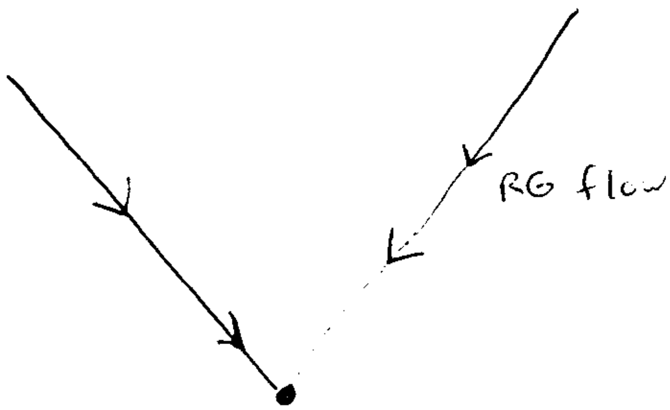
For $N_f = \frac{3}{2}N_c$, M is a free field.

For $N_f \leq \frac{3}{2} N_c$ the theory is in a new phase which can not be described in terms of the unconfined quarks Q & $\tilde{Q}$ & gluons.

<u>Duality</u>: $\frac{3}{2} N_c < N_f < 3N_c$

"electric" $SU(N_c)$ with N_f flavors $Q, \tilde{Q}$

dual "magnetic" theory



IR fixed point = Interacting SCFT

Non-Abelian Coulomb phase

Electric non-Abelian Coulomb phase $\xleftrightarrow{\text{duality}}$ Magnetic non-Abelian Coulomb phase

The dual magnetic theory:

$\mathcal{N}=1$ $SU(N_f - N_c)$ gauge theory with N_f matter flavors q^f, $\tilde{q}^{\tilde{g}}$ and N_f^2 dimension 1 gauge singlets $M^{(m)}_{f\tilde{g}}$ with a superpotential

$$W = M^{(m)}_{f\tilde{g}}\, q^f\, \tilde{q}^{\tilde{g}}.$$

What is $M^{(m)}$

UV electric: $M_{f\tilde{g}} = Q_f \tilde{Q}_{\tilde{g}}$
(dimension 2)

UV magnetic
$M^{(m)}_{f\tilde{g}}$ = elementary field
(dim 1)

$$D(M_{f\tilde{g}}) = 3\left(\frac{N_f - N_c}{N_f}\right)$$

$M^{(m)}_{f\tilde{g}}$ is related to the M of the electric description via a scale μ, $M = \mu M'$

So the superpotential of the magnetic $SU(N_f - N_c)$ theory is

$$W = \frac{1}{\mu} M_{f\tilde{g}}\, q^f \tilde{q}^{\tilde{g}}$$

The magnetic thy. has scale $\tilde{\Lambda}$ related to Λ by

$$\Lambda^{3N_c - N_f}\, \tilde{\Lambda}^{3(N_f - N_c) - N_f} = (-1)^{N_f - N_c} \mu^{N_f}$$

$\Rightarrow$ The magnetic thy. becomes weaker as the electric thy. becomes strong.

The fact that a fixed pt. theory can have descriptions in terms of theories with different classical Lagrangians is known as "quantum equivalence".

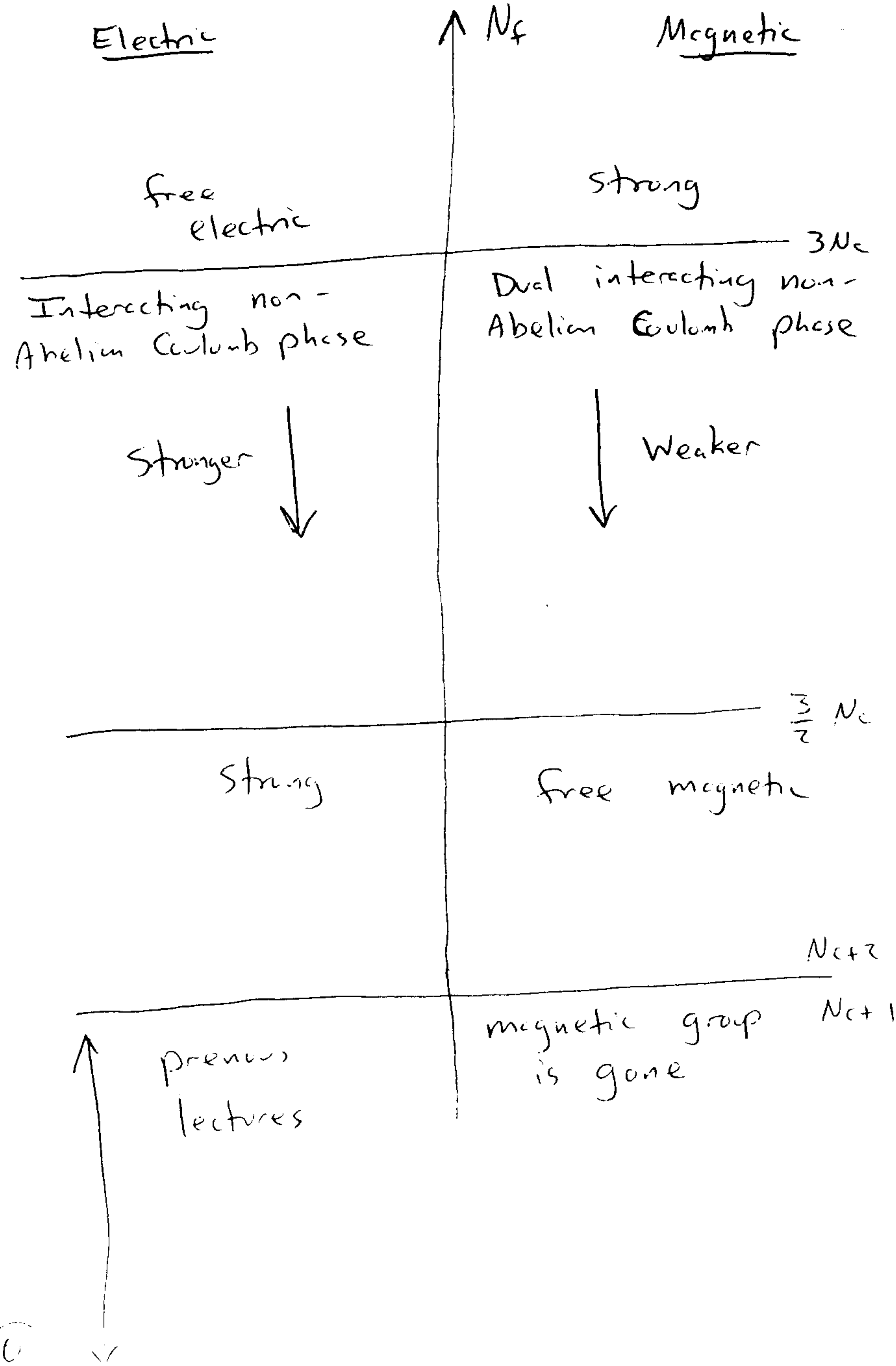
There is no experimental way to know whether the Coulomb potential is due to the electric or the magnetic interacting dynamics

For $N_c + 2 \le N_f \le \frac{3}{2} N_c$ the magnetic $SU(N_f - N_c)$ theory with N_f flavors is not asymptotically free. Thus for this range the theory is in a "free magnetic" phase

$$\oplus \xleftrightarrow{R} \ominus \qquad V(R) \sim \frac{1}{R \log(R\Lambda)}$$

The low energy spectrum = the particles in the dual magnetic Lagrangian: M and the magnetic quarks q & $\tilde{q}$ and the $SU(N_f - N_c)$ magnetic gluons. These are composites in terms of the original electric degrees of freedom

For $N_f \le N_c + 1$ there is no magnetic gauge symmetry left since $SU(N_f - N_c)$ is meaningless for $N_f \le N_c + 1$.


Electric
N_f
Magnetic
free electric
Strong
$3N_c$
Interacting non-Abelian Coulomb phase
Dual interacting non-Abelian Coulomb phase
Stronger
Weaker
$\frac{3}{2} N_c$
Strong
free magnetic
N_c+2
N_c+1
previous lectures
magnetic group is gone

Note

Electric		**Magnetic**
$SU(N_c)$ w/	$\xrightarrow{\text{duality}}$	$SU(\tilde N_c)$ $\quad \tilde N_c = N_f - N_c$
N_f $\quad Q, \tilde Q$	$\xleftarrow[\text{duality}]{}$	N_f $\quad q, \tilde q$ & M
$W = 0$		$W = \frac{1}{\mu} M q \tilde q$

The duality is a $\mathbb{Z}_2$ operation

$$\tilde{\tilde N}_c = N_c, \qquad \tilde\Lambda = \Lambda$$

Dualizing the magnetic theory, in addition to the singlets M_{fg} we have singlets $N^{f\tilde g} \sim q^f \tilde q^{\tilde g}$ & N_f flavors d_f & $\tilde d_{\tilde g}$

$$W = \frac{1}{\mu} M_{f\tilde g} N^{f\tilde g} - \frac{1}{\mu} N^{f\tilde g} d_f \tilde d_{\tilde g}$$

M_{fg} eqn of motion $\Rightarrow$ N^{fg} is massive & can be integrated out.

$N^{f\tilde g}$ equation of motion $\Rightarrow$ $M_{f\tilde g} = d_f \tilde d_{\tilde g}$

i.e. $Q_f \sim d_f$ & $\tilde Q_{\tilde g} \sim \tilde d_{\tilde g}$.

The dual of the dual = original theory (up to very massive fields)

Operator mapping There should be a mapping of gauge invariant primary fields. of the electric theory to those of the dual.

E.g electric mesons $M_{f\tilde{g}} = Q_f \tilde{Q}_{\tilde{g}}$ $\longleftrightarrow$ magnetic singlets $M_{f\tilde{g}}$.

In the electric theory we can also form gauge invariant baryon operators

$$B_{f_1 \cdots f_{N_c}} = Q_{f_1 c_1} \cdots Q_{f_{N_c} c_{N_c}} \epsilon^{c_1 \cdots c_{N_c}}.$$

In the magnetic thy. we can likewise form

$$b^{g_1 \cdots g_{\tilde{N}_c}} = q^{g_1}_{c_1} \cdots q^{g_{\tilde{N}_c}}_{c_{\tilde{N}_c}} \epsilon^{c_1 \cdots c_{\tilde{N}_c}}$$

$(\tilde{N}_c \equiv N_f - N_c)$.

The baryons of the electric theory are mapped to baryons of the magnetic theory as

(SU(N_f) ϵ tensor)

$$B_{f_1 \cdots f_{N_c}} \longleftrightarrow \epsilon_{f_1 \cdots f_{N_c} g_1 \cdots g_{\tilde{N}_c}} b^{g_1 \cdots g_{\tilde{N}_c}}$$

Global symmetries

The electric & magnetic theories have different gauge symmetries. As long as they're interacting, that is fine because gauge symmetries really have to do with redundancy rather than with symmetry. On the other hand, global symmetries are physical and should be the same in the electric & magnetic theories. The global symmetry of the electric theory is

		$SU(N_f)$	$SU(N_f)$	$U(1)_B$	$U(1)_R$
N_c	Q	$\square$		$1/N_c$	$\frac{N_f-N_c}{N_f}$
N_c	$\tilde{Q}$		$\square$	$-1/N_c$	$\frac{N_f-N_c}{N_f}$

Taking the singlets $M_{f\tilde{g}}$ of the magnetic theory to transform as $Q\tilde{Q}$ and the baryons $\epsilon_{f_1 \ldots f_{N_c} g_1 \ldots g_{\tilde{N}_c}} b^{g_1 \ldots g_{\tilde{N}_c}}$ to transform as the electric baryons $B_{f_1 \ldots f_{N_c}}$ in order to preserve our operator mapping, the magnetic theory respects the above global symmetry if the fields transform as:

		$SU(N_f)$	$SU(N_f)$	$U(1)_B$	$U(1)_R$
$\hat{N}_c$:	q	$\overline{\square}$		$1/\tilde{N}_c$	$\frac{N_f - \tilde{N}_c}{N_f}$
$\tilde{N}_c$:	$\hat{q}$		$\overline{\square}$	$-1/\tilde{N}_c$	$\frac{N_f - \tilde{N}_c}{N_f}$
	M	$\square$	$\square$	0	$2\left(\frac{N_f - N_c}{N_f}\right)$

It is a non-trivial check that these symmetries are indeed anomaly free in the $SU(\hat{N}_c) = SU(N_f - N_c)$ gauge theory.

We can also check the 't Hooft anomaly matching conditions.

e.g. $SU(N_f)_L^2\, U(1)_R$

electric: $N_c \left(\frac{N_f - N_c}{N_f} - 1 \right) \leftarrow$ from Q

magnetic: $\tilde{N}_c \left(\frac{N_f - \tilde{N}_c}{N_f} - 1 \right) + N_f \left(2\left(\frac{N_f - N_c}{N_f} \right) - 1 \right)$

from q — from M

$$-N_c^2 = -\tilde{N}_c^2 + N_f - 2N_c \quad \checkmark$$

Another example: $U(1)_R^3$

Electric: $2N_c N_f \left(-\frac{N_c}{N_f}\right)^3 + N_c^2 - 1$

↑ glvinos

↑ quarks

Magnetic: $2\tilde{N}_c N_f \left(-\frac{\tilde{N}_c}{N_f}\right)^3 + N_f^2 \left(1 - \frac{2N_c}{N_f}\right)^3$

↑ dual quarks ↑ singlets

$+ \tilde{N}_c^2 - 1$ ← magnetic gluinos

- they're equal!

There are many non-trivial conditions to check

$SU(N_f)^3$, $SU(N_f)^2 U(1)_B$, $U(1)_R$, $U(1)_B^2 U(1)_R$, ...

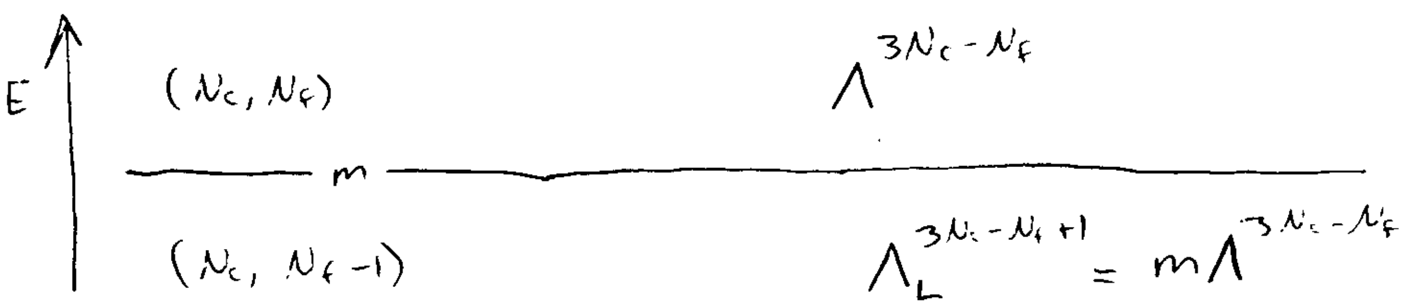
They all work!

This is a very non-trivial check on duality.

Deformations: Under electric $\leftrightarrow$ magnetic, Higgs $\leftrightarrow$ confinement.

Consider starting from the electric (N_c, N_f) theory and giving a mass to the N_f-th flavor by adding

$$W_{tree} = m M_{N_f \tilde{N}_f}.$$



The low energy electric theory is at stronger coupling.

In the magnetic theory, adding W_{tree} gives

$$W = \frac{1}{\mu} M_{f\tilde{g}}\, q^f \tilde{q}^{\tilde{g}} + m M_{N_f \tilde{N}_f}.$$

$M_{N_f N_f}$ eqn of motion $\frac{\partial W}{\partial M_{N_f N_f}} = 0 \Rightarrow \langle q^{N_f} \tilde{q}^{\tilde{N}_f} \rangle = -\mu m$

$\langle q^{N_f} \tilde{q}^{\tilde{N}_f} \rangle = -\mu m$. breaks the magnetic $SU(\tilde{N}_c)$ theory with N_f flavors to $SU(\tilde{N}_c - 1)$ with $N_f - 1$ flavors by the Higgs mechanism.

E

$(\tilde{N}_c, N_f)$ $\qquad\qquad$ $\tilde{\Lambda}^{3\tilde{N}_c - N_f}$

———— $\langle q^{N_f} \tilde{q}^{\tilde{N}_f} \rangle$ ————

$(\tilde{N}_c - 1, N_f - 1)$ $\qquad\qquad$ $\Lambda_L^{3(\tilde{N}_c - 1) - (N_f - 1)} = \dfrac{\Lambda^{3N_c - N_f}}{\langle q^{N_f} \tilde{q}^{\tilde{N}_f} \rangle}$

The low energy magnetic theory is at weaker coupling and is the dual of the low energy electric theory.

Note: $\Lambda_L^{3N_c - (N_f - 1)} \; \tilde{\Lambda}_L^{3(\tilde{N}_c - 1) - (N_f - 1)} = (-1)^{(N_f - 1) - N_c} \mu^{N_f -}$

Duality is preserved under the deformation.

A <u>special case</u>. Consider starting from the electric theory with $N_f = N_c + 2$ & giving a mass to the N_c+2^{th} flavor.

In the magnetic theory we have

E

$(\tilde{N}_c = 2,\ N_f = N_c+2)$

$\langle q^{N_f} \tilde{q}^{N_f} \rangle = -\mu m$

The magnetic gauge group is broken

Get N_c+1 fields $\hat{q}^f$. & N_c+1 fields $\hat{\tilde{q}}^{\tilde{g}}$ with $W = \frac{1}{\mu} \hat{M}_{f\tilde{g}}\, \hat{q}^f \hat{\tilde{q}}^{\tilde{g}}$

coming from the components of the high energy q^f & $\tilde{q}^{\tilde{g}}$ satisfying

$$q^f \langle \tilde{q}^{\overline{N_c+2}} \rangle = \tilde{q}^{\tilde{g}} \langle q^{N_c+2} \rangle = 0.$$

But we also get a contribution to W from an instanton in the broken magnetic $SU(2)$

$$W_{inst} = \frac{\tilde{\Lambda}_{2,1}^{5}}{\langle q^{N_f+2}\tilde{q}^{N_f+2}\rangle} = \frac{\tilde{\Lambda}_{2,N_f+2}^{6-(N_f+2)}\det \hat{M}}{\mu^{N_c+1}\langle q\tilde{q}\rangle}$$

$$= -\frac{\tilde{\Lambda}_{2,N_c+2}^{6-(N_c+2)}\det \hat{M}}{\mu^{N_c+2}\, m}$$

$$= -\frac{\det \hat{M}}{m\Lambda_{N_c,N_c+2}^{3N_c-(N_c+2)}} = -\frac{\det \hat{M}}{\Lambda_{N_c,N_c+1}^{3N_c-(N_c+1)}}$$

so $W_{N_c,N_c+1} = \frac{1}{\mu} M_{f\tilde{g}}\, q^f \tilde{q}^{\tilde{g}} - \frac{\det M}{\Lambda^{3N_c-N_f}}$

$$= \frac{1}{\Lambda^{3N_c-N_f}}\left(M_{f\tilde{g}}\, B^f \tilde{B}^{\tilde{g}} - \det M\right)$$

of previous lecture! The quantum massless baryons there are components of the magnetic quarks!

(13/4)

We can also deform the electric theory along a D-flat direction, e.g. $\langle Q_{N_f} \rangle = \langle \tilde{Q}_{\tilde{N}_f} \rangle =$ large, breaking

$$(N_c, N_f) \xrightarrow[\langle Q \rangle, \langle \tilde{Q} \rangle]{} (N_c - 1, N_f - 1).$$

(N_c, N_f) $\qquad\qquad$ $\Lambda^{3N_c - N_f}$

——— $\langle Q_{N_f} \tilde{Q}_{N_f} \rangle$ ———

$(N_c - 1, N_f - 1)$ $\qquad\qquad$ $\Lambda_L^{3(N_c-1)-(N_f-1)} = \dfrac{\Lambda^{3N_c - N_f}}{\langle Q_{N_f} \tilde{Q}_{N_f} \rangle}$

The low energy electric theory is at weaker coupling.

In the magnetic theory $\langle M_{N_f \tilde{N}_f} \rangle$ gives a large mass to the flavor $q^{N_f}\, \tilde{q}^{N_f}$,

$$W = \frac{1}{\mu} \langle M_{N_f \tilde{N}_f} \rangle\, q^{N_f} \tilde{q}^{\tilde{N}_f} + \cdots$$

$$\Rightarrow \quad q^{N_f} \tilde{q}^{\tilde{N}_f} \text{ gets a mass } \frac{1}{\mu} \langle M_{N_f \tilde{N}_f} \rangle$$

So in the magnetic theory

$E\uparrow$

$(\tilde N_c, N_f)$ $\qquad\qquad$ $\tilde\Lambda^{3\tilde N_c - N_f}$

——— $\frac{1}{\mu}\langle M_{N_f \tilde N_f}\rangle$ ———

$(\tilde N_c, N_f - 1)$ $\qquad\qquad$ $\tilde\Lambda_L^{3\tilde N_c - N_f + 1} = \frac{1}{\mu}\langle M_{N_f \tilde N_f}\rangle \tilde\Lambda^{3\tilde N_c - N_f}$

The low energy magnetic theory is at stronger coupling and is the dual of the low energy electric theory.

$$\Lambda_L^{3(N_c-1)-(N_f-1)} \; \tilde\Lambda_L^{3\tilde N_c - N_f + 1} = (-1)^{N_f - N_c} \mu^{N_f - 1}.$$

$$\begin{tikzcd}
\begin{array}{l}(\text{elec.})\to (N_c, N_f)\\ (\text{mag})\to [\tilde N_c, N_f]\end{array} \arrow[r, "(\text{conf.}/\text{Higgs})"] \arrow[d, "(\text{Higgs}/\text{conf.})"'] \arrow[dr, phantom, "(\text{commutative})"] & \begin{array}{c}(N_c, N_f - 1)\\ [\tilde N_c - 1, N_f - 1]\end{array} \arrow[d] \\
\begin{array}{c}(N_c - 1, N_f - 1)\\ [\tilde N_c, N_f - 1]\end{array} \arrow[r] & \begin{array}{c}(N_c - 1, N_f - 2)\\ [\tilde N_c - 1, N_f - 2]\end{array}
\end{tikzcd}$$

In the electric theory there is a classical constraint $\text{rank}\langle M\rangle \le N_c$.

"How does this arise in the dual?"

By strong coupling effects in the dual theory!

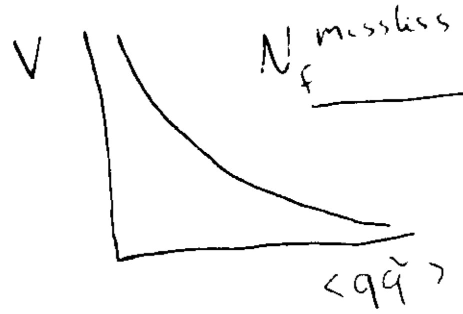
$$W = \frac{1}{\mu} M_{f\bar{g}}\, q^f \bar{q}^{\bar{g}}$$

$$\frac{\partial}{\partial M_{f\bar{g}}} + SU(\tilde{N}_c) \text{ D terms} \Rightarrow \langle q^f\rangle = \langle \tilde{q}^{\tilde{g}}\rangle = 0.$$

The magnetic quarks must sit at the origin. The number of flavors of massless magnetic quarks is

$N_f^{massless} = N_f - \text{rank}\langle M\rangle$. From the previous lectures we know:

$\underline{N_f^{massless} < \tilde{N}_c}$



No vacuum at $\langle q\bar{q}\rangle = 0$ for $N_f^{massless} < \tilde{N}_c$.

$$\Longrightarrow \text{rank}\langle M\rangle \le N_c \checkmark$$