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EXCERPT FROM "REPORT OF THE DPF LONG RANGE PLANNING WORKING GROUP ON QCD"

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1 Lattice QCD

1.1 Introduction

Many important features of QCD lie outside the reach of perturbation theory. In order to study them one must resort to non-perturbative techniques. In particular, one must be able to regularize and renormalize the theory in a non-perturbative manner. The formulation of QCD on a lattice in Euclidean space-time, introduced by Wilson in 1974, provides such a regularization and establishes a powerful framework for studying the non-perturbative properties of QCD and other quantum field theories. Since its introduction, lattice QCD has formed the basis for a very large number of investigations of hadron properties. Of special importance has been the fact that the lattice regularization permits the application of numerical simulation techniques to the analysis of the quantum fluctuations. These have been used successfully to derive several quantitative predictions from the first principles of QCD[1].

Lattice QCD does not hold the unique key to the study of the non-perturbative properties of hadrons, even if we add the challenging constraint of a meaningful regularization of the ultraviolet divergences. For instance, a calculation based on the expansion into quantum fluctuations around a semiclassical solution of $O(1/g)$, even if the expansion takes a perturbative form, would embody non-perturbative effects. However, because of the very special role that the lattice formulation has played in the study of non-perturbative QCD phenomena and because of the many results that have been obtained through its application, this entire section will be dedicated to it.

1.2 Lattice QCD in Euclidean Space-Time

Methodology

The most important aspect of lattice QCD is that it provides a gauge regularization of the ultraviolet divergences which does not require a gauge fixing. This is accomplished by taking finite elements of the gauge group, rather than the gauge potentials $A^i_\mu(x)$ which are elements of the gauge algebra, as dynamical variables. These finite elements of the gauge group $U^{c,c'}_\mu(x)$ are color $SU(3)$ matrices and are defined over the oriented links of a lattice in Euclidean space-time[2]. In most applications this is a hypercubical lattice with lattice spacing a. Matter fields (the quark fields ψ, $\bar\psi$ in QCD) are defined over the sites of the lattice. The gauge field variables $U_\mu(x)$ and the quark fields are combined into gauge invariant expressions, which form the building blocks of the discretized space-time action. This consists of a pure gauge term $S_g(U)$, which reduces to $\int \frac{1}{4} F_{\mu\nu} F^{\mu\nu} d^4x$ in the continuum limit, and of a matter field term $S_q(\psi, \bar\psi, U)$, which discretizes the Dirac term of the continuum action. In terms of these variables the quantum expectation value of any observable is given by

$$< O >= Z^{-1} \int dU\, d\bar\psi\, d\psi\, O(U, \bar\psi, \psi)\, e^{-S_g - S_q} \qquad (1)$$

with

$$Z = \int dU\, d\bar\psi\, d\psi\, e^{-S_g - S_q} \qquad (2)$$

If one considers a system of finite space-time volume V at first, letting $V \to \infty$ at the end of the calculations, the integrals in the two equations above are integrals over a finite, albeit very large, number of variables. These integrals are either over a compact domain (for the group elements U) or over Grassman variables ($\bar\psi$, ψ), and thus they represent mathematically well-defined, finite quantities. Since O can be any observable, Eqs. 1, 2 provide in principle the description of all QCD phenomena. Of course, in principle is the keyword. Although the integrals are well defined, they are quite complex and calculating them, even in an approximate manner, is a formidable task. Moreover, at the end of the calculation the regulator given by the finite lattice spacing a must be removed in order to obtain continuum results. This is done by readjusting the coupling constant g which appears in S_g and determines the strength of quantum fluctuations of the gauge field. g plays the role of a bare coupling constant. In the process of renormalization g and a are sent simultaneously to zero, with a functional relation $a = a(g)$ determined in its leading orders by asymptotic freedom, in such a way that all physical observables tend to a finite limit[3].

The regularization of QCD given by Eqs. 1, 2 is non-perturbative and permits the implementation of many calculational techniques, frequently similar to techniques used in statistical mechanics, which are not available in the more conventional perturbative schemes of renormalization. Of particular importance is the possibility of applying powerful computational methods to an approximate calculation of the quantum expectation values.

The computational analysis of lattice QCD proceeds first through the integration over the quark fields $\bar{\psi}, \psi$, which can be done explicitly because the matter part of the action S_q is bilinear in the quark fields. This leads to integrals over the gauge variables only

$$< O >= Z^{-1} \int dU \; < O >_U \; e^{-S_{eff}} \tag{3}$$

where $< O >_U$ stands for the average of O over the quark field fluctuations alone, in the background provided by the gauge field U, and

$$S_{eff} = S_g - \log \det[D(U)] \tag{4}$$

$D(U)$ being the lattice Dirac operator that appears in S_q.

Because of γ_5 invariance, $\det[D(U)]$ is a positive, semidefinite quantity. $e^{-S_{eff}}$ can therefore be taken as a measure factor in the space of the gauge variables $U_\mu(x)$ and the integrals giving $< O >$ can be approximately calculated by numerical simulation techniques. This is the essence of the computational methods underlying the majority of the numerical studies of QCD performed in the past, or envisioned for the future. There are, however, some important remarks which must be appended even to the most concise description of the methodology of lattice QCD.

i) Numerical simulations techniques proceed by averaging over a very large number of "configurations" of the system (in our case the collection of all $U_\mu(x)$) distributed according to the desired measure. These are obtained through repeated "upgrades" of the dynamical variables $U_\mu(x)$, in which these are either individually or collectively replaced by new values, according to some definite stochastic or deterministic algorithm. Since the number of dynamical variables is huge and the number of upgrades required for reasonably accurate averages can also be very large, it is crucial that the upgrades be done by the computer as rapidly as possible. With a measure factor that involves only couplings between neighboring variables, such as the exponential of the pure gauge part of the action e^{-S_g}, an individual upgrade requires a small number of arithmetic operations (these can range in the thousands, but this is still a small number with respect to the typical number of dynamical variables and to the overall scale of the computation) and, in any case, independent of the volume of the system. But this is no longer the case when the non-local $\det[D(U)]$ is incorporated in the measure. Algorithms to account for the effects of the fermionic determinant, either in an approximate manner or exactly, have been introduced and are routinely applied. They require a few orders of magnitude ($10^2 - 10^4$) more arithmetic operations than are needed with a local measure alone. All of this has prompted the use of an approximation, called the quenched or valence approximation, whereby the gauge field configurations are generated according to the pure gauge measure factor e^{-S_g}. Since in field theoretic terms the fermionic determinant accounts for the creation and annihilation of virtual quark-antiquark pairs, the quenched approximation consists in neglecting $q - \bar{q}$ vacuum polarization effects. Various arguments can be given to support the validity of such approximation. Also, there is a considerable effort in lattice QCD investigations to go beyond the quenched approximation. It is a fact, however, that many computational analyses of QCD, especially those aiming at the largest lattices or smallest quark masses, have been or are currently based on the quenched approximation.

ii) There are some notorious problems in the lattice discretization of the continuum Dirac operator. It is not possible to define a lattice Dirac operator with the formal chiral properties of the continuum one[4]. There are formulations of the lattice Dirac operator which permit meaningful simulations of QCD, but one pays with either an explicit breaking of chiral symmetry (Wilson formulation)[2], which must be recovered through the careful tuning of a mass counterterm, or with a breaking of flavor symmetry, which is restored only in the continuum limit, and limitations on the possible number of flavors (Kogut-Susskind or staggered formulation)[5][6].

iii) The γ_5 invariance which guarantees the reality of $\log \det[D(U)]$ and makes it possible to incorporate the fermionic determinant in the measure is no longer true in presence of a quark chemical potential. Thus numerical studies of QCD at finite baryon number density, while not impossible, are computationally much more demanding and the results much more approximate.

iv) Perturbative techniques can also be applied to the lattice formulation of QCD. The perturbation expansion is more complicated on the lattice than in the continuum because of the loss of Lorentz invariance, but can still be carried out. Lattice perturbative calculations are important and have been done to determine crucial renormalization parameters and to establish a bridge to the more conventional perturbative results[7][8].

Hadron spectroscopy

Among the non-perturbative observables of QCD hadron masses occupy a very prominent role. Accordingly, through the years many lattice calculations have been devoted to the calculation of the hadron spectrum. One considers an observable $O(t)$ with non-vanishing matrix elements between the vacuum and the states with the quantum numbers of the hadron whose mass is being sought. O can therefore act as source for creation of the hadron, $\bar{O}$ as a sink for its annihilation. Typically O

will consist of a quark-antiquark bilinear for the calculation of meson masses, a product of three quark fields for baryon masses or some expression involving the gauge fields for the study of glueballs. Also, it is convenient to project over states of zero spatial momentum by including into the definition of O a sum over spatial sites (a projection over definite, non-zero spatial momentum can also be easily implemented). One uses then simulation techniques to evaluate the Euclidean correlation function (or Green function) $< \bar{O}(t)O(t') >$. On general grounds this is given by

$$< \bar{O}(t)O(t') >= \sum | < \phi|O|0 > |^2 e^{-E(\phi)|t-t'|} \qquad (5)$$

where the sum ranges over all physical states with the quantum numbers of O and the exponential fall-off is due to the fact that one is considering Green functions in Euclidean space-time. From a numerical determination of the leading exponential behavior(s) one can then derive the energy (mass, if one has performed a projection over zero space momentum) of the lowest state(s).

The basis for such calculations was established in the early eighties[9][10]. The intervening years have brought, however, very important refinements in the construction of the source (sink) operators O ($\bar{O}$), by which crucial enhancements of the matrix elements between the vacuum and the desired hadron states have been obtained, as well as constant improvements in the scope and accuracy of the calculations[11]. The actual precision is limited by the statistical nature of the calculations as well as by a variety of systematic errors. The latter are due to the finite volume of the lattice, the finite lattice spacing, practical limitations on the quark mass (the rate of convergence of the algorithms for calculating quark propagators becomes prohibitively slow for small quark masses) and to the finite extent in Euclidean time over which one can calculate the Green function with sufficient accuracy (this limits the precision in the calculation of the masses due to mixing with higher states). In regard to this last point, it is to be noticed that the calculation of Green functions for operators built out of quark fields proceeds through an initial calculation of the quark propagators, which are then combined as appropriate and averaged over several gauge field configurations. Of the overall fall off of the Green functions, a large fraction is then due to the fall off of the quark propagators themselves, and only part to the averaging procedure. This is to be contrasted to the case of the Green functions for gluonic operators, where the entire fall off comes from cancellations among quantities which are typically of order one. As a consequence, the masses of states whose Green functions are given by connected quark lines can be determined with better accuracy than the masses of purely gluonic states (glueballs). Even worse is the situations for states that would involve disconnected quark lines, such as admixtures of q-$\bar{q}$ states and glueballs, for which up to now it has been possible to do very little with lattice techniques.

Currently, large scale calculations of the hadron spectrum in the quenched approximation involve lattices with a spatial extent ranging up to 32^3 sites, time extent up to 64 sites, ultraviolet cutoffs ranging up to $a^{-1} \approx 3GeV$ and spatial volumes ranging up to $(3-4\,Fm)^3$ [11]. The lowest values of the quark mass is best characterized by the corresponding value of the pseudoscalar mass, as it emerges from the calculation. One obtains ratios m_π/m_ρ down to ≈ 0.3 as opposed to the experimental value 0.175. Because of the current algebra relation $m_{ps}^2 \sim m_q$ the square of the pseudoscalar mass is a better indicator of the quark mass and this gives a current value $(m_{ps}/m_{vect})^2 = 0.09$ versus a target 0.03. A lot of attention is being paid to the effects of the finite lattice spacing, finite volume and other sources of systematic effects. Extrapolations based on analyses done for several values of these parameters are used to reach the physical domain. One recent study[12], based on a very careful set of extrapolations over volume, lattice spacing and quark mass, has produced a set of results in excellent agreement with the experimental data (e.g. $m_N/m_\rho = 1.216(104)$ (experimental value 1.222); $m_\Delta/m_\rho = 1.565(122)$ (exp. 1.604); $m_\phi/m_\rho = 1.333(32)$ (exp. 1.327); $f_\pi/m_\rho = 0.106(14)$ (exp. 0.121). In this study m_π, m_K are used to determine the bare quark masses, while m_ρ sets the scale for the lattice spacing. The same study found a value $1740 \pm 71 MeV$ for the 0^{++} glueball mass. A slightly lower but not inconsistent value of $1625 \pm 92 MeV$ has recently been found by another group[13]. These values pertain to the pure gauge system and do not account for possible mixing with $q\ \bar{q}$ states which, as mentioned above, are much harder to calculate. Figures 1 and 2 illustrate the results obtained for hadron masses and for the scalar glueball decay constant in the study mentioned above[12][14].

There are also many investigations which do not rely on the quenched approximation (full QCD simulations or simulations with dynamical quarks). Since including $\det[D(U)]$ (and thus the effects of virtual q-$\bar{q}$ pairs) in the measure is algorithmically very costly, such calculations are typically limited to lattice sizes about one half of the corresponding quenched calculations. Even more important is the fact that the simulation algorithms can be effectively implemented only with rather large quark masses leading to $(m_{ps}/m_{vect})^2 \gtrsim 0.25$, versus 0.09 for the quenched approximation and the experimental value of 0.03 for light quarks. With such large quark masses the whole effect of the fermionic determinant appears to be limited to the renormalization of the bare coupling constant and, moreover, the ratio between the masses of the nucleon and the vector meson stays very close to the heavy quark limit of 3/2. This situation is reminis-

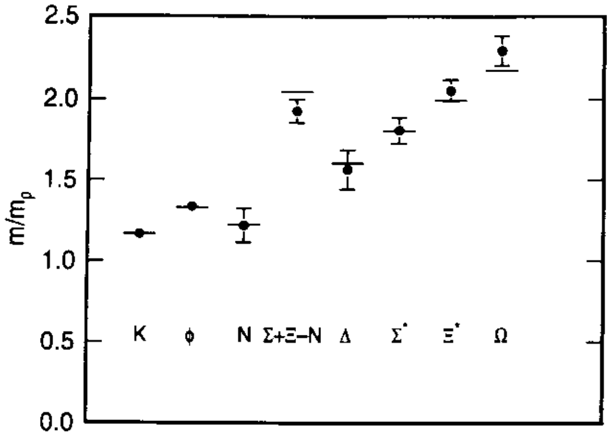

Figure 1: Masses of several hadronic states in units of m_ρ as obtained in a recent large scale quenched calculation. The horizontal lines represent the experimental values.

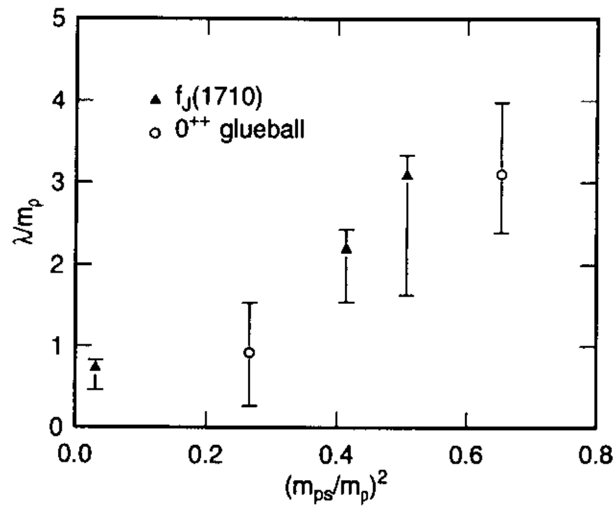

Figure 2: Decay constants of the f_0 (1720) in comparison to lattice results for the scalar glueball.

cent of the earlier quenched calculations, where because of the more modest computer resources and in absence of the recent algorithmic improvements, one was similarly limited to large quark masses. It is very likely that in the near future the progress in non-quenched calculations will parallel the advances achieved by quenched spectrum calculations during the last few years.

Another set of spectral data of great interest in QCD are the masses of states containing heavy quarks. These are too large for a direct lattice calculation based on the formalism outlined above, but can still be calculated with good accuracy either by using the lattice to evaluate the potential binding the heavy quarks (and the spin dependent potentials) or by developing an effective theory to describe the degrees of freedom of the heavy quarks in a non-relativistic approximation. These approaches give origin to interesting issues of renormalization, where substantial progress has recently been made[15]. The splittings among different states in the heavy quark families can be calculated with precision, and these results can in turn be used to determine the value of the coupling constant α_S. Recently values for $\alpha^{(5)}_{\overline{MS}}$ at M_Z clustering around 0.110, with errors ± 0.008, have thus been found, with the major element of uncertainty coming from the corrections one has to make to the quenched approximation to include short-distance quark polarization effects[16]. It is to be noticed that these calculations of the strong coupling constant are already competitive with those based on perturbative QCD and that the lattice may soon provide the way to produce the most precise determinations of α_S. A compilation of values of α_S, obtained by perturbative and by lattice methods, is presented in Figure 3 [16].

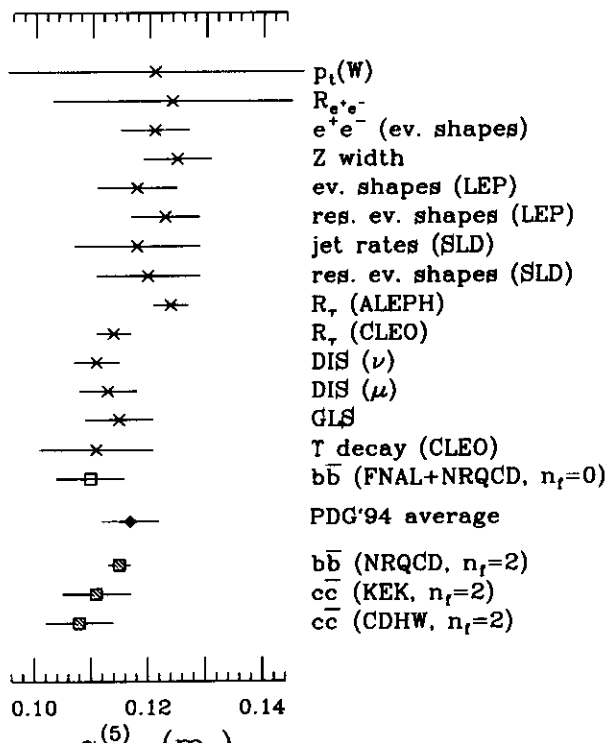

Figure 3: Compilation of results for α_S. The crosses denote results based on perturbative QCD, the boxes results based on lattice QCD. The shaded boxes represent very recent results which are not part of the PDG'94 average.

High temperature QCD

The demonstration that quenched QCD undergoes a deconfining transition at a temperature of approximately $200MeV$ was one of major successes of lattice QCD. The study of QCD at a finite temperature T proceeds through a path integral formulation of the thermal average

$$< O >_T = Tr(Oe^{-H/T}) \tag{6}$$

The exponential is interpreted as a propagation factor for an Euclidean time $t = 1/T$ and one is thus lead to consider a system quantized in an Euclidean domain of infinite extent in the spatial directions but finite extent $1/T$ in the temporal direction, where periodic (antiperiodic) boundary conditions are imposed on bosonic (fermionic) fields to implement the trace in Eq. 6. In practice this system is simulated on a lattice of N_s sites in the space directions and $N_t \ll N_s$ sites in the time direction. The temperature is related to lattice spacing and temporal extent by $T{=}1/(N_t a)$.

The properties of the deconfining transition in quenched QCD have been by now rather well established[17]. The critical temperature, the order of the transition (weakly first order) and other observables, such as the surface tension of nucleating hadrons, have been determined. Many of the current efforts are being devoted to simulating hot QCD with dynamical quarks[18][19][20]. As one would expect, the creation and annihilation of virtual q-$\bar{q}$ pairs has strong effects on the dynamics of the thermal fluctuations. These are felt even for moderately large quark masses and lattice simulations have shown that they can alter the nature of the transition. For large quark masses, of course, one expects little departure from the results of quenched QCD. For intermediate quark masses, full QCD simulations indicate a weakening of the transition, which appears to change to a rapid cross over from the hadronic medium to a quark-gluon plasma with no discontinuities. Also, the temperature of the transition is lowered with respect to the quenched case. At exactly zero quark mass there are theoretical arguments for a transition driven by the restoration of chiral symmetry, which is of the second order for two quark flavors and of the first order with $N_f > 2$. The interesting case is, of course, the one of light, but not vanishing quark masses and current investigations have been focusing on this situation. There is good numerical evidence that a first order transition persists into the domain of finite (non-zero) quark mass with four flavors of light quarks. For the more realistic case of two light quarks and one quark of intermediate mass the results are still inconclusive, but it is realistic to expect that substantial progress will soon be made.

Weak matrix elements

Hadronic matrix elements of weak operators are very important for extracting the parameters of the electroweak theory from the experimental data and, more generally, for testing the predictions of any fundamental theory of weak interactions against experiment. Lattice techniques can be used to calculate many of these matrix elements[21][22][23].

The difficulty in calculating these observables, and consequently the precision which can be achieved, depends a lot on the type of matrix element under consideration. If we recall that individual hadronic states are isolated on the lattice by the "filtering ability" of the propagation in Euclidean time, which enhances the contribution from the lightest states, it will be clear that matrix elements between the vacuum and a single particle state, such as those encountered in the calculation of pseudoscalar decay constants, are much easier to calculate that those involving two or three external particle states. Additional difficulties lattice calculations have to contend with, beyond the general need of correcting for finite lattice spacing, finite volume etc., come from the fact that the weak interactions frequently involve scales much larger than the lattice momentum cutoff. This can be taken care of by an operator product expansion of the interaction followed by renormalization down to the scale of lattice momenta. Problems of renormalization thus play a very important role in the lattice determination of weak matrix elements and much progress has been made in this field. Perturbative and non-perturbative methods of renormalization have been developed and are routinely used to relate the quantities calculated on the lattice with their continuum counterparts[21][22][23].

Pseudoscalar decay constants for the light mesons (f_π, f_K) are calculated together with the masses in the hadronic spectrum and the results are in good agreement with experiment. Recently much attention has been paid to the calculation of the decay constants for heavy-light mesons, f_B and f_D. Since the mass of the b meson is larger that the lattice cutoffs which can be reached in present calculations, the determination of f_B can proceed either through the use of a static approximation for the heavy meson or via an extrapolation of results obtained for lighter mesons. Earlier calculations showed a marked discrepancy between the results obtained by the two methods, but a better understanding of the static approximation and of various renormalization factors has brought the two sets of results in much better agreement. The values found for f_B thus tend to cluster around $200MeV$, with quoted errors of the order of $10 - 15\%$ and variances between the results obtained by different groups of about as much. Values $f_D \approx 210MeV$ and ratios $f_{B_s}/f_B \approx 1.1$, $f_{D_s}/f_D \approx 1.1$ are also found[22].

Quantitatively meaningful results have begun to appear for semileptonic form factors in the decays $D \to K$, $D \to K^*$, $B \to K^*\gamma$ and for the Isgur-Wise function. Another quantity for which very substantial progress has been made is B_K, the $K^0 - \bar{K}^0$ mixing parameter. It is possible to quote today a lattice value $B_K(2GeV) = 0.616 \pm 0.020 \pm 0.017$ ($\hat{B}_K = 0.825 \pm 0.027 \pm 0.023$)[24].

Many other matrix elements have been considered in the literature, including those governing the non-leptonic decays $K \to \pi\pi$, trying in particular to find a computational explanation for the $\Delta I = 1/2$ rule. As implied above, these are very challenging and for the moment cannot be calculated with confidence, but with the expected improvements in algorithms and computational resources they will also become calculable within the next few years.

Hadron structure and other observables

There are many more QCD observables which can be calculated by lattice techniques. Reasons of space prevent us to go at any depth into their list. Many of these observables are discussed in detail in [1]. First steps have been taken toward the calculation of structure functions. Charge density correlations within hadrons have been determined. A very interesting recent calculation has shown that these are left almost unchanged if one uses so-called cooling techniques in the simulation to suppress short range quantum fluctuations, leaving only long range instanton excitations. This points to an intriguing role played by topologically non-trivial structures in the dynamics of hadrons.

The properties of the QCD vacuum have been the subject of many investigations. Lattice techniques have been used to evaluate observables such as the magnitude of the fluctuations of the topological charge and the gluon condensate. They have helped clarify the effects of monopoles in the maximally Abelian gauge and investigate the gauge fixing ambiguities encountered for large fields. Altogether, the lattice formulation of QCD is much more than a tool for the numerical determination of experimental observables. Suitably used, it can provide valuable insights into the whole dynamics of strong interactions.

Discussion of the errors

Since lattice QCD calculations are based on sampling techniques, the results are affected by statistical errors. In general it is rather straightforward to estimate the magnitude of the statistical errors (exceptions are the cases where metastabilities make it difficult to reach statistical equilibrium) and these are universally quoted together with the results. Somehow more difficult is the estimate of the systematic errors coming from finite lattice spacing, finite volume, the quenched approximation (if used) and all other approximations required to implement the numerical simulations. A lot of attention is generally paid to these sources of error and various procedures, such as repeating the calculations with different lattice sizes and different values of the bare coupling constant (which, through the renormalization relation $a = a(g)$ implies different lattice spacings), are used to estimate the magnitude of the systematic effects and, if possible, to correct for them.

Nevertheless, although these (statistical and systematic) errors can be quantified, there are other elements of uncertainty which depend to a large extent on the questions which are being asked and on what one is willing to assume. This is what makes the often heard question "when will lattice calculation produce a result accurate to (say) 5% for the ratio m_ρ/m_N?" difficult to answer. Depending on what theoretical assumptions one is willing to accept, such a rate of precision has already been achieved or may be still several years far away. The recent calculation of the spectrum considered above is a case in point. Large samples of configurations have been used to reduce the statistical errors and very careful extrapolations in lattice volume and lattice spacing have been made. Still the calculation could only be performed for quark masses larger or equal to approximately one half the strange quark mass and in the quenched approximation. Recent theoretical studies, based on chiral perturbation theory, of the quenched approximation indicate that the limit of zero quark mass is singular. Taken per se this would seem to invalidate completely the extrapolation in quark mass that was used to derive the masses of hadrons made of u and d quarks: on theoretical grounds one would not trust a linear extrapolation for the quenched approximation. At the very least, one would want to see the values it produces with much lighter quark masses. But then the effects of $q - \bar{q}$ vacuum polarization effects are expected to become important and one would not trust the quenched approximation anyway. This road leads to the conclusion that the only reliable results would be those of full QCD simulations done with light dynamical quarks. Such simulations are certainly several years away.

But one can look at things from a different perspective. One can give theoretical arguments in support of the fact that hadron masses should exhibit a smooth behavior as function of the masses of the quarks. From this point of view, one can then assume the legitimacy of a linear extrapolation in m_q (using squared masses, on current algebra arguments, for the lightest pseudoscalars), which finds confirmation in the experimental data. Notice that even with this assumption, the slopes and intercepts of the linear fits remain as important, and quite non-trivial, non-perturbative observables of QCD. The lattice calculation of the spectrum, done within a range of values for m_q where the quenched approximation is expected to be valid, provides then a quantitative determination of these observables. This is a major accomplishment, for which one would have held little hope prior to the advent of lattice QCD.

1.3 Alternative Discretization Techniques

Null-plane quantization

The null-plane quantization is a well established, alternative method of defining a quantum field theory where one of the light cone coordinates, e.g. $x^+ = (x^0 + x^3)/\sqrt{2}$, replaces the time coordinate x^0 as the evolution variable. It offers some important advantages over the more conventional $x^0 = const$ quantization, such as better properties of the vacuum state, explicit invariance under Lorentz boosts in the x^3 direction and a more direct relationship between deep inelastic structure functions and the wave-functions of quarks within hadrons. Computational techniques based on the null plane quantization have been introduced and studied during the past few years[25]. In this approach one focuses directly on the wave-functions of the hadronic components. In the restricted two dimensional space spanned by the $x^+, x^- = (x^0 - x^3)/\sqrt{2}$ coordinates the gauge field interaction produces a linear potential, which gives origin to confinement. The extension to four dimension can be accomplished by discretizing the space of transverse coordinates x^1, x^2. The challenge is then to show that confinement survives this extension of the degrees of freedom and to incorporate all appropriate renormalization effects. As a computational technique, the null-plane quantization of QCD has not been as widely studied as the Euclidean lattice formulation, but it constitutes a quite different approach with the potential of producing valuable complementary results.

Hamiltonian QCD and other approaches

In the Hamiltonian approach to lattice QCD one discretizes the space coordinates, but maintains a continuous time variable. The gauge dynamical variables are finite group elements associated with the oriented links of the spatial lattice (very much like in the Euclidean formulation) and their conjugate momenta, which are the components of the chromoelectric field. The evolution is in real time and is generated by a well-defined Hamiltonian operator[5]. Indeed, if one considers a system of finite volume, this is a many-body Hamiltonian with a finite number of degrees of freedom. One tries then to find good approximations for the wave function of the vacuum and of the hadrons, and for the energy levels of these states, typically by using variational techniques. The major difficulty in this approach is the need of incorporating a very large number of components in the wave functions, a problem which is bypassed in Euclidean lattice QCD by simulating directly the quantum fluctuations. Thus, unless one succeeds in producing extremely good *Ansätze* for the wave functions, there are serious limitations to the accuracy which can be achieved.

Several other computational techniques, for example methods based on the derivation of equations relating the expectation values of transport factors (Wilson loops), have been proposed and studied. In addition, one should mention the large body of analytical work that has been and is been done in the context of lattice QCD. Research in this field is indeed far from being exhausted by the numerical simulations. Perturbative calculations, done by analytic expansion techniques, play a crucial role in defining various renormalizations that must be made to bridge the gap to the continuum. Analytic methods have been used to study finite size effects, to study gauge fixing ambiguities and their implications, to calculate the spectrum of QCD in a small box, to perform large N_c and strong coupling expansions etc. Very much like what is happening in other fields of physics, in lattice QCD one is also finding that analytical and computational methods complement each other and together provide a very powerful tool for deriving quantitative predictions.

1.4 Expected Progress

Computational resources

Lattice QCD calculations are very demanding computer applications. The size of the lattices one can consider, as well as other important parameters such as the values of the quark mass, depend in a crucial manner on the number of variables one can store and on total number of arithmetic operations one can perform. For a computation of a reasonably limited duration, the latter converts in number of floating point operations per second (flops). Indeed, scope and accuracy of lattice QCD calculations have steadily increased over the years as computers have gained in memory capacity and speed. The pace of progress in computer technology is forecast to continue for years to come and thus one can correspondingly foresee very substantial, hardware driven improvements in lattice QCD calculations.

While the advance in computer technology is obviously quite independent of lattice QCD applications, which can thus ride the wave of commercial development, the very special computational features of such applications has stimulated the design and construction of dedicated computers, to be used exclusively (or mostly) as a laboratory for the numerical simulation of QCD[26]. The rationale behind such developments is that the highly organized structure of data and communications in QCD applications permits an optimal utilization of parallelism, so that one can gain in economy and efficiency by designing and building a supercomputer targeted to these calculations. Dedicated machines capable of sustained speeds of several Gigaflops have been built and used successfully in the US and abroad[26].

At present there are two projects within the US for dedicated QCD supercomputers capable of reaching into the Teraflops domain:

i) A project pursued by a group at the Columbia University in collaboration with researchers from several other institutions plans to use digital signal processors and a rather streamlined communications architecture to achieve a peak speed of 0.8 Teraflops and a sustained speed of 0.5 Teraflops[27]. The total cost of this project is estimated at M$ 3.

ii) The QCD Teraflops project plans to enhance a commercially available machine with special multiprocessor boards carefully designed to take advantage of the locality features exhibited by QCD calculations (and of many other large scale applications as well)[28]. This supercomputer, with a peak speed of 1.6 Teraflops and an estimated sustained speed in excess of 1 Teraflops, would anticipate the pace of commercial development by a few years and at a fraction of the cost (estimated cost M$ 10 development, M$ 25 construction).

The two projects are quite different and, to a large extent, complementary. The Columbia project is for a rather rigid machine, designed to implement the currently available algorithms in an outstandingly efficient and economical manner. The QCD Teraflops project is for a much more general purpose and easier to program supercomputer, which could be fruitfully used also for a wide range of non QCD applications. The importance that the development and implementation of new algorithms are likely to play for the progress of lattice QCD speaks of course in favor of the flexibility of the QCD Teraflops machine, but the Columbia project has on its side its substantially lower cost.

Since either project would require a substantial allocation of funds, issues of access become important. To formulate these in terms familiar to particle physicists, the question is whether a dedicated machine should be considered more like an accelerator, i.e. a facility to serve several groups of experimenters, or like a detector, where the group who built it is entitled to take and analyze the data in an exclusive manner. Given the expectation that the funds allocated to a QCD machine may, directly or indirectly, reduce the total amount of computer resources otherwise available, many researchers within the lattice community have expressed a strong sentiment that any such machine should be operated as a facility. However, the physicists who design and build a special purpose computer can legitimately expect to see their efforts rewarded by some kind of priority in the use of machine. These are important issues, which will require careful consideration.

Algorithms

Advances in supercomputer technology alone are not sufficient for the progress of lattice QCD. The development of Teraflops supercomputers will bring an increase in computer speed and memory of one to two orders of magnitude with respect to what is available today. The number of operations required by a lattice simulation obviously contains the volume of the lattice as a factor. This implies that a sheer increase of computer power, even into the Teraflop domain, can produce little more than a doubling of the size of the largest lattices that can be studied. As a matter of fact, the situation is worse than that. A major motivation for considering larger lattices is to reduce the lattice spacing, coming closer to the continuum limit. One also wants to be able to consider smaller quark masses. But smaller lattice spacings and smaller quark masses both imply a decrease in computational efficiency, through the phenomenon of critical slowing down. The algorithms for calculating quark propagators (a crucial component of almost all QCD simulations) are based on iterative procedures, whose rate of convergence decreases dramatically as the quark mass (or, better, its value in lattice units $m_q a$) is reduced. Thus, in absence of progress leading to more efficient computational procedures, one cannot expect, from hardware developments alone, even the gains that a naive scaling of the number of degrees of freedom would suggest.

The lattice community has always been aware of the importance of algorithm development and the progress in accuracy of lattice QCD calculations has been accompanied, indeed made possible, by crucial advances in computational techniques. Examples of this progress are all the techniques that have been developed to incorporate fermionic degrees of freedom in the simulations, with the discovery of the "hybrid Monte Carlo" algorithm topping the list of the most important breakthroughs[29]. Another example is given by the refinement in the source and sink operators used for spectroscopy and matrix element calculations. Here, indeed, the line between what should be considered algorithm development and what ought to be considered theoretical progress becomes blurred, but correctly so, because the development of better computational techniques typically finds its roots in a better understanding of the physics of the phenomena under investigation.

Current areas of algorithmic research include the development of better methods for calculating quark propagators, which may overcome, or at least moderate, critical slowing down. Multigrid methods[30] as well as other techniques are being studied. Some progress has been made, but more progress will require a better understanding of the properties of the lattice Dirac operator in presence of fluctuating gauge fields.

Another promising direction of progress consists in the computational use of improved actions[31]. Renormalization group ideas have recently been applied to the definition of a "perfect" action, an action which remains unaltered in the renormalization leading to the continuum limit. Actions approximating the properties of perfect actions may permit to recover the features of the continuum limit working with coarser lattices and therefore with smaller number of dynamical variables. The increased computational power (of the computers of the next generation) could then be applied to an improvement of the accuracy of the simulations and to an expansion of their scope.

Observables

The increase in computer power and the progress in algorithms which are anticipated to occur in a time of three to four years will permit substantial improvements in the accuracy of QCD lattice calculations and a widening of their scope.

One expects that it will be possible to perform quenched calculations of the spectrum for light quark masses close to the experimental value (more properly, for values of m_{ps}/m_{vect} close to m_π/m_ρ). For full QCD, barring unanticipated progress in the algorithms for simulating dynamical fermions, one will probably be able to perform calculations with quark masses half way between m_s and m_u, m_d. This should be sufficient to see the $q-\bar{q}$ vacuum polarization effects go beyond a mere renormalization of the bare coupling constant. One should also be able to see genuine departures from the quenched approximation. Precise determinations of α_S from heavy quark spectroscopy can be expected.

The nature of the transition to a quark-gluon plasma will probably be resolved and progress will be made toward a precise calculation of several thermodynamic observables.

One may expect a rather accurate determination of weak matrix elements (perhaps with errors of $\pm 5\%$) for which one is beginning to get quantitative results, as well as an extension of the calculations to matrix elements which cannot be evaluated today.

More observables will become calculable. These may range from phenomenological parameters, such as the coupling constants in effective chiral Lagrangians, to scattering lengths, to moments of structure functions, to quantities relevant to the interface between perturbative and non-perturbative lattice QCD. While this list of observables is potentially very rich and interesting, it would futile to try to define it too exactly now: its overall span will depend on the detailed progress of the methods of lattice QCD as well as on the ingenuity of the scientists who will apply them.

1.5 Long Range Outlook

Looking farther ahead into the future, given the rapid progress of computer technology and the theoretical and algorithmic developments which are likely to occur, it is to be expected that, five to ten years from now, lattice QCD will be able to produce an accurate determination of a wide range of observables, which will provide stringent tests for the underlying theory of QCD and valuable input for theories describing non-strong interactions. However, the actual rate of progress of lattice QCD will depend on decisions which reflect the policies of the entire particle physics community. This leads us to the following conclusions.

Computer resources - The progress of lattice QCD is heavily dependent on the availability of ever more powerful means of computation. Fortunately the development of supercomputer technology is receiving a lot of support at the policy making level, and thus lattice QCD automatically benefits from the expansion of computer resources that this support generates. Lattice QCD applications, because of the huge volumes of data that they manipulate and the extremely large number crunching capability they require, have been acknowledged as one of the driving forces of supercomputer development. Thus there is the possibility of obtaining funding for QCD applications (either as direct funds for the development of dedicated machines, or in the form of increased allocations of supercomputer time) from non-HEP sources because of the impact that QCD applications may have on supercomputer technology. However, this also requires a strong endorsement from the particle physics community of the value of this mode of research, including the willingness of allocating funds to support the necessary hardware and algorithm developments. While any such investment can be leveraged by appreciable funding from non-HEP sources, it would be difficult to expect the latter to occur if particle physicists, first, do not recognize the value of QCD applications.

Exchange of information - Lattice QCD calculations are not a black box which turns out numbers affected by smaller or larger errors. The details of the calculations are frequently as significant as the final results. On the other hand, lattice QCD investigations are of marginal value if pursued in isolation, without exposure to the whole problematic of strong interactions. Thus, it would be valuable if there were better contacts between scientists working on lattice QCD and other areas.

Experiments - Lattice QCD has the potential of producing one day very accurate results, derived entirely from first principles, on many hadronic observables. It is conceivable, for instance, that it will be possible to predict spectroscopic data with a precision sufficient to put stringent tests to the validity of QCD. However, this would be of little value in absence of experimental data to compare with. As the experimental frontier moves to ever higher energies and smaller distances, it is important not to neglect those energy domains, which do not lie at the boundary of technology, but where a lot of very valuable information can still be collected.

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