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Lecture I

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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AN INTRODUCTION TO THE STANDARD MODEL

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Please note: These are preliminary notes intended for internal distribution only.

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An introduction to the Standard Model

and to (some of) its most relevant

Predictions.

C. Verzegnassi / Lecce.

Lemma:

For every beautiful theory there exists at least one "special" experimental test.

The latter is supposed to be able to verify whether one is dealing with a satisfactory "effective" description or with something "more".

Definition:

A beautiful theory is one that makes definite unambiguous predictions that can be experimentally tested ("provando e riprovando")

Illustrative example: QED and the muon's (g-2).

The main features of the theory are supposed to be known.
The relevant coupling $= \alpha(0)$ is defined and measured from a previous independent experiment (e.g. Thomson scattering / or Hall... or other ~ zero momentum transfer measurements). This fixes $\alpha(0)$:

e.g. $$\alpha(0) = \frac{1}{137.03599979(32)}$$

The muon's $(g-2)$ is defined from:

$$M \sim \bar{u}(p_2)\Lambda_\sigma u(p_1) . A^\sigma(q)$$

$$\Lambda_\sigma = F(q^2)\gamma_\sigma - \frac{1}{2m_\mu}\sigma_{\sigma\lambda} q^\lambda G(q^2)$$

$$a_\mu = F(0) = \frac{1}{2}(g-2)_\mu$$

Experiment (CERN 1977):

(J. BAILEY et al; F. J.M. FARLEY + E. PICASSO.)

$$a_{\mu^+} = 1.165.937(12)\ 10^{-9}$$

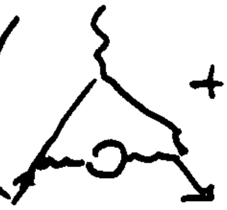
$$a_{\mu^-} = 1.165.911(11)\ 10^{-9}$$

Guess: $a_\mu \Rightarrow 1.165.924(\sim 10)\ 10^{-9}$

Theory (T. KINOSHITA ..... ENDLESSLY)

QED: (e, μ, τ)

$a_\mu = 0$ () $+ \frac{\alpha}{2\pi}$ () $+$

$+ 0.76578223 \left(\frac{\alpha}{\pi}\right)^2$ ( $+ \ldots$)

$+ \ldots\ldots$

$$\Longrightarrow \left(\frac{\alpha}{\pi}\right)^4$$

($\left(\frac{\alpha}{\pi}\right)^5$ estimated in 1990....)

Summing up the available QED (= leptons) contributions gives:

$10^9 a_\mu^{QED}$ $\quad$ 0

1. 1.61417 $\quad \alpha$

$\sim \boxed{4}\square\square\square \quad \alpha^2$

$\sim \boxed{4}\square\square \quad \alpha^3$

$\sim \boxed{3}\square \quad \alpha^4$

$\overset{?}{\Longrightarrow} \sim \square \quad \alpha^5$

$$10^9 a_\mu^{QED} = 1.165.847(00?)$$

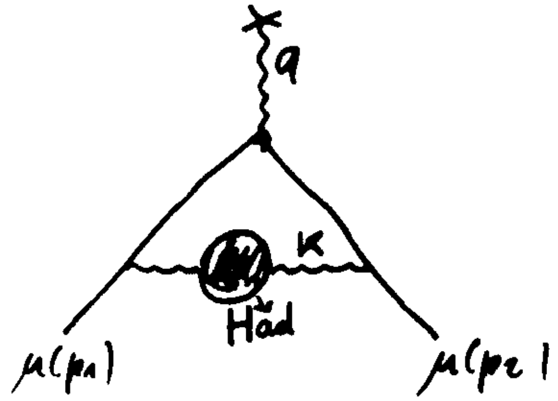
$$10^9 a_\mu^{EXP} = 1.165.924(10)$$

One still needs some ~ 70-80 10^{-9} pieces!

QED α^6 unlikely .... Z,W(H) can give ~

Luckily, hadrons contribute!

Main hadronic contribution:



$\equiv O(\alpha^2)$

 $\Rightarrow (K_\mu K_\nu - g_{\mu\nu} K^2)\Pi$

$$a^{(2)}_{\mu,\mathrm{Had}} \simeq \alpha^2 \int dK \; (F \ldots) \mathrm{Re}\,\Pi^{\mathrm{Had}}(K^2)$$

$$\Rightarrow \mathrm{Re}\,\Pi^{\mathrm{Had}}(K^2) = \frac{K^2}{\pi} \int_{4m_\pi^2}^{\infty} \frac{\mathrm{Im}\,\Pi^{\mathrm{Had}}(t)\,dt}{t(t-K^2)}$$

$$\mathrm{Im}\,\Pi^{\mathrm{had}}(t) \equiv R = \frac{\sigma(e^+e^- \to \mathrm{had})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

Therefore

$$a^{(2)}_{\mu,\mathrm{Had}} = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} dt\, f(t)\, R(t)$$

$$f(t) = \frac{1}{t} \int_0^1 \frac{dx \; x^2(1-x)}{x^2 + (1-x)\,t/m_\mu^2} \sim \begin{array}{ll} 1/t^2 & t \to \infty \\ 1/t & t \to 0 \end{array}$$

(<u>very</u> low-energy part $-\sim \rho, \omega -$ of the integrand dominates)

TABLE III. Hadronic contributions to the muon anomalous magnetic moment arising from Fig. 1(a). The first error is statistical and the second is systematic.

Contributing process and energy range	Contribution to $10^{10}a_\mu$	Reference
$\rho,\omega\to\pi^+\pi^-$ $(2m_\pi \le \sqrt{s} \le 1.1976$ GeV)	586.39(2.15)(15.0)	14,29,33,34,35
$\omega\to 3\pi$ $(3m_\pi \le \sqrt{s} \le 2.0$ GeV)	46.64(4.75)(1.49)	36
ϕ $(3m_\pi \le \sqrt{s} \le 2.0$ GeV)	40.17(1.00)(1.39)	36
$J/\psi(3.100)$	5.64(71)(08)	37
$\psi(3.685)$	1.47(21)(20)	37
$\psi(3.770)$	0.18(4)(4)	36
$\Upsilon,\Upsilon',\Upsilon'',\Upsilon'''$	0.085(3)(5)	38
	Background	
$e^+e^-\to\pi^+\pi^-$ $(1.1976 \le \sqrt{s} \le 3.0$ GeV)	3.85(28)(31)	33,39
$e^+e^-\to\pi^+\pi^-\pi^0$ $(0.8432 \le \sqrt{s} \le 1.002$ GeV)	2.92(81)(<81)	40
$e^+e^-\to K^+K^-$ $(1.05 \le \sqrt{s} \le 3.0$ GeV)	4.32(32)(46)	33,34,41,42
$e^+e^-\to K_S{}^0K_L{}^0$ $(1.088 \le \sqrt{s} \le 2.15$ GeV)	0.98(47)(10)	34,43,44
$e^+e^-\to p\bar{p}$ $(1.9 \le \sqrt{s} \le 2.2375$ GeV)	0.17(3)(<3)	41,45
$e^+e^-\to\pi^+\pi^-\pi^0$ $(1.42 \le \sqrt{s} \le 2.05$ GeV)	0.96(7)(10)	41
$e^+e^-\to K_S{}^0K^\pm\pi^\mp$ $(1.4415 \le \sqrt{s} \le 2.05$ GeV)	1.12(9)(<9)	46
$e^+e^-\to\pi^+\pi^-\pi^0\pi^0$ $(0.99 \le \sqrt{s} \le 2.05$ GeV)	23.95(79)(3.00)	47
$e^+e^-\to\pi^+\pi^-\pi^+\pi^-$ $(0.986 \le \sqrt{s} \le 2.05$ GeV)	14.02(35)(1.12)	34,41,47
$e^+e^-\to K^+K^-\pi^+\pi^-$ $(1.45 \le \sqrt{s} \le 2.05$ GeV)	1.39(9)(18)	48
$e^+e^-\to\pi^+\pi^-\pi^+\pi^-\pi^0$ $(1.282 \le \sqrt{s} \le 2.05$ GeV)	1.75(15)(21)	34,49
$e^+e^-\to\pi^+\pi^-\pi^+\pi^-\pi^0\pi^0$ $(1.44 \le \sqrt{s} \le 2.05$ GeV)	5.05(46)(1.00)	50
$e^+e^-\to\pi^+\pi^-\pi^+\pi^-\pi^+\pi^-$ $(1.45 \le \sqrt{s} \le 2.05$ GeV)	0.43(4)(12)	51
$e^+e^-\to$ more than two hadrons $(2.05 \le \sqrt{s} \le 3.15$ GeV)	31.43(81)(4.33)	52
$e^+e^-\to$ hadrons		
$(3.15 \le \sqrt{s} \le 7.8$ GeV)	19.81(27)(2.77)	53
$(7.8 \le \sqrt{s} \le 30.8$ GeV)	4.27(26)(26)	54
$(\sqrt{s} \ge 30.8$ GeV) (Asymptotic freedom with 6 quarks)	0.4	
Total	707.4(5.9)(16.4)[a]	

[a] The second error is obtained by treating systematic errors by the least-squares method. Simple addition of these errors will give a value of about 34.

Note that for the contributions from the processes $e^+e^-\to\pi^+\pi^-\pi^0$ (0.8432 GeV $\le\sqrt{s}\le$ 1.002 GeV), and $e^+e^-\to K^+K^-$ (1.05 GeV $\le\sqrt{s}\le$ 3.0 GeV), we have subtracted the Breit-Wigner tails of ω and ϕ resonances, and the ϕ resonance itself, respectively, in order to avoid double counting. Finally, the contribution from the region $\sqrt{s}\ge 30.8$ GeV was estimated by the lowest-order QCD with six quarks. The results are summarized in Table III. The total contribution from this diagram [Fig. 1(a)] is given by (1.4).

APPENDIX B: CLARIFICATION OF THE PARAMETRIC METHOD IN THEORIES WITH DERIVATIVE COUPLING

In this appendix we present a minor simplification of the parametric method in theories with derivative cou-

Numerically:

$$a^{(2)}_{\mu,\mathrm{Had}} \simeq [71 \pm (2?\,3?)]\,10^{-9}$$

Therefore:

$$10^9 [a_\mu^{QED} + a^{(2)}_{\mu,\mathrm{Had}}] = 1.165.918(03?)$$
$$10^9 \quad a_\mu^{exp} \quad = 1.165.924(10)$$

(extra(+) copper from Z, W, $a^{(4)}_{\mu,\mathrm{Had}}$ -?-)

$\Longrightarrow$ O.K. for QED.

Two important facts:

I) The contribution from $\Pi^{\mathrm{Had}} \Rightarrow$ $\sigma_{e^+e^- \to \mathrm{had}}$ is essential (~ 7 times larger than the experimental error).

I) The dominant theoretical error comes from the experimental error on $\sigma_{e^+e^- \to \mathrm{had}}$ ~ in the ρ, ω region.

Can the measurement of a_μ be considered a "predictive" one?

Not for the moment (<u>but</u> a spectacular reduction of the experimental error, and a suitable reduction of the theoretical one, perhaps ....)

The interpretation <u>would</u> have been different if hadrons (in particular, $\equiv \pi^+ (\overline{\pi^+})$) had not yet been known.

Still, one can say that: (CAREFUL!)

the consistency between QED and the muon's $(g-2)$ <u>requires</u> the existence of ~ a couple of "quarks" (pions) of "effective" mass ~ a few $\sqrt{q^2} \simeq m_\mu$, and of a "loop" term ~ 

Question:

do (shall) we have similar tests for the

Minimal Standard Model?
(MSM)

To answer the question, a (quick) review of the main properties of the MSM is necessary.

The MSM at tree level.

The MSM is a renormalizable quantum field theory of strong and electroweak interactions whose Lagrangian is invariant with respect to local gauge transformations belonging to the group $SU(3)_c \times SU(2)_L \times U(1)_Y$:

$$\mathcal{U} \equiv e^{i\sum_{a=1}^{8} \eta_a(x) T_a^{(3)}} e^{i\sum_{j=1}^{3} \varepsilon_j(x) I_j} e^{i\omega(x) Y}$$

where T_a, I_j and Y are the generators of $SU(3)_c, SU(2)_L, U(1)_Y$ and

$$Q \equiv I_{3L} + \frac{Y}{2}.$$

The particle content of the model is the following:

1) 3 families of left-handed and right-handed leptons: $\left(f_{L,R} \equiv \frac{1}{2}(1 \mp \gamma_5) f\right)$

$$L_L \equiv \begin{vmatrix} \nu_L \\ e_L \end{vmatrix} \begin{matrix} \Rightarrow I_{3L} = +1/2 \\ \Rightarrow I_{3L} = -1/2 \end{matrix} \Rightarrow y = -1$$

$$e_R \; (I_{3L} = 0; \; y = -2), \; \underline{\text{NO } \nu_R}. \qquad \underline{\text{COLOR} = 0}.$$

2) 3 families of left-handed and right-handed quarks:

$$Q_L \equiv \begin{vmatrix} u_L \\ d_L \end{vmatrix} \begin{matrix} \Rightarrow I_{3L} = +1/2 \\ \Rightarrow I_{3L} = -1/2 \end{matrix} \Rightarrow y = \frac{1}{3}$$

$$u_R, d_R \; (I_{3L} = 0; \; y = \tfrac{4}{3}, -\tfrac{2}{3}).$$

<u>COLOR = "3"</u>.

(In each family), quarks are <u>requested</u> to make

$$\sum_i Q_{f_i} = \underset{e_L}{-1} \; \underset{e_R}{-1} + 3\left[\underset{u_L}{\tfrac{2}{3}} - \underset{d_L}{\tfrac{1}{3}} + \underset{u_R}{\tfrac{2}{3}} - \underset{d_R}{\tfrac{1}{3}}\right] =$$

$= 0$ (to avoid <u>anomalies</u>)
(next order....).

⇒ 12 Gauge vector bosons:

$$8 \text{ gluons} \Rightarrow SU(3)_c$$
$$3\ W_s \Rightarrow SU(2)_L$$
$$1.\ B \Rightarrow U(1)_Y$$

to produce gauge-invariance.

With a), b), c) one could make a gauge-invariant theory of <u>massless</u> leptons, quarks, gauge-bosons.

⇒ Magic trick: to introduce

1) a $SU(2)_L$ doublet of complex (Higgs) scalar fields

$$\Phi \equiv \begin{vmatrix} \phi^+ \\ \phi^0 \end{vmatrix} \begin{matrix} \Rightarrow I_{3L} = +1/2 \\ \Rightarrow I_{3L} = -1/2 \end{matrix} \Rightarrow Y = 1.$$

The role of the Higgs doublet in the MSM is that of Santa Klaus:

it gives mass to whomever asks for it ...

(to the charged leptons, to the quarks, to three not colored gauge bosons).

and leaves massless those who so wish

(the gluons, the photon, the neutrinos(?))

This operation requires the simultaneous presence (and collaboration) of two distinct tricks:

a) The spontaneous symmetry breaking

b) The Higgs mechanism

The spontaneous symmetry breaking

This happens when a symmetry of the Lagrangian is <u>not</u> a symmetry of the minimum energy state ("vacuum")
(i.e. no "Wigner-type" symmetry)

Old classical example: ferromagnetism near the Curie temperature T_c.

For T close to T_c one (Ginzburg-Landau) expands the free energy in powers of the magnetization $\vec{M}$:

$$u(\vec{M}) \propto (\partial_i \vec{M})^2 + V(\vec{M})$$

$$V(\vec{M}) = \alpha_1(T)\, \vec{M}.\vec{M} + \alpha_2 (\vec{M}.\vec{M})^2 \quad (\alpha_2 > 0)$$

(rotational invariance)

$$\alpha_1 \equiv \alpha(T - T_c) \qquad (\alpha > 0)$$

Min. $\frac{\partial V}{\partial M_i} = 0 \implies \vec{M}(\alpha_1 + 2\alpha_2 \vec{M}.\vec{M}) = 0$

Vacuum: $\alpha_1 > 0$ $(T > T_c) \Rightarrow \vec{M} = 0$ (rot. inv.)

$\alpha_1 < 0$ $(T < T_c) \Rightarrow |\vec{M}| = \sqrt{\frac{-\alpha_1}{2\alpha_2}}$ (<u>no</u> rot. inv.)

$(\partial^2 V > 0 \ldots)$ $(\mathcal{R}\,|0\rangle \neq |0\rangle)$

In QFT, one has the Goldstone theorem:

Spontaneous breakdown of a <u>global</u> continuous symmetry implies the existence of massless spinless particles.

(true to all orders of perturb. theory).

$U \equiv e^{i\varepsilon_\alpha A^\alpha}$ (element of sym. group)

$U|0\rangle \neq |0\rangle \Rightarrow$ <u>at least one $A_\alpha|0\rangle \neq 0$.</u>

In correspondence to <u>that A^α</u>:

$\sim A^\alpha \Rightarrow \phi_i \;:\; \langle 0|[A^\alpha, \phi_i]_-|0\rangle \neq 0$

and there exists a massless physical state $|n_\alpha\rangle$ such that: ($A^\alpha \overset{?}{\sim} \int d\vec{x}\, J_0^\alpha(\vec{x},t)$)

$\langle 0| J_0(0)|n_\alpha\rangle \neq 0$ ($\Rightarrow$ spinless)

$\langle 0| \phi_i(0)|n_\alpha\rangle \neq 0$ ($\Rightarrow \phi_i$ ")

Also, if $[A^\alpha, \phi_i]_- = i\, t^\alpha_{ij}\, \phi_j$

<u>at least one</u> ϕ_j must exist such that

$$\langle 0|\phi_j(0)|0\rangle \equiv v_j \neq 0.$$

Rule: (for <u>global</u> cont. symm.)

to every generator of the symm. group that does <u>not</u> annihilate the vacuum there corresponds a massless spinless particle (Goldstone boson).

This is a <u>physical</u> state

(~ the pion in the σ- model ...).

If the spont. broken symmetry is a (local) gauge symmetry, this is no longer true since the <u>Higgs phenomenon</u> appears.

The Higgs phenomenon.

This is typical of spontaneously broken gauge symmetries.

Simplest example: an Abelian U(1) gauge theory

$$L = (D_\mu \phi)^\dagger D^\mu \phi - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$D_\mu \equiv (\partial_\mu - i g A_\mu), \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$(\phi \to \phi' = e^{-i\alpha(x)} \phi \ \ldots\ldots)\left(A'_\mu = A_\mu - \frac{1}{g} \partial_\mu \alpha(x)\right)$$

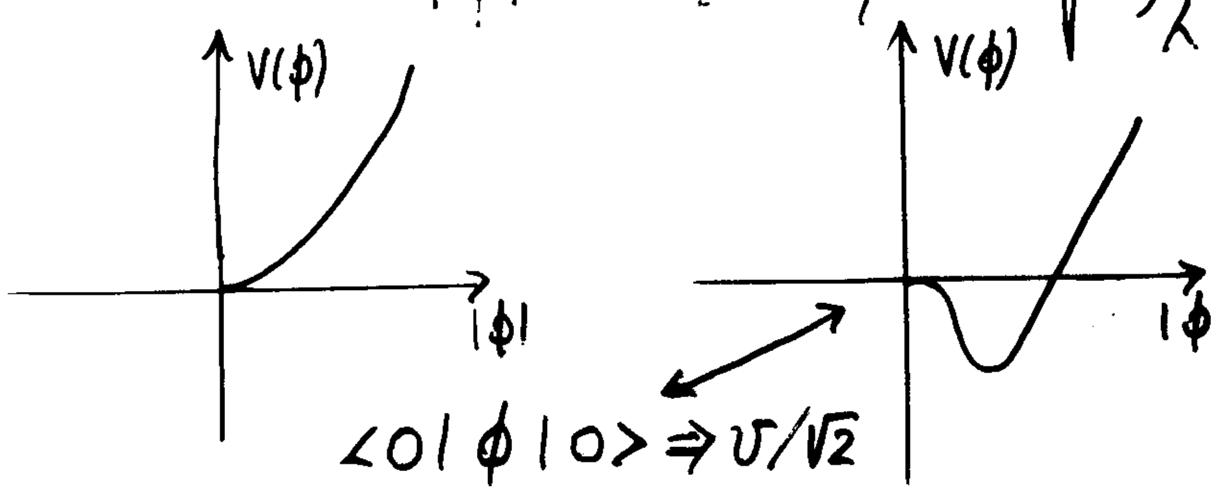
$$\left[(D_\mu \phi)' \equiv e^{-i\alpha(x)} D_\mu \phi\right]$$

The potential

$$V(\phi) = +\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \; ; \; \lambda > 0$$

for $\mu^2 < 0$ has a minimum at

$$|\phi| = v/\sqrt{2}, \quad v = \sqrt{\frac{-\mu^2}{\lambda}}$$



Normal choice : $\left(\phi \equiv \frac{\phi_1 + i\phi_2}{\sqrt{2}}\right)$

$$\langle 0|\phi_1|0\rangle \equiv v; \quad \langle 0|\phi_2|0\rangle = 0.$$

Then : "shifted" fields $\phi_i' \equiv \phi_i - \langle 0|\phi_i|0\rangle$

$$\phi_1' \equiv \phi_1 - v; \quad \phi_2' = \phi_2$$

The Lagrangian becomes :

$$L = \underset{\sim\sim\sim}{\mu^2 \phi_1'^2} + \frac{\mu^2}{4v^2}\left[O(\phi_i'^3, \phi_i'^4)\right]$$
$$- \frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \left[O(g^2 A_\mu^2 \phi_i'^2, g A_\mu \phi_i \phi_j', (\partial\phi'^2))\right]$$
$$- g v A^\mu \left[g A_\mu \phi_1' \right]$$
$$+ \underset{\sim\sim\sim}{\frac{g^2 v^2}{2} A^\mu A_\mu} \qquad \underset{\sim\sim\sim}{- g v A^\mu \partial_\mu \phi_2'}.$$

$\Longrightarrow$ ϕ_2' is massless (expected)

$\phi_1' \Rightarrow m_1^2 = -2\mu^2 \; (>0)$.

<u>A_μ has acquired a mass $M_A = gv$</u>

<u>a very disturbing term</u> $\sim A^\mu \partial_\mu \phi_2$ has appeared.

The disturbing term $\sim A^\mu \partial_\mu \phi_2'$ can be removed if ϕ_2' is washed out (drastic solution).

This can be done parametrizing the fields as

$$\phi(x) = \frac{1}{\sqrt{2}} [\eta(x) + v] \, e^{i \xi(x)/v}$$

$(\eta \sim \phi_1', \ \xi \sim \phi_2' \ \ldots.)$

and performing a gauge transf.

$$\phi''(x) = e^{-i \xi(x)/v} \phi = \frac{1}{\sqrt{2}} [\eta(x) + v]$$

$$B_\mu(x) = A_\mu - \frac{1}{gv} \partial_\mu \xi$$

that fixes the choice of the gauge ($\equiv$ Unitary gauge)

In this gauge we are left with:

One massive scalar $\eta \to \phi_1'$, $m_\eta^2 = -2\mu^2$

One massive g.b. B_μ, $M_B^2 = (gv)^2$.

no more ϕ_2' ("eaten" by B_μ).

The (poor) ϕ_2' is called "would-be Goldstone boson".

N.B. After ϕ_2' has been "swallowed" the overall number of degrees of freedom does not change.

A scalar has disappeared $(\Delta DG^{(s)} = -1)$

DG = degree of freedom

A vector has acquired a mass $\Rightarrow$

$\Rightarrow$ a new helicity $(\Delta DG^{(v)} = +1)$

Overall $\Delta DG = 0$. / O.K.

This trick can be generalized to a situation with a larger non abelian gauge group with <u>N</u> generators $[I_i, I_j] = i f_{ijk} I_k$ and e.g. fermions sitting in irreducible representations of the group. One then introduces a certain "Higgs multiplet" ($\in$ a certain irred. repr. of the group) that contains m real fields ($m \geq N$).

Then (<u>theorem</u>):

If $M \leq N$ is the number of "<u>broken</u>" generators, $I_i |0\rangle \neq 0$, $i=1\ldots M$

M <u>will also be</u> the number of "would-be" Goldstone bosons, to be "eaten" by the corresponding M gauge bosons to make them massive.

Question:

What has all this to do with the SM (= Standard Model)?

To answer the question, one should (quickly) go back to the

BSM

("Before Standard Model")
(for most of the audience
BSM ≃ Paleozoic...)

here.

In fact, old(er) people still remember that:

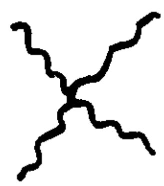
Once upon a time, there was the "Fermi theory" of weak interactions, where ~ everything was explained by the Lagrangian:

$$L_F(x) = -\frac{G_F}{\sqrt{2}} \left[J_\lambda(x) J^{\lambda +}(x) + h.c. \right]$$

with the <u>charged</u> weak current:

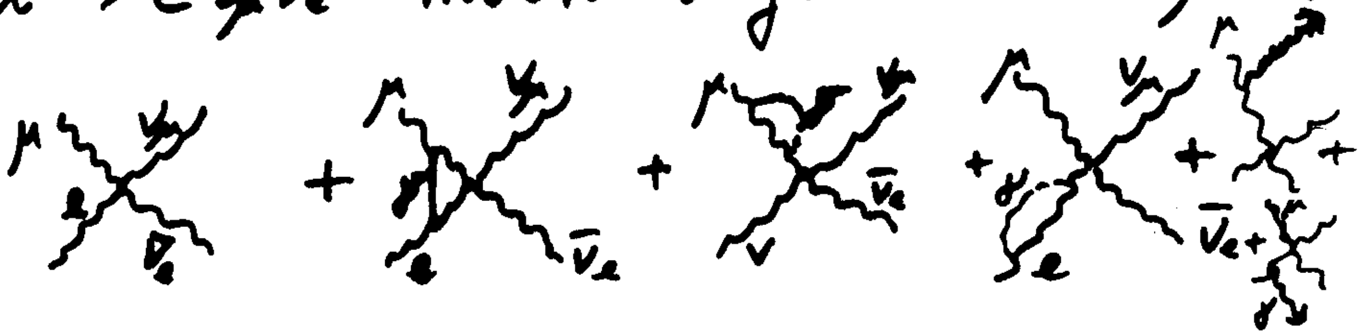
$$J_\lambda = J_{\lambda,\ell} + J_{\lambda,h}$$

and e.g. $J_{\lambda,\ell} = \bar{\psi}_{\nu_e} \gamma_\lambda (1-\gamma_5) \psi_e + \dots$

("four fermion" interaction ⇒ )

(<u>purely left-handed</u> fermions, $\psi_L = \frac{(1-\gamma_5)}{2} \psi$).

The coupling constant $G_F \to G_\mu$ was <u>defined</u> from the measured $\mu \to e \nu_\mu \bar{\nu}_e$ muon lifetime $= \tau_\mu$:



$$\Rightarrow \frac{1}{\tau_\mu} \equiv G_\mu^2 \frac{m_\mu^5}{192\pi^3}\left[1 - \frac{8 m_e^2}{m_\mu^2} + O_\mu(\alpha)\right]$$

(<u>definition</u> of G_μ...)

where $O_\mu(\alpha)$ is a (small) known <u>finite</u> R.C. (radiative correction) that describes the effect of the last 3 diagrams. $\left[O_\mu = \frac{\alpha}{2\pi}\left(\frac{25}{4} - \pi^2\right)\left(1 + \frac{2\alpha}{3\pi}\ln\frac{m_\mu}{m_e}\right)\right.$

Numerically, one has:

$$G_\mu = \underline{1}.\underline{1}66389\,(22)\ 10^{-5}\ \text{GeV}^{-2}$$

with $\frac{\delta G_\mu^{exp}}{G_\mu} \simeq 2.\,10^{-5}$

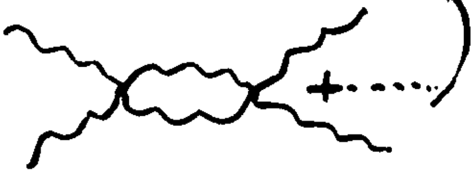
(not as good as $\frac{\delta\alpha(0)}{\alpha} \simeq 2.10^{-8}$, still..

An impressive number of weak interactions phenomena was correctly described by the Fermi charged current Lagrangian. From the very beginning, though, this was affected by two major deseases:

I) Lack of renormalizability:

G_F has dimension $[\text{mass}]^{-2}$

$\Longrightarrow$ meaningless at higher orders

(NO  + ...)

II) Violation of unitarity:

already at Born level, for certain processes, high energy σ: BAD.

E.g. $\nu_\mu e \to \mu \nu_e$

$$\sigma(s) \xrightarrow[s\to\infty]{\simeq} G_F^2 s \quad (s \equiv 2 m_e E_{\nu_\mu}^{LAB})$$

But (unitarity) $\sigma \lesssim \frac{1}{s} \Longrightarrow$ Limit (Low energy theory) $\sqrt{s} \lesssim \frac{1}{\sqrt{G_F}} \simeq 300\,\text{GeV}$

The massive $W^{\pm}$ alternative.

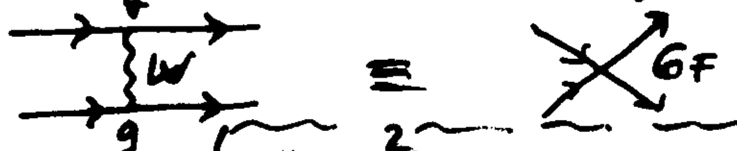
An immediate temptation would have been that of imitating the familiar QED treatment by introducing a charged massive field W_μ^+ (W_μ^-).

The Fermi interaction would then become

$$L(x) \equiv \text{"}g\text{"} \left[J_\lambda W^\lambda + h.c. \right]$$

(J_λ same as before)

This would reproduce the low-energy ($\equiv$ zero momentum transfer) predictions of the Fermi Lagrangian <u>if</u>:

( , e.g. $M_{FI}(\mu \to \nu_\mu e \bar{\nu}_e)$)

$$\boxed{\frac{\text{"}g\text{"}^2}{(8) M_W^2} = \frac{G_F}{\sqrt{2}}}$$

FUNDAMENTAL !!!

However: even if g dimensionless, the W propagator is

$$\Delta_{\mu\nu}(k) = -i\left[\frac{g_{\mu\nu} - k_\mu k_\nu / M_W^2}{k^2 - M_W^2}\right]$$

that behaves like a constant when $k \to \infty$,
$\Longrightarrow$ again, <u>NON RENORMALIZBLE</u>.

(also, unitarity violation remains e.g. $\nu\bar{\nu} \to W^+ W^-$).

$\Rightarrow$ Apparently, $M_W \neq 0$ (<u>required</u> e.g. by the previous argument $G_F/\sqrt{2} = {}$"$g^{2}/M_W^2 (8)$") catastrophic.

<u>Unless</u> a tricky way is found of introducing gauge boson mass(es) without destroying renormalizability

(for $M_W = 0$, $\Delta_{\mu\nu}(k) \sim -\dfrac{g_{\mu\nu}}{k^2}$ O.K.)

$\Longrightarrow$ Spontaneous Sym. breaking ???