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Lecture II

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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AN INTRODUCTION TO THE STANDARD MODEL

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Please note: These are preliminary notes intended for internal distribution only.

From the previous discussion: (4)
a possibly promising approach
might be that of introducing a
gauge symmetry and of breaking
it spontaneously to generate masses
for the needed massive gauge bosons.

First problem: choice of a proper
electroweak Gauge group (for SI: SU₃(C) U₁)
(not to be discussed here)

Consider the oversimplified case
of one family of fermions (e, ν_e, u, d).

The charged lepton current to be
coupled to $W^\pm$ is then:

$$J_{\chi, e} \equiv \bar{\psi}_e \gamma_\chi (1 - \gamma_5) \psi_e$$

and the component of the candidate
Lagrangian contains both J_χ and $J_\chi^\dagger$.

If one wants to introduce a gauge symmetry group G , this will have to contain at least two charges (generators) associated to these currents (and to the charged bosons W^+, W^-):

$$T_+ \equiv \frac{1}{2} \int d\vec{x} J_0(x)$$

$$T_- \equiv T_+^\dagger \equiv \frac{1}{2} \int d\vec{x} J_0^\dagger(x).$$

No meaningful groups are known with two generators. The "smallest" one that is available has the mathematical structure of $SU(2)$

$$G \doteq SU(2)$$

and it has three generators, with an algebra

$$[T_+, T_-] = 2 T_3.$$

$\Rightarrow$ at least one more ^(neutral) gauge boson $\Rightarrow T_3$!

Could this third "minimal" gauge boson be the photon? (3)

$$(L \equiv L_w + L^{e.m.}, L^{e.m.} \equiv e J_A A, e \approx g)$$

NO. Because $[T_+^e, T_-^e] = 2 T_3^e =$

$=$ (from canonical a.m. rules)

$$2. \quad \frac{1}{4} \int d\vec{x} [\bar{\psi}_\nu (1 - \gamma_5) \psi_\nu - \psi_e^\dagger (1 - \gamma_5) \psi_e]$$

$$\neq 2 Q^e !$$

$\Rightarrow$ one solution: (initially, not the only one — extra charged lepton E^+ also —)

Introduce another gauge boson $k/3$ different from the photon but (coupled to $T_3 \Rightarrow \tau_3 \dots$) electrically neutral.

In this case, a Lagrangian describing both weak and electromagnetic interactions ("electroweak" interactions) should contain at least four different gauge bosons $\Rightarrow$ a minimal gauge group G should have four generators.

A simple group G with four generators is not available.

But one can consider the minimal non simple group

$$G \doteq SU(2) \times U(1)$$

(mathematical structure)

where the condition $(U(1) \Rightarrow Y)$

$$[Y, T_i] = 0 \quad (i = +, -, 3)$$

implies that also the $U(1)$ g. boson $= B$ _{cannot be the photon!}

The prescription to ensure that the fermionic Lagrangian has G as a (symmetry) gauge group is to replace systematically (generalizing QED)

$$\partial_\mu \psi_i \Rightarrow D_\mu \psi_i$$

where ψ_i is the i -th independent "structure", having certain transformation properties under $SU(2)$ and $U(1)$, and

$$D_\mu \psi_i \equiv \left[\partial_\mu - ig \vec{T}^{(i)} \cdot \vec{W}_\mu - ig' \frac{Y^{(i)}}{2} B_\mu \right] \psi_i$$

with $\vec{T}^{(i)} \equiv$ matrices representing the 3 $SU(2)$ generators $\vec{T}$ in the irred. repr. provided by ψ_i , whose Y value is $Y^{(i)}$.

In practice, either $SU(2)$ doublets e.g. $\psi_L \equiv \begin{pmatrix} \nu \\ e \end{pmatrix} \dots \Rightarrow \vec{T} = \frac{\vec{\tau}}{2}$, or singlets e, ν ($\vec{T}=0$), (same for u, d).

A number of purely phenomenological considerations now appears, that will influence

DRASTICALLY

the construction of the fermionic/g. boson sector of the candidate Lagrangian.

I) To reproduce correctly the known low-energy features of weak interactions, the charged part of the interaction $\Rightarrow T_{\pm} (\Rightarrow W^{\pm})$ must only involve left-handed fermions (e_L, ν_L, u_L^w, d_L^w) ($q^w \neq q^{SU(3)}$).

$\Rightarrow$ the covariant derivative "belonging" to $SU(2)$, that only acts on the $SU(2)$ doublets, must find in these doublets left-handed fermions

$$\Rightarrow \psi_1 \equiv \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad \psi_2^{(c)} \equiv \begin{pmatrix} u_L^{w,c} \\ d_L^{w,c} \end{pmatrix} \quad (c \Rightarrow \text{color}) \\ (c=1,2,3)$$

For this reason, the "electroweak" $SU(2)$ was called $SU(2)_L$.

I) If one wants to incorporate e.m. interactions in the game, the Lagrangian must contain right-handed fermions as well (photons interact equally with f_L and f_R ...). But these must not be affected by the $SU(2)_L$ covariant derivative $\Rightarrow$ must be $SU(2)_L$ singlets.

~~$\psi_3 = \nu_R$~~ $\psi_4 = e_R$, $\psi_5 = u_R$, $\psi_6 = d_R$
(quarks have $SU(3)$ color, $q = q^{(c)}_{(1,2,3)}$)
(ν_R "not wanted"....)

II) One still has to fix the values of $Y^{(i)}$ for the i -th "structure".
Request: Y must commute with $\vec{T}$.
Discovery: $(Q^{e.m.} - T_3)$ commutes with $\vec{T}$

$\Rightarrow$ natural proposal

$$U(1) \Rightarrow -\frac{Y}{2} \equiv [Q - T_3] \Rightarrow U(1)_Y$$

In this way, the relation

$$Q = T_3^{\text{strong}} + \frac{1}{2} Y^{\text{strong}}$$

is retained at the e.w. level:

$$Q = T_3^{\text{e.w.}} + \frac{1}{2} Y^{\text{e.w.}}$$

with

$$G \equiv SU(2)_L + U(1)_Y.$$

This fixes $Y^{(i)}$ for the i -th structure.

$$\text{E.g. } \psi_1 \equiv \begin{pmatrix} \nu_L (T_3 = +1/2) \\ e_L (T_3 = -1/2) \end{pmatrix} \Rightarrow Y^{(1)} = -1$$

$$\psi_2^{(c)} \equiv \begin{pmatrix} u_L^{(c)} (T_3 = +1/2) \\ d_L^{(c)} (T_3 = -1/2) \end{pmatrix} \Rightarrow Y^{(2)} = 1/3$$

and so on.

So the fermion content is fixed: 15 two-components fields $(\nu_L, e_L, e_R, u_L^{(c)}, d_L^{(c)}, u_R^{(c)}, d_R^{(c)})$ with known transformation properties.

V) No ν_R in the original ("Minimal") Standard Model: only "phenomenological".

Once the group has been fixed, what follows?

From the request of "Wigner-symmetry" one derives that:

$$M_{\vec{w}, B} = 0 \quad (\text{like in QED})$$

$$M_{fi} = 0$$

since $\bar{\psi}\psi = \bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L$

(no $SU(2)$ invariance).

Back to page 21", 1st Lecture:

"spontaneous breaking" of the G

($N=4$) symmetry can generate
as many masses as one wishes
for gauge bosons.

And also, as one will see, for
fermions!

$\Rightarrow$ one only must decide
how many gauge bosons must
acquire a mass.

(remember that $SU(3)_c$ is also involved

Generalization to the MSM:

no need of the trick in the $SU(3)_c$ sector
(gluons remain massless).

In fact, $SU(3)_c$ can be separated away and treated "canonically"
($SU(3)_c$ commutes with $SU(2)_L \times U(1)_Y$)

(only quark masses "borrowed"
from $SU(2)_L \times U(1)_Y$ Sp. Sy. Br.)

From now on: concentrate on
the E.W. component $SU(2)_L \times U(1)_Y$.

Of the four g.bosons, only one
(the photon) must remain massless.

Three must acquire a mass.....

$\Rightarrow$ four real Higgs fields
(two complex fields)

wills do the job.

("bonus": fermions too will become massive)

The $SU(2)_L \times U(1)_Y$ Lagrangian

This is made of four pieces:

$$L = L_1 + L_2 + L_3 + L_4$$

$$L_2 = -\frac{1}{4} F_{\mu\nu}^i F^{i\mu\nu} - \frac{1}{4} G_{\mu\nu} G^{\mu\nu}$$

$$F_{\mu\nu}^i = \partial_\mu A_\nu^i - \partial_\nu A_\mu^i + g \epsilon^{ijk} A_\mu^j A_\nu^k \Rightarrow SU(2)_L$$

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \Rightarrow U(1)_Y$$

$$L_1 = \sum_j \bar{\psi}_j i \gamma^\mu D_\mu \psi_j$$

$j \Rightarrow (L)_{L,R}, Q_L^{1,2,3}, (u,d)_{R,L}^{1,2,3}$

$$D_\mu \psi_j = (\partial_\mu - i g_0 \vec{T}^j \cdot \vec{A}_\mu - i g_0' \frac{Y^j}{2} B_\mu) \psi_j$$

$$(D_\mu L_L = (\partial_\mu - i g_0 \frac{\vec{\tau}}{2} \cdot \vec{A}_\mu + i g_0' \frac{1}{2} B_\mu) L_L)$$

$$L_4 = \sum_i \left[\bar{L}_L^i \Phi e_R^i h_e^i + \bar{Q}_L^i \Phi d_R^i h_d^i + \bar{Q}_L^i \tilde{\Phi} u_R^i h_u^i + h.c. \right]$$

$$\Phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad \tilde{\Phi} \equiv i \tau_2 \Phi^* \Rightarrow Y_{\tilde{\Phi}} = -1.$$

$$L_3 = (\mathcal{D}_\mu \Phi)^\dagger (\mathcal{D}_\mu \Phi) - V(\Phi)$$

$$\mathcal{D}_\mu \Phi = \left(\partial_\mu - \underbrace{\frac{i}{2} g \vec{\tau} \cdot \vec{A}_\mu}_{SU(2)_L} - \underbrace{\frac{i}{2} g' B_\mu}_{U(1)_Y} \right) \Phi$$

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

$$\mu^2 < 0 \Rightarrow S_p, S_y, B_n. (SSB)$$

$$SSB: \langle 0 | \Phi | 0 \rangle = \begin{vmatrix} 0 \\ v/\sqrt{2} \end{vmatrix} \neq 0, v = \sqrt{\frac{\mu^2}{\lambda}}$$

$$\text{Then: } \Phi(x) = e^{-i \left[\vec{\xi}(x) \cdot \frac{\vec{\tau}}{v} \right]} \begin{vmatrix} 0 \\ \frac{v + \eta(x)}{\sqrt{2}} \end{vmatrix}$$

$$t_b \Rightarrow T_b: T_b |0\rangle \neq 0$$

$\xi_{1,2,3}$ are the "would-be" Golds. bosons that can be washed away (Unitary gauge).

The physical spectrum is then made by the following particles:

E) One physical massive scalar
(the Higgs) $\Rightarrow \eta(x)$:

$$m_{\eta}^2 = -2\mu^2$$

I) 3 families of fermions with
a massless neutrino and
massive electron and quarks:

$$m_{oe} = h_e v/\sqrt{2}$$

$$m_{ou,d} = h_{u,d} v/\sqrt{2}$$

II) 3 massive and one massless
($\equiv$ photon) gauge bosons.

The derivation of the g.b.
masses is a "key-point" of
the MSM. It contains
predictions and features that
will affect both the tree- and the

In the unitary gauge

$$\Phi(x) \Rightarrow \frac{(v + \eta(x))}{\sqrt{2}} \chi; \quad \chi \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The derivative-term in $\mathcal{L}_3$ produces:

$$\begin{aligned} \mathcal{L}_{3,M} &= \frac{v^2}{2} \chi^\dagger \left(\frac{g_0}{2} \vec{\tau} \cdot \vec{A}'_\mu + \frac{g_0'}{2} B'_\mu \right)^2 \chi = \\ &= \frac{v^2}{8} \left\{ g_0^2 [(A'^1_\mu)^2 + (A'^2_\mu)^2] + [g_0 A'^3_\mu - g_0' B'_\mu]^2 \right\} \end{aligned}$$

$$W^\pm = \frac{1}{\sqrt{2}} [A'^1 \mp i A'^2]$$

$$\mathcal{L}_{3,M} = M_{W^2} [W^\pm_\mu W^\mp{}^\mu] + \frac{1}{2} M_{Z^2} Z_\mu Z^\mu$$

where $M_{W^2} = g_0^2 v^2 / 4$

and $\frac{1}{2} M_{Z^2} Z_\mu Z^\mu = \frac{v^2}{8} [g_0 A'^3_\mu - g_0' B'_\mu]^2 =$

$$= \frac{v^2}{8} \begin{vmatrix} A'^3_\mu & B'_\mu \end{vmatrix} \begin{vmatrix} g_0^2 & -g_0 g_0' \\ -g_0 g_0' & g_0'^2 \end{vmatrix} \begin{vmatrix} A'^3_\mu \\ B'_\mu \end{vmatrix} =$$

$$\equiv \frac{1}{2} \begin{vmatrix} Z_\mu & A'^\mu_\nu \end{vmatrix} \begin{vmatrix} M_{Z^2} & 0 \\ 0 & 0 \end{vmatrix} \begin{vmatrix} Z^\mu \\ A'^\mu_\nu \end{vmatrix}$$

The physical states are obtained diagonalizing the "mathematical" states mass matrix via the Θ_0 -rotation:

$$\tan \Theta_0 \equiv g'/g_0$$

$$\sin \Theta_0 \equiv s_0 = \frac{g'_0}{\sqrt{g_0^2 + g'^2_0}}$$

$$\cos \Theta_0 \equiv c_0 = \frac{g_0}{\sqrt{g_0^2 + g'^2_0}}$$

Thus, by construction:

$$Z_\mu = c_0 A'^3_\mu - s_0 B'_\mu$$

$$A^\mu_\gamma = s_0 A'^3_\mu + c_0 B'_\mu$$

und

$$M_{02}^2 = \frac{v^2}{4} (g_0^2 + g'^2_0)$$

$$(M_\gamma^2 = 0)$$

$$\{ \Theta_0 \equiv \text{Weinberg-Salam angle} \} \quad (\Rightarrow \Theta_w \dots)$$

In terms of the physical states Z_μ, A_μ , the part of L_1 that describes the neutral interactions of fermions can be written as:

$$L_{1,nc} \equiv e_0 \overline{\psi}_\mu \gamma^\mu \psi A^\mu + \underbrace{e_0 \overline{\psi}_\mu \gamma^\mu \psi Z^\mu}_{\substack{\text{with } e_0 \equiv g_0 s_0 \equiv g_0' c_0}}$$

$$\text{and } \overline{\psi}_\mu \gamma^\mu \psi \equiv \overline{\psi}_\mu \gamma^\mu \psi^{(I_{3L})} - s_0^2 \overline{\psi}_\mu \gamma^\mu \psi^{e.m.}$$

Thus, the Z couplings to fermions will be:

$$g_{L,R}^f = (I_{3L})_{L,R} - s_0^2 (Q_f) \\ (Q_f \equiv e/|e|)$$

$$\text{with } \overline{\psi}_\mu \gamma^\mu \psi \equiv \sum_f [\overline{g}_{0L}^f \overline{f}_L \gamma^\mu f_L + \overline{g}_{0R}^f \overline{f}_R \gamma^\mu f_R]$$

$$(\text{equivalently } \equiv \sum_f \overline{f} [\overline{g}_{0V}^f \gamma^\mu - \overline{g}_{0A}^f \gamma^\mu \gamma_5] f;$$

$$g_{0V} \equiv \frac{1}{2} (g_{0L} + g_{0R}); \quad g_{0A} \equiv \frac{1}{2} (g_{0L} - g_{0R})$$

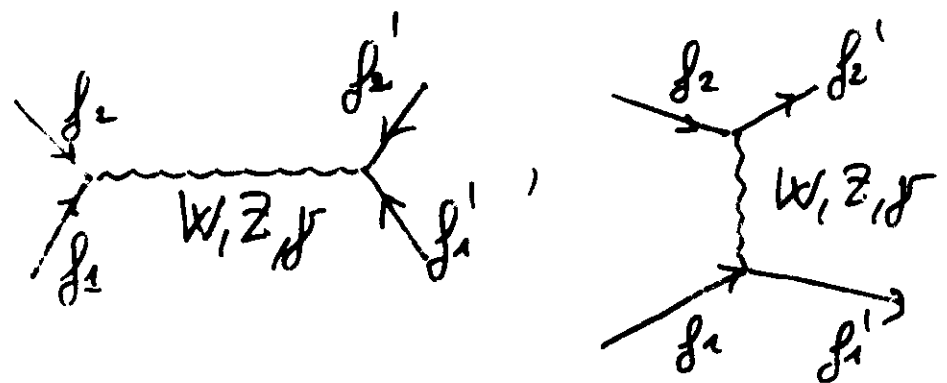
Analogously, one can write for the charged component:

$$L_{1,cc} = \frac{g_0}{\sqrt{2}} (J_\mu^+ W^{+\mu} + J_\mu^- W^{-\mu})$$

where $J_\mu^+ = J_\mu^1 + i J_\mu^2 =$

$$= \sum_f \left[\frac{1}{2} \bar{u}' \gamma_\mu (1 - \gamma_5) e' + \frac{1}{2} \bar{u}' \gamma_\mu (1 - \gamma_5) d' \right]$$

The fermion - g.bosons interactions to lowest order, corresponding to four fermion processes of the kind



will depend at this order

on e_0, g_0, M_{0Z}, M_{0W}

(we assume that in the unitary gauge "naïve" Feynman rules are valid,

But: from the expressions of the g.b. masses one has

$$\frac{M_{0W}^2}{M_{0Z}^2} = \frac{g_0^2}{g_0^2 + g_0'^2} \equiv c_0^2$$

that is more conveniently written

$$\frac{M_{0W}^2}{c_0^2 M_{0Z}^2} = 1.$$

Therefore one can eliminate one of the 3 parameters e.g. $c_0 \Rightarrow s_0$ and retain, to describe 4-fermion processes at tree level, the set (e.g.) e_0, M_{0W}, M_{0Z} (at the time t_0)

The elimination of one parameter is due to the formal identity

$$\rho_0 \equiv \frac{M_{0W}^2}{c_0^2 M_{0Z}^2} = 1$$

that is peculiar of the MSM. In fact, it is not completely accidental.

A better way of understanding it is to consider the "mathematical" (1,1) element of the non-diagonal 2×2 mass matrix in the (A_μ^1, B_μ^1) basis $\equiv M_{03}^2 \equiv \frac{v^2}{4} g_0^2 \equiv c_0^2 M_{02}^2$.

Then one can alternatively define

$$\rho_0 \equiv \frac{M_{02}^2}{M_{03}^2} = 1$$

and this is a consequence of a residual $SU(2)$ global "custodial" symmetry of the scalar potential after SSB (in this case, reflected into the fact that $(A_\mu^1)^2, (A_\mu^2)^2, (A_\mu^3)^2$ appear symmetrically in $L_{3,H}$).

(a more general proof exists).

SM predictions: tree level

From the previously defined SM Lagrangian one can already make a number of "tree level" predictions, e.g. for M_W, M_Z !

Since several unknown parameters are involved, one has to choose a specific "optimal" strategy.

The latter in turn is, almost unavoidably, related to the available experimental facilities.

~ 20 years ago, the approach was the following:

I) Consider the "fundamental" equality ("tree level" SM quantities)

$$\frac{G_F}{\sqrt{2}} = \left[\frac{g^2}{8 M_W^2} \right]^{tree} \left(\Rightarrow G_F \rightarrow \text{Feynman diagram: } \nu_e \rightarrow \nu_e \text{ via } W \text{ exchange} \right)$$

"lowest order" SM parameters implied)
as a "prediction" for M_W :

$$M_W^{(tree)} = \sqrt{\frac{\sqrt{2} g^2 (tree)}{8 G_F}} = \left(\begin{array}{l} e \equiv g S_W \\ (S_W \equiv \sin \theta_W) \end{array} \right)$$

$$= \sqrt{\frac{\sqrt{2} e^2 (tree)}{8 G_F}} \cdot \frac{1}{S_W^{(tree)}}$$

At this "lowest order" calculation
 $e^{(tree)} \equiv e = \sqrt{4\pi\alpha_{QED}}$, and one gets:

$$M_W^{(tree)} = \frac{37.2804 \dots \text{GeV}}{S_W^{(tree)}}$$

(error from G_F negligible.....).

As a byproduct of the previous derivation:

$$\frac{G_F}{\sqrt{2}} \equiv \frac{g^2}{8M_W^2} \equiv \frac{1}{2v^2}$$

$$\Rightarrow v = \frac{1}{2^{1/4}} \frac{1}{\sqrt{G_F}} \simeq 250 \text{ GeV}$$

($\sim$ "scale" of $SU(2)_L \times U(1)_Y$ BREAKING).

II) Consider the tree level

identity:

$$\left[\frac{M_W}{M_Z} \right]^{(tree)} = \cos \theta_W^{(tree)} \equiv C_W^{(tree)} \equiv$$

$$\equiv \left| \frac{g^2}{g^2 + g'^2} \right|^{(tree)}$$

as a "prediction" for $M_Z^{(tree)}$:

$$M_Z^{(tree)} = \frac{M_W^{(tree)}}{C_W^{(tree)}} =$$

$$= \frac{37.2804 \dots \text{GeV}}{(S_W C_W)^{(tree)}}$$

III) Measure S_W (C_W) from a suitable experiment.

IV) Give numbers for $M_W, M_Z \Rightarrow$

⇒ then

Find W, Z at
the predicted masses.

("spectacular" $\equiv$ *** prediction)

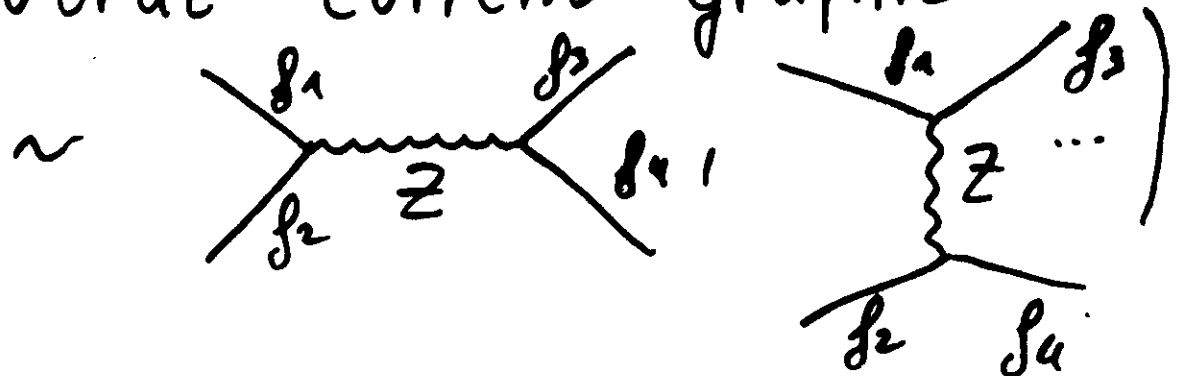
Fundamental for this original
approach: choose a suitable
process to "measure" S_W .

Which process was suitable
 ~ 26 years ago?

Intuitively, a process that was
described by Fermi th. and SM

At tree level, the relevant difference comes from virtual Z exchange e.g. in a four fermion process $f_1 + f_2 \Rightarrow f_3 + f_4$.

("neutral" current graphs



In particular : the vertex ν is not appearing in Fermi + QED.

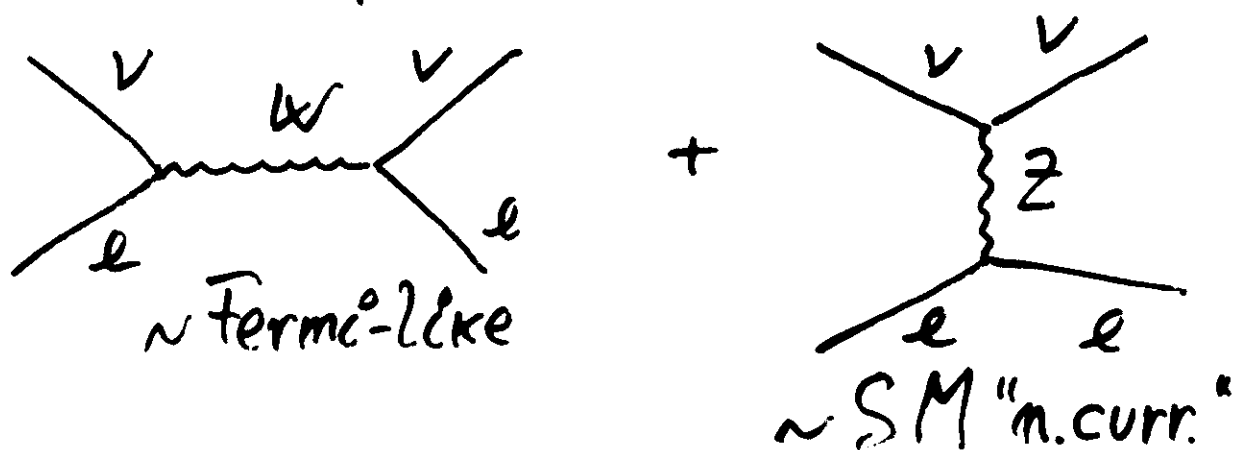
$\Rightarrow$ typically : the process

$\nu_\mu e^- \Rightarrow \nu_\mu e^-$ is allowed by SM,
but NOT by Fermi + QED.

Also, a process like

$$\nu_e e^- \rightarrow \nu_e e^-$$

is differently parametrized
in a Fermi-like (charged
current) picture and in the SM:



In fact, the first historical
(memorable) indirect evidence
for a "SM-like" picture
came from the observation
of neutral current events in
 ν -hadrons at CERN by Gargamelle
(Lacort et al 1973)

Then, five years of
confusion

(negative result in search
of neutral current effects
in atoms - Atomic Parity Violation,
APV - at Clarendon Lab., Oxford)

until 1978 when

C. Prescott et al.

(SLAC, Stanford)

by scattering of LONGITUDINALLY POLARIZED

← N.B.

electrons on deuterons
identified neutral current effects
beyond any doubt. (Clarendon in error.)

N.B.

Longitudinal $e^- (e^+)$ polarization is a key-point in the SM since e_L couple differently to the Z (and equally to the γ)
 $\Rightarrow$ in the SM there will be non zero long. polar. asymmetries

$$\left(\sim \frac{\overline{\nu_{eL}} - \overline{\nu_{eR}}}{\overline{\nu_{eL}} + \overline{\nu_{eR}}} \right)$$

From 1978 to 1983:
several n. curr. experiments
leading to an average value:

$$\sin^2 \theta_w (1983) \simeq 0.22 \pm 0.02$$

Thus, at the beginning of 1983,
one had the predictions:

$$\left\{ M_W^{(tree)} \simeq (79 \pm 3) \text{ GeV} \right\}$$

(approximate.....)

$$\left\{ M_Z^{(tree)} \simeq (90 \pm 3) \text{ GeV} \right\}$$

(approximate.....)

In 1983, at the end of a memorable search, the UA1 collaboration led by C. Rubbia at CERN found experimental evidence for the existence of W, Z with a mass (first values):

$$\left\{ \frac{M_W}{\text{GeV}} = (81.0 \pm 5.0) (\text{Phys. Lett. B } 122, 103) \right\}$$

$$\left\{ \frac{M_Z}{\text{GeV}} = (95.2 \pm 2.5) (\text{Phys. Lett. B } 126, 493) \right\}$$

$\Rightarrow$ SM at "tree" level

undeniably impressive.

To make more enthusiastic
claims:

calculation (and verification)
of next order perturbative
"electroweak" effects

(analogy with $(g-2)_\mu \dots$)

compulsory.