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# MINIMAL SUPERSYMMETRIC STANDARD MODEL AND GRAND UNIFICATION

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# Minimal supersymmetric standard model and grand unification*

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ABSTRACT

We give an introduction to the minimal supersymmetric extension of the $SU(3)\times SU(2)\times U(1)$ standard model of strong and electroweak interactions. In contrast to the standard model itself, this model might have a simple and realistic grand unified extension. Theoretical and phenomenological properties of supersymmetric grand unified theories are discussed in detail.

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## 1. THE STANDARD MODEL

In these lectures we shall discuss in detail the supersymmetric extension of the standard model. I assume that through the previous lectures you are familiar with the structure and phenomenology of the standard model[1]. Nonetheless, mainly to fix notations, let us review the basics of this model.

The standard model is based on the gauge interactions of the strong and electroweak forces with gauge group $SU(3)\times SU(2)\times U(1)$. It thus contains 12 spin 1 gauge bosons: 8 gluons of $SU(3)$, 3 $SU(2)$ weak gauge bosons and the hypercharge gauge boson of $U(1)$. The photon will be a particular combination of the neutral $SU(2)$ gauge boson and the hypercharge boson. The fermions of the theory consist of three generations of quarks and leptons, where we assume the existence of the top quark for which direct experimental evidence ist still lacking. The spin-1/2 fermions of a family have the following transformation properties with respect to $SU(3)\times SU(2)\times U(1)$:

$$\begin{aligned} U^a &= \begin{pmatrix} u \\ d \end{pmatrix} = (3,2,1/6) \\ \bar{u} &= (\bar{3},1,-2/3) \\ \bar{d} &= (\bar{3},1,1/3) \\ L^a &= \begin{pmatrix} \nu_e \\ e \end{pmatrix} = (1,2,-1/2) \\ \bar{e} &= (1,1,1) \end{aligned} \tag{1.1}$$

where $a=1,2$ is an $SU(2)$ index and the first two entries in the brackets denote the dimensions of the $SU(3)\times SU(2)$ representations while the last entry denotes $U(1)$ hypercharge. Electric charge is given by $Q=T_3+Y$. Thus the up-quark, for example, has $Q(u)=1/2+1/6=2/3$ whereas for the down quark we obtain $Q(d)=-1/3$.

The so-called Higgs sector contains a scalar $SU(2)$-doublet

$$h=\begin{pmatrix} h^0 \\ h^- \end{pmatrix} = (1,2,-1/2) \tag{1.2}$$

with potential $V=\mu^2(h^\dagger h)+\lambda(h^\dagger h)^2$ and one also introduces Yukawa couplings for the interactions of the scalars with the fermions

$$L_Y = g_d U h \bar{d} + g_e L h \bar{e} + g_u U h^\dagger \bar{u} \tag{1.3}$$

in all combinations that are allowed by $SU(3) \times SU(2) \times U(1)$ gauge symmetry. A spontaneous breakdown of $SU(2) \times U(1)$ occurs for negative $\mu^2$ and the neutral component of $h$ receives a vacuum expectation value (vev)

$$< h >= \frac{1}{\sqrt{2}} \begin{pmatrix} v \\ 0 \end{pmatrix} \tag{1.4}$$

where $v = (-\mu^2/\lambda)^{1/2}$. $SU(2) \times U(1)_Y$ is broken to $U(1)_Q$ and three gauge bosons become massive

$$\begin{aligned} M_{W^\pm} &= \frac{1}{2} g_2 v \\ M_Z &= \frac{1}{2} v \sqrt{g_1^2 + g_2^2} \end{aligned} \tag{1.5}$$

where $g_1$ and $g_2$ are the coupling constants of $SU(2)$ and $U(1)$, respectively. The $U(1)$ gauge coupling constant is given by

$$e = g_2 \sin\theta_W = g_1 \cos\theta_W \tag{1.6}$$

where $\theta_W$ denotes the weak mixing angle. The mass of the physical Higgs-scalar is given by $\sqrt{-2\mu^2}$. Yukawa couplings then allow, in presence of the spontaneous breakdown of $SU(2) \times U(1)$, mass terms for the fermions. The term $g_d h U \bar{d}$, e.g. leads to $g_d v d\bar{d} = m_d d\bar{d}$. The masses and mixings for the three families of quarks and leptons are parametrized by the $3 \times 3$ Kobayashi-Maskawa[2] matrix.

Let us now count the parameters of the model. We have three gauge couplings $g_1, g_2$ and $g_3$ usually parametrized by $\alpha_{e.m.}, \alpha_{strong}$ and $\sin\theta_W$. In the gauge sector we have in addition a $\Theta$ -parameter multiplying a $F^{\mu\nu} F^{\rho\sigma} \epsilon_{\mu\nu\rho\sigma}$ in the action. Its actual value seems to by very close to zero as can be deduced from the absence of the electric dipole moment of the neutron. Nonetheless we have to treat $\Theta$ as an arbitrary parameter and it still has to be understood why its value is so small.

In the Higgs sector we have introduced two parameters $\mu^2$ and $\lambda$ of which one combination defines the scale of $SU(2) \times U(1)$ breakdown while the other determines the Higgs mass. The 9 fermion masses (not including the possibility for neutrino-Majorana masses) are parametrized by the Yukawa couplings. The same applies to quark mixing consisting of 3 angles and one phase in the Kobayashi-Maskawa matrix, the latter giving rise to CP-violation. We do not know yet whether there is a corresponding mixing in the lepton sector. In any case we can conclude that the above mentioned quantities are completely free parameters in



the standard model. Any attempt to understand their specific values will require a generalization of the model. Apart from these questions we have eventually also to address the more fundamental puzzles out of which I shall mention some in the following. Why is the gauge group $SU(3) \times SU(2) \times U(1)$, why is $SU(2)$ broken and why at a scale of 100 GeV and not at the Planck mass? Why is the mass of the proton 1 GeV and is this scale related to other physical scales? Why do we have this repetition of families, why 3 families and why does a family not contain exotic representations of $SU(3) \times SU(2) \times U(1)$ (like e.g. a 3 of $SU(2)$)? Why are neutrinos massless (are they?) and why is the electron mass so small compared to the $W$-mass? These and many more related questions are the subject of discussions of the physics beyond the standard model.

One important property of the standard model is the chirality of the fermion spectrum. Fermion masses are protected by $SU(2) \times U(1)$, i.e. they can be nonzero only after $SU(2) \times U(1)$ breakdown. Thus all fermion masses are proportional to the vev of the Higgs-field (1.4) and this explains why fermion masses cannot be very large compared to $M_W$. It does, of course, not explain why the mass of the electron is so small compared to $M_W$ and also the smallness of neutrino masses remains a mystery. Only the top quark seems to be as heavy as allowed by $SU(2) \times U(1)$. We will regard this chirality of fermions as a very important property of the standard model and will therefore in the course of these lectures only discuss extensions that share these remarkable properties.

Another important symmetry of the standard model is baryon (B)- and lepton (L)- number conservation. From the requirement of gauge invariance and renormalizability (i.e. absence of nonrenormalizable terms in the action) the model has automatic B and L conservation. Among other things this implies the stability of the proton. Possible violations could come from higher dimensional (nonrenormalizable) terms as e.g. the one displayed in Fig. 1.1. This operator has dimension 6 and therefore the coefficient $1/M_x^2$ has the dimension of inverse (mass)$^2$. $M_x$ denotes the scale of the new physics that is responsible for proton decay. From the long lifetime of the proton we conclude that $M_x$ must be larger than $10^{15}$ GeV, a very large scale. For other processes, like lepton number violation, the corresponding scale could still be in the TeV region. It is a central question in all

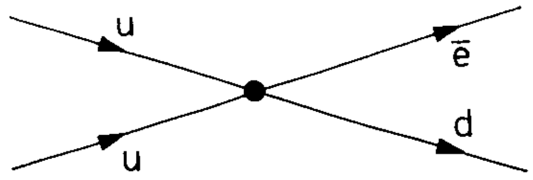


Fig. 1.1: A dimension 6 operator that could lead to proton decay.

discussions of the physics beyond the standard model to isolate these new processes and discuss the corresponding scales.

During these lectures we shall focus on the supersymmetric extension of the standard model. In the next section we give a motivation for the introduction of supersymmetry in particle physics and argue that the intrinsic scale should be in the TeV-region. Later we shall also discuss the grand unified version of the supersymmetric model.

## 2. THE PROBLEM OF THE WEAK SCALE

The standard model contains a dimensionful scale of the order of 100 GeV, represented by the masses of the intermediate gauge bosons. All parameters of dimension mass in the model are related to the vev of the scalar field that is responsible for the breakdown of $SU(2) \times U(1)$. If this would be the only scale in physics we could regard this scale then as *the* input parameter in the model and derive all mass parameters from it. There are reasons to believe, however, that there exist other fundamental scales in physics such as the Planck scale around $10^{19}$ GeV related to the gravitational interactions or a hypothetical grand unified scale of $10^{16}$ GeV in connection with the possible unification of strong and electroweak interactions. Compared to these scales the weak scale is tiny, in fact so tiny that one would think that one should find an explanation for this fact. Such a reason could be a symmetry as we encountered in the discussion of fermion masses, where chiral symmetry protected the masses. Chiral symmetry cannot forbid scalar masses and can therefore not explain the smallness of the weak scale.



Let us discuss this situation in detail. Recall the Higgs potential

$$V(h) = \mu^2|h|^2 + \lambda|h|^4. \tag{2.1}$$

The Higgs mass is $m = \sqrt{-2\mu^2}$ and $M_W = g_2 < h > \approx 80$ GeV. Experimental bounds on $m$ come from LEP $m \geq 60$GeV while an upper bound of 1 TeV can be argued from unitarity constraints. Observe that the mass scale of the standard model $M_W$ is solely set by the parameters $\mu^2$ and $\lambda$ in the Higgs sector.

Theoretically the model is very appealing; it is not just based on an effective Lagrangian, like e.g. the Fermi theory of weak interactions, but it is a renormalizable field theory. This has drastic consequences for the possible range of validity of the model; would it be nonrenormalizable it necessarily would only be defined with a cutoff $\Lambda$ (of dimension of a mass) and its region of validity would be bounded from above by $\Lambda$. Above $\Lambda$ one expects new things to happen which are not described by the model. Since the standard model is renormalizable it could however, be valid in a much larger energy range. Strangely enough this very nic[e] property of the model constitutes one of its problems. The mass scale of 100 GeV is put in by hand and there is no understanding of its origin: it is a completel[y] free input parameter. In a more complete theory one would like to understand th[e] origin of $M_W$ in terms of more fundamental parameters like e.g. the Planck scal[e] $M_P \sim 10^{19}$ GeV, but such a complete theory would need more structure tha[n] present in the standard model.

A reconfirmation of the statement that $M_W$ is a completely free parameter i[s] found in the discussion of perturbation theory. The parameter $\mu^2$ in (2.1) receive[s] a contribution due to the graph of Fig. 2.1 which is quadratically divergent. Ther[e] is nothing wrong with quadratic divergencies as they do not spoil the consistency o[f] the theory; we regularize them and define the theory in terms of the renormalize[d] parameters. The actual correction to $\mu^2$ depends on the regularization schem[e] and the renormalized quantity is an arbitrary parameter even if we would hav[e] understood its value at the tree level. This is true for all quadratically diverger[nt] quantities. These divergences introduce a new mass scale in the theory which ha[s] nothing to do with the scales already present; it is an arbitrary parameter whic[h] we can choose at our will. To understand the origin of these masses the quadrati[c] divergencies have to be absent i.e. they have to be cut off at a larger scale by a new physical structure. With such a *physical* cutoff $\Lambda$ we would have

$$\delta\mu^2 \sim \lambda\Lambda^2 \tag{2.2}$$

and to understand the order of magnitude of $\mu^2$ it would not be appropriate to have $\Lambda$ of the order of the Planck mass $M_P$ but rather in the TeV region. An understanding of the order of magnitude of $M_W$ would therefore require new physics in the TeV-region.

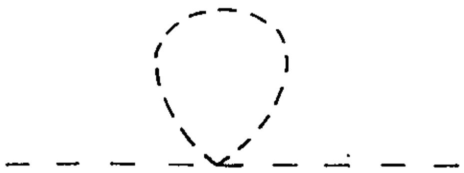

Fig. 2.1: Quadratically divergent contribution from the scalar self interaction.

Having agreed that the standard model might have this subtle theoretical problem one has to look for ways out. The presence of quadratic divergencies is originated by the existence of fundamental scalar particles. One way out is to remove these scalars from the theory. Since we have to break $SU(2) \times U(1)$ spontaneously (and want to maintain Lorentz invariance) some scalar objects have to exist; they could be composite as postulated in the technicolour approach[3]. A new gauge interaction becomes strong in the region of a few hundred GeV; leading to the formation of condensates and many composite bound states. This is the new physics in the TeV-region.

But this is not the only possible solution and we could try to insist to live with fundamental scalar particles. Remember for this purpose the situation with spin 1 particles. Models containing spin 1 particles have usually serious theoretical problems unless there is a gauge symmetry that makes these fundamental spin 1 particles acceptable. Observe that this gauge symmetry also stabilizes the mass of these spin 1 particles; in the symmetric limit they have to vanish. Could we also have such a situation for scalar masses? In the standard model, of course, such a



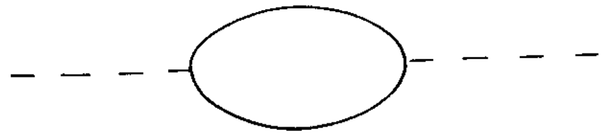

Fig. 2.2: A contribution to $\mu^2$ from supersymmetric partners

situation is not present. We can take the limit $\mu^2 \to 0$ and this does not enhance the symmetry of the action.

The only known way to protect scalar masses is supersymmetry. This symmetry relates bosons and fermions and therefore makes bosons as well behaved as fermions which implies the absence of quadratic divergencies. Supersymmetry provides us with the physical cutoff discussed earlier. In addition to the contribution to $\mu^2$ given in Fig. 2.1 we have now a contribution of Fig. 2.2 with the supersymmetric partner of the Higgs boson in the loop. In the supersymmetric limit these two contributions cancel exactly. If supersymmetry is broken the masses of the boson-fermion multiplet are split. We get a contribution

$$\delta\mu^2 \approx \lambda(m_B^2 - m_F^2) \tag{2.3}$$

and we would require the quantity on the right-hand side to be in the TeV range. If we would remove the partner with mass $m_F$ from the theory we would again recover the quadratic divergence of the standard model. Thus to solve the Higgs problem we have to consider new structure in the TeV-region.

## 3. THE PARTICLE CONTENT OF A SUPERSYMMETRIC STANDARD MODEL

Let us now start to construct a supersymmetric generalization of the standard model. I shall assume that the reader is familiar with the concept of global supersymmetry and I refer to the lectures of D.R.T. Jones [4] or a previous review [5].

Let us recall the particle content of the standard model. Apart from the gauge bosons $G^a_\mu$, $W^i_\mu$, $B_\mu$ in the adjoint representation we have quarks and leptons in three families with quantum numbers

$$\begin{aligned} Q &= \begin{pmatrix} u \\ d \end{pmatrix} = (3, 2, 1/6) \\ \bar{u} &= (\bar{3}, 1, -2/3) \\ \bar{d} &= (\bar{3}, 1, 1/3) \\ L &= \begin{pmatrix} \nu_e \\ e \end{pmatrix} = (1, 2, -1/2) \\ \bar{e} &= (1, 1, 1) \end{aligned} \tag{3.1}$$

together with a Higgs doublet

$$h = \begin{pmatrix} h^0 \\ h^- \end{pmatrix} = (1, 2, -1/2) \tag{3.2}$$

The spectrum of this model is not supersymmetric and we have to add new degrees of freedom. There are no fermions in the adjoint representation of $SU(3) \times SU(2) \times U(1)$ and we thus have to add gauge fermions (gauginos), which together with the gauge bosons form a massless vector superfield $V = (V_\mu, \lambda, D)$. Quarks and leptons require spin 0 partners in chiral superfields e.g. $\bar{E} = (\varphi_{\bar{e}}, \bar{e}, F_{\bar{e}})$ where $\varphi_{\bar{e}}$ is a complex scalar with $\bar{e}$ quantum numbers. Next observe that the lepton doublet has the same quantum numbers as the Higgs: could it be that $\varphi_e = h^-$? Unfortunately it does not work. One reason is the absence of lepton number violation and other reasons will become clear in a moment. We thus have to add scalar partners to all quarks and leptons. To the Higgs scalar we have to join the partner spin 1/2 fermions. With these fermions $SU(2) \times U(1)$ is no longer anomaly free and we have to add a second Higgs chiral superfield $\bar{H} = (1, 2, +1/2)$. In short, every particle in the standard model requires a new supersymmetric partner and one has to add a second Higgs superfield.



To construct the Lagrangian we first write the kinetic terms and the gauge couplings in the usual supersymmetric way. We still have to discuss the superpotential which contains mass terms and the supersymmetric generalization of the Yukawa couplings. If we write the most general superpotential consistent with the symmetries and renormalizability it will contain two sets of terms

$$g = g_w + g_u. \tag{3.3}$$

Let me first discuss the term

$$g_w = \mu H \bar{H} + g_E^{ij} L_i^a H^b \epsilon_{ab} \bar{E}_j + g_D^{ij} Q_i^a H^b \epsilon_{ab} \bar{D}_j + g_U^{ij} Q_i^a \bar{H}_a \bar{U}_j \tag{3.4}$$

where $i, j = 1, \ldots 3$ is a family index and $a, b$ are $SU(2)$ indices (colour indices are suppressed). It is not really clear whether we want $\mu$ from a theoretical point of view but we need it to break certain global symmetries that might be problematic. I will come back to this point later. Observe that we really need two Higgs superfields to give masses to all quarks and leptons. We can here no longer couple the up-type quarks to $h^*$ as we did in the nonsupersymmetric case. It is then also clear that in the breakdown of $SU(2) \times U(1)$ both Higgses have to aquire a vev to provide masses to all quarks and leptons.

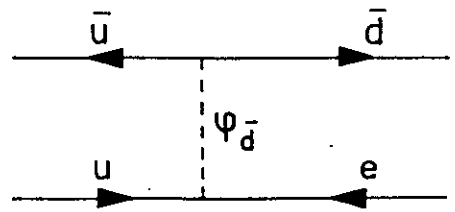


Fig. 3.1: Proton decay through exchange of the scalar partner of $\bar{d}$.

Unlike in the standard model where the requirement of gauge symmetry and renormalizability automatically led to baryon and lepton number conservation we are here not in such a nice situation. This comes from the fact that the Higgs and the lepton doublet superfields have the same $SU(3) \times SU2) \times U(1)$ quantum numbers. Consequently we have additional terms in (3.3) that we can write as (forgetting family indices)

$$g_u = Q^a L^b \epsilon_{ab} \bar{D} + L^a \bar{E} L^b \epsilon_{ab} + \bar{U}\bar{D}\bar{D}. \tag{3.5}$$

These terms violate baryon and lepton number explicitly and lead to proton decay at unacceptable rates through a process as shown in fig. 3.1 (as long as we assume the partner of the d-quark to be lighter than the grand unification scale). The terms in (3.5) have to be forbidden and we want to achieve this with help of a symmetry. We can turn the question the other way around. Suppose we drop (3.5) from the superpotential; does the symmetry increase? In fact it does. The new symmetry is a global symmetry that, however, does not commute with supersymmetry (called $R$-symmetry[6]). Different components in the same supermultiplet have different charges. The concept of $R$-symmetry can best be explained in superspace. Suppose we have a symmetry that transforms $\theta$ to $e^{i\alpha}\theta$; so $\theta$ has charge $R = 1$. Suppose we have a chiral superfield $\phi$ transforming also with $R = 1$. Then it is obvious that the scalar component transforms as

$$\varphi \rightarrow e^{i\alpha}\varphi \tag{3.6}$$

with $R = 1$. But what happens to the fermion? Since $R(\phi) = 1$ we have

$$(\theta\psi) \rightarrow e^{i\alpha}(\theta\psi) \tag{3.7}$$

but the phase comes already from the $\theta$ transformation and obviously $R(\psi) = 0$. The $F$-component of the superfield has $R(F) = -1$. Invariance of the Lagrangian requires $\int d^2\theta g$ to have $R = 0$ whereas $d^2\theta$ transforms with $R = -2$. In the given example only the term $\phi^2$ is allowed in the superpotential. So far our discussion of the implication of R-symmetry on chiral superfields. The vector superfield is real and consequently $R = 0$. From this we conclude

$$\begin{aligned} R(V_\mu) &= 0 \\ R(\lambda) &= 1 \end{aligned} \tag{3.8}$$

and this is a general and important statement. Gauginos transform nontrivially under any $R$-symmetry. The R-symmetry, in particular, forbids Majorana masses for the gauge fermions.



Let us now go back to the superpotential (3.4) and (3.5). There is an $R$-symmetry with e.g. $R(\theta) = 1$ and

$$\begin{aligned} R(H, \bar{H}) &= 1 \\ R(Q, L, \bar{U}, \bar{D}, \bar{E}) &= 1/2 \end{aligned} \tag{3.9}$$

which leaves $g_w$ in (3.4) as the most general superpotential. In other words this means that if we drop the terms in (3.5) a continuous global $R$-symmetry appears. To forbid these terms in principle a smaller symmetry like $R$-parity

$$R_p = (-1)^{3B+L+2S} \tag{3.10}$$

(where $B$, $L$ are baryon, lepton number and $S$ is the spin) would be sufficient, but here a continuous $R$-symmetry appears. This continuous $R$-symmetry is somewhat problematic since it forbids gaugino Majorana masses and at least for the case of the gluino we might have experimental evidence that its mass cannot vanish. Thus the $R$-symmetry has to be broken. Since only a spontaneous breakdown of this symmetry is acceptable, this then would lead to an embarrassing Goldstone boson. Actually in our case it will be an axion since the $R$-symmetry is anomalous[7]. This then tells us that this spontaneous breakdown cannot happen at an energy scale like 100GeV. The breakdown scale of the R-symmetry has to be larger to make the axion invisible[8], i.e. a breakdown scale of something like $10^{10}$ to $10^{11}$GeV. In a simple way this can, however, only be realized if also the supersymmetry breakdown scale $M_S$ is large. Now remember that the splitting of the multiplets is given by $\Delta m^2 \sim g M_S^2$ where $g$ is the coupling to the goldstino. We thus need small couplings to have the supersymmetric partners of quarks and leptons in the TeV-range to provide us with a physical cutoff that stabilizes $M_W$. These couplings have to be really small, compare them e.g. with the gravitational coupling constant $\kappa$. We have

$$\delta m \sim \kappa M_S^2 = M_S^2/M_P \tag{3.11}$$

which is in the TeV-region for $M_S = 10^{11}$GeV. Actually if we assume that all particles couple universally to gravity our requirement of the mass splittings implies $M_S$ to be approximately $10^{11}$GeV. It is thus natural to assume that the small coupling required from our discussion about $R$-symmetry is actually the gravitational coupling constant[17].

We consider this as a hint to include gravity in our framework. This will lead us to the local version of supersymmetry which includes gravity automatically. It will turn out that such considerations avoid some problems connected with the breakdown of global supersymmetry and their desastrous consequences for model building. We shall not discuss this here in detail and refer the reader to ref. [5] for a review.

Local supersymmetry[9] will also resolve the paradox concerning the nonzero cosmological constant in models of spontaneously broken global supersymmetry. We shall see that one can have $E_{\rm vac} = 0$ in models of spontaneously broken local supersymmetry.

## 4. LOCAL SUPERSYMMETRY (SUPERGRAVITY)

In local supersymmetry the transformation parameter is no longer constant but depends on space-time[10]. We have already acquired some experience in the framework of gauge symmetries: the local form of ordinary global symmetries; and for supersymmetry we proceed in the same way. In usual symmetries we had a scalar transformation parameter $\Lambda$. The requirement of local invariance then leads to the introduction of a gauge field $A_\mu$ with transformation property $\delta A_\mu = \partial_\mu \Lambda$. In supersymmetry we have a spinorial parameter $\epsilon_\alpha$. Local supersymmetry then requires the introduction of a gauge particle $\Psi_{\mu\alpha}$ (the gravitino) with transformation property $\delta\Psi_{\mu\alpha} = \partial_\mu \epsilon_\alpha(x)$. Thus the gauge particle of local supersymmetry is a spin 3/2 particle and for reasons that will become clear in a moment it is called the gravitino. These statements can also be made plausible when we discuss the Higgs effect. In ordinary global symmetries a spontaneous breakdown implied the existence of Goldstone bosons. In the local version these bosons then supply the gauge bosons with the missing degrees of freedom to make them massive. In supersymmetry the goldstone particle is a spin 1/2 fermion. This then can provide the two degrees of freedom in the transition of a massless to massive spin 3/2 particle: the super-Higgs effect.

The next point to discuss shows a conceptual difference between ordinary symmetries and supersymmetry. While in ordinary theories it was sufficient for the local symmetry to introduce a spin 1 gauge boson in supersymmetry this is not the case. The gauge particle is a spin 3/2 fermion and supersymmetry requires



a bosonic partner. The construction of local supersymmetry has shown that this partner is a spin 2 boson that has to have all the properties of the graviton. This then implies that local supersymmetry necessarily includes gravity. We could have guessed that already from the algebra

$$[\epsilon(x)Q, \bar{Q}\bar{\epsilon}(x)] = 2\epsilon(x)\sigma_\mu\bar{\epsilon}(x)P^\mu . \tag{4.1}$$

On the right hand side we have a space-time translation that differs from point to point, a general coordinate transformation.

We have now to discuss explicit Lagrangians containing chiral matter and gauge fields coupled to the $(2,\frac{3}{2})$-supergravity multiplet. In general this requires a lot of tedious calculations which I shall not repeat here. Also the general form of the Lagrangian is quite lengthy and I refer to the literature for the complete expression[11]. I will instead concentrate on an analysis of the scalar potential of these theories which we need for our further discussion.

Remember that in the global case the most general Lagrangian was defined by three functions of the superfields: the gauge kinetic terms $W^2$, the matter field kinetic terms $S(\phi^* \exp(gV)\phi)$ and the superpotential $g(\phi)$. In the local case the most general action can be defined by $f_{\alpha\beta}(\phi)W^\alpha W^\beta$ (with indices $\alpha$, $\beta$ labeling the adjoint representation of the gauge group) and the Kähler potential

$$G = 3\log\left(-\frac{S}{3}\right) - \log(|g|^2). \tag{4.2}$$

The kinetic terms of the scalar particles $z_i$ are then given by

$$G^i_j D_\mu z_i D^\mu z^{j*} = \frac{\partial^2 G}{\partial z_i \partial z^{j*}} D_\mu z_i D^\mu z^{j*} \tag{4.3}$$

where $z_i$ is the lowest component of a chiral superfield $\phi_i$. The scalar potential reads

$$V = -\exp(-G)[3 + G_k (G^{-1})^k_l G^l] + \frac{1}{2} f^{-1}_{\alpha\beta} D^\alpha D^\beta . \tag{4.4}$$

In these lectures I will use what is called minimal kinetic terms

$$G^i_j = -\delta^i_j . \tag{4.5}$$

This simplifies all our formulas considerably and allows us nonetheless to see all the essential properties of the potential. The Kähler potential can therefore be written as

$$G = -\frac{z_i z^{i*}}{M^2} - \log \frac{|g|^2}{M^6} \tag{4.6}$$

where we have explicitly written out the mass scale $M$ related to the gravitational coupling constant $\kappa$:

$$M = \frac{1}{\kappa} = \frac{M_{\rm Planck}}{\sqrt{8\pi}} \approx 2.4 \times 10^{18} {\rm GeV}. \tag{4.7}$$

The first derivative of the Kähler potential is then given by

$$G^i = -\frac{z^{i*}}{M^2} - \frac{g^i(z_i)}{g(z)} \tag{4.8}$$

and we can rewrite the potential in terms of the superpotential $g(z)$ as

$$V = \exp\left(\frac{z_i z^{i*}}{M^2}\right)\left[\left|g^i + \frac{z^{i*}}{M^2}g\right|^2 - \frac{3}{M^2}|g|^2\right]. \tag{4.9}$$

Contrary to the case of global supersymmetry the potential is no longer semi-positive definite. I still have to tell you under which conditions supersymmetry is spontaneously broken. As in the global case this breakdown is signaled by a vacuum expectation value of an auxiliary field. There we had the auxiliary field $F$ given as the derivative of the superpotential; here we have an additional term

$$F^i = g^i + \frac{z^{i*}}{M^2}g \tag{4.10}$$

where in the limit $M \to \infty$ we recover the global result. Supergravity is now spontaneously broken if and only if an auxiliary field receives a vev. The supergravity breakdown scale is found to be

$$M_S^2 = < F > \exp\left(\frac{z_i z^{i*}}{M^2}\right). \tag{4.11}$$

Observe that the vacuum energy is no longer an order parameter. We can have unbroken supergravity with $E_{\rm vac} < 0$ (Anti de Sitter) or $E_{\rm vac} = 0$ (Poincare supersymmetry) and $E_{\rm vac} > 0$ always implies broken supergravity. The most important observation is, however, that we can have broken supergravity with



vanishing vacuum energy (cosmological constant), a situation that could not occur in the framework of global supersymmetry. Here we need

$$\sum_i F^i F_i^* = \frac{3}{M^2}|g|^2 \tag{4.12}$$

and we will assume this to be fulfilled. In all cases I know of this is an ad hoc adjustment of the cosmological constant to zero. If (4.12) is fulfilled and if $M_S \neq 0$ the gravitino becomes massive through the super-Higgs effect

$$m_{3/2} = M \exp(-G/2) = \frac{g}{M^2} \exp\left(\frac{z_i z^{i*}}{M^2}\right) \tag{4.13}$$

and we therefore have the relation

$$m_{3/2} = \frac{M_S^2}{\sqrt{3}M} \tag{4.14}$$

valid in the case of vanishing cosmological constant.

Let us now discuss some simple specific models with spontaneous supersymmetry breakdown. As a warm-up example consider one field $z$ and a constant superpotential $g = m^3$. The potential is then given by

$$V = m^6 \exp\left(\frac{zz^*}{M^2}\right)\left[\frac{|z|^2}{M^4} - \frac{3}{M^2}\right] \tag{4.15}$$

which has stationary points at $z = 0$ and $|z| = \sqrt{2}M$. At $z = 0$ supersymmetry is unbroken but this is a local maximum of the potential. The minima with broken supersymmetry and $E_{\rm vac} < 0$ are at $z = \pm\sqrt{2}M$.

Next we want to give an example with broken supersymmetry and $E_{\rm vac} = 0$. We consider a superpotential

$$g(z) = m^2(z + \beta) \tag{4.16}$$

A nonvanishing vev of

$$F = \frac{\partial g}{\partial z} + \frac{z^*}{M^2}g = m^2\left(1 + \frac{z^*(z+\beta)}{M^2}\right) \tag{4.17}$$

would signal a spontaneous breakdown of supergravity. The equation

$$M^2 + zz^* + z^*\beta = 0 \tag{4.18}$$

has the solutions

$$z = -\frac{\beta}{2} \pm \frac{1}{2}\sqrt{\beta^2 - 4M^2}. \qquad (4.19)$$

Since (4.18) only allows real solutions (we assume $\beta$ to be real) (4.19) implies that supersymmetry is broken as long as $\beta < 2M$. Thus we can arrange for a supersymmetry breakdown but we still have the annoying task to fine tune the vacuum energy. Let us therefore first consider the case $\beta = 0$ in which the potential is proportional to

$$(M^2 + |z|^2)^2 - 3M^2|z|^2 \qquad (4.20)$$

which is positive definite with minimum at $z = 0$. Increasing $\beta$ implies decreasing the vacuum energy and also $z$ aquires a nonvanishing vev. We can now increase $\beta$ until the potential just touches zero. This is found to happen at $\beta = (2 - \sqrt{3})M$ with a vev of $(\sqrt{3} - 1)M$ for the $z$-field. The potential is semipositive definite with $E_{vac} = 0$ and, since $|\beta| < 2M$, supersymmetry is broken and we have found the desired example. The super-Higgs effect occurs. The gravitino swallows the fermion in the chiral superfield and has a mass

$$m_{3/2} = \frac{m^2}{M} \exp\left(\frac{(\sqrt{3}-1)^2}{2}\right) \qquad (4.21)$$

and the two remaining scalars have masses

$$\begin{aligned} m_1^2 &= 2\sqrt{3}m_{3/2}^2 \\ m_2^2 &= 2(2 - \sqrt{3})m_{3/2}^2. \end{aligned} \qquad (4.22)$$

Supersymmetry is broken and $E_{vac}$ remains zero. Observe that such a situation is not possible in the framework of global supersymmetry. Observe also, that in the present example we had to perform an explicit fine-tuning to obtain $E_{vac} = 0$.

Before closing this chapter let us discuss two more examples which we shall need later. The first is supersymmetry breakdown through gaugino-condensation. Consider a *pure* supersymmetric gauge theory, just a gauge theory with fermions (the gauginos) in the adjoint representations of the gauge group. Such a theory is asymptotically free, the gauge coupling becomes strong at small energies and we assume, in analogy to QCD, that this leads to confinement and that gaugino bilinears condense. For a detailed discussion see ref.[12]. To see whether this



leads to supersymmetry breakdown we have to consider the auxiliary fields of supergravity including the gaugino bilinears

$$F_i = \exp(-G/2)(G^{-1})_i^j G_j + \frac{1}{4} f_{\alpha\beta k}(G^{-1})_i^k(\lambda^\alpha\lambda^\beta) + \ldots \qquad (4.23)$$

where $\lambda^\alpha$ are the gauginos, $f_{\alpha\beta}$ the socalled gauge-kinetic function that multiplies $W^\alpha W^\beta$ and $f_{\alpha\beta k} = \partial f_{\alpha\beta}/\partial z^k$. A nontrivial vev $< \lambda\lambda > \neq 0$ thus breaks supersymmetry provided that the gauge kinetic function is nontrivial[13]. The supersymmetry breakdown scale is given by

$$M_S^2 \sim \frac{< \lambda\lambda >}{M} \qquad (4.24)$$

leading to a gravitino mass of order $< \lambda\lambda > /M^2$. Observe that the value of $M_S$ in (4.24) vanishes in the global limit $M \to \infty$. Models in which supersymmetry breakdown is induced by gaugino condensation have recently attracted revived attention because of their appearance in the low energy limit of string theories. They are also interesting because of the fact that for a nontrival $f$ the value of the gauge coupling constant $g^2 \sim 1/f$ is a dynamical parameter. In string theories it is related to the vev of the dilaton field[14].

Up to now we have for the sake of simplicity only discussed models with minimal kinetic terms for the scalar fields. Models with nonmininal kinetic terms can have interesting structure. Consider e.g.

$$G = 3\log(\phi + \phi^*) - \log|g|^2 \qquad (4.25)$$

and take a constant superpotential. If you compute the potential as given in (4.4) you will find that it vanishes identically. Nontheless the quantity

$$e^{-G} = \frac{|g|^2}{(\phi + \phi^*)^3} \qquad (4.26)$$

does not vanish and supersymmetry is broken. Such so-called no-scale models[15] might also have applications in the low energy limit of string theories.

## 5. LOW ENERGY SUPERGRAVITY MODELS

As we discussed in chapter 3 we should consider models that consist of two sectors: a hidden sector and an observable sector which are only coupled weakly through gravitational interactions. The observable sector consists of the fields discussed in chapter 3 which we will collectevely denote by $y_a$. The hidden sector is responsible for the breakdown of supersymmetry at a scale $M_S \sim 10^{11}$GeV and leads to a gravitino mass in the TeV region. Its fields will be denoted by $z_i$ and we chose a superpotential

$$\tilde{g}(z_i, y_a) = h(z_i) + g(y_a). \tag{5.1}$$

Let us parametrize a general hidden sector by assuming that at the minimum

$$\begin{aligned} < z_i > &= b_i M \\ < h > &= m M^2 \\ < h_i > &= < \partial h / \partial z_i > = a_i^* m M \end{aligned} \tag{5.2}$$

while all abserable sector fields $y_a$ should have vanishing vev's. In the example of last chapter we had e.g. $b = \sqrt{3} - 1$. The potential is given by

$$V = \exp\left(\frac{|z_i|^2 + |y_a|^2}{M^2}\right)\left[\left|h_i + \frac{z_i^* \tilde{g}}{M^2}\right|^2 + \left|g_a + \frac{y_a^* \tilde{g}}{M^2}\right|^2 - \frac{3}{M^2}|\tilde{g}|^2\right]. \tag{5.3}$$

The vacuum energy vanishes provided that

$$\sum_i |a_i + b_i|^2 = 3 \tag{5.4}$$

and the gravitino mass is given by

$$m_{3/2} = \exp\left(\frac{1}{2}|b_i|^2\right) m, \tag{5.5}$$

thus $m$ sets the scale of the gravitino mass. We furthermore define[16]

$$A = b_i^*(a_i + b_i) \tag{5.6}$$

which will turn out to be an important parameter besides the gravitino mass. In the previous example we had $A = 3 - \sqrt{3}$. The potential given in (5.3) is



complicated but we have $m \ll M$ and we can simplify the expressions enormously by neglecting subleading terms. Formally this means that we take the limit $M \to \infty$ keeping, however, $m_{3/2}$ fixed. We then replace the hidden sector fields by there vev's and obtain the following potential for the observable sector fields

$$V = \left|\frac{\partial g}{\partial y_a}\right|^2 + m_{3/2}^2 |y_a|^2 + m_{3/2}\left[y_a \frac{\partial g}{\partial y_a} + (A - 3)g + \text{h.c.}\right]. \tag{5.7}$$

Thus the spontaneous breakdown of supergravity in the hidden sector manifests itself as explicit breakdown of global supersymmetry in the low energy limit of the observable sector. The first term in (5.7) is the usual potential of a globally supersymmetric theory while the other terms are soft breaking terms.

The second term gives universal scalar masses to all the partners of quarks and leptons. The supertrace formula is here given in general by[11]

$$\text{STr}M^2 = 2(N - 1)m_{3/2}^2 \tag{5.8}$$

where $N$ is the number of chiral superfields. This avoids the mass relations obtained in the globally supersymmetric models and its desastrous consequences for model building. The universality property of the mass terms is needed to ensure the absence of flavour changing neutral currents. It appears here because of the choice of minimal kinetic terms for the scalar fields.

The term $(A-3)g$ is of equal importance since it breaks all $R$-symmetries of the model. This implies that there are no problems with potential axions and that also gaugino Majorana masses are allowed (recall our discussion in chapter 3). This breakdown of $R$-symmetry is a direct consequence of the coupling to gravity.

One more technical remark. In general we will deal with a superpotential $g = g_3 + g_2$ where $g_3$ denotes the trilinear and $g_2$ the bilinear terms. The last term in (5.7) then reads $A m_{3/2} g_3 + (A - 1) m_{3/2} g_2$. Apart grom the gaugino mass $m_0$ we find that $m_{3/2}$ (which sets the scale for the soft scalar masses) and $A$ are the important parameters parametrizing the effects of supersymmetry breakdown in this class of models. In some cases one can also consider a new parameter $B$ as the coefficient of the bilinear terms in the superpotential. In the simplest example $B = A - 1$, but this need not be the case in general.

A remark about the mechanism of SUSY-breakdown is in order here. The example of one scalar field with superpotential (4.16) should, of course, only be considered as a toy example and existence proof for such a mechanism. The true mechanism of SUSY-breakdown will certainly look different, already because of the fact that the scale of $10^{11}$ GeV has to be put in by hand. Nowadays the most discussed mechanism for SUSY-breakdown is based on the mechanism of gaugino condensation[12]. Here the SUSY breakdown scale can be understood dynamically as a consequence of a new strong gauge coupling, in a similar way as we can understand the mass of the proton through the scale of QCD. One should also remark that a model based on SUSY breakdown through gaugino condensation initiated the construction of hidden sector models based on broken supergravity[17]. Later it was found that such a mechanism fits very well in the framework of models derived from heterotic string theory[18]. Therefore this mechanism of SUSY-breakdown is very popular at present.

Let us now discuss the superpotential

$$g = \mu H\bar{H} + g_E HL\bar{E} + g_D HQ\bar{D} + g_U \bar{H}Q\bar{U}. \tag{5.9}$$

The parameter $\mu$ has to be different from zero since otherwise we would have problems with a light higgsino (the supersymmetric partner of the Higgs-scalar) or axions. The value of $\mu$ is not directly related to the supersymmetry breakdown scale but one can construct models [19] where $\mu$ is related to $m_{3/2}$ and we shall assume that also $\mu$ is in the TeV range.

Let us now address the question of $SU(2) \times U(1)$ breakdown. We have two Higgs multiplets and members of both have to receive nonvanishing vev's to give masses to all quarks and leptons, according to (5.9). The relevant part of the Higgs potential reads[20]

$$V = m_1^2|h|^2 + m_2^2|\bar{h}|^2 + m_3^2(h\bar{h} + h^*\bar{h}^*) + \frac{g_1^2+g_2^2}{8}(|h|^2 - |\bar{h}|^2)^2 \tag{5.10}$$

where the last term corresponds to the $SU(2) \times U(1)$ $D$-term and $g_2$ and $g_1$ denote the respective coupling constants. From (5.7) and (5.9) we obtain

$$\begin{aligned} m_1^2 &= m_2^2 = m_{3/2}^2 + \mu^2 \\ m_3^2 &= -B\mu m_{3/2} \\ B &= A - 1 \end{aligned} \tag{5.11}$$



The potential consists of quadratic and quartic terms. The quartic terms have a positive coefficient such that the potential at infinity is well behaved, with the exception, however, of a flat direction for $|h| = |\bar{h}|$. To have the potential bounded from below we therefore have to impose a constraint on the coefficients of the quadratic terms

$$m_1^2 + m_2^2 \geq 2|m_3^2|. \tag{5.12}$$

Next we have to discuss the requirement of $SU(2) \times U(1)$ breakdown. Since there are no trilinear terms in (5.10) a stationary point at $h = \bar{h} = 0$ has to be unstable, i.e. the mass matrix at this point has to have a negative eigenvalue. The requirement for a nontrivial $SU(2) \times U(1)$ breaking absolute minimum is therefore

$$|m_3|^4 \geq m_1^2 m_2^2. \tag{5.13}$$

With the input parameters (5.11) we observe now that the constraints (5.12) and (5.13) can only be fulfilled in the limiting case

$$m_{3/2}^2 + \mu^2 = B\mu m_{3/2} \tag{5.14}$$

i.e. at most we can arrive at a flat direction where $SU(2) \times U(1)$ breaking and nonbreaking minima are degenerate. We would then have to look for radiative corrections to see whether $SU(2) \times U(1)$ breaking minima can be reached at all within this approach. This is actually a nice feature of the model. It tells us again that we have not put in $SU(2) \times U(1)$ breaking by hand. Instead this breakdown will be intrinsically related to the supersymmetry breakdown and the dynamics of the model. But we still have to see whether it works. In addition we have to observe that all our input parameters are defined at a very large scale $M$. The value of the parameters in the 100 GeV region has still to be computed using renormalization group improved perturbation theory in the same way as we have to compute the evolution of the gauge coupling constants in a grand unified model. This we would have also to do if our input parameters would already allow a $SU(2) \times U(1)$ breakdown at the tree level. In the evolution from $M$ to 100 GeV the parameters will change substantially and it would not be clear at all whether $SU(2) \times U(1)$ could not be restored. Before we do this calculation, however, let me give you a simple argument how an $SU(2) \times U(1)$ breakdown can

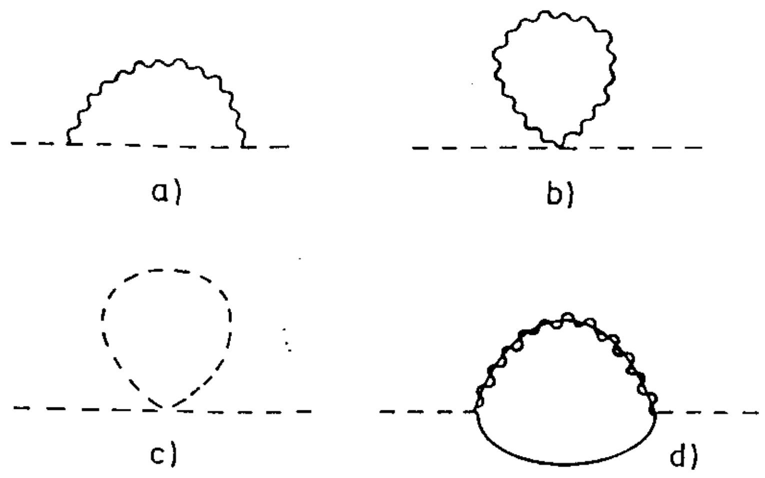


Fig. 5.1: Corrections to scalar masses. Wavy, dashed, solid and wavy-solid lines correspond to gauge bosons, scalars, fermions and gauginos respectively.

be induced by radiative corrections[21]. This argument is not complete and has later to be backed up by the real calculation but it exhibits the essential points of the mechanism quite nicely.

Let us therefore look at the radiative corrections to the masses of the scalar particles. One way to see whether there is a chance to have $SU(2) \times U(1)$ breakdown is to see whether a $m^2$ of a Higgs scalar can become negative and at the same time the $m^2$ of all others scalars in the theory should remain positive. The contributions to the masses of the scalar particles can be classified as in Fig. 5.1. The sum of the two gauge contributions as well as the scalar self interaction contribution are positive. In the supersymmetric limit these contributions are exactly cancelled by the remaining graph in Fig. 5.1 which contains a gauge fermion exchange. So if supersymmetry is exact nothing happens. Let us now suppose that supersymmetry is broken and for definiteness let us take $m_0$ (a gauge fermion mass) as the only source of supersymmetry breakdown. With this mass the contribution of the last graph will be suppressed and the cancellation will no longer be complete. Since a negative contribution is suppressed all scalar particles in the



theory will receive a positive contribution to their $m^2$. For the partners of quarks the dominant contribution comes from the strong interactions

$$\delta m^2 \sim \alpha_3 m_0^2 \tag{5.15}$$

where $\alpha_3$ denotes the $SU(3)$ coupling constant. Higgses and partners of leptons will receive a smaller contribution

$$\delta m^2 \sim \alpha_W m_0^2 \tag{5.16}$$

where $\alpha_W$ denotes a combination of $SU(2) \times U(1)$ coupling constants. No indication for an induced $SU(2) \times U(1)$ breakdown whatsoever. But let us look more closely. With these corrections in particular the partners of quarks become heavy and this will reduce the contribution of the graph with the scalar quadrilinear interaction. This now leads to an asymmetry between the mass corrections for Higgses and for the scalar partners of leptons. Going back to (5.7) and (5.9) we see that the partners of quarks couple to the Higgses but not to the partners of leptons. The suppression of this positive contribution then gives a negative contribution to the Higgs mass and in total we have

$$\delta m^2 \sim \alpha_W m_0^2 - \alpha_Y(\alpha_3 m_0^2) \tag{5.17}$$

where $\alpha_Y$ denotes the Yukawa coupling responsible for the Higgs-squark quartic interaction. The corresponding quantity for the partners of leptons is

$$\delta m^2 \sim \alpha_W m_0^2 - \alpha_Y(\alpha_W m_0^2) \tag{5.18}$$

and will under reasonable circumstances stay positive. The partner of quarks were already heavy enough in the first case. With a large enough Yukawa coupling (but still small enough to trust perturbation theory) the contributions in (5.17) could become negative and thus induce a breakdown of the weak interactions. A candidate for such a Yukawa coupling would be the one that is responsible for the mass of the heaviest particle in the model: the top quark. We can thus conclude two things from this simplified discussion of radiative corrections:

1) $SU(2) \times U(1)$ breakdown can in principle be induced in a desirable way. Observe that it is nontrivial to have the situation that only the $m^2$ of a Higgs becomes

negative while other scalars keep positive $m^2$, i.e. everything could go wrong but it does not;

2) the mass of the top-quark $m_{top}$, i.e. the top-quark Yukawa coupling is a crucial input parameter in the model (unlike in the standard model where it just parametrizes $m_{top}$). Knowing $m_{top}$ would tell us a lot more about the model and its predictions.

So far our qualitative discussion of these issues. We have now to go on and compute. As we said already, the parameters in (5.11) are defined at a large scale and we have to compute their evolution down to a scale of something like 100 GeV to discuss $SU(2) \times U(1)$ breakdown. For the whole model this then requires the integration of some 25 coupled renormalization group equations[22] and you can imagine that this cannot be done analytically. Let us first specify our boundary conditions. We should actually start our evolution at a scale $\bar{\mu} = M \sim 2 \times 10^{18}$GeV, but we do not really know the spectrum of this theory at such large scales. There could be a grand unified sector and this could change the results. Let us therefore assume that the input parameters as given in (5.11) are valid at a scale $\bar{\mu} = M_x \sim 3 \times 10^{16}$GeV where $M_x$ is the grand unification scale in our model. If we assume that below this scale our model gives the complete spectrum we observe that the $SU(3) \times SU(2) \times U(1)$ couplings constants $g_3$, $g_2$ and $g_1$, once properly chosen at a scale $\bar{\mu} \sim 100$ GeV meet at the scale $\bar{\mu} = M_x$ with magnitude $\alpha(M_x) \sim 1/24$. This we will then take as our starting point. Let us now look at some of the renormalization group equations more closely and the most important ones are certainly those for the masses of the scalars. Here I will give you the equations for $\bar{h}$, $\varphi_t$ and $\varphi_{\bar{t}}$ (the partners of $t$ and $\bar{t}$-quark) in the approximation that only the top quark Yukawa coupling ($g_t$) is different from zero. In the calculation the effects of the other couplings have also been included

$$\bar{\mu}\frac{\partial}{\partial\bar{\mu}}m_{\bar{h}}^2 = \frac{3}{8\pi^2}g_t^2(m_{\bar{h}}^2 + m_{\varphi_t}^2 + m_{\varphi_{\bar{t}}}^2 + m_{3/2}^2|A_t|^2 - \frac{1}{2\pi^2}[\frac{3}{4}|\tilde{m}_2|^2 g_2^2 + \frac{1}{4}|\tilde{m}_1|^2 g_1^2] \tag{5.19}$$

$$\bar{\mu}\frac{\partial}{\partial\bar{\mu}}m_{\varphi_{\bar{t}}}^2 = \frac{2}{8\pi^2}g_t^2(m_{\bar{h}}^2 + m_{\varphi_t}^2 + m_{\varphi_{\bar{t}}}^2 + m_{3/2}^2|A_t|^2 - \frac{1}{2\pi^2}[\frac{4}{3}|\tilde{m}_3|^2 g_3^2 + \frac{4}{9}|\tilde{m}_1|^2 g_1^2] \tag{5.20}$$



$$\bar{\mu}\frac{\partial}{\partial\bar{\mu}}m_{\varphi_t}^2 = \frac{1}{8\pi^2}g_t^2(m_{\bar{h}}^2 + m_{\varphi_t}^2 + m_{\varphi_{\bar{t}}}^2 + m_{3/2}^2|A_t|^2 - \frac{1}{2\pi^2}[\frac{4}{3}|\tilde{m}_3|^2 g_3^2 + \frac{3}{4}|\tilde{m}_2|^2 g_2^2 + \frac{1}{36}|\tilde{m}_1|^2 g_1^2] \tag{5.21}$$

where the $\tilde{m}_i$ denote the $SU(3) \times SU(2) \times U(1)$ gaugino masses respectively and $A_t$ is the $A$ parameter that comes with the term in the superpotential that contains the top-quark scalar. Observe that although we started with these $A$'s to be universal they will no longer stay degenerate once we include the radiative corrections.

We can now look more closely at these equations and recover the qualitative behaviour we found in the simple example discussed above. The first term in (5.19) has a positive sign. This implies that the mass of $\bar{h}$ decreases if we lower $\bar{\mu}$ from $M_x$ down to 100GeV. If the top quark mass (i.e. $g_t$) is large enough we could even imagine $m_{\bar{h}}^2$ to become negative, a sufficient (but not necessary) condition to have spontaneously broken $SU(2) \times U(1)$. But a lot of things could go wrong. The evolution equations of the partners of the top quark also have this first term with a positive coefficient and the $m^2$ of these particles should remain positive to keep $SU(3)_{colour} \times U(1)_{e.m.}$ unbroken. The reason why the model works is that the coefficients of these terms are 3 : 2 : 1 in (5.19-21). Observe that these coefficients are not free parameters which we can choose arbitrarily. They are an intrinsic property of the model and if they would have come out differently (like e.g. 1 : 2 : 3) there would be no way for this model to be correct. The second terms in (5.19-21) depend on the effects of gaugino masses. They have a negative coefficient and so increase $m^2$ with decreasing $\bar{\mu}$. At first sight they therefore do not favor an induced breakdown of $SU(2) \times U(1)$ (remember our simple example above). But indirectly they help. The terms in (5.20) and (5.21) contain the gluino contribution with the strong coupling constant $g_3$ which is not present in (5.19). When we now lower $\bar{\mu}$ this could give a big contribution to $m_{\varphi_{t,\bar{t}}}^2$ but not to $m_{\bar{h}}^2$. The equations, however, are coupled and these large contributions enter the first term in (5.19) and speed up the evolution of $m_{\bar{h}}^2$ to small and possibly negative values. This is exactly the behaviour we had already guessed from our simplified discussion above. The discussion, however, also shows that it will be difficult (technically) to arrive at quantitative results. We have to solve all these coupled renormalization group equations numerically i.e. equations for the gauge couplings, gaugino masses, Yukawa couplings, scalar masses, $A$-parameters etc.. We have also to determine e.g. the value of the Yukawa couplings at $M_x$ such that at $\tilde{\mu} \sim 100$GeV they have the correct values to parametrize the masses of quarks and leptons.

The parameters relevant for the breakdown of $SU(2) \times U(1)$ have been identified before. They are $m_{3/2}$, $\mu$, $m_0$, $A$ and $g_t$. They have to fulfill one constraint to give $SU(2) \times U(1)$ breakdown with the correct value for $M_W$ and $M_Z$. To give you a feeling about this relation let me first discuss a simplified situation in which $\mu = m_0 = 0$[23]. We know already from our discussion before that a model with $\mu = 0$ has problems but here we just want to exhibit the mechanism in a simplified case. In addition this case will illustrate what happens in models where $\mu$ is small.

Fig. 5.2 shows the result of a numerical integration of the renormalization group equations in this simplified case $\mu = m_0 = 0$. Only a part of the parameter space in $A$, $m_{3/2}$ and $m_{top}$ can lead to a breakdown of $SU(2) \times U(1)$, as we had to expect from our discussion above. Before we discuss the figure in detail, let me mention that with $\mu = m_0 = 0$ the sign of $A$ is unphysical, only $|A|$ matters. The allowed region for $SU(2) \times U(1)$ breakdown is bounded at the left-hand side in Fig. 5.2, because $m_{top}$ (i.e. $g_t$) is simply too small to drive $m_{\tilde{h}}^2$ negative enough to induce the breakdown. The actual value of this lower bound for $m_{top}$ depends on $A$ and in general large $A$ allows smaller $m_{top}$. Nonetheless we obtain a significant lower bound for $m_{top}$ in this configuration $\mu = m_0 = 0$ of something like 100 GeV. We will later see that large $m_{top}$ is usually required in models with moderately small $\mu$ (at least compared to $m_0$ and $m_{3/2}$). The bound on the right hand side of Fig. 5.2 cannot yet be understood from our discussion up to now. It simply comes from the evolution equation of the Yukawa coupling $g_t$

$$\tilde{\mu}\frac{\partial}{\partial\tilde{\mu}} g_t = \frac{3}{8\pi^2} g_t^3 - \frac{1}{8\pi^2} g_t \left(\frac{8}{3} g_3^2 + \frac{3}{2} g_2^2 + \frac{13}{18} g_1^2\right). \tag{5.22}$$

This shows that for large $m_{top}$ we need a large Yukawa coupling at $M_x$. If $g_t$ is large, however, the first term in (5.22) will be dominant. It has a positive coefficient and will reduce the magnitude of $g_t$ when we lower $\tilde{\mu}$. This then gives us an upper bound on $m_{top}$ of 200 GeV. Even if we choose $g_t$ at $M_x$ to be infinitely large the evolution (which goes with $g_t^3$) will reduce it to a "small" value at $\tilde{\mu} \approx 100$ GeV. Of course, the approximation on which (5.22) is based will break down for large $g_t$



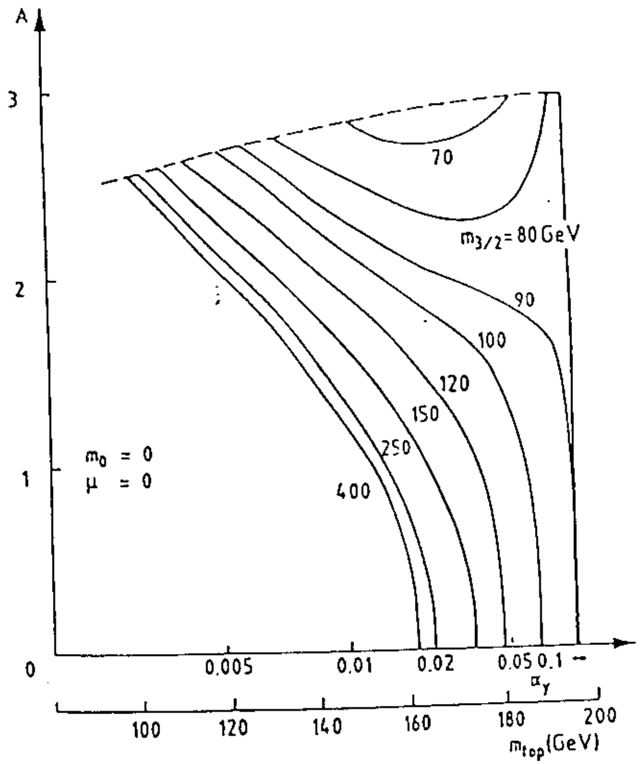


Fig. 5.2: Breakdown of $SU(2) \times U(1)$ for $\mu = m_0 = 0$ from ref.[23].

but we see that in the minimal model $m_{top}$ is bounded from above by something like 200 GeV, the number depending on the exact value of $\alpha_s$.

Last we have to discuss the bound in the upper part of Fig. 5.2 related to the parameter $A$. We have already seen that large $A$ makes it easier to induce the breakdown of the weak interactions, i.e. the breakdown is possible for smaller values of $m_{top}$. This comes from the appearance of $|A|^2$ in the first term in (5.19). One could actually think that by increasing $A$ sufficiently one could induce the $SU(2) \times U(1)$ breakdown for arbitrarily small $m_{top}$ as long as $g_t A_t$ stays large enough. This is true, but for large $A$ other unpleasant things happen and we have to discuss this now in detail[24]. For this purpose consider again the potential (5.7) but for simplicity with just one Yukawa coupling:

$$V = \left|\frac{\partial g}{\partial y_a}\right|^2 + m_{3/2}^2\left(|h|^2 + |\varphi_e|^2 + |\varphi_{\bar{e}}|^2\right) + A m_{3/2} g_E \left(h\varphi_e\varphi_{\bar{e}} + \text{h.c.}\right). \quad (5.23)$$

We also dropped the $\frac{1}{2}D^2$ term because it is irrelevant for our discussion. For small $A$ it is evident that the minimum of this potential will be at $h = \varphi_e = \varphi_{\bar{e}} = 0$ with $V = 0$. For large $A$ the trilinear terms dominate in a certain range and the minimum will be at $h = \varphi_e = \varphi_{\bar{e}} \neq 0$ breaking weak interactions but also electromagnetic and strong interactions and this has to be avoided. Thus $A$ has to be small enough. In the special potential (5.23) (as well as in the general case (5.7) with universal $A$) the critical value is $A = 3$ because in this case the potential reads:

$$V = \left|\frac{\partial g}{\partial y_a} + m_{3/2} y_a^*\right|^2 \geq 0 \quad (5.24)$$

and we need $A \leq 3$. Including the radiative corrections in our model the $A$'s will no longer stay universal and there will be separate bounds on $A$ required by the absence of $SU(3) \times U(1)_{\text{e.m.}}$ breaking minima:

$$A_E^2 < 3\left(\frac{m_h^2 + m_{\varphi_e}^2 + m_{\varphi_{\bar{e}}}^2}{m_{3/2}^2}\right) \quad (5.25)$$

and similar expressions for $A_D$, $A_U$ and all three families separately. These questions have to be carefully checked in all models explicitely.

Curves similar to Fig. 5.2 can now be produced for arbitrary values of the parameters to explore the phenomenological consequences of the model. Apart from the parameters $m_0$, $m_{3/2}$, $A$ and $\mu$, we have seen that the value of $m_{\text{top}}$ is very important. Since we know by now that $m_{\text{top}}$ is quite heavy let us give another illustration of the behaviour of the model in this part of parameter space (fig. 5.3), where we have chosen $m_{3/2} = 100$ GeV. There is an approximate scaling law. If you want to know the behaviour for different values $am_{3/2}$, just scale $m_0$ and $\mu$ by the same factor $a$ while $A$ should be scaled by $a^{-1}$ (at least in large parts of parameter space). The masses of the supersymmetric particles, of course, depend strongly on these parameters. Although they have to be computed in the explicit



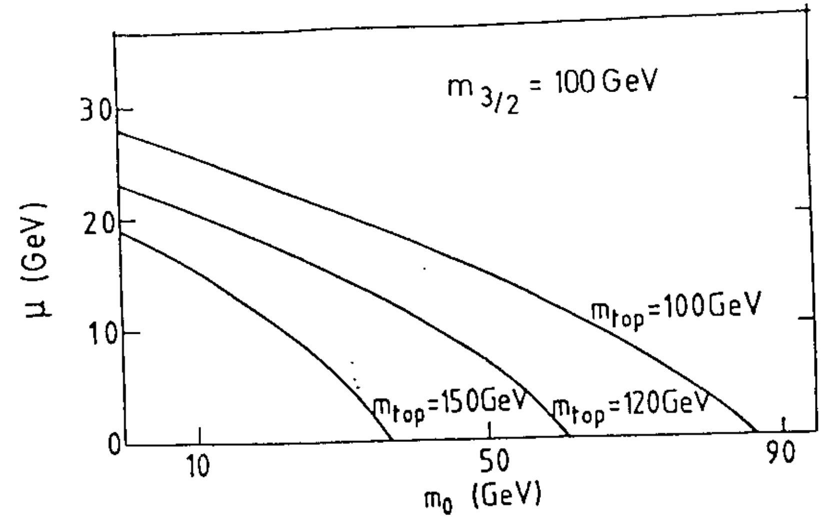


Fig. 5.3: The situation in the case of large top quark masses.

models under consideration one can use as a rule of thumb that the mass of the photino is $\sim \frac{1}{2}m_0$ while the mass of the gluino is larger: approximately $3m_0$. The masses of the quark partners are given by

$$m_{\tilde{q}}^2 \approx m_{3/2}^2 + 7.4 m_0^2 \quad (5.26)$$

while the lepton partners are less sensitive to the gaugino mass

$$m_{\tilde{l}}^2 \approx m_{3/2}^2 + 0.14 m_0^2. \quad (5.27)$$

Of course, the masses can be computed exactly in a given model once the set of input parameters is specified.

Let us now discuss the spectrum more thoroughly and start with the Higgs system[20]. The model contains five physical Higgs bosons, two charged and three neutral ones, one of which is a pseudoscalar. The mass of the charged Higgs bosons is given by

$$m_{\pm}^2 = m_1^2 + m_2^2 + M_W^2 \quad (5.28)$$

thus heavy since $m_1^2 + m_2^2$ has to be positive. The mass of the pseudoscalar is

$$m_{PS}^2 = m_1^2 + m_2^2 \quad (5.29)$$

while the mass of the scalars is given by

$$m_S^2 = \frac{1}{2}\left(m_{PS}^2 + M_Z^2\right) \pm \frac{1}{2}\sqrt{(m_{PS}^2 + M_Z^2)^2 - 4m_{PS}^2 M_Z^2 \frac{(v_1^2 - v_2^2)^2}{(v_1^2 + v_2^2)^2}} \qquad (5.30)$$

which gives the only absolute prediction of the minimal supersymmetric extension of the standard model. *One Higgs scalar is always lighter than the Z boson.* The bound is reached in the case where the ratio of the vev's of the Higgs fields is very large. If in contrast $v_1 = v_2$ we see that one of the scalars is massless. A crucial experimental test of the model will therefore come from Higgs searches. The above formulae and bounds are only valid at the tree level. Radiative corrections could change these results[25]. These corrections depend strongly on the mass of the top quark and become large[26] if $m_{top}$ exceeds a value of 130 GeV. For such high values also the limit on the mass of the lightest Higgs-boson moves up beyond the Z-boson mass.

Let us next consider the gauginos. The gluinos only feel $m_0$ with $m_g \approx 3m_0$. The so-called charginos are combinations of charged gauginos and higgsinos with mass matrix

$$\begin{pmatrix} \tilde{m}_2 & \sqrt{2}M_W \\ \sqrt{2}M_W \frac{v_1}{v_2} & \mu \end{pmatrix} \qquad (5.31)$$

and the spectrum depends strongly on $m_0$ and $\mu$. Observe that one of the states is massless in the limit $\mu = 0$ since there also one finds $v_1 = 0$. The neutral gauginos $\tilde{W}^0$, $\tilde{B}^0$ mix with the neutral higgsinos leading to a complicated mass matrix which we shall not discuss here in detail[5,27]. Among these particles one usually expects to find the lightest supersymmetric particle (LSP) which is stable as long as $R$-parity remains unbroken. This could be (and is over a wide range of parameter space) the photino

$$\tilde{\gamma} = \sin\theta_W \tilde{W}^0 - \cos\theta_W \tilde{B}^0 \qquad (5.32)$$

but there remain other possibilities, like a higgsino if $\mu$ is small or e.g. also a scalar partner of a neutrino in the case where $m_{3/2}$ is small compared to $m_0$ and $\mu$.

The masses of the scalar partners of quarks and leptons are essentially determined by $m_{3/2}$ and $m_0$ (see (5.26-27)) with squarks feeling a stronger influence of $m_0$. One would then conclude that the righthanded slepton is the lightest of these



particles, but this is not necessarily true. There could be an influence of quark masses $m_q$ on the squark masses $m_{sq}$ in case of a large $A$

$$\begin{pmatrix} m_{sq}^2 + m_q^2 & Am_q m_{sq} \\ Am_q m_{sq} & m_{sq}^2 + m_q^2 \end{pmatrix} \qquad (5.33)$$

and it could very well be that one of the partners of the top-quark is the lightest squark. So far our first discussion of the minimal supersymetric extension of the standard model (also called the minimal low energy supergravity model). It depends on several parameters, those in the superpotential ($\mu$ and the Yukawa couplings) and those parametrizing the breakdown of supersymmetry ($m_{3/2}$, $m_0$ and $A$). The magnitude of the dimensionful quantities is supposed to lie in order of magitude in the 100 GeV to TeV region.

## 6. SUPERSYMMETRIC GRAND UNIFICATION - BASICS

The general idea of grand unification has already been discussed at this school and we shall assume that the reader is acquainted with the subject. We shall therefore concentrate here on those special points that are important in the supersymmetric case. This concerns the scale $M_x$, a discussion of the superpotential, the question of the triplet-doublet splitting and proton decay via dimension 5 operators. We shall exclusively stay within the $SU(5)$ framework, with $\bar{5} + 10$ for a quark-lepton family.

In this first chapter on supersymmetric grand unification we give the basic structure of these theories. A more careful discussion of the models including the results of recent precision measurements will be postponed to the next chapter. If we very roughly assume a value of $\alpha_3 \sim 0.1$ and $\alpha \approx 1/128$ at a scale of 100 GeV we obtain in the nonsupersymmetric model a scale $M_x$ of approximately $5 \times 10^{14}$ GeV and desastrous proton decay. The supersymmetric model, however, has more light particles and as such the evolution of coupling constants changes[28]. The most important contribution comes from the gauginos implying a slow-down of the evolution. As a result we observe a larger $M_x \sim 3 \times 10^{16}$ GeV roughly 60 times larger than in the corresponding nonsupersymmetric model. Since proton decay is suppressed with the fourth inverse power of $M_x$ there are no problems with proton stability in the supersymmetric $SU(5)$ model. For a long time the experimental uncertainties concerning the value of the gauge coupling constants did not allow a distinction between the supersymmetric and nonsupersymmetric models. But more recently this situation has changed. In fact, when I was here at TASI90, Paul Langacker showed me the result given in Fig. 6.1. A precision analysis of electroweak data indicated that the supersymmetric model (with two Higgs doublets and a supersymmetry breakdown scale in the TeV-region) gives, in constrast to nonsupersymmetric $SU(5)$ the correct prediction for $\sin^2\theta_W(M_Z)$ [29]. Now three years later we have, of course, even more precise experimental results. These will be discussed in the next chapter.

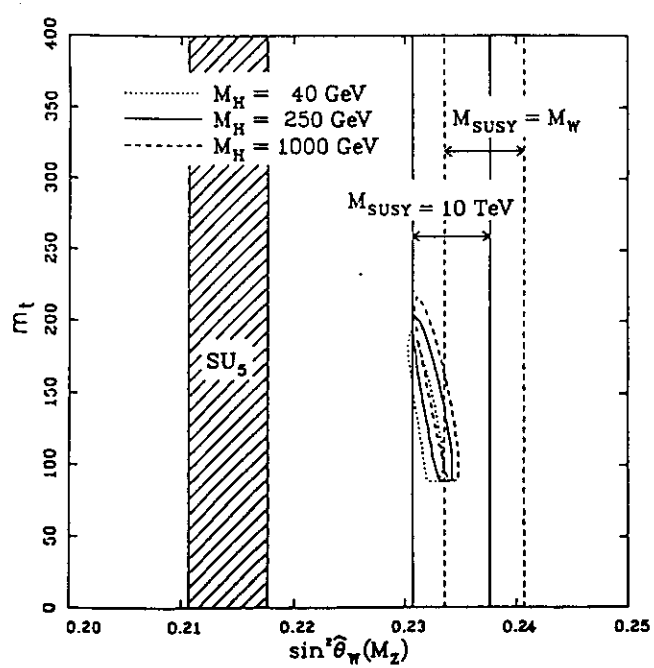


Fig.6.1: $\sin^2\theta_W$ versus $m_t$ and the predictions from grand unified models[29].

Let us here first examine the superpotential and the question of $SU(5)$ breakdown. We denote the quark superfields $X_i(10)$, $Y_i(\bar 5)$ $i = 1,2,3$ and the Higgs



superfields $H(5)$, $\bar H(\bar 5)$ and $\Phi(24)$. The superpotential can then be writen as

$$g = g_{ij}X_iX_jH + h_{ij}X_iY_j\bar H + \lambda_1 H\Phi\bar H + \lambda_2\Phi^3 + M\Phi^2 + M'H\bar H \tag{6.1}$$

where $g_{ij}$ determines the masses of up-type quarks and $h_{ij}$ those of down-type quarks and leptons. The discussion of the spontaneous breakdown of $SU(5)$ is similar to the one in nonsupersymmetric $SU(5)$ models. The auxiliary fields read

$$\begin{aligned} -F_\Phi^* &= \lambda_1 H\bar H + 3\lambda_2\Phi^2 + 2M\Phi \\ -F_H^* &= \lambda\Phi\bar H + M'\bar H + g_{ij}X_iX_j \\ -F_{\bar H}^* &= \lambda_1\Phi H + M'H + h_{ij}X_iY_j \end{aligned} \tag{6.2}$$

and a minimum with $SU(5)$ broken to $SU(3)\times SU(2)\times U(1)$ can be found with $<H>=<\bar H>=<X_i>=<Y_i>=0$,

$$<\Phi>= v\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -\frac{3}{2} & 0 \\ 0 & 0 & 0 & 0 & -\frac{3}{2} \end{pmatrix} \tag{6.3}$$

and vanishing vacuum energy. Since we have not discussed here the breakdown of supersymmetry there are degenerate minima with gauge group $SU(5)$ and $SU(4)\times U(1)$. Also the breakdown of $SU(2)\times U(1)$ has finally to be induced by the effects of supersymmetry breakdown along the lines discussed in the last chapter.

Again a fine-tuning has to be performed to keep the Higgs-doublets light. Here it amounts to

$$M' = \frac{3}{2}v\lambda_1. \tag{6.4}$$

This is similar to the nonsupersymmetric case but here we could argue that the fine-tuning concerns only parameters in the superpotential and is therefore not disturbed by radiative corrections. If we now would be able to find a reason why (6.4) should be valid at tree level we could claim to have solved the fine-tuning problem. There have been several interesting attempts in this direction. As a first we discuss the mechanism of a sliding singlet[30]. Take a gauge singlet superfield $Z$ and add a term $\lambda HZ\bar H$ to the superpotential. The $H$ auxiliary field reads now

$$-F_H^* = \bar H(\lambda_1\Phi + \lambda Z + M'). \tag{6.5}$$

In the full theory, including supersymmetry breakdown, the doublet component of the scalar of $\bar{H}$ should receive a vev (in contrast to the $SU(3)$-triplet component). The vev of $Z$ is undetermined and it can adjust its vev to have $F = 0$ for the doublet component, thus it slides to make

$$-\frac{3}{2}\lambda_1 v + \lambda z + M' = 0 \tag{6.6}$$

and the Higgs-doublet remains light. This looks nice, but also this mechanism has some problems. We do not understand why the allowed $Z^2$ and $Z^3$ terms are absent and also we cannot rule out the possibility that the absolute minimum of the potential occurs for large vev's of both the triplet and the doublet. Moreover, there are usually problems with a small supersymmetry breakdown scale in the presence of light singlets[31].

A second mechanism to be discussed here is the one of the missing partner. $H$ and $\bar{H}$ contain $(3,1)+(\bar{3},1)$ and $(1,2)+(1,\bar{2})$ of $SU(3)$ and $SU(2)$ respectively. Try to find now a new representation which only contains a (3,1) but not a (1,2). The former could then pair up with the $(\bar{3},1)$ in $\bar{H}$ while $(1,\bar{2})$ would remain massless. A simple example[32] is a 50 of $SU(5)$. It decomposes with respect to $SU(3)\times SU(2)$ as $(\bar{6},1)+(8,2)+(1,1)+(3,2)+(6,3)+(\bar{3},1)$ and as a cross term in the superpotential we could imagine $50\times 5\times 75$ with $75=(1,1)+(3,1)+(3,2)+(\bar{3},1)+(\bar{3},2)+(\bar{6},2)+(6,2)+(8,1)+(8,3)$. Fortunately a vev of 75 can break $SU(5)$ to $SU(3)\times SU(2)\times U(1)$ thus avoiding the presence of $\Phi$ in (6.1). Instead we choose now for the superpotential

$$\begin{aligned} g = \lambda 75\times 75\times 75 + M75\times 75 + \lambda_1 50\times 75\times \bar{50} \\ + \lambda_2 50\times 75\times 5 + \lambda_3 \bar{50}\times 75\times \bar{5} + \tilde{M} 50\times \bar{50} \end{aligned} \tag{6.7}$$

and as a mass matrix for the triplets we obtain

$$\begin{pmatrix} 0 & \lambda_2 v \\ \lambda_3 v & \tilde{M} \end{pmatrix} \tag{6.8}$$

(where $v$ is the vev of 75), while the doublets remain light. Of course, one still has to explain why we have omitted a direct $5\times\bar{5}$ mass term in (6.7) and the question of a complete solution of the fine tuning problem remains open.

We had seen at the beginning of this chapter that $M_x$ is quite large in supersymmetric grand unified models and that therefore proton decay via gauge boson



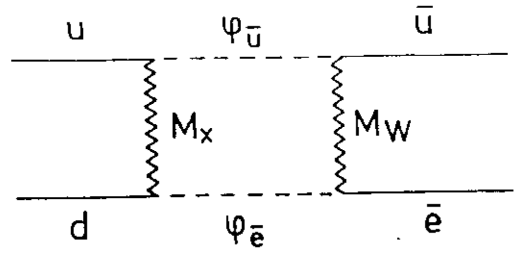


Fig. 6.2: Proton decay through dimension-5 operators.

exchange is sufficiently suppressed. This, however, is not the last word about proton decay in supersymmetric grand unified models. Remember, that in the supersymmetric version of the standard model we already had to suppress proton decay via dimension-4 operators by introducing an $R$-symmetry (see chapter 3). Here we have to worry about proton decay via dimension five operators[33] leading to proton decay as shown in Fig. 6.2. The first step couples two fermions to two bosons (therefore the name dimension-5 operator) and has a propagator suppression of $1/M_x$ and the second step involves only light particles. Instead of $1/M_x^2$ in the amplitude we have now $1/M_x M_W$ and there is a potential danger of fast proton decay. A careful investigation of the dimension 5-operators has therefore to be performed. Out of the possible terms we need only consider those which are invariant under the $R$-symmetry discussed earlier and these are the two $F$-terms $(QQQL)_F$ and $(\bar{U}\bar{U}\bar{D}\bar{E})_F$ . The latter reads in components

$$\bar{U}_{ia}\bar{U}_{jb}\bar{D}_{kc}\bar{E}_l\epsilon^{abc} \tag{6.9}$$

where $a$, $b$, $c$ are $SU(3)$ indices and $i$, $j$, $k$, $l$ are generation indices. All fields above are scalar superfields and should obey Bose-statistics. The two $\bar{U}$'s are antisymmetrized in $a$ and $b$ and therefore $i \neq j$ and one of the $\bar{U}$'s has to come from the second generation. Since the charmed quark is heavier than the proton the presence of the term in (6.9) does not constitute a problem. The other possibility reads

$$Q^a_{ir}Q^b_{js}Q^c_{kt}L_{lu}\epsilon_{abc}\epsilon^{rs}\epsilon^{tu} \tag{6.10}$$

where $r$, $s$, $t$, $u$ are $SU(2)$-indices. Here we can have $i = j = 1$ but then we need $k = 2$ which leads to

$$\begin{pmatrix} c \\ s \end{pmatrix}_t \begin{pmatrix} \nu \\ e \end{pmatrix}_u \epsilon^{tu} \qquad (6.11)$$

thus $ce$ or $s\nu$. Proton decay therefore is only possible with the $(uds\nu)_F$ operator The dominant decay mode is proton to $K^+$ and antineutrino, a quite unique prediction of supersymmetric grand unified models. The rate is faster than the one from dimension-6 operators but it is not desastrously fast since $p \to K^+\bar{\nu}$ involves Yukawa couplings in graphs like Fig. 6.2 as compared to gauge couplings in the process with dimension-6 operators. At the moment $p \to K^+\bar{\nu}$ seems to be at the border of observability and further experimental results are eagerly awaited.

## 7. SUPERSYMMETRIC GRAND UNIFICATION RECENT DEVELOPMENTS

In figure 6.1 we had already seen that according to the data available in 1990 coupling constants in the standard model did not behave in a way consistent with grand unification. There is an indication that the grand unified supersymmetric extension of the standard model, however, gives the correct prediction for the weak mixing angle $\sin^2\theta_W$, provided that the mass scale of the supersymmetric partners is in the 100GeV to TeV region. Meanwhile with more data this trend has been confirmed as shown in fig. 7.1.

A more artistic view of this result is displayed in fig. 7.2. Comparing fig. 6.1 and 7.1 we observe that precision has increased within the last three years and we also see that nowadays the mass scale of the supersymmetric partners $M_{SUSY}$ appears to be at a somewhat lower scale. To understand this evolution it is important to know that the uncertainties in these plots are dominated by the experimental error bars in the determination of the strong coupling constant $\alpha_s$ (which is obvious from fig. 7.2). In addition as we shall see soon in detail, the value of $\alpha_s$ is correlated with $M_{SUSY}$: larger $\alpha_s$ implies smaller $M_{SUSY}$. And this has happened in the last three years: we have now a more precise value of $\alpha_s$, but the central value has increased. While in 1990 one considered central values $\alpha_s(M_Z) = 0.108$ now values could be as high as 0.125. See Fig 7.3 for an update[36].



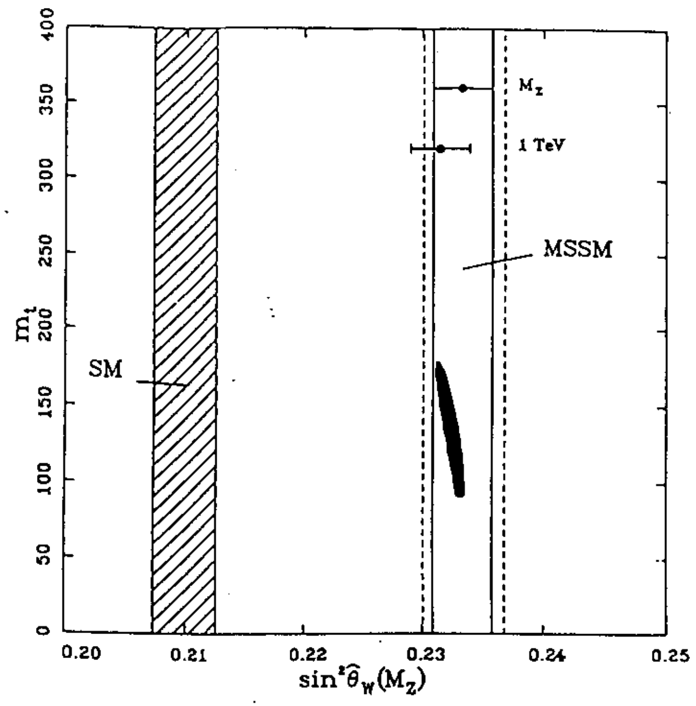


Fig.7.1: $\sin^2\theta_W$ versus $m_t$ and the predictions from grand unified models[34].

There is still a debate concerning the evolution of $\alpha_s$ below $M_Z$ in the standard model. This might lead to a revival of the light gluino hypothesis[37] and we need more precise data to settle these questions. We do not have the time to discuss this in detail in these lectures. We are here more concerned with the evolution of the coupling constants above $M_Z$. There you should remember that large values of $\alpha_s$ imply smaller values of $M_{SUSY}$ and vice versa.

We have now to take a closer look at the definition and the role of $M_{SUSY}$. It is understood that between $M_Z$ and $M_{SUSY}$ one should use the renormalization group equations of the standard model while above $M_{SUSY}$ the evolution equations of the supersymmetric extension of the standard model should be applied. If we consider e.g. a model where all the supersymmetric partners like the gauginos,

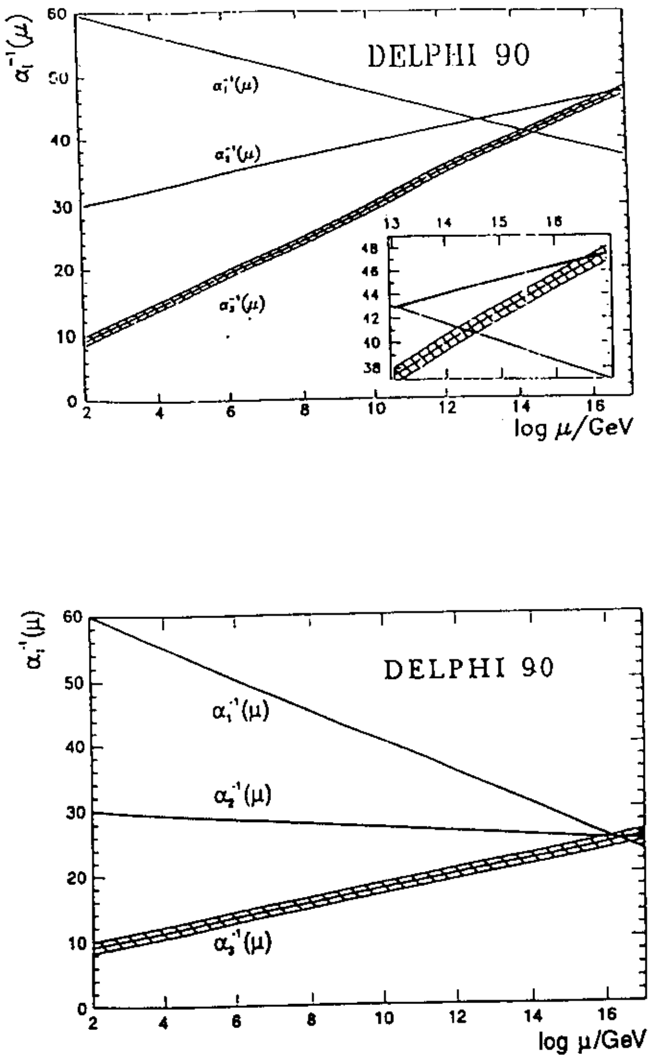


Fig.7.2: The evolution of coupling constants[35]



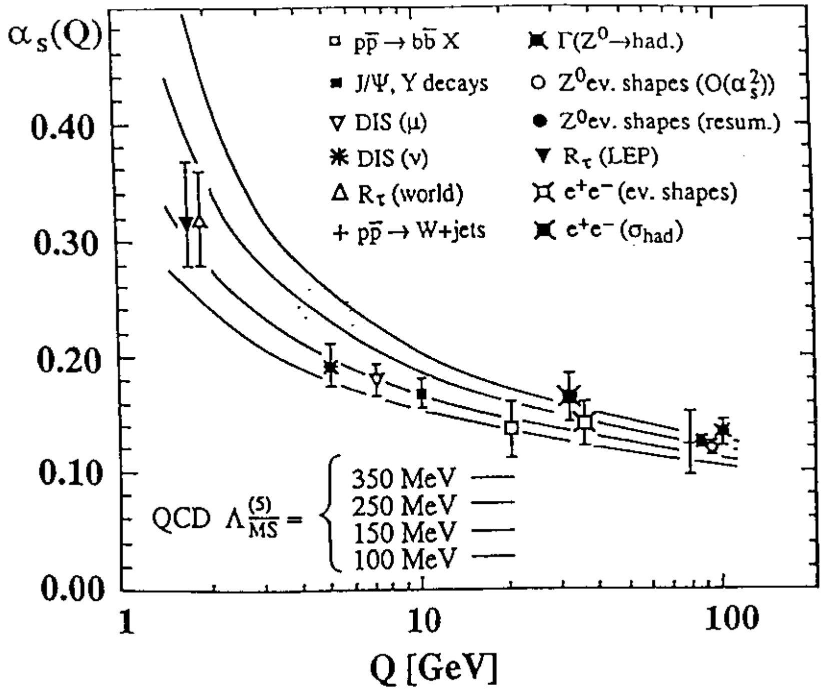


Fig.7.3: The strong coupling constant measurements[36].

the higgsinos, the squarks and the sleptons are degenerate with mass $m$, then $M_{SUSY} = m$; this in fact would then mean, that $M_{SUSY} = m \geq M_Z$.

A more realistic spectrum of supersymmetric partners, however, might look different. We have seen in the earlier chapters, that usually the gluino is heavier than the photino or the sqarks heavier than the sleptons; in any case one would expect a nondegenerate spectrum. Some averaging proceedure should then be performed. It turned out that strange things happen in this proceedure. It was observed[38] that even with nondegenerate partners *all in mass above* $M_Z$ the effective scale $M_{SUSY}$ can become *smaller* than $M_Z$. We shall therefore (following ref.[39]) call this effective scale $T_{SUSY}$ and still keep the notation $M_{SUSY}$ for the physical mass scale of supersymmetric partners. The relation between the two parameters is in spirit similar to the relation between temperature and wind chill factor. In fact in the degenerate case (no wind) $M_{SUSY} = T_{SUSY}$.

This effect of the averaging proceedure for a nondegenerate spectrum has been explained in ref. [34]. Let us here follow this discussion and use the evolution equations at the one-loop level. The qualitative features are valid also if we include the two-loop contribution, but the formulae become too complicated to be discussed here. Here we obtain the following relation:

$$19\log\left(\frac{T_{SUSY}}{M_Z}\right) = -25\log\left(\frac{M_1}{M_Z}\right) + 100\log\left(\frac{M_2}{M_Z}\right) - 56\log\left(\frac{M_3}{M_Z}\right), \quad (7.1)$$

where $M_1$, $M_2$ and $M_3$ in some way represent the average mass of particles with $U(1)$, $SU(2)$ and $SU(3)$ quantum numbers, respectively[34]. At the moment it is not necessary to understand these masses in detail; we will shortly give a more detailed explanation. It is important to realize first that in fact the whole spectrum can be described by *one* effective scale $T_{SUSY}$ that represents all information about these threshold corrections for the supersymmetric particles. Secondly we observe that the right hand side of (7.1) contains positive as well as negative signs. And here we now understand the strange behaviour mentioned above: if we increase the mass of the gluino (contributing only to $M_3$) while keeping all other masses fixed we lower the effective scale $T_{SUSY}$. This also makes clear that it is possible to have $T_{SUSY} < M_Z$. The threshold effects that take place above $M_Z$ and which might come from a complicated spectrum can be summarized with this one effective scale $T_{SUSY}$.

Let us now examine more closely the effect of the various particles on $T_{SUSY}$:

$$\begin{aligned} -19\log\left(\frac{T_{SUSY}}{M_Z}\right) &= 3\log\left(\frac{M_{squarks}}{M_Z}\right) + 28\log\left(\frac{M_{gluino}}{M_Z}\right) \\ &\quad - 3\log\left(\frac{M_{slepton}}{M_Z}\right) - 32\log\left(\frac{M_{wino}}{M_Z}\right) \\ &\quad - 12\log\left(\frac{M_{higgsino}}{M_Z}\right) - 3\log\left(\frac{M_{Higgs}}{M_Z}\right), \end{aligned} \quad (7.2)$$

which allows you to compute $T_{SUSY}$, once you know the masses of the particles in the supersymmetric standard model. It is clear that even with all the supersymmetric partners heavy, still $T_{SUSY}$ might be small. Observe also, that the contribution from squarks and sleptons cancel if they are degenerate. The terms



with the gauginos have quite large coefficients. If one considers models with a universal gaugino mass at the large scale, there is also the tendency that the winos partially cancel a big gluino contribution. In general, however, threshold corrections due to the nondegeneracy of the supersymmetric spectrum are quite important. This remains true after the inclusion of two-loop effect in the evolution equations which we have not discussed here. An account of the difference between one- and two-loop results on these questions can be found in [40].

As can be seen from fig. 7.1. the experimental values of $\alpha_s$, $m_{top}$ and $\sin^2\theta_W$ are consistent with grand unification for a large range of values for $T_{SUSY}$ in the desired range. More detailed information can only come from new experimental input. Remember e.g. that a more precise value of $\alpha_s$ would lead to important restrictions. As we have seen the value of $\alpha_s$ is strongly anticorrelated with the value of $T_{SUSY}$. Another parameter which we could learn something about in the near future is $m_{top}$ which is strongly correlated with $\sin^2\theta_W$ as discussed in [34]

$$\sin^2\theta_W(M_Z) = 0.2324 \pm 0.0003 - 1.03\times10^{-7}\mathrm{GeV}^{-2}(m_{top}^2 - 138^2\mathrm{GeV}^2), \quad (7.3)$$

i.e. large $m_{top}$ corresponds to rather small values of the weak mixing angle. Thus knowledge of $m_{top}$ would help a lot.

In fact, the question concerning fermion masses turns out to be more interesting than expected. We remember that a discussion of the quark and lepton masses in the standard model as well as its supersymmetric extension usually consisted of the statement that one has to adjust the Yukawa-couplings to obtain the correct spectrum. In grand unified models, however, due to the larger gauge symmetry we also have to consider *Yukawa coupling unification*. In our example based on the group $SU(5)$ we have, according to equation (6.1), only one Yukawa coupling for the charged leptons and the down quarks, as long as we assume that they obtain their mass through the vev of the same Higgs-scalar. The complete fermion mass matrix is very complicated, and in order to understand it completely one would most probably need more than just one of these scalars. It is, however, tempting to assume that for the heaviest generation just one scalar is responsible for b-quark and $\tau$-lepton mass. This then implies $h_\tau(M_X) = h_b(M_X)$ for the b- and $\tau$-Yukawa-couplings at the GUT-scale. Of course, at low energies, $h_\tau$ and $h_b$ differ because of the renormalization effects. The one-loop equations for the Yukawa-couplings are given by (assuming $h_t \gg h_b,\ h_\tau$):

$$\bar{\mu}\frac{\partial}{\partial\bar{\mu}}h_\tau = -\frac{1}{8\pi^2}h_\tau\left(\frac{3}{2}g_2^2+\frac{3}{2}g_1^2\right) \tag{7.4}$$

$$\bar{\mu}\frac{\partial}{\partial\bar{\mu}}h_b = -\frac{1}{8\pi^2}h_b\left(\frac{8}{3}g_3^2+\frac{3}{2}g_2^2+\frac{7}{18}g_1^2\right)+\frac{1}{16\pi^2}h_t^2h_b \tag{7.5}$$

$$\bar{\mu}\frac{\partial}{\partial\bar{\mu}}h_t = -\frac{1}{8\pi^2}h_t\left(\frac{8}{3}g_3^2+\frac{3}{2}g_2^2+\frac{13}{18}g_1^2\right)+\frac{3}{8\pi^2}h_t^3 \tag{7.6}$$

in the notation of chapter 5. Since the b-quark has strong interactions in contrast to the $\tau$-lepton $h_b$ evolves faster than $h_\tau$ giving rise to a larger b-mass at low energies in agreement with experimental results. This is a well known result and it was considered a great success that the $m_b/m_\tau$ ratio could be explained by this fact[41]. In [42] it was pointed out, that for a large value of $h_t$ (comparable in size to the gauge coupling constants) its effects could be quite important. This comes from the last term in (7.5) with the opposite sign, thus reducing the $m_b/m_\tau$ ratio. This ratio thus depends strongly on $\alpha_s$ and $h_t$. For a long time $\alpha_s$ was so poorly known that no conclusion could be drawn from these facts. With the more precise value of $\alpha_s$ and the knowledge of the $m_b/m_\tau$ ratio now, however, we can obtain information on the size of the top-quark Yukawa coupling $h_t$ [43]. This leads to the statement, that $h_t$ should be close to its infrared quasi fixed-point [44] which is obtained in case of a vanishing right hand side of (7.6), thus with $Y_t = h_t^2/4\pi$

$$8\alpha_s(m_t) \sim 9Y_t(m_t), \tag{7.7}$$

evaluated at the low energy scale, here chosen to be the mass of the top-quark. This value of $h_t$ close to the infrared quasi fixed point leads to rather large values of the top-quark mass:

$$m_t(m_t) = h_t(m_t)v\sin\beta, \tag{7.8}$$

where $\tan\beta$ is the previously defined ratio of the vevs of the two Higgs-fields. We have already seen in chapter 5 (compare (5.22) with $g_t \equiv h_t$) that at this fixed point the top-quark mass obtains its largest possible value within this framework: $(m_t)_{max} \simeq 200$ GeV. The exact value depends, of course, on the exact value of $\alpha_s$. We also observe that (7.8) implies a strong correlation between $m_t$ and $\tan\beta$. Up



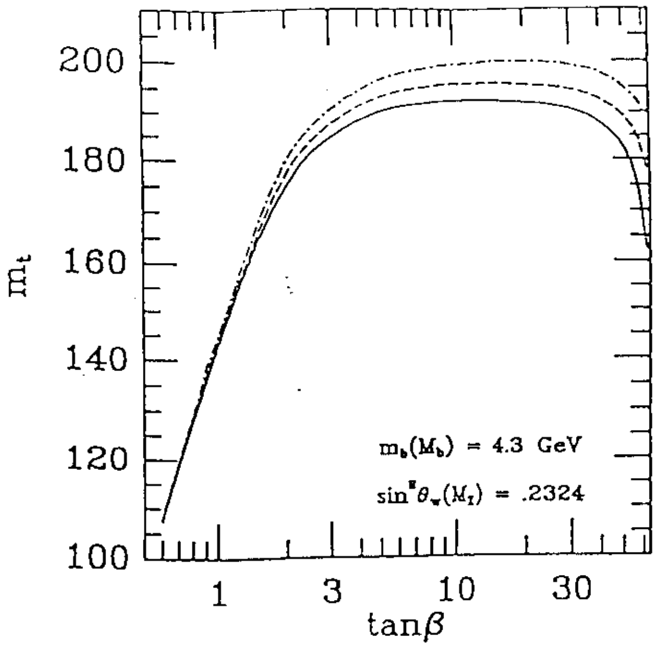


Fig.7.4: The correlation between $m_{top}$ and $\tan\beta$ [39].

to now we have neglected the effects of a nonvanishing $h_b$ in these calculations. For large values of $\tan\beta$ they become important. For an example see fig. 7.4.

Observe also, that in models with radiative symmetry breakdown one has $\tan\beta \geq 1$ and thus $m_t \geq 140$ GeV approximately. The assumption of *Yukawa coupling unification* for the $b-\tau$ system gives strong restrictions on $m_t$. A detailed discussion of these and related questions can be found in the literature[39,45].

More detailed restrictions from grand unification can be obtained in specific models. For example in models with a light gluino one has a bound $0.123 \leq \alpha_s \leq 0.132$ with $m_{top} \geq 145$ GeV[46]. Such models are only consistent with LEP results for small $\tan\beta \leq 2$ and there is a similar bound on $\alpha_s$ for this range of $\tan\beta$ even in the case of a heavy gluino.

As we discussed in the last chapter there can be constraints from proton decay via dimension-5 operators. The graph in fig. 6.2 contains e.g. the down-quark Yukawa coupling and given the d-quark mass we see that this coupling is proportional to $\tan\beta$. The experimental results might therefore lead to an upper bound[47] on $\tan\beta$, but the exact value of this bound is still under debate[48]. If proton decay via dimension-5 operators is not found one might also consider models where some discrete symmetries[49,50] (as alternatives to R-parity) prohibit this mechanism. In these cases we would, however, expect new sources of lepton number violation.

An upper bound on $\tan\beta$ might become important in those models based on an $SO(10)$ grand unified gauge group where the heaviest generation receives a mass from a single Higgs representation. There $h_t = h_b$ at $M_X$ and therefore $\tan\beta \sim 60$. These issues will certainly be covered in the lectures of L. Hall in these proceedings.

The simplest supersymmetric grand unified model is consistent with the value of $\sin^2\theta_W$. Given this success, we can thus test more specific models, like the assumption of Yukawa-coupling unification discussed above. Another more specific scenario is the one based on the induced radiative breakdown of $SU(2) \times U(1)$. Here we obtain strong restrictions on the parameters[51], especially in models that also exhibit Yukawa coupling unification[52]. At the moment we can just try and study the full parameter space of the model. New data has then to decide which part of it might be selected. A lot of work has been done in this field recently which we do not have the time to present in detail. For more information and a more complete list of references I refer the reader to existing reviews[53-57].

We should, however, be aware of the fact that in all grand unified models there are inherent uncertainties at the grand scale that we cannot control. These are e.g. threshold corrections due to heavy particles. While in minimal $SU(5)$ they are usually rather mild[34], they could become quite important in more complicated models like those with a 75-representation discussed earlier[58]. Other uncertainties include heavy thresholds in the evolution of Yukawa-couplings, the presence of nonminimal gauge kinetic terms[59] or just a more general set of boundary conditions for the soft breaking terms at the grand scale.



This brings us to the central question: should we believe in the reality of supersymmetric grand unified theories? After all some ten years ago many people believed in normal grand unified theories. Then proton decay was not found and now we also know that the coupling constants in a nonsupersymmetric theory do not match at a single scale. Could history repeat itself? Of course, we cannot answer this question. Nonetheless it might be useful to keep this possibility in mind. If the GUT idea were true, however, we could then ask the question how well we can determine the grand unified scale $M_X$ with our present experimental knowledge. That seems to be easy: just take the precisely known values of $\alpha_1$ and $\alpha_2$ at $M_Z$ and then determine the value where they cross. This would give something like $M_X \sim 3 \times 10^{16}$GeV. But we cannot control heavy threshold effects and they might strongly influence the value of $M_X$. In fact, grand unified models with a complete description of the fermion mass spectrum turn out to lead to a complicated spectrum of heavy particles[60] and significant heavy threshold effects might be a genuine property of realistic grand unified models. Also $M_X$ tends to be only two orders of magnitude smaller than the Planck scale. How sure can we be that $M_X \ll M_{\rm Planck}$ since gravitational effects might also influence $M_X$.

We cannot answer these questions at the moment and one way to proceed is to compare SUSY-GUTs with alternative models. One of them is the embedding of the supersymmetric standard model within the framework of string theory, called *string unification*. Such theories contain one scale $M_{\rm string} \approx 1.7 \times 10^{18}$GeV related to the Planck scale. Many heavy particles can act as sources for threshold effects. There is usually a fixed relation between the gauge coupling constants but they need not necessarily all coincide at a single scale. The models in general do not contain a grand unified gauge group like $SU(5)$ or $SO(10)$ although such groups might be present. This could relieve somewhat the problem of splitting doublet and triplet in grand unified $SU(5)$ since the Higgs-doublet does not neccesarily have an $SU(5)$ partner. It also implies that Yukawa couplings like $h_b$ and $h_\tau$ need not be equal at the grand scale.

Let us now examine string unification in more detail. At the tree level the gauge couplings are determined by the vev of the scalar dilaton field:

$$k_3 g_3^2 = k_2 g_2^2 = k_1 g_1^2 = g_{\rm string}^2 \equiv g^2 \tag{7.9}$$

where the coefficients $k_i$ (the so-called Kac-Moody levels) are rational numbers. One could now try to see which choice for the $k_i$ leads to models consistent with observed values of the coupling constants. From the experience with model building we know that it is very hard to obtain realistic models with $k \neq 1$ for nonabelian gauge groups and one would choose $k_3 = k_2 = 1$ leaving $k_3$ as a free parameter. In SUSY-GUTs the relation between the coupling constants would be fixed, but $M_X$ would be the free parameter while in the other approach $M_X$ is fixed through $M_{string}$. For a discussion see ref. [61]. The usual normalization of the $U(1)$ gauge coupling corresponds to $k_1 = 5/3$.

The evolution of coupling constants requires a loop-calculation and apart from the usual evolution the gauge couplings become moduli-dependent (i.e. a function of scalar fields $T_i$) and this can be understood as the influence of heavy particles. In simple models such a functional dependence can be estimated [62-65] while in more realistic models such a calculation turns out to be quite complicated[66]. One can write

$$\frac{16\pi^2}{g_a^2(\mu)} = k_a \frac{16\pi^2}{g_{string}} + b_a \log(M^2/\mu^2) + \Delta_a, \qquad (7.10)$$

where in the simplest cases[64] the threshold $\Delta \sim \log[\mathrm{Im}T(\eta(T))^4]$ is a function of one modulus $T$ which is related to the overall size of compactified space and $M$ is chosen in such a way to be closest to a point that could play the role of the grand unified scale[63]

$$M = (2\pi)^{-1/2} \exp[(1-\gamma)/2] 3^{-3/4} M_{string} \approx 0.216 M_{string} \approx 3.6 \times 10^{17}\mathrm{GeV}. \quad (7.11)$$

Thus $M$ turns out to be a factor of 20 larger than $M_X$. If we now, hypothetically take $M$ as the unification scale and assume that all the particles lighter than $M$ are those of the minimal supersymmetric standard model we can determine $\sin^2\theta_W \approx 0.218$ in conflict with the measured value. This calculation, however, is purely academic, since such a string model might not exist. Usually such models contain more particles below $M$ and thresholds might be important. Just consider $\Delta(T)$ in its simplest form with a value of $T \approx 10^{16}$GeV: one could have effects big enough to reproduce the correct value of $\sin^2\theta_W$[67]. Of course, also such a calculation might be academic since this simple threshold function is valid only in very simple models with unbroken $E_6$ gauge group. It indicates, however, the potential importance



of heavy thresholds. This is confirmed in more realistic models; see ref. [66] for a detailed discussion. One way to distinguish string unification and grand unification could be related to the question of Yukawa couplings. While in many grand unified models with a simple Higgs sector we expect also some group theory relations between Yukawa-couplings (like e.g. $h_b = h_\tau$), this needs not necessarily be the case in string theory. We have to see how the experimental situation develops, before we can make some more definite statements.

## 8. THE $\mu$-PROBLEM

In this chapter we want to look more closely at the Higgs mass term in the superpotential:

$$\mu H\bar{H}. \qquad (8.1)$$

As we have seen in the earlier chapters one usually argues that the parameter $\mu$ is of the order of $m_{3/2}$, the scale of supersymmetry breakdown in the observable sector. Such a term, however, is allowed by supersymmetry and one might therefore ask the question why this term should be small compared to $M_X$ or $M_{Planck}$. The lack of a theoretical argument for small $\mu$ is called the $\mu$-problem. In grand unified theories this problem is obviously related to the question of doublet-triplet splitting.

According to our notion of naturalness we can split the discussion into two steps. In a first step one should find a reason that explains why $\mu$ vanishes, e.g. if the term in (8.1) is forbidden by a symmetry. The second step would then be the generation of a small $\mu$-term of the order of the soft SUSY-breaking terms. Let us start with the minimal supersymmetric extension of the standard model and consider the case $\mu = 0$. In fact the model has an additional $U(1)$ symmetry which, however, is anomalous. Vanishing $\mu$ then implies the presence of a Peccei-Quinn (PQ) symmetry[7]. Of course, $\mu = 0$ has phenomenological problems and just imposing this PQ-symmetry cannot be the final answer, but it could give a hint for a solution. In a situation where the strong CP-Problem is solved through the PQ-mechanism[68] we would expect a relation to the $\mu$-problem.

One simple way to generate $\mu$ could be achieved through the introduction of a singlet field $S$ and a term $SH\bar{H}$ in the superpotential[16]. If $S$ receives a nontrivial vev this would generate a $\mu$-term. This, however, just rephrases the problem. We would then have to understand why $S$ has such a small vev and not a vev of the order of $M_{\rm Planck}$. A possible solution to the $\mu$-problem might come through a mechanism that links the $\mu$-problem to the strong CP-problem[19]. Consider a model with an invisible axion[68]. Astrophysical and cosmological arguments lead to an axion decay constant in the range $f_a \approx 10^{10} - 10^{12}$GeV[69]. This scale coincides with the scale $M_S \approx 10^{11}$GeV of SUSY breakdown as discussed earlier. Remember that the gravitino mass is given by $m_{3/2} \sim M_S^2/M_{\rm Planck}$. To implement an axion within the supersymmetric framework one could introduce a singlet field $S$ with a term

$$\frac{1}{M_{\rm Planck}} S^2 H\bar{H} \tag{8.2}$$

in the superpotential, whereas terms like $\mu H\bar{H}$ and $SH\bar{H}$ would be forbidden by the PQ-symmetry. A vev of $S$ of order of $10^{11}$GeV would then simultaneously solve the two problems. $f_a \approx < S >$ would be of the correct order of magnitude and $\mu \approx < S >^2 / M_{\rm Planck} \approx$TeV, as desired. A consistent model along these lines has been constructed in ref. [19]. Note that in this model we were forced to introduce an intermediate scale at $10^{11}$GeV, but this has to appear in any model based on spontaneously broken supergravity in the hidden sector. The presence of the small number $M_S/M_{\rm Planck} \approx 10^{-8}$ is crucial for the solution of the problem. The model has been generalized to the case of a dynamical breakdown of supersymmetry, avoiding the introduction of singlets[70]. In a model where SUSY is broken through gaugino condensation $< \lambda\lambda >= \Lambda$ one in general obtains an effective coupling

$$\frac{< \lambda\lambda >}{M^2_{\rm Planck}} H\bar{H} \tag{8.3}$$

which also leads to $\mu \sim m_{3/2}$ and a composite invisible axion. For details see [70].

As we have seen, a solution of the $\mu$-problem requires:

- a reason why $\mu = 0$,
- the generation of a small $\mu$.

The actual value of $\mu$ is then related to the scale of breakdown of that symmetry that would enforce $\mu = 0$. In the previous example the symmetry was a PQ-symmetry spontaneously broken at the scale of $f_a \sim 10^{11}$GeV and $\mu$ was generated by a (nonrenormalizable) term in the superpotential. In supergravity theories there



are of course various other types of nonrenormalizable terms. Ref. [71] investigated a situation where heavy ($z$) and light ($y$) fields couple in nonrenormalizable kinetic terms:

$$\Lambda(z,z^*)yy^* + (\Gamma(z,z^*;y) + \text{h.c.}), \tag{8.4}$$

where $\Gamma = \sum_m c_m(z,z^*)y^m$. The presence of these terms could lead to an *effective superpotential* at low energies $g(y) = g^{(3)}(y) + \mu y^2$, where for vevs of $z$ of order of the Planck mass one obtains $\mu \sim m_{3/2}$. This would be another possibility to generate an acceptable $\mu$-term. One difficulty with this approach might be the fact that now the PQ-symmetry is broken spontaneously at the large scale $< z >\approx M_{\rm Planck}$ with the well known cosmological problem[69]. Reducing the vev of $z$ to the allowed window around $10^{11}$GeV leads to a very small value of $\mu \sim 10^{-8} m_{3/2}$. One should, however, keep in mind that there again could be nonrenormalizable terms in the superpotential $(1/M_{\rm Planck})z^2 H\bar{H}$ that would lead to $\mu \approx M_{\rm Planck}$ for a $< z >$ of order of the Planck scale. Thus the contribution of the nonrenormalizable terms in the kinetic terms can be safely neglected with respect to the nonrenormalizable terms in the superpotential (unless one finds a reason why the latter might be forbidden).

The consideration of nonrenormalizable terms is of crucial importance in connection with any discussion of the $\mu$-problem. If one considers only renormalizable terms or picks certain specific nonrenormalizable terms one should make sure that they give the leading contribution to $\mu$. Otherwise one has to require that the dangerous leading terms are forbidden and this is then just a reformulation of the problem.

In string theory one might argue that a term $\mu H\bar{H}$ does not appear at the tree level. It was argued that in a theory with a trilinear superpotential $g_{(3)}$ one might then consider additional nonrenormalizable terms $g_{(3)}H\bar{H}$ in the superpotential[72]. As we have learned earlier a vev of the superpotential $< g_{(3)} >\approx m_{3/2}$ leads to a $\mu$ of the same order of magnitude. In this approach some $SU(3) \times SU(2) \times U(1)$ singlet fields receive large vev's and it remains to be understood why a coupling of these fields to $H\bar{H}$ in trilinear or quadrilinear terms should be forbidden, since these terms would give the dominant contribution to $\mu$.

In grand unified theories the $\mu$-problem is, of course, closely related to the doublet-triplet splitting. The missing partner mechanism as discussed in chapte

6 (with superpotential as given in (6.7)) might provide a solution. There might be more explicit examples of the mechanism [73] and the following discussion will be independent of the special group theoretical details.

Actually here we could consider two choices of such a model. A first possibility might use an exact symmetry (continuous or discrete) that would forbid the $\mu$-term. We would then have the problem to understand the breakdown of that symmetry in order to generate a nonvanishing $\mu$. A second possibility would consider a more accidental reason for $\mu$ to vanish. Again we can distiguish between two cases. In a first case we just put $\mu = 0$. This would mean that we ignore the $\mu$-problem. With unbroken supersymmetry, the nonrenormalization theorems would assure a vanishing $\mu$ in any order of perturbation theory. In the case of broken SUSY one would expect a nonvanishing $\mu$ proportional to the size of SUSY-breakdown in the observable sector. The second case might consider an accidental symmetry valid for the renormalizable terms in some part of the theory. As an example one might consider a theory where the (renormalizable) superpotential has a higher symmetry than the full theory[74]. This symmetry might forbid a $\mu$-term or such a symmetry might be spontaneously broken, leading to a massless Higgs-doublet as a Goldstone boson. Since this symmetry is not a symmetry of the full theory one expects a generation of $\mu$ in those cases, again proportional to the SUSY breaking terms for the same reason as above. The question remains, why the superpotential should have a higher symmetry than the full theory. An accidental symmetry is not so easy to achieve if one considers also nonrenormalizable terms in the superpotential. These higher terms might induce large $\mu$ values, if some heavy fields receive nonvanishing vacuum expectation values.

All in all it seems that a satisfactory solution of the $\mu$-problems requires the introduction of a small parameter in the theory. In the example with the axion this parameter was $f_a/M_{\mathrm{Planck}} \sim 10^{-8}$ and a satisfactory $\mu$ term was generated through nonrenormalizable terms in the superpotential. It worked because $f_a \sim M_S$ thus giving $\mu$ the desired order of magnitude. Of course, another solution of the $\mu$-problem would be to ignore it!?



# 9. OUTLOOK AND CONCLUSIONS

We have seen that the supersymmetric model provides an interesting framework for physics beyond the standard model. In contrast to the standard model itself it might even have a simple grand unified extension. Unfortunately up to now the model remains a theoretical dream. No sign of supersymmetry has been detected yet. We did not have the time here to discuss the experimental limits for the various superpartners. Such a discussion can be found in [75] with the yearly updates given in the big conferences. Of course, still plenty of parameter space remains unexplored and we have to keep in mind, that also the Higgs boson of the standard model has not been found. So we have to wait and see.

On the more theoretical side there could come some progress as well. I had no time to discuss these developments in the lectures at this school and will give an account of these issues elsewhere[76]. Among the much discussed subjects is the embedding of the supergravity models in the framework of string theory. This might lead to more detailed information on the nature and the size of the soft breaking terms, also in connection with the mechanism of supersymmetry breakdown via gaugino condensates. Stringy symmetries like so-called targed space duality could play an important role in this process. For review and a list of references see ref. [77,78].

In these lectures, I concentrated on the simplest model with an exact R-parity. This leads to a stable particle that might have cosmological relevance. You have heard about that during this school. But there are alternatives[79]. Of course, such models then necessarily will have some amount of L(epton-number)-violation in dimension four operators and it is not clear whether we would like to have those. Alternative choices of discrete symmetries might avoid a possible problem with the dimension-5 operators in grand unified models [49,50] at the expense of L-violation. May be this could be relevant in connection with the solar neutrino problem[80] as well as many particle physics and cosmological phenomena.

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