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III

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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BASICS OF QCD

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HINCHLIFFE ICTP July 1995 (1)

Basics of QCD

(1) Basics, renormalization, μ and R Scheme

dependence of rates and physical processes
- total cross section $e^+e^- \to$ hadrons
  Jets in e^+e^- annihilation

(2) Parton Model GLAP and BFKL structure functions
and evolution.
- FACTORIZATION THEOREM

(3) Preasymptotic effects - Higher twist
- Renormalons
- Heavy Quark effective theory

(4) Examples and applications.

(5) Current Issues
(1) Value of α_s
(2) Top quark mass
(3) "Pomeron".

References E. Reya Perturbative QCD Phys Rep 69C 195

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Handbook of QCD Rev Mod Phys. 67 157 (1995)

K Ellis at TASI 94
A. Martin.
B Grinstein Heavy Quark effective theory
Ann. Rev. Nucl. Part. Sci Vol 42

M Shifman Theory of Preasymptotic effects in QCD. UMN-TH-1253/94.

(2)

(1) Basic Structure

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} + \sum_{\substack{\text{flavours}\\ i}} \bar{q}^i_a (i\not{D} - m)_{ab}\, q^i_b + \theta \tilde{F} F$$

space time index ($\mu\nu$); color label (a)

θ: Neglect ~~de~~ term.

$$D_\mu = \partial_\mu - i g T_a A^a_\mu \quad : \quad g^{\alpha\beta} = \mathrm{diag}(1,-1,-1,-1)$$

$$[T_a, T_b] = i f_{abc} T_c$$

f_{abc}: structure constants of SU(3)
see Particle Data Book for list

T^a is representation dependent. $n \times n$ matrix acting on n dimensional rep.

$$\mathrm{Tr}\,[T^a T^b] = T(R)\, \delta_{ab}$$

$T(3) = 1/2$ (quarks) $T(8) = N$ (gluons)

For $\bar{3}$ rep $T^a = \frac{\lambda^a}{2}$ ← Gell-Mann matrix

For $\bar{8}$ rep $(T^a)_{bc} = f^{abc}$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f^{abc} A_\mu^b A_\nu^c$$

Gauge transformation

$$q_{\dot{a}}(x) \to \exp\left[i T_{\hat{a}} \cdot \alpha^a(x)\right]_{\dot{a}\dot{b}} q_{\dot{b}}(x)$$

$$= (U(x))_{ab}\, q_b(x)$$

$$D_\alpha q \to U D_\alpha q$$

$$t \cdot A_\mu \to U (t \cdot A_\mu) U^{-1} - \frac{1}{ig} (\partial_\mu U) U^{-1}$$

or $$T^A \cdot F_{\mu\nu}^A \to U(x)\, T^A F_{\mu\nu}^A\, U^{-1}(x)$$

8 parameters in Bare Lagrangian
- 6 quark masses
- 1 coupling constant
- θ.

Need to Fix gauge to quantize.

COVARIANT GAUGE $$\mathcal{L}_{gauge} = -\frac{1}{2\lambda} (\partial^\mu A_\mu^a)(\partial_\mu A_a^\mu)$$

$$(1 \leq \lambda \leq \infty)$$

~~Axial gauge~~

$\lambda = 1$ is Feynman gauge

$$\mathcal{L}_{ghost} = \left(\partial_\mu \eta^{a\dagger} D_{ab}^\mu \eta_b\right)$$

η^a is octet ghost field.

Aside why $N = 3$?

$\pi^0 \to \gamma\gamma$

$$M \propto \langle 0 | J_\mu(x) \, J_\nu(y) \, \phi(0) | 0 \rangle \, \epsilon^1_\mu \, \epsilon^\nu_2$$

$$= \frac{1}{2 f_\pi m_\pi^2} \langle J_\mu(x) \, J_\nu(y) \, \partial_\alpha A_\alpha \rangle$$

From quark graph $\Gamma(\pi^0 \to \gamma\gamma) = \left(\frac{\alpha_{em}}{\pi}\right)^2 \frac{1}{64\pi} \frac{m_\pi^3}{f_\pi^2} \; \overset{N}{3} \left[\left(\frac{2}{3}\right)^2 - \left(\frac{1}{3}\right)^2 \right]^2$

$= 7.6 \text{ eV}$

exp 7.7 ± 0.6 eV.

Theory now has extra parameter λ.

$\boxed{\text{AXIAL GAUGE}}$

$$\mathcal{L}_{gauge} = -\frac{\lambda}{2} (n \cdot A)^2 \qquad \lambda \to \infty$$

propagator
$$G^\mu_{\ \nu}(p) = \frac{i}{k^2 - i\epsilon} \left[-g^{\mu\nu} + \frac{k^\mu n^\nu + k^\nu n^\mu}{n \cdot k} - n^2 \frac{k^\mu k^\nu}{(n \cdot k)^2} \right]$$

as $p^2 \to 0$
$$p^\alpha G^\alpha_{\ \nu}(p) \to i \left[\frac{n^\nu}{n \cdot p} \mp \frac{n^2 k^\nu}{(n \cdot k)^2} \right]$$
$$n^\alpha G^\alpha_{\ \nu}(p) \to 0$$

$\Rightarrow$ only 2 propagating states since no pole at $p^2 = 0$

$\Rightarrow$ No ghost needed.

Special gauge is Light cone gauge $n^2 = 0$

choose polarization state ϵ such that $(n\cdot\epsilon) = (k\cdot\epsilon) = 0$

Parton picture is simple in this gauge, but loop calculations are difficult because poles at $(n\cdot p) = 0$ need regularization.

~~Regular.~~

$$\boxed{\begin{matrix}\text{REGULARIZATION}\\ \text{RENORMALIZATION}\end{matrix}}$$

Bare and renormalized quantities are related by

$$A = \frac{1}{\sqrt{Z_3}} A_0 \qquad \lambda = \lambda_0 / Z_3$$

$$q = \frac{1}{\sqrt{Z_2}} q_0 \qquad m_0 = m Z_m$$

Coupling constant can be defined from  

or . All related by Ward Identity

$$g_0 = g\mu^{\epsilon} Z_1 / Z_3^{3/2} = g\mu^{\epsilon} Z_F / Z_3\sqrt{Z_2} = g\mu^{\epsilon} Z_4^{1/2} / Z_3$$

Here I have used dimensional regularization.

$$d = 4 - 2\epsilon \qquad \frac{d^4k}{(2\pi)^4} \rightarrow \frac{d^d k}{(2\pi)^d}$$

Note coupling constant has dimension if $d \neq 4$ hence μ^{ϵ}.

Dirac rules $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$

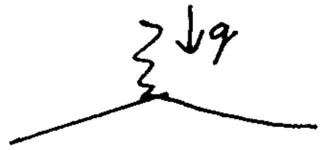
$$\gamma^\mu\gamma_\mu = d\, I \;, \quad \mathrm{Tr}\, I = 4$$

$$\mathrm{Tr}(\gamma^\mu\gamma^\nu) = 4g^{\mu\nu}, \quad \gamma^\mu\gamma^\alpha\gamma_\mu = -2(1-\epsilon)\gamma^\alpha$$

RENORMALISATION SCHEME

Specified by defining g in terms of a physical quantity

recall in QED

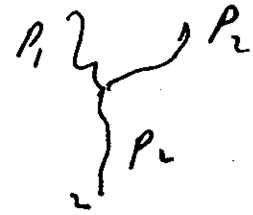


$e \equiv$ value of vertex [electron charge] at $q = 0$ on mass shell for electron.

Does not work in QCD (i) infra red problem
(ii) quark masses are zero for light quarks.

(A) Momentum scheme

$g \equiv$ value of



at $p_1^2 = p_2^2 = p_3^2 = \mu^2$

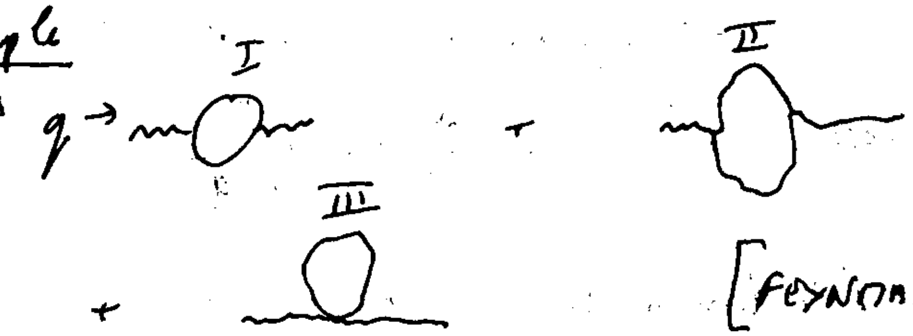
Arbitrary, inconvenient and unphysical.

(B) $\overline{MS}$ scheme drop all $\frac{1}{(d-4)}$, γ_E and $\log 4\pi$

Arbitrary, well defined, easy to use, unphysical

(c) others.
Result depends on arbitrary scale μ.

Example



[FEYNMAN GAUGE]

$$\Pi^{\mu\nu}_{AB}(q^2) = \left(g^{\mu\nu} q^2 - q^\mu q^\nu\right) \Pi_{AB}(q^2)$$

I $$\Pi^1_{AB} = -\frac{4}{3} g^2 T_R \delta_{AB} \mu^{2\epsilon} \left[B(q,m,m)\left(1 - \frac{\epsilon}{3} + \frac{2m^2}{q^2}\left(1 + \frac{2\epsilon}{3}\right)\right) - \frac{2A}{q^2}\left(1 - \frac{\epsilon}{3}\right) \right] + O(\epsilon^2)$$

II $$= \frac{5}{3} \delta^{AB} N g^2 \, B(q,0,0) \left[1 + \epsilon/15\right] \mu^{2\epsilon}$$

III $$= 0$$

$$A = \int \frac{d^\kappa k}{(2\pi)^\kappa} \frac{1}{(k^2 - m^2)} \; ; \quad B = \int_0 \frac{d^\kappa k}{(2\pi)^\kappa} \frac{1}{(k^2 - m^2)} \frac{1}{(k+q)^2 - m^2}$$

$$= \frac{i m^2}{16\pi^2}\left[D + 1 - \log m^2\right]; \quad = \frac{i}{16\pi^2}\left[D - \int_0^1 dx \log\left(m^2 - x(1-x)q^2\right)\right]$$

$$D = \frac{1}{\epsilon} + \log 4\pi - \gamma_E \rightarrow 0 \text{ in } \overline{MS} \text{ scheme}$$

8)

Note $\overline{MS}$ result depends on $\log(q^2/\mu^2)$

μ is arbitrary $\Rightarrow$ Physical result cannot depend on μ.

Physical process $\quad P^N(\alpha_s, Q^2, \mu) = \alpha_s^P \sum_{k=0,N} a_k \alpha_s^k$

Suppose that P is dimensionless process at energy scale Q

e.g. $\quad \dfrac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$.

$$P^N(\alpha_s, Q^2/\mu^2)$$

Exact result $P^\infty \equiv P$ must be independent of μ.

$$\frac{d}{d\mu} P = 0 \Rightarrow \mu^2 \left[\frac{\partial}{\partial\mu^2} + \frac{\partial\alpha_s}{\partial\mu^2}\frac{\partial}{\partial\alpha_s}\right] P \equiv 0$$

$$\beta(\alpha_s) \equiv \mu^2 \frac{\partial\alpha_s}{\partial\mu^2}$$

Define $t = \log\mu^2$

$$\left(-\frac{\partial}{\partial t} + \beta(\alpha_s)\frac{\partial}{\partial\alpha_s}\right) P = 0$$

$$P(Q^2/\mu^2, \alpha_s) = P(1, \alpha_s(Q^2))$$

$$t \equiv \int_{\alpha_s}^{\alpha_s(Q)} \frac{dx}{\beta(x)} \quad \text{defines } \alpha_s(Q).$$

$$\beta(\alpha) = -\alpha \sum \beta_n \left(\frac{\alpha}{4\pi}\right)^n$$

$$\beta_0 = 11 - 2n_f/3 \quad , \quad \beta_1 = 102 - \frac{38 n_f}{3}$$

are scheme dependent n_f = # flavors of quark

$$\beta_2^{\overline{MS}} = \frac{2857}{2} - \frac{5833}{18} n_f + \frac{325}{54} n_f^2 .$$

Can solve for $\alpha_s(Q^2)$ as power series in $1/\log Q^2$

$$\alpha_s(Q^2) = \frac{12\pi}{33-2n_f}\left[1 - \frac{6(153-19n_f)}{(33-2n_f)^2}\frac{\log(\log Q^2/\Lambda^2)}{\log Q^2/\Lambda^2}\right]\frac{1}{\log Q^2/\Lambda^2}$$

[neglecting β_2]. This <u>DEFINES</u> Λ.

Note $\alpha_s(\mu_0)$ is the physical parameter
Need to specify value of $\alpha_s(\mu_0)$, μ_0 and scheme.

Can use Λ instead of $\alpha_s(\mu_0)$

But <u>note</u> ~~a~~. $\Lambda(n_f)$

Need a matching condition to relate $\Lambda(5)$ to $\Lambda(4)$
etc. Problem is best avoided by quoting

$$\alpha_s^{\overline{MS}}(M_Z)$$

Use R.G.E. to evolve all other measurements to this value.

(10)

[More on Renormalization scheme]

(P has several energy scales p_i)

$$Q = f(p_i)$$

Example R depends on $\sqrt{s}$. Use $Q^2 = s, s/2$??

Choice of Q is arbitrary result will not depend on it.

$$\frac{dP^{\infty}}{dQ} = 0.$$

But at Fixed order $\frac{dP^N}{dQ} \neq 0$.

$$P = \alpha_s^A(Q)\left[1 + a_1(Q)\,\alpha_s(Q) + a_2(Q)\,\alpha_s^2(Q) \;--\right]$$

$$= \alpha_s^A(Q_0)\left[1 + a_1(Q_0)\,\alpha_s(Q_0) + a_2(Q_0)\,\alpha_s^2(Q_0)\right]$$

Note $a_1(Q) \neq a_1(Q_0)$ etc

What is the optimal choice for Q_0?

There isn't one, it's a silly question.

If only a_1 is known [~~NLO~~: Next to Leading order: NLO calculation]

Q_0 is equivalent to scheme choice.

At Next to Next to leading order [NNLO]

Need to specify Q_0 and scheme.

The value of β_2 can be used to fix the scheme

[~~$\beta_2 = 0$ is the~~ $\beta_n = 0$ $n \geq 2$ is t'Hooft scheme].

Popular choices

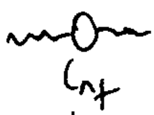
fix Q in a given scheme

(1) PMS $\quad \dfrac{\partial}{\partial Q} \rho^N(Q) = 0$

(2) fastest apparent convergence $\quad a_1 = 0$

(3) BLM effective charge – choose Q so that n_f does not appear in a_1

argument is that n_f is associated with fermion loops



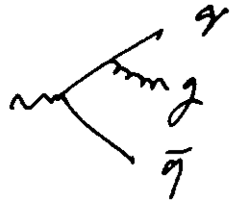
and should be scaled into coupling constants

NB a leading order prediction is no prediction at all since Q dependence is maximal.

$$R_Z = \frac{\Gamma(Z \to \text{hadrons})}{\Gamma(Z \to e^+e^-)} = \frac{3\sum_q (a_q^2 + v_q^2)}{a_\mu^2 + v_\mu^2}$$

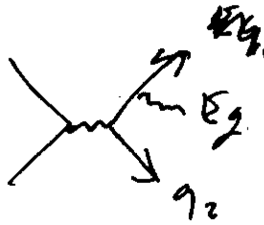
$$Z^\mu \bar{\psi}(v\gamma_\mu + a\gamma_\mu\gamma_5)\psi.$$

next order in α_s



(12)

$$\sigma(e^+e^- \to \text{hadrons}) = \sigma_0 =$$



$$= \frac{8}{3\pi\,\Gamma_Z^2}\,\frac{2G_F^2 M_Z^4}{}\,(v_e^2+a_e^2)\,(v_q^2+a_q^2)$$

Next order $\sigma(e^+e^- \to Z \to \text{hadrons})$

$$= \sigma_0\,(v_q^2+a_q^2)\int dx_1\,dx_2\,\frac{4}{3}\,\frac{\alpha_s}{2\pi}\,\frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)}$$

$x_1 = 2E_{q_1}/M_Z$

$0 \le x_1 \le 1,\ 0 \le x_2 \le 1.$

Divergences appear when $x_1 = 1$ or $x_2 = 1$

$x_1 = 1 \Rightarrow$ either $E_g \to 0$ in which case $x_2 = 1$

or $g \parallel q_2$

[kinematics $(1-x_1) = 2E_g x_2 (1-\cos\theta_{2g})$]

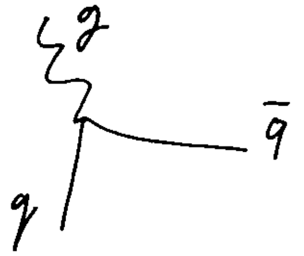
Soft divergence $E_g \to 0 \quad x_1, x_2 \to 1$

Collinear divergence $g \parallel 1 \Rightarrow x_2 \to 1$

or $g \parallel 2 \Rightarrow x_1 \to 1.$

Divergence is unphysical because unobservable experimentally

$\Longrightarrow$ 2 jets

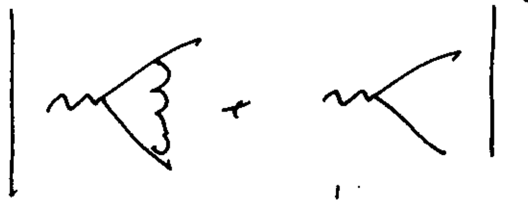


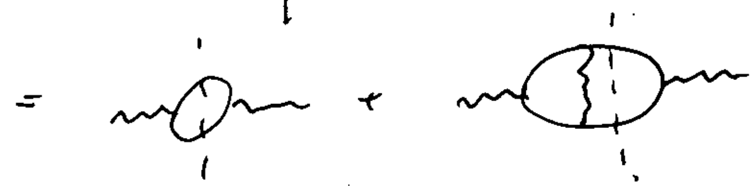
$\Rightarrow$ 3 jets

e  $\Rightarrow$ 2 jets.

Exclusive quantity needs jet definition.

Inclusive does not

Both need virtual graph 



Need a regulator for Infra Red as well as UV divergence. Easiest is Dimensional Reg since it preserves gauge invariance.

d dimensional dirac algebra and phase space gives

$$\sigma_{q\bar{q}g} = \sigma_0 \int dx_1\, dx_2\, A(\epsilon)\, \frac{x_1^2 + x_2^2 - \epsilon(2 - x_1 - x_2)}{(1-x_1)^{\epsilon+1}(1-x_2)^{\epsilon+1}} \frac{2\alpha_s}{3\pi}$$

$$A(\epsilon) = \frac{3(1-\epsilon)^2}{(3-2\epsilon)\Gamma(1-2\epsilon)}$$

$$\Rightarrow \sigma_{gq\bar{q}} = \sigma_0 \frac{2\alpha_s}{3\pi} A(\epsilon) \left[+\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} + O(\epsilon) \right]$$

14

Virtual graph $\sigma = \sigma_0 \frac{2\alpha_s}{3\pi}\left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2\right]$

$$\Rightarrow \quad R = R_0\left[1 + \frac{\alpha_s}{\pi}\right]$$

at next order

$$R = R_0\left[1 + \frac{\alpha_s(\mu^2)}{\pi} + \sum_{n\geq 2} C_n\left(s/\mu^2\right)\left(\frac{\alpha_s(\mu^2)}{\pi}\right)^n\right]$$

$$C_2 = 1.986 - 0.115 n_f - \frac{\beta_0}{4}\log s/\mu^2$$

$$C_3 = -6.637 - 1.2 n_f - 0.005 n_f^2 - 1.24 X + \frac{\beta_0^2}{16}\log^2(s/\mu^2) - \left[\left(\frac{\beta_1}{16} + \frac{\beta_0}{2}(1.986 - 0.115 n_f)\right)\log s/\mu^2\right]$$

$$X = \frac{\left(\sum_q v_q\right)^2}{3\sum_q (v_q^2 + a_q^2)} \quad \text{at} \quad s = m_Z^2$$

Note s/μ^2 dependence in coefficients.

Prediction is much more stable w.r.t. variation of μ

[FIGURE]

Choices for μ

$$\mu_{FAC} \approx 0.7\sqrt{s}$$

$$\mu_{pms} \approx 0.587\sqrt{s}$$

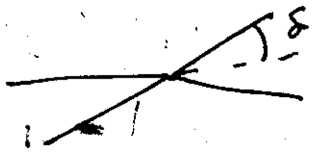
$$\mu_{BLM} = 0.71\sqrt{s} \quad \text{for } n_f = 5$$

Jet $\boxed{\text{JET CROSS SECTIONS}}$

Need a jet definition.

Oldest "Sterman Weinberg"

Fraction of events where all but ϵ of total energy is contained in 2 cones of ½ angle δ defines F_2 the fraction of 2 jet events. This is physical but not easy to extend to large #'s of jets



$$f_2 = 1 - \frac{16\alpha_s}{3\pi}\Big[\log(1/\delta)\Big[\log\Big(\frac{1}{2\epsilon}-1\Big) - \frac{3}{4} + 3 + \frac{\pi^2}{12} - \frac{7}{16} - \epsilon + \frac{3}{2}\epsilon^2 + O(\delta^2 \log \epsilon)\Big]$$

$$f_3 = 1 - f_2$$

You can get this from $\int \frac{dx_1\, dx_2}{(1-x_1)(1-x_2)}(x_1^2 + x_2^2)$ by restricting the integral.

(b)

Fixed angle cone is poor because

as $\sqrt{s}\Uparrow$ non-perturbative effects $\downarrow$

$\Rightarrow$ Smaller cones $\delta/\approx$ ~~$\frac{1}{Q}$~~ $1/\sqrt{s}$ become resolvable.

JADE algorithm

Find smallest $M_{ij} = (p_i + p_j)^2$

$$= 2E_iE_j(1-\cos\theta_{ij})$$

If $M_{ij}^2 < sy$ then combine $i+j \to k$.

Iterate until process stops

Then # of "clusters" $\equiv$ # of jets

Algorithm can apply both to ~~jets~~ partons and particles detected in an experiment.

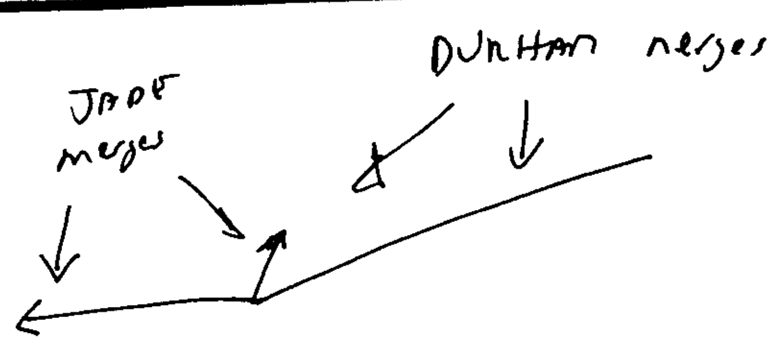
$$f_3 = \frac{2\alpha_s}{3\pi}\Big[(3-6y)\log(y/1-2y) + 2\log^2(y/1-y) + 5/2 - 6y + \frac{9y^2}{2} + 4\,Li_2(y/1-y) - \pi^2/3\Big]$$

$f_2 = 1 - f_3$. To order α_s.

(FIGURE)

A Better algorithm is DURHAM

$$M_{ij} = 2\min(E_i^2, E_j^2)[1-\cos\theta_{ij}]$$

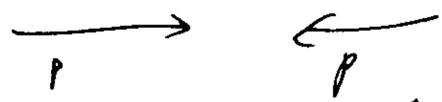


Durham ~~merges~~ merges slow one with the **nearest** one = more physical.

other jet properties not discussed.

(i) gluon vs quark jets

(ii) Jets in ep or pp usually defined by a cone algorithm and $p_\perp$ threshold



gather energy in a cone of size $\Delta R = \sqrt{\Delta\phi^2 + \Delta\eta^2}$

and $p_\perp > p_\perp|_{min}$.

(iii) determination of α_s from jet events at LEP.

~~Proto~~

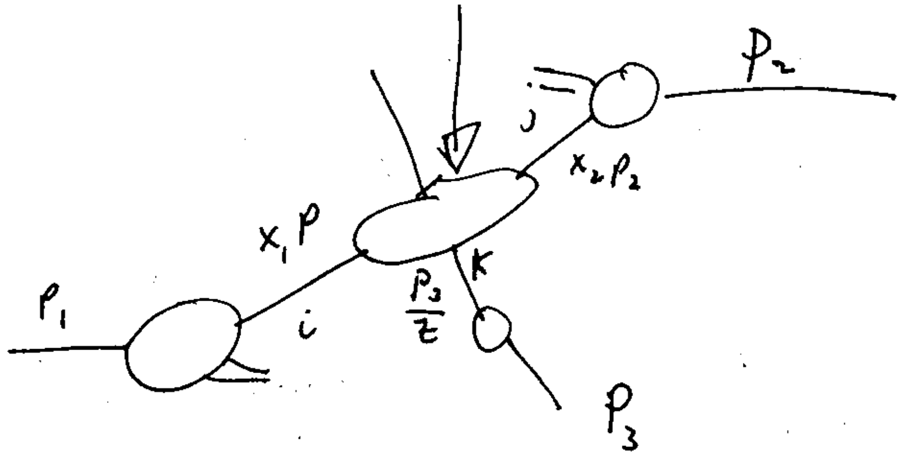
PARTON MODEL

Basic idea

hadron + hadron $\longrightarrow$ hadron + X

$$E\frac{d^3\sigma}{d^3p_3} = \int dx_1\, dx_2\; g_i(x_1)\; g_j(x_2)\; dz\, D_k(z)$$

$$E\frac{d^3\sigma}{d^3p}\bigg|_{i+j\to k+X}$$



Note (i) Not obviously right

(ii) Probabilistic picture

$g_i(x)$ = prob to find parton of type i with momentum fraction x.

(iii) Assumes partons move // parent hadrons

[limited $P_\perp$]

19

Physical intuition is based on argument about scales.

Hadronic size $\sim R$ $\left(1/_{300\,\text{MeV}} \right)$

Energy transfer in parton collision $\sim Q$

$\Rightarrow$ σ(partonic) might be computable in terms of $\alpha_s(Q)$.

$q(x)$ will not be computable

Various "derivations" of the model.

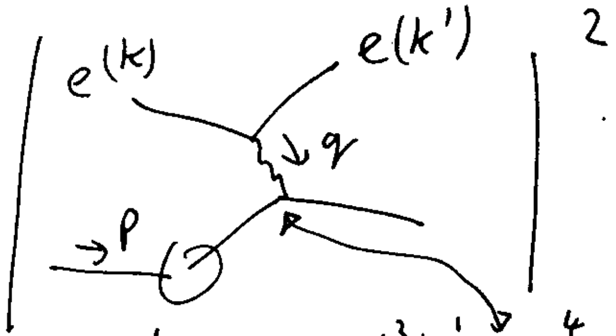
Note that "intuition" is frame dependent.

Basic idea $Q \gg 1/R$ partons inside a hadron cannot interact with each other during the parton scattering — IMPULSE APPROXIMATION.

$\boxed{\text{Best analysed by example}}$

Deep inelastic scattering

$e + p \to e + X$



$$d\sigma = \frac{d^3k'}{2s\,|k'|}\, \frac{g_v^4\, L^{\mu\nu}(k,q)}{(q^2 - m_v^2)^2}\, W^{\mu\nu}(p,q)$$

Lepton current is trivial $L^{\mu\nu}(k,q) = \frac{1}{2} \mathrm{Tr}\left[\not{k}\,\gamma^\mu (\not{k}-\not{q})\,\gamma^\nu\right]$

Hadron current is <u>not</u> trivial — Most general form.

$$W^{\mu\nu} \equiv \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) W_1(x, Q^2) + \left(p - q\,\frac{p\cdot q}{q^2}\right)\left(p - q\,\frac{p\cdot q}{q^2}\right) W_2(x, Q^2)$$

$$+\ i\,\epsilon_{\mu\nu\alpha\beta}\, \frac{p^\alpha q^\beta}{M_p^2}\, W_3(x, Q^2)$$

[Assumes spin average and zero quark masses]

$Q^2 = -q^2$, $\nu = p\cdot q / m_p$, and $x \equiv Q^2 / 2\nu M$

(21)

$W_3^{\gamma}(x, Q^2) = 0$ [parity is conserved]

$$\left.\begin{aligned} F_1 &= W_1 \\ F_2 &= \frac{\nu}{M} W_2 \\ F_3 &= \frac{\nu}{M} W_3 \end{aligned}\right\}$$ Most common forms.

Analyse in terms of photon helicity is useful

In rest frame of target $\cancel{q \Rightarrow t q}$

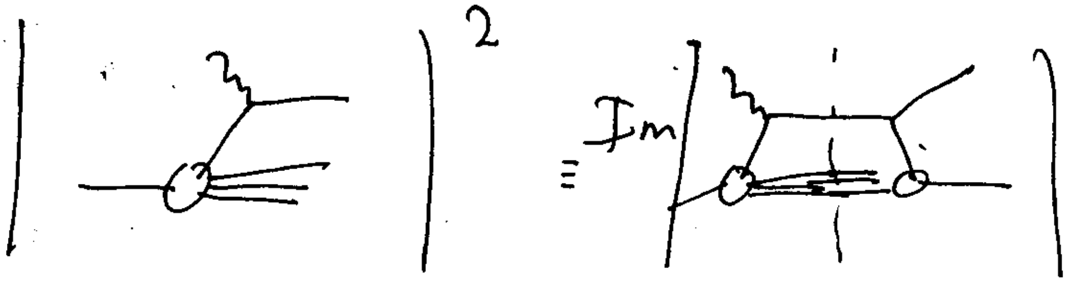
$$\left.\begin{aligned} \epsilon_1 &= \frac{1}{\sqrt{2}}(0, 1, -i, 0) \\ \epsilon_2 &= \frac{1}{\sqrt{2}}(0, 1, i, 0) \end{aligned}\right\} \text{TRANSVERSE}$$

$$\epsilon_3 = \frac{1}{\sqrt{Q^2}}\left[\sqrt{Q^2+\nu^2}\;\; 0, 0, \nu\right]$$

$$W_{\mu\nu} = \sum_{\lambda} \epsilon_{\lambda}^{*\mu} \epsilon_{\lambda}^{\nu} F_{\lambda}$$

$$F_{\mp} = F_1 \pm F_3 \quad \text{and} \quad F_L = \frac{F_2}{2x} - F_1$$

Now calculate in Parton Model

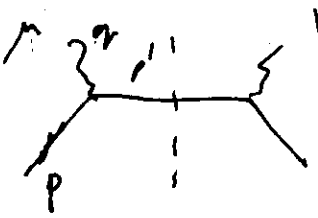


Feynman graph

$$\sigma(x) = \sum_i \int \frac{d\xi}{\xi}\, q_i(\xi)\, \sigma_{i\gamma}(x/\xi)$$

or $$F_i \approx \sum q_i(x)\, \sigma(q+\gamma \to X)$$

$$W^{\mu\nu}[\text{parton}] = \frac{1}{8\pi} \int \frac{d^3p'}{(2\pi)^3} \frac{1}{2E_{p'}} Q^2\, \mathrm{Tr}\left[\gamma^\mu (\not p + \not q) \gamma^\nu \not p\right]$$



$$(2\pi)^4 \delta^4(p - p' - q)$$

$$F_2^{\text{parton}}(x) = 2F_1^{\text{parton}}(x) = Q^2 \delta(1-x)$$

$$\Rightarrow 2xF_1^{\text{hadron}} = F_2^{\text{hadron}} = \sum_i Q_i^2 x q_i(x)$$

<u>Note Independent of Q^2</u>

Can derive sum rules from probability interpretation

$$\int dx \sum x\, q_i(x) = 1 \quad [\text{Momentum must add up}]$$

$$\int dx \sum (q_i - \bar q_i) = 3 \quad [\text{Baryon number}]$$

(23)

$$\int dx \, \left((u-\bar{u}) - d - \bar{d}\right) = 1/2 \quad [\text{Isospin}]$$

$$\int dx \, (s-\bar{s}) = 0 \quad [\text{proton has no strangeness}]$$

These can be related to physical quantities

$$\int \frac{dx}{x}\left(F_2^{ep} - F_2^{en}\right) = \frac{1}{3}\int dx \left[(u-\bar{u}) - (d-\bar{d}) + \frac{2}{3}(\bar{u}-\bar{d})\right]$$

Gottfried sum rule

$$= 1/3 \quad \text{if } \bar{u} = \bar{d}$$

ADLER

$$\int \frac{dx}{x}\left(F_2^{\bar{\nu}p} - F_2^{\nu p}\right) = 2\left(u-\bar{u} - (d-\bar{d})\right)$$

$$= 2$$

Gross Ll smith.

$$\frac{1}{2}\int\left(F_3^{\nu p} + F_3^{\bar{\nu}p}\right) = (u-\bar{u}) + (d-\bar{d})$$

$$= 3$$

These are approximately true

Bjorken

$$\int F_1^{\nu p} - F_1^{\bar{\nu}p} = 1$$

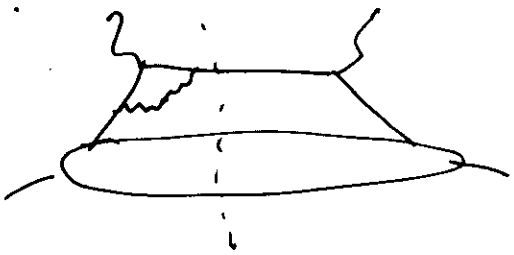
What about QCD?

Compute corrections to parton model.

If model is inconsistent with QCD we will get garbage.

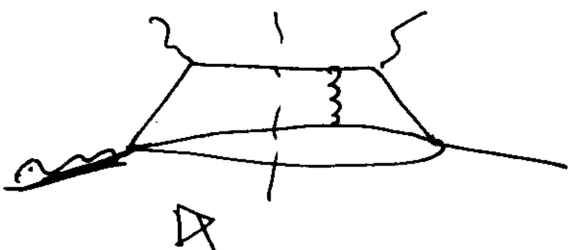
2 types of QCD corrections

(A)



correction to σ (parton)

(B)

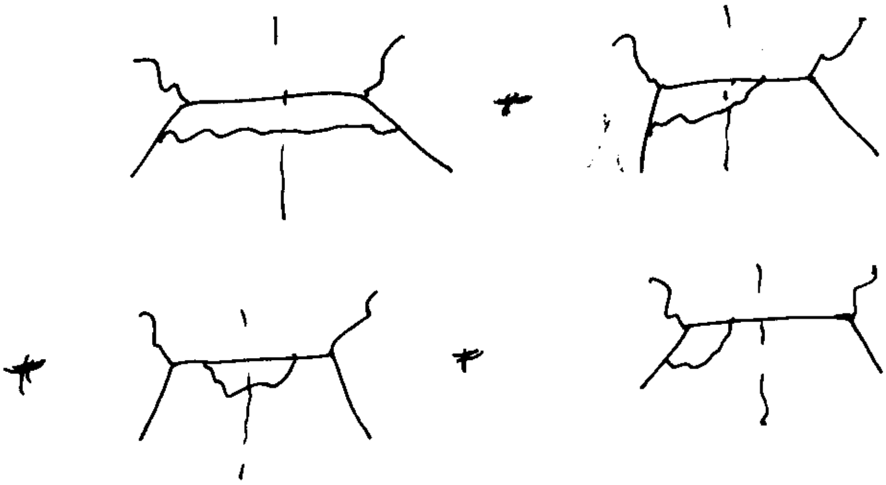


NO place in parton model.

Interference graph Supressed by $\frac{1}{Q^2}$

"Higher twist" effect

look at (A)



(25)

Computing $\gamma q \to X$

$$\sigma = \int |M|^2 \, PS$$

lowest order

$$M^\mu = -ie\,\bar{u}(p+q)\,\gamma^\mu u(p)$$

$$|M|^2 = 4e^2 \quad \text{and} \quad PS = 2\pi\,\delta\big((p+q)^2\big)$$

Next order   Real emission graphs

$$\gamma(q) + q(p) \to g(r) + q(\ell)$$

$$PS \sim d^4r\, d^4\ell \;\; \delta^4(p+q-r-\ell)\;\delta(r^2)\;\delta(\ell^2)$$

$$\equiv \frac{1}{(2\pi)^2}\int d^4k \;\; \delta\big((p-k)^2\big)\;\delta\big((k+q)^2\big)$$

$$\left[\;k = p - r = \ell - q\;\right]$$

Go to the infinite momentum frame

$$\cancel{p^\mu = (P, 0, 0, P)}$$

Define

$$P^\mu = (P,\ 0,\ 0,\ P)$$

$$N^\mu = \left(\frac{1}{2P},\ 0,\ 0,\ \frac{-1}{2P}\right)$$

$$p^\mu = P^\mu + \frac{M^2}{2} N^\mu$$

$$q^\mu = \nu N^\mu + q_\perp^\mu$$

Neglect M and take P large

$$k^\mu \equiv \xi P^\mu + \frac{k_\perp^2 - |k|^2}{2\xi} N^\mu + k_\perp$$

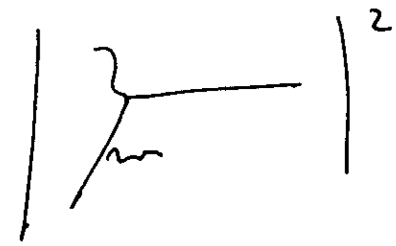
Define Sudakov parameters ξ and $k_\perp$.

$$(p-k)^2 = (1-\xi)\frac{|k|^2}{\xi} - \frac{k_\perp^2}{\xi}$$

$$(k+q)^2 = 2\xi\nu - Q^2 - |k|^2 + 2 q_\perp \cdot k_\perp$$

$$PS = \frac{1}{16\pi^2\nu}\int d\xi\, dk^2\, d^2k_\perp\, \delta\left(k_\perp^2 - (1-\xi)k^2\right)\, \delta\left(\xi - x - \frac{|k|^2 + 2 q_\perp \cdot k_\perp}{2\nu}\right)$$

consider



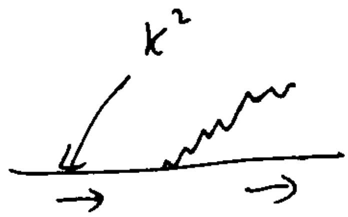
$$|M|^2_{\mu\nu} = \frac{1}{2} e^2 g^2 \sum_{\substack{pol \\ gluon}} \frac{C_F \, Tr\left[\gamma^\mu (\not k + \not q) \gamma^\alpha \not k \not\epsilon \not p \not\epsilon \not k\right]}{|k|^4}$$

Need to sum over polarizations

Care needed

Note as $|k| \to 0$ g is // to quark

(27)



Helicity conserved at vertex

<u>PHYSICAL</u> gluon polarization is $\underset{\rightarrow}{\sim}$ or $\underset{\leftarrow}{\sim}$

$\Rightarrow |M|^2 \propto k^2$ as $k^2 \to 0$.

$$\sum_{pol} |M|^2 = \frac{8 e^2 \alpha_s}{|k|^2} \xi P(\xi)$$

$$P(\xi) = \frac{4}{3} \frac{(1+\xi^2)}{(1-\xi)}$$

$$F_2 \approx \frac{\alpha_s}{2\pi} x P(x) \int_0^{2\nu} \frac{d|k|^2}{|k|^2}$$

Note (1) divergent at lower end

(2) all range contributes equally

Divergence come from on-shell intermediate state

and is an infra-red problem

$\Rightarrow$ Result is ∞ $\Rightarrow$ QCD corrections are large
✓ (yes) ✓ (yes)

$\Rightarrow$ Parton Model collapses
(No)

Introduce a regulator

(i) quark mass $|M|$ or

(ii) dimensional regularization.

<u>Aside</u> other graphs do not have the divergence
(check this)

You <u>must</u> use polarization sum over physical polarizations. If you use $\sum \epsilon^\mu \epsilon^\nu = -g^{\mu\nu}$ then <u>all</u> graphs must be added to get the right answer.

No single graph dominates.

Aside II in the frame I am using: (∞ mom frame)

$$k^\mu = \xi p^\mu + \frac{k_\perp^2 - |k|}{2\xi} N^\mu + k_\perp^\mu$$

$k_\perp^2 \simeq k^2$ since $p^2 = N^2 = 0$

Back to $\mathcal{F}$! Add to lowest order piece

$$F_2 = x \sum_q \int_x^1 dy\, q(y)\, e_q^2 \left[\delta\left(1 - \frac{x}{y}\right) + \frac{\alpha_s}{2\pi} \left\{ P\left(\frac{x}{y}\right) \log \frac{Q^2}{\mu^2} + R\left(\frac{x}{y}\right) \right\} \right]$$

later.

(29)

Roter

Define

$$q(x, M^2) = q_0(x) + \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[P\left(\frac{x}{y}\right) \log \frac{M^2}{\mu^2} + R_2\left(\frac{x}{y}\right) \right] q_0(y)$$

Then

$$F_2(x) = \sum_q \int \frac{dy}{y} q(x; M^2) \left[\delta(1 - x/y) + \frac{\alpha_s}{2\pi} \left[P(x/y) \log Q^2/M^2 + R_1\left(\frac{x}{y}\right) \right] \right]$$

eqn
$\times\times\times$

Results ① $F_2(x) \to F_2(x, Q^2)$ - SCALING VIOLATED

② Take $\mu \to 0$ by choosing

$q_0(x, \mu)$ so that $q(x, M^2)$ finite

like renormalization but for IR
Called MASS FACTORIZATION

Note R_2 is arbitrary can choose it so that

$R_1 \equiv 0$. as R_1 R_2 and R_ϕ are

related.

This is a scheme ambiguity in the definition of

$q(x, M^2)$.

Scheme where $R_1 \equiv 0$ is $\overline{DIS}$ scheme

In this scheme F_2 directly determines

$q(x, M^2)$

Easiest calculational scheme is $\overline{MS}$. Again use

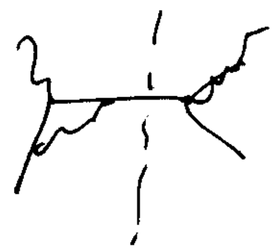
Dim reg for $\int \frac{dk^2}{k^2}$ then drop $\frac{1}{\epsilon}$ into $q(x, M^2)$

Note $q(x, M^2)$ are different by finite calculable differences in different schemes.

One more trouble $P(x) \sim \frac{1}{1-x}$

$$\Rightarrow \int_0^1 dx \; P(x) \quad \text{diverges.}$$

This is removed by virtual graph prob to $\delta(1-x)$



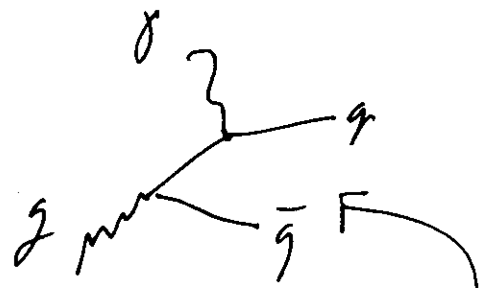
But $\int_0^1 dx \; q(x) = 1$ [1 quark]

require that $\int_0^1 q(x, M^2) \, dx = 1$

Then $P_{qq}(x) \longrightarrow \frac{4}{3} \frac{1+x^2}{(1-x)_+} + 2\delta(1-x)$

by definition $\displaystyle\int_0^1 dx \frac{f(x)}{(1-x)_+} \equiv \int_0^1 \frac{dx\,[f(x)-f(1)]}{(1-x)}$

More complicated because of mixing



Same order in α_s

P_{qq} $\quad P_{qg}$ $\quad P_{gq}$ $\quad P_{gg}$ $\quad$ mixed problem

relating Bare $q(x), g(x)$ and

renormalized quantities $q(x,M), g(x,M)$

M is arbitrary – physics does not depend on it

[remember μ]

$$\frac{d}{d}\ \frac{dF_2}{dM^2} = 0$$

General parton result $\sigma^{\text{hadron}} = \sigma^{\text{parton}} \otimes q(x,M)$

denoted convolution over parton momentum fraction

So far we have a leading log result.

We can use the renormalization group to sum up

$$\sum_n \alpha_n \log^n M^2 \quad \text{all leading logs.}$$

[just like we did in $R(e^+e^-)$].

Note that M is totally arbitrary

$$\frac{d}{dM^2} F_2(Q^2, M^2) \equiv 0$$

$\Rightarrow$

Best seen by taking moment $g(j, M^2) \equiv \int_0^1 dx\, x^{j-1} g(x)$

$$\frac{1}{g(j)} \frac{\partial}{\partial \log M^2} g(j, M^2) \equiv \gamma(j, \alpha_s) \qquad (A)$$

where <u>by definition</u> $\gamma(j, \alpha_s) = \frac{\alpha_s}{2\pi} \int P_{gg}(x)\, dx\, x^{j-1}$

or remember $F_2 = \sigma \times g$ [previous page XXX]

$$\Rightarrow \frac{\partial \log \sigma(j)}{\partial \log M^2} = -\gamma(j)$$

Hence γ is a scaling dimension [ANOMALOUS DIMENSION]

Can solve (A) to get full evolution

go General solution in x space

$$\frac{\partial g(x, M^2)}{\partial \log M^2} = \frac{\alpha_s(M^2)}{2\pi}\int_x^1 \frac{dy}{y}\left[P_{gq}\left(\frac{x}{y}\right) q(y, M^2) + P_{gg}\left(\frac{x}{y}\right) g(y, M^2)\right]$$

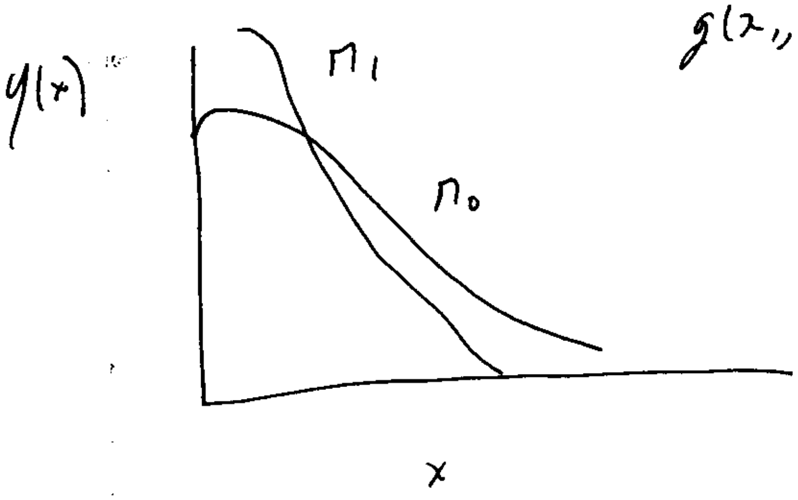
GRIBOV-LIPATOV, DOKSHITZER, ALTARELLI-PARISI

$\equiv$ $\boxed{\text{GLAP evolution}}$

$g(x_0, M_0^2) \longrightarrow g(x_1, M^2)$

$g(x_0, M_1^2) \longrightarrow g(x_1, M^2)$

For $M^2 > M_0^2$ note only $x_1 < x_0$ feeds $g(x_1, M^2)$ Evolution



Need data at some μ_0, then can solve for all μ^2.

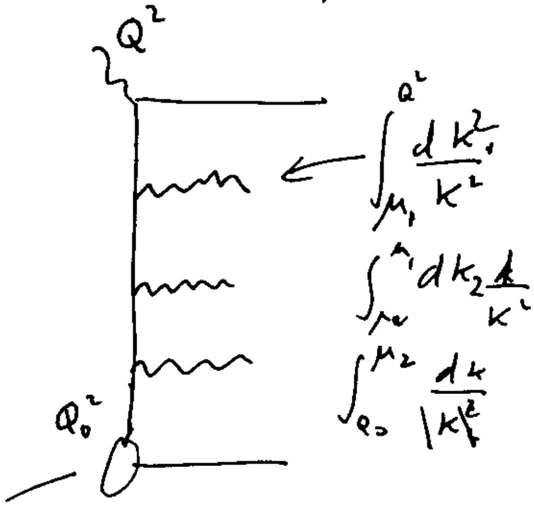
$\boxed{\text{SUMMARY}}$

UV divergences $\Rightarrow$ $\alpha_s(Q^2)$

Initial state singularities $\Rightarrow$ $f(x, Q^2)$

Final state singularities cancel [inclusive process]

Remember picture



Integrals are divergent $\Rightarrow$ (1) k_i^2 or $k_\perp^2$ is

<u>strongly</u> ordered

-- $Q^2 \gg k_1^2 \gg k_2^2 \gg k_3^2 \dots \gg Q_0^2$

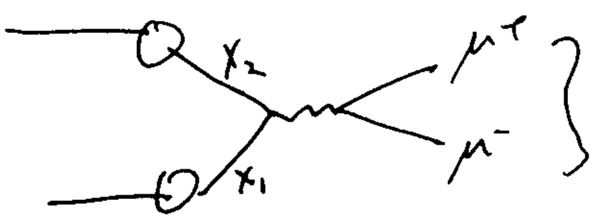
We now have another place to TEST <u>QCD</u>.

PARTON MODEL AND UNIVERSALITY

Take another process in parton model

eg $\quad p\bar{p} \to \underbrace{\mu^+\mu^-}_{\text{large invariant mass } M} + X$

Naive parton model

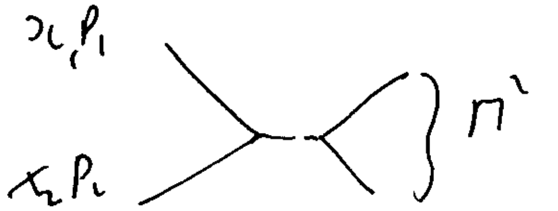


invariant mass M

$$\frac{d\sigma}{dM^2} = \int \frac{dx_1}{x_1}\frac{dx_2}{x_2}\, q(x_1)\,\bar{q}(x_2)\,\frac{d\sigma}{dM^2}(q\bar{q}\to\mu\mu)$$

$$= \frac{4\pi\alpha_{em}^2}{9M^2 s}\int dx_1 dx_2\, q(x_1)\,\bar{q}(x_2)\left[e_i^2\,\delta\!\left(x_1 x_2 - \frac{M^2}{s}\right) + (1 \rightleftarrows 2)\right]$$

$\delta\left(x_1 x_2 - M^2/s\right)$ is kinematical



$$M^2 = (x_1 P_1 + x_2 P_2)^2$$

$$= \cancel{x_1^2 P_1^2} + \cancel{x_2^2 P_2^2}$$

$$+ \; x_1 x_2 \underbrace{(2 P_1 \cdot P_2)}_{s}$$

$$= x_1 x_2 s.$$

Next order in α_s

integrate over gluon emission

$$\frac{d^4 k}{(2\pi)^3} \; \frac{\delta(k^2 - m^2) \quad \epsilon^\mu(g)}{(x_1 P_1 - k)^2}$$

Singularity is the same as that which appeared in ep scattering: define $z \equiv M^2/s x_1 x_2$

$$\frac{d\sigma}{dM^2} = \frac{4\pi\alpha_s}{9M^2} \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} \; g(x_1)\, \bar{q}(x_2)$$

$$\left[\delta(1-z) + \theta(1-z) \frac{\alpha_s}{2\pi} \left[2 P_{qq}(z) \log M^2 + f'(z) \right] \right]$$

(37)

But now must replace $g(x)$ by

$g(x, M^2)$

[i.e. calculate g to order α_s]

$$\frac{d\sigma}{dM^2} = \frac{4\pi\alpha_s}{9M^4} \int dx_1 dx_2\, g(x_1, M^2)\, \bar{g}(x_2, M^2)$$

$$\left[\delta(x_1 x_2 - M^2/s) + O(\alpha_s) \right]$$

No terms of $\log M^2$

Very powerful result called $\boxed{\text{FACTORIZATION}}$

This implies that all partonic processes use the same structure functions.

Use one process [deep inelastic scattering]

$\Rightarrow$ extract $g(x, M_0^2)$ [$g(x, M^2)$]

Then predict everything else.

In order for calculation to be reliable

1. Energy scale must be large so $\alpha_s(Q)$ small

2. All scales must be similar.

[This is because — 2 scale process E_1, E_2

$$P = f\left(\alpha_s(E_1),\ E_1/E_2\right)$$

$$= \alpha_s(E_1)^n \left[1 + \alpha_s(E_1) \log\left(E_1/E_2\right)\right.$$

Expansion collapses if $\alpha_s \log\left(E_1/E_2\right) \sim 1$

must now sum to all orders $\sum_n \alpha_s^n \log^n\left(E_1/E_2\right)$

Sometimes this works — sometimes not

Drell Yan

$p\bar{p} \to \underbrace{\mu^+\mu^-}_{M,\ p_\perp} + X \quad (p_\perp \ll M)$

$p\bar{p} \to b\bar{b} + X$

logs(s/m_b^2)