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Lecture III

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SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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AN INTRODUCTION TO THE STANDARD MODEL

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Please note: These are preliminary notes intended for internal distribution only.

The SM at one loop.

I) Can one make calculations at higher pt. orders in the SM (in particular, in the E.W. sector)? (this was, after all, ~ the main motivation.....).

Answer: YES (t'Hooft!)

(Highly non trivial → separate course)

Fundamental: specification of "renormalizable" gauges via a "gauge-fixing" parameter ξ.

Purpose ('t Hooft): to work in (I)
a general renormalizable gauge
with gauge-fixing conditions:

$$f_i(A^i_\mu, \phi_i) \equiv (\partial_\mu A^{\mu i} + \xi_i M^i_0 \phi_i) = 0$$

($i \Rightarrow i$-th massive g. boson; $\phi_i \equiv$
related w.be G. boson)
($\equiv R_\xi$ gauges).

The gauge fixing condition does not "eat" the would-be G.bosons, but it gets rid of the disturbing mixing terms $\sim g v A^{\mu i} \partial_\mu \phi_i$.

$\Longrightarrow$ Feynman rules contain ξ_i
(gauge dependence).

But physical observables
Must be ξ_i ($\equiv$ gauge) INDEPENDENT
[First/main rule!!]

Briefly : the W, Z propagators become at lowest order:

$$\Delta^{(i)}_{\mu\nu}(k) = \frac{-i}{(k^2 - M_{0i}^2)}\left[g_{\mu\nu} + \frac{(\xi^i - 1)k_\mu k_\nu}{k^2 - \xi^i M_{0i}^2}\right]$$

($i = W, Z$; M_0^2 = "bare" mass^2, ≃ ≃ "pole" of the (lowest order propagator).

Calculations at higher orders are usually performed in the $\xi^i = \underline{1}$ ("t'Hooft") gauge.

($\xi \to \infty$ ≡ "Unitary gauge")

<u>Warning</u> : only gauge invariant quantities (≃ "physical observables") to be calculated.

(This point is <u>not</u> harmless!)

Feynman rules for the propagators

$$W^{\pm} \Longrightarrow D^{W}_{\mu\nu}(q) = \frac{-i}{q^2 - M_{0W}^2 + i\varepsilon}\left[g_{\mu\nu} + (\xi_W - 1)\frac{q_\mu q_\nu}{q^2 - \xi_W M_{0W}^2}\right]$$

$$Z \Longrightarrow D^{Z}_{\mu\nu}(q) = \text{(Same, } W \leftrightarrow Z\text{)}.$$

$$\gamma \Longrightarrow D^{\gamma}_{\mu\nu}(q) = \frac{-i}{q^2 + i\varepsilon}\left[g_{\mu\nu} + (\xi_\gamma - 1)\frac{q_\mu q_\nu}{q^2}\right]$$

$$\phi^{\pm} \Longrightarrow D^{\pm}(q) = \frac{i}{q^2 - \xi_W M_{0W}^2 + i\varepsilon}$$

$$\phi_Z \Longrightarrow D^{Z}(q) = \frac{i}{q^2 - \xi_Z M_{0Z}^2 + i\varepsilon}$$

"would-be G.bos.

$$\phi \Longrightarrow D^{\phi}(q) = \frac{i}{q^2 + 2\mu^2 + i\varepsilon}$$

(Ph. Higgs)

$$\omega^{\pm, Z} \Longrightarrow D^{\omega^{\pm,Z}} = \frac{-i}{q^2 - \xi M_{0W,Z}^2 + i\varepsilon}$$

$$\omega^{\gamma} \Longrightarrow D^{\omega^{\gamma}} = \frac{-i}{q^2 + i\varepsilon}$$

Ghosts

+ "canonical" ξ_i-indepdt. fermion prop

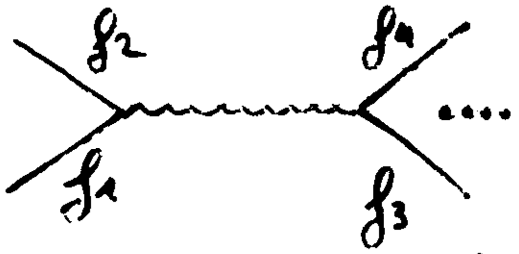
In practice: for $SU(2)_L \times U(1)_Y$

"Higher orders" means: <u>next after tree level</u> order, also said

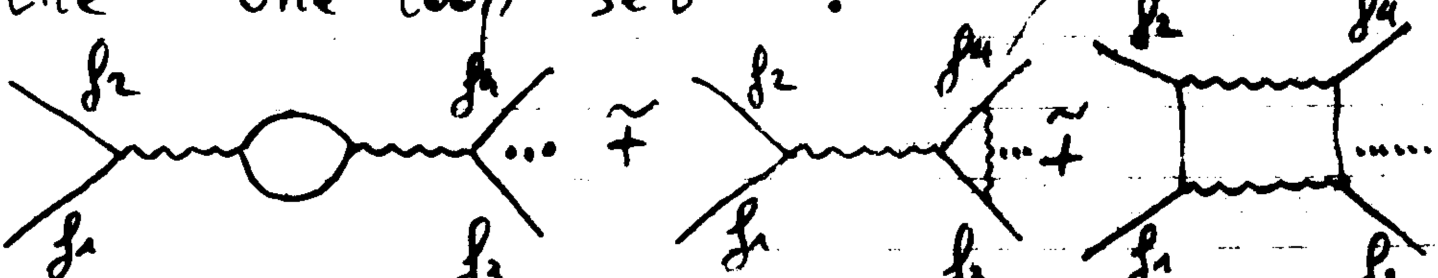
<u>ONE LOOP</u>.

('Z physics at LEP1', CERN 89-08, G. Altarelli, R. Kleiss, C.V.)

Considering the (most) relevant case of "four fermion" processes, this means to add to the tree level graphs:



the "one loop set":



(Self Energy, S.E.) (Vertices, V.) (Boxes, B.)

(+"QED"!)

(In <u>proper</u> g. invariant combinations!).

Are One Loop calculations Relevant?

YES!!

In fact, "tree level" predictions generally "wrong" (= outside experimental error bars at some probabilistic level .....).

This is good if One Loop "Corrections" restore the requested theoretical-experimental agreement!

Best" experiment to provide examples of this (remarkable) fact: e^+e^- annihilation on top of Z resonance.

Claim:

For the SM, e^+e^- annihilation on top of Z resonance (i.e. for e^+e^- energy ≡ "physical" Z mass) is ~ the analogue of the muon's (g−2) measurement for QED.

Proof of the claim: requires a brief illustration of some experimental facts.

e^+e^- annihilation on top of Z resonance: experiment.

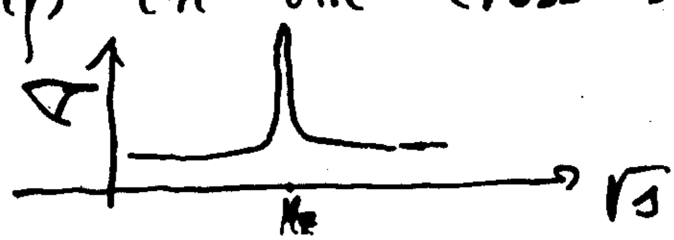
After the discovery of Z, <u>two</u> big experiments were designed to study in more detail the Z properties $\Longrightarrow$ SM predictions.

"Almost" obvious idea: to produce Z by e^+e^- scattering at the "correct" energy $\sqrt{s} = M_Z$, $s \equiv (P_{e^+} + P_{e^-})^2$, knowing that M_Z should have been "around" 90 GeV.

(For W, less simple .... see later).

Clean experimental signature:

BIG bump in the cross section

$\sim$ 

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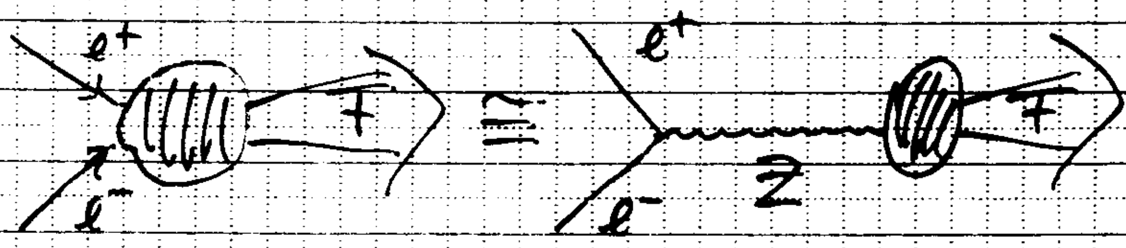
Diagrammatically, on Z resonance the picture is essentially "Z-exchange" (for $f\bar{f} \neq e^+e^-$):



and F can be any Z decay product (l^+l^-, $q\bar{q}$ ....).

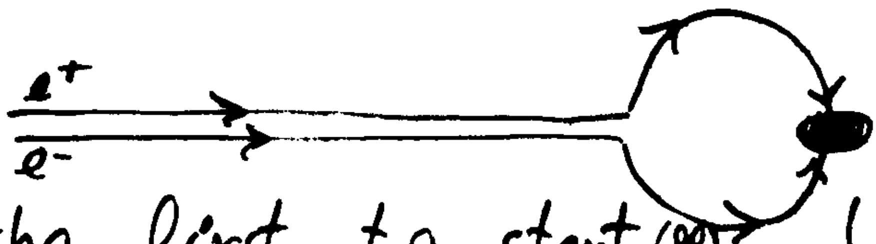
In the SM, each Z decay is predicted ⟹ accurate measurements of separate F production provide tests ⟹ to provide high accuracy tests, a lot of F, i.e. a lot of Z, must be produced

(in the exp. error, a statistical term $\sim \frac{1}{\sqrt{N_F}}$ appears ....).

$N_F \Rightarrow$ number of produced $F \Rightarrow Z$

The two big e^+e^- colliders have the following shape and main features:

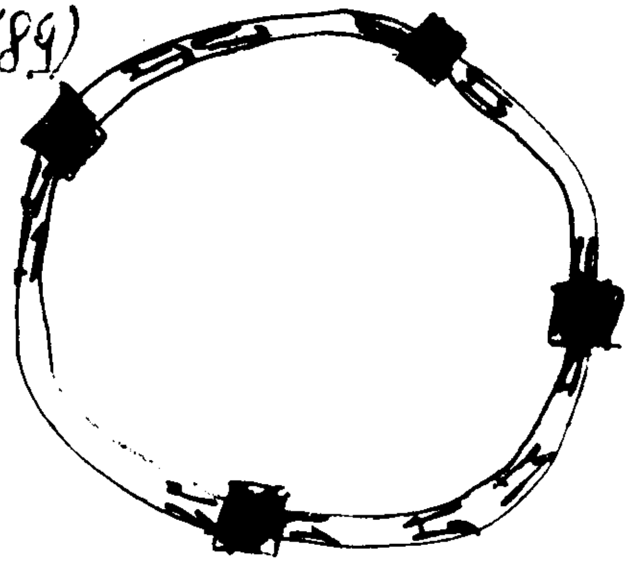
I) SLC, the Slac Linear Collider



was the first to start (88). Until now, it has "seen" a number of Z $N_Z \simeq 160.000$

II) LEP1, the CERN Large Electron-Positron (phase 1) collider

started later (89)



Until now:

$N_Z > 11.000.000$

(see Table)

		ALEPH	DELPHI	L3	OPAL	LEP
$q\bar{q}$	'90-'91	451	356	416	454	1677
	'92	680	697	678	733	2788
	'93 prel.	631	677	658	646	2612
	'94 prel.	(a)	(a)	1362	1524	2886
	total	1762	1730	3114	3357	9963
$\ell^+\ell^-$	'90-'91	55	37	40	58	190
	'92	82	69	58	88	297
	'93 prel.	78	71	62	82	293
	'94 prel.(b)	(a)	54	88	184	326
	total	215	231	248	412	1106

Table 1: The LEP statistics in units of 10^3 events used for the analysis of the Z line shape and lepton forward-backward asymmetries.
(a)No preliminary result quoted yet.
(b)DELPHI muon pairs, L3: electron and muon pairs.

As a consequence of the much higher statistics, ~ almost every measurable Z property (SM test) is provided by LEP1 nowadays, with one <u>remarkable</u> exception:

the measurements of quantities produced by longitudinally polarized electrons
("easy" for SLC,
at the moment, impossible for LEP1)

See forthcoming discussion...

(M

How "wrong" are the SM tree level predictions?

Simple and *particularly* *instructive* example: the gauge boson mass ratio

$$M_W / M_Z .$$

Today: LEP1 $\Rightarrow \frac{M_Z}{GeV} = 91.1887 \pm 0.0022$

Relative error $\simeq 2.4 \times 10^{-5}$ (!!!)

(Original 1983 value 95.2000 ± 2.5000
relative error $\simeq 2.6 \times 10^{-2}$)

For M_W, less accurate (FermiLab $p\bar{p}$) value:

$$\frac{M_W}{GeV} = 80.26 \pm 0.16 \quad (1983:\ 81.0 \pm 5.0)$$

This leads to the value

$$\left[\frac{M_W}{M_Z}\right]^{exp} = 0.8802 \pm 0.0018$$

At tree level, one has

$$\left[\frac{M_W}{M_Z}\right]^{(0)} = \cos\theta_W^{(0)} = \sqrt{1-\sin^2\theta_W^{(0)}}$$

At the same level, one would have several other Z-observables directly related to $\sin^2\theta_W^{(0)}$, and therefore immediately related to $\frac{M_W}{M_Z}$ ($\Longrightarrow$ predictions).

Simplest example: the (famous) quantity (Lynn...) $A_{LR}(M_Z^2) \equiv A_{LR} =$
$\equiv$ longitudinal polarization asymmetry.

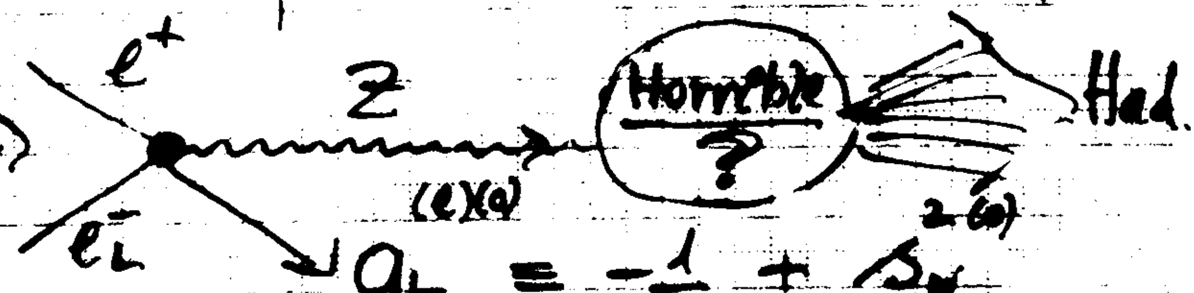
A_{LR} is defined as:

$$A_{LR} \equiv \left. \frac{\sigma_{e^+e^-_L \Rightarrow had.} - \sigma_{e^+e^-_R \Rightarrow had}}{\sigma_{e^+e^-_L \Rightarrow had} + \sigma_{e^+e^-_R \Rightarrow had}} \right|_{s=M_Z^2}$$

(initial electron longitudinally polarized, total hadronic production considered where statistics much higher...).

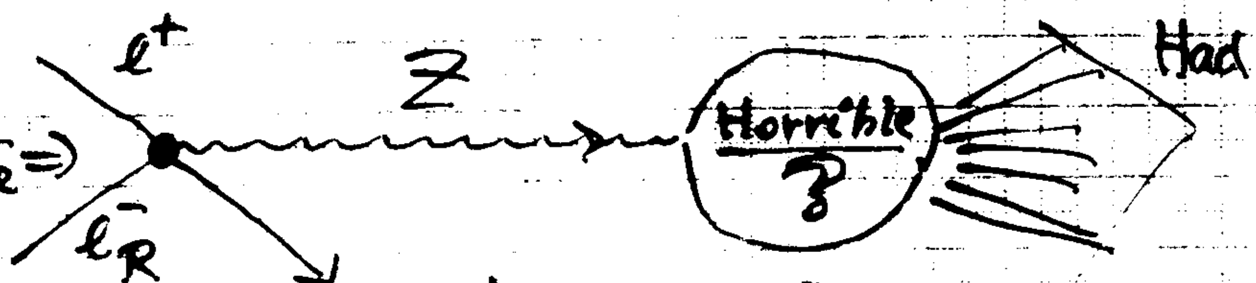
Graphically, one has the two different possibilities ($SU(2)_L \times U(1)_Y$ "tree")

L) $\sigma_L \Rightarrow$



$$g_L^{(e)(0)} = -\frac{1}{2} + s_W^{2(0)}$$

R) $\sigma_R \Rightarrow$



$$g_R^{(e)(0)} \equiv s_W^{2(0)}$$

("Horrible" may contain Strong interactions...)

In the (L) case, one produces a Z by left-handed electrons, with an "intensity" $\sim g_L^2$.

Then, this Z will decay into "all" hadrons in a "Horribly" complicated way.

In the (R) case, one produces a Z by right-handed electrons, with an "intensity" $\sim g_R^2$.

Then, this Z will decay into "all" hadrons in a "Horribly" complicated way.

But the two "Horrible" ways are identical.

$\Rightarrow$ DISAPPEAR IN $A_{LR} = \dfrac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}$!!

$$\Rightarrow A_{LR}^{(0)} = \frac{g_{L,0}^{(e)2} - g_{R,0}^{(e)2}}{g_{L,0}^{(e)2} + g_{R,0}^{(e)2}}$$ (indepd. of FINAL STATE !!)

In terms of $s_W^{2\,(0)} \equiv \sin^2\theta_W^{(0)}$:

$$A_{LR}^{(0)} = \frac{2\left(1-4 s_W^{2\,(0)}\right)}{1+\left[1-4 s_W^{2\,(0)}\right]^2}$$

If "tree" predictions were correct, the same s_W^2 should be derivable from A_{LR} and $\frac{M_W}{M_Z}$.

$\Rightarrow$ from the measurement of A_{LR}(SLC)

$\Rightarrow s_W^2 \Rightarrow c_W^2 \Rightarrow c_W$

CONSISTENT WITH PAGE 12.

In a (only slightly) less immediate way, one also relates, at tree level, $s_W^{2\,(0)}$ to several other LEP1 Z-observables (always at tree level).

For example: at this level, the lepton forward-backward asymmetry,

$$A_{FB,l} = \frac{\sigma_F(e^+e^- \to l^+l^-) - \sigma_B(e^+e^- \to l^+l^-)}{\sigma(e^+e^- \to l^+l^-)}$$

(F, B = Forward, Backward)

would be simply given by

$$A^{(0)}_{FB,l} = \frac{3}{4} A^{2(0)}_{LR} =$$

$$= \frac{3\left[1 - 4 s^{2(0)}_W\right]^2}{\left[1 + (1 - 4 s^{2(0)}_W)^2\right]^2}.$$

Assuming this formula to reproduce "reality", from the measured $A_{FB,l}$ LEP1 would give:

$$s^{2(0)}_W(A_{FB,l}) = 0.2310 \pm 0.0007$$

This would give:

$$\cos\vartheta_W^{(0)}(A_{FB,e}) \equiv$$

$$\equiv c_W^{(0)}(A_{FB,e}) = 0.8769 \pm 0.0004$$

Using the full set of LEP1 leptonic observables would produce

$$c_W^{(0)}(\text{LEP1, leptons}) = 0.8765 \pm 0.0003$$

while (page 12) $c_W^{(0)}(M_W/M_Z) = 0.8802 \pm 0.0018$

(not "fully" satisfactory).

N.B. If one had considered the two separate tree level predictions for M_W, M_Z – pages 22, 23 lecture 2 – the disagreement would have been catastrophic but much less interesting – see later –).

Clear conclusion:

to calculate "RC" in the MSM

Necesse est!

The (most) relevant Radiative Corrections in the SM.

Preliminary statement:

I) Only one loop considered

I) Only "interesting" effects retained

("interesting" effects $\Longrightarrow$

$\Longrightarrow$ hadronic contributions to $(g-2)_\mu$).

Example: M_W/M_Z at one loop

and A_{LR} at one loop.

Aem:

SHOW THAT, AT 1 LOOP:

$$\left[\frac{M_W^2}{M_Z^2}\right]^{(1)} = C_W^{2(1)} \quad (\underline{\text{definition of } C_W})$$

$$A_{LR}^{(1)} = \frac{2\left(1-4 s_{LR}^{2(1)}\right)}{\left[1+\left(1-4 s_{LR}^{2(1)}\right)^2\right]} \quad (\underline{\text{definition}} \text{ of } s_{LR}^2)$$

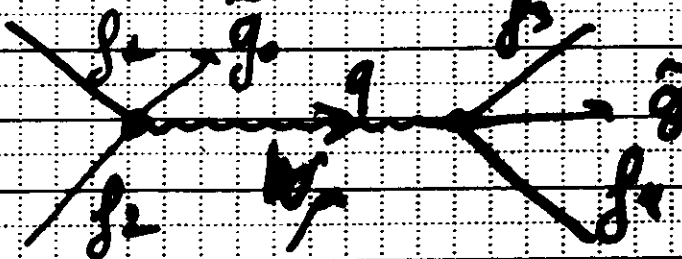
and
$$\underbrace{\left.\begin{aligned} s_{LR}^2 &= 1 - C_{LR}^2, \\ C_{LR}^{2(1)} &\neq C_W^{2(1)} \end{aligned}\right\}}$$

$\underline{\left(\text{at tree level, } C_{LR}^{2(0)} = C_W^{2(0)}\right).}$

To prove the statement, start from the calculation of $\left[\frac{M_W}{M_Z}\right]^{(4)}$.

Only necessary ingredient: definition of physical masses.

Start from W, Z exchange in a four fermion process at tree level:

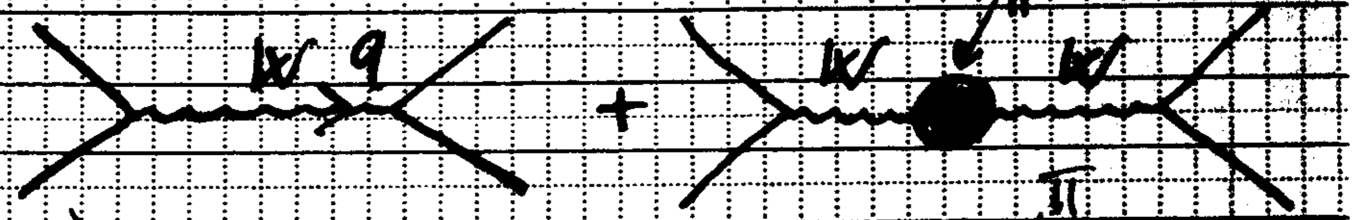


$$\Longrightarrow \frac{\tilde{g}_0^2}{q^2 - M_{0W}^2}$$ ("bare" mass <u>and</u> coupling)

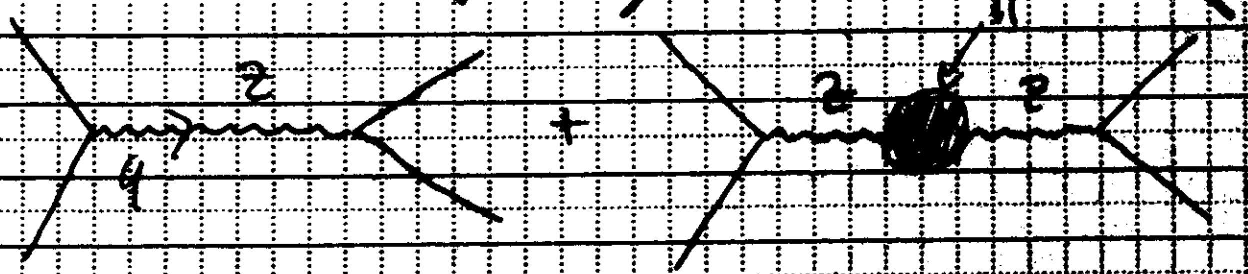
($\tilde{g}_0 \longrightarrow W$ or Z ...)

Adding the "diagonal" self-energy:

e.g.



or



$$\Longrightarrow \frac{\tilde{g}_0^2}{q^2 - M_{0W,Z}^2} \xrightarrow{\text{1 loop!}} \frac{\tilde{g}_0^2}{q^2 + A_{W,Z}(q^2) - M_{0W,Z}^2}$$

(∼ massless fermions)

where $\Pi_{\mu\nu}(q^2) \simeq g_{\mu\nu}\, A(q^2)$

$$\boxed{A(q^2) \equiv A(0) + q^2 F(q^2)}$$

($A \equiv$ "transverse" part of the S.E.)

Definition of "physical" mass:

$$M_0^2 \equiv \underbrace{M^2}_{\text{physical}} + \delta M^2_{ph.}$$

$\delta M^2_{ph.} \equiv \mathrm{Re}\, A(M^2)$ ("on shell" scheme)

(zero of the real part of in prop.,

$$\Longrightarrow \frac{\tilde{g}_0^2}{q^2 - M_0^2} \Longrightarrow$$

$$\Longrightarrow \frac{\tilde{g}_0^2}{[q^2 - M^2] + \mathrm{Re}[A(q^2) - A(M^2)] + i\,\mathrm{Im}\,A(q^2)}$$

$i M \Gamma(q^2)$

$\Rightarrow$ for $q^2 = M^2$

$$|\quad|^2 \Longrightarrow \left|\frac{\tilde{g}_0^2}{M\Gamma}\right|^2$$ (<u>bump</u> in σ...)

In general, $\mathrm{Re}\,A(q^2)$, $\mathrm{Re}\,A(M^2)$

DIVERGENT

(<u>Dirac</u> : SMALL, BUT INFINITE)

(F. BOUDJEMA...)

(dimensional regularization <u>essential</u>)

<u>But</u> : in general, also $\overbrace{\tilde{g}_0^2}$

small but infinite

Expectation:

PHYSICAL QUANTITIES

FINITE

$\Rightarrow$ Infinities must cancel.

defining "physical" couplings

$$\tilde{g}_0^2 \equiv \tilde{g}^2 + \delta\tilde{g}^2.$$

In practice: only couplings

$$\alpha_0 \equiv \alpha + \delta\alpha$$

$$\left(\frac{\delta\alpha}{\alpha} \equiv F_{\gamma}(0)\right) \quad (QED \ldots)$$

$$s_0^2 \equiv 1 - c_0^2 \equiv 1 - \frac{M_{0W}^2}{M_{0Z}^2}$$

....

$\Longrightarrow$ possible definition of "physical" c_w^2:

$$\frac{M_{0W}^2}{M_{0Z}^2} \equiv c_0^2 \equiv c_w^2 + \delta c_w^2$$

$$c_w^2 \equiv \frac{M_W^2}{M_Z^2}\Bigg|_{\text{physical}}$$

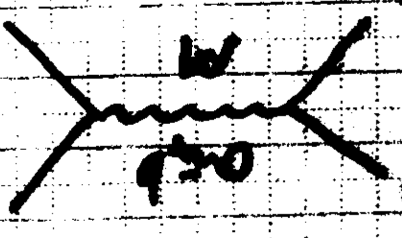
$$\Longrightarrow \quad \delta c_w^2 = \frac{M_{0W}^2}{M_{0Z}^2} - \frac{M_W^2}{M_Z^2} =$$

$$= c_w^2\left[\frac{\delta M_W^2}{M_W^2} - \frac{\delta M_Z^2}{M_Z^2}\right]$$

$\Longrightarrow$ everything ready...

Tree level equality:

$$\frac{G_\mu^{(0)}}{\sqrt{2}} \Longrightarrow \sim T^{(0)}_{\mu e \ldots}(q^2 \approx 0) = \text{[illegible]} =$$

$$= \frac{\alpha_0 \pi}{2 s_0^2 M_{0W}^2}$$ (= 

)

W

we

def

Then

one

$T_{eq}^{(sa)} = \frac{\Pi}{2}$

The bare

replaced by

C

Then one can write:

$$\sim T_{\mu e}^{(S.E.)(1)}(q^2=0) = \frac{e^2 M_Z^2}{8 M_W^2 (M_Z^2 - M_W^2)} \left[1 + \Delta r^{(S.E.)} \right] \equiv \frac{G_\mu}{\sqrt{2}}$$

where

$$\Delta r^{(S.E.)} = \left[2\frac{\Delta e}{e} - \frac{\delta M_W^2}{M_W^2} + \frac{1}{(M_Z^2 - M_W^2)} \left(\delta M_W^2 - c_W^2 \delta M_Z^2 \right) + \frac{A_W(0)}{M_W^2} \right]^{**}$$

and the definition (valid to all orders of pt. th) of c_W^2 is

$$\boxed{c_W^2 \equiv \frac{M_W^2}{M_Z^2}}$$

By construction, all quantities in the bracket are g. invariant, with the only possible exception of

$A_W(0)$.

In fact, $A_W(0)$ is not g. invariant:

$\Longrightarrow$ add vertices, boxes $\Longrightarrow$

$$\frac{G_\mu}{\sqrt{2}} = \frac{e^2}{8M_W^2 s_W^2}\left[1 + \Delta r\right] \quad ; \; s_W^2 = 1 - \frac{M_W^2}{M_Z^2}$$

SIRLIN'S FORMULA

where

$$\Delta r = \Delta r^{(S.E.)} + (\text{vertices, boxes.....})$$

and

$$\Delta r^{(S.E.)} = \mathcal{R}\left[F_\gamma(0) + \frac{c_W^2}{s_W^2}\left(\frac{A_W(M_W^2)}{M_W^2} - \frac{A_Z(M_Z^2)}{M_Z^2}\right) + \frac{A_W(0) - A_W(M_W^2)}{M_W^2}\right]_{\xi_i = 1} \left(\equiv \underline{\text{finite!}}\right)$$

($\mathcal{R}$=real part)

The contribution from vertices and boxes is small:

$$\Delta r\,(\text{vertices, boxes}) = \simeq \frac{\alpha}{4\pi s_W^2}\left(6 + \frac{7 - 4s_W^2}{2 s_W^2}\ln c_W^2\right) \simeq 6 \cdot 10^{-3}$$

In the MSM the most interesting and peculiar part of Δr is $\Delta r^{(S.E.)}$.

It is convenient to use the notation

$$A(q^2) = A(0) + q^2 F(q^2)$$

for the various s.energies, with

$$A_\gamma(0) = 0 \quad \text{always}$$

and $\quad A_{z\gamma}(0) = 0 \quad$ with some trick.

Then, after some reshuffling, $\Delta r^{(S.E.)}$ becomes

$$\Delta r^{(S.E.)} \equiv \mathcal{R}\Big\{ \Big[F_\gamma(0) - F_\gamma(M_z^2)\Big] - \frac{c_W^2}{s_W^2}\Big[\frac{A_z(0)}{M_z^2} - \frac{A_W(0)}{M_W^2}\Big]$$

$$+ \frac{(c_W^2 - s_W^2)}{s_W^2}\Big[F_W(M_W^2) - c_W^2 F_z(M_z^2) -$$

$$- s_W^2 F_\gamma(M_z^2) - 2 s_W c_W F_{z\gamma}(M_z^2)\Big]$$

$$+ 2\Big[-c_W^2 F_z(M_z^2) + c_W^2 F_\gamma(M_z^2) +$$

$$+ \frac{c_W}{s_W}(1 - 2 s_W^2) F_{z\gamma}(M_z^2)\Big]\Big\} \equiv$$

$$\equiv \mathcal{R}\Big\{\Big[\Delta\alpha(M_z^2)\Big] - \frac{c_W^2}{s_W^2}\Big[\Delta\rho(0)\Big] +$$

$$+ \frac{(c_W^2 - s_W^2)}{s_W^2}\Big[\Delta_2\Big] + 2\Big[\Delta_3\Big]\Big\}$$

1) $\Delta\alpha(M_Z^2) \ldots [\Pi_\gamma(0) - \Pi_\gamma(M_Z^2)]$

effect of the "running" of

$$\Rightarrow \Delta\alpha(M_Z) = \frac{\alpha(\ldots)}{\ldots}$$

Numerically only fermions ... t.

Leptonic contribution fo...

$$\Delta^l_\alpha(M_Z^2) = \frac{\alpha}{3\pi} Q_l^2 \left[\ln \ldots \right.$$

If $m_l \gg M_Z$, one has

$$\Delta^l_\alpha(M_Z^2) = \frac{\alpha}{3\pi} Q_l^2 \ldots$$

This is a ...

Carazzone's Decoupling ...

(same if the ...

$\Rightarrow$ No ...

For the overall fermionic contrib.

$$\Delta\alpha^{(f)}(M_Z^2) = 0.0602 \pm 0.0009$$

Not small!

2) $\Delta\rho(0) \equiv \frac{A_Z(0)}{M_Z^2} - \frac{A_W(0)}{M_W^2}$

Here, <u>two</u> peculiar phenomena are met:

2a) <u>Non decoupling</u>. Due to the <u>axial</u> $\neq 0$ components of the (W, Z) currents, the A.C. decoupling is <u>not guaranteed</u> $\Rightarrow$ one expects effects from heavy particles (both fermions <u>and</u> bosons). In principle, they could be "soft" (e.g. logarythmic) or "hard" (e.g. quadratic).

2b) <u>Breaking of custodial SU(2)</u>. At tree level, $\rho_0 \equiv \frac{M_{0W^+}^2}{M_{0W_3}^2} = \underline{1}$ from c. SU(2)

Thus, if at the one loop level contributions arise from sectors where clearly $SU(2)$ is violated, one expects that the corresponding definition of ρ deviates from one "proportionally" to $SU(2)$ breaking "indicator" ($\sim m_u - m_d$ for a quark doublet).

Writing as usually

$$\rho_0 \equiv 1 = \rho + \Delta\rho$$

one finds

$$\Delta\rho = \mathcal{R}\left[\frac{A_Z(M_Z^2)}{M_Z^2} - \frac{A_W(M_W^2)}{M_W^2}\right]$$

$\Rightarrow$ the effect on $\Delta\rho(0)$ of a "very" heavy quark doublet will be like that on $\Delta\rho$ $\Rightarrow$ in fact one finds

$$\Delta\rho(0)(u,d) \simeq \frac{\alpha}{\pi}\frac{(m_u^2 - m_d^2)}{M_Z^2}$$

so from (t,b) one gets the <u>famous</u> term:

(Veltman, Maiani...) $\Delta\rho^{(t,b)}(0) \simeq \alpha/\pi \; m_t^2/M_Z^2$.

For $m_t \simeq 2M_Z$ this gives

$$\Delta\rho(0)^{(m_t=2M_Z)} \simeq 0.009$$

that would strongly reduce Δr.

What about the Higgs contribution to $\Delta\rho(0)$? A priori, it has no reasons for being decoupled. But it is "screened" at one loop (Veltman's screening) $\Longrightarrow$

$$\Delta\rho(0)^{(\mathrm{Higgs})} \xrightarrow[m_H \gg M_Z]{} \sim -\frac{3\alpha}{16\pi}\frac{1}{c_W^2}\ln\frac{m_H^2}{M_Z^2}.$$

For $m_H \simeq 2M_Z$ this gives

$$\Delta\rho(0)^{(m_H=2M_Z)} \simeq -0.001$$

3), 4) <u>In the MSM</u>, the contribution from Δ_2, Δ_3 is small. <u>In particular</u>:

Δ_3 has a logaryth. dependence on <u>both</u> m_t and m_H, in the limit of large $m_{t,H}$.

Δ_2 has a logaryth. dependence on m_t and <u>no</u> m_H dependence in the limit.

Numerycally, one can (only) give the Δr values at variable m_t, M_H

Going back to Sirlin's formula, page ..

$$\frac{G_\mu}{\sqrt{2}} = \frac{4\pi\alpha}{8M_W^2(1-M_W^2/M_Z^2)}\left[1+\Delta r\right] \quad \left(\alpha\equiv\alpha(0) \simeq 1/137\right)$$

and this becomes also

$$M_W^2 = M_Z^2\left[\frac{1}{2}+\frac{1}{2}\sqrt{1-\frac{4\mu_0^2}{M_Z^2}(1+\Delta r)}\right]$$

where $\mu_0^2 \equiv \frac{\alpha(0)\pi}{\sqrt{2}G_\mu} = (37.28\,\mathrm{GeV})^2$

Since the <u>definition</u> (to all orders) of c_W^2 is $c_W^2 = M_W^2/M_Z^2$, the previous expression provides the one-loop value of $c_W^2 \equiv c_W^{2(1)} =$

$$= c_W^{2(1)} = \frac{1}{2}+\frac{1}{2}\sqrt{1-\frac{4\mu_0^2}{M_Z^2}(1+\Delta r)}$$

and, correspondingly, of the (often met) quantity:

$$s_W^{2(1)} \equiv 1 - c_W^{2(1)} =$$

$$= \frac{1}{2} - \frac{1}{2}\sqrt{1 - \frac{4\mu_0^2}{M_Z^2}(1+\Delta r)} \Rightarrow$$

$$\left\{ s_W^{2(1)} = \frac{1}{2} - \frac{1}{2}\sqrt{1 - \frac{4\mu_0^2}{M_Z^2(1-\Delta r)}} \right\}$$

($1+\Delta r \Rightarrow \frac{1}{1-\Delta r}$ takes care of next leading logs from $\Delta\alpha$...).

To one loop, this can still be rewritten as

$$s_W^{2(1)} = s_1^2 \left[1 + \frac{c_1^2}{1-2s_1^2} \Delta r \right]$$

where $s_1^2 = \frac{1}{2}\left[1 - \sqrt{1 - \frac{4\mu_0^2}{M_Z^2}}\right] \simeq 0.2122.$

From the previous expression we see that the <u>physical</u>, <u>gauge invariant</u> quantity s_W^2 can be thought of as built up by a <u>sum</u> that also contains self-energies, tadp., vert., boxes that make up $\Delta r \Rightarrow s_W^2 \equiv s_1^2 + s_W^{2(SS)} + \ldots$

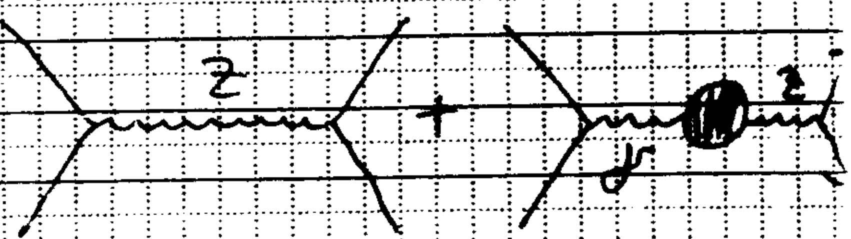
By an essentially analogous procedure one can now derive a quantity to be called

$$s^{2(1)}_{LR}(M_Z^2).$$

(Super) simplified approach: calculate $A = A_{LR}$ at one loop.

On Z resonance, the S.E. contribution will be simple:

$$A^{(1)(S.E.)} \Longrightarrow$$



(other corrections to the Z propagator disappear in the ratio)

By conventional Feynman rules:

$$A^{(1)(S.E.)} = \frac{2v_0}{1+v_0^2}\left[1 - \frac{4c_0 s_0 (1-v_0^2)}{v_0(1+v_0^2)} \frac{A_{\gamma Z}(M_Z^2)}{M_Z^2}\right]$$

where $v_0 \equiv 1 - 4s_0^2$.

One can still use the tree level definition $c_0^2 = \frac{M_W^2}{M_{0Z}^2}$, $s_0^2 = 1 - c_0^2$ and the quantity $s_W^2 = 1 - \frac{M_W^2}{M_Z^2}$ defined in Lecture II. Then $s_0^2 = s_W^2 + \delta s_W^2$ (g.inv decomposition) and $\delta M_i^2 = \mathrm{Re}\, A_i(M_i^2)$ (tadpoles already included).

Replacing s_0^2 by s_W^2 leads then to an expression where the only not g.i. term is $\sim \frac{A_{\gamma Z}(M_Z^2)}{M_Z^2}$. This gauge dependence will be cancelled by that of the Z vertex (with virtual W's, Sirlin's notation) (Appendix)

Rather than s_w^2, one can use at one loop the quantity s_1^2 (page 35, lecture 3). This introduces $\underline{\Delta r}$ and makes the s.e. component of $A^{(1)}$ to become, after some reshuffling:

$$A^{(1)(S.E.)} = \frac{2(1-4s_1^2)}{1+(1-4s_1^2)^2}\left[1+\Delta p\right]$$

where

$$\Delta p = \frac{-32 s_1^4}{(1-4s_1^2)(1+v_1^2)}\left\{c_1^2\left[\Delta_\alpha(M_Z^2)\right] - c_1^2\left[\Delta_\rho(0)\right] + \left[\Delta_3\right]\right\} \text{ (finite!)}$$

$$v_1 \equiv 1-4s_1^2 \quad **$$

NB

) Δp contains three $\{\Delta_\alpha, \Delta_\rho, \Delta_3\}$ quantities that make up Δr, without Δ_2.

) Again, like for G_μ, the one-loop effect is to multiply a "ren tree" by $[1+\Delta p]$

The last point leads in a rather natural way to the idea of <u>defining</u> the quantity s^2_{LR} as:

$$A^{(1)} \equiv \frac{2(1-4s^2_{LR})}{1+(1-4s^2_{LR})^2}$$

(at one loop, this definition is universal, from the $A^{(1)}$'s properties).

The <u>physical</u>, <u>gauge invariant</u> quantity s^2_{LR} will be built up by a sum that contains those self-energies (with tadpoles...), vertices, boxes that appear in $A^{(1)}$, in full analogy with the decomposition of s^2_W, page 25, Lecture 2.

In particular, the self-energy content of s^2_{LR} can be easily determined.

From the previous discussion:

$$A^{(1)(S.E.)} = \frac{2(1-4s_1^2)}{1+(1-4s_1^2)^2}(1+\Delta p) = \frac{2(1-4s_{LR}^{2(S.E.)})}{1+(1-4s_{LR}^{2(S.E.)})^2}$$

one finds

**

$$s_{LR}^{2(S.E.)} = s_1^2 - \frac{1}{4}\, v_1 \frac{(1+v_1^2)}{(1-v_1^2)}\, \Delta p$$

From this equality, one can derive the (more interesting) relationship between s_{LR}^2 and s_W^2 (s.e. components). Introducing the approximation of neglecting systematycally terms of $O(\alpha v_1)$, this reads:

$$s_{LR}^{2(S.E.)} = s_W^{2(1)(S.E.)} \left\{ 1 + 3\left[\Delta_{\rho(0)}\right] - 3\left[\Delta_2\right] - 3\,\frac{s_1^2}{c_1^2}\left[\Delta_3\right] \right\}$$

This shows some rather important things, in particular:

1) At one loop, the "sinsquared" measured from the $\frac{M_W}{M_Z}$ ratio and from (LEP 1) asymmetries are no longer equal (we expect). In particular, s^2_{LR} is (much) less $\Delta\rho(0)$ (i.e. m_t) dependent; neglecting the remaining terms one has in fact:

$$s_W^{2(1)}(top) \simeq \frac{-c_1^4}{1-2s_1^2}\Delta\rho(0) \simeq -1.08\,\Delta\rho(0).$$

$$s^2_{LR}(top) \simeq \frac{-8\,s_1^4 c_1^2}{1-v_1^2}\Delta\rho(0) \simeq -0.28\,\Delta\rho(0)$$

2) The ratio $\frac{s^2_{LR}}{s_W^{2(1)}}$ (s.e.) is $\neq 1$

only because of the presence of the "genuine" ($\neq \Delta_\rho(0)$) electroweak SM RC. Thus, a (very) high precision measurement of s_W^2 and s^2_{LR} would provide a test of the corresponding SM [illegible]

What does one measure at LEP1, SLC?

Essentially

I) M_Z $\left(\frac{\delta M_Z}{M_Z} \simeq 10^{-5} \,! \right)$

II) From a lot of independent quantities (unpolarized and polarized asymmetries + cross sections $\Rightarrow$ Z total and partial widths):

$$\sim s^2_{LR}(M_Z^2) \quad \text{(FIGURE)}$$

(now called $\sin^2\theta_{eff}^{lept.} = 0.2318 \pm$ [illegible])

III) N.B. In hadronic quantities $\alpha_s(M_Z)$ enters! (HINCHLIFFE)

From ...

LEP1 "needs" ...

contribution of 

corresponding ...

$$m_t^{(\ldots)} = 17\ldots$$

($\alpha_s(M_Z) = 0.125 \pm 0.006 \ldots$) (60 GeV $\lesssim$ ...)

($\sim$ "prediction" !)

The very recent CDF/D0 value (FermiLab) ...

$$\frac{m_t^{(exp)}}{\text{GeV}} = \begin{cases} 17\ldots \\ \ldots \end{cases}$$

Undeniably, impressive

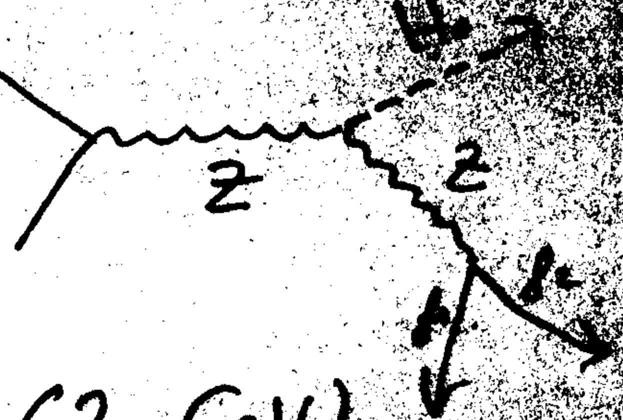
($\sim$ "better" than $(g-2)$)

What ... next? ...

Immediate answer:

THE HIGGS

Not found at LEP1



($M_{H^0} \gtrsim 62$ GeV)

"Good" (?) hopes at LEP2
(A. Masiero)

$\sqrt{q^2} \underset{\sim}{\Longrightarrow} 205$ GeV (?)

($M_{H^0} \simeq 100$ GeV ...

Virtual effects?
Weak ("screened").

<u>If</u> m_t definitely fixed at CDF

from $\delta M_W \lesssim 30$ MeV

"some" indication by comparing

$$s_W^2 \equiv M_W^2 / M_Z^2$$

with s_{LR}^2

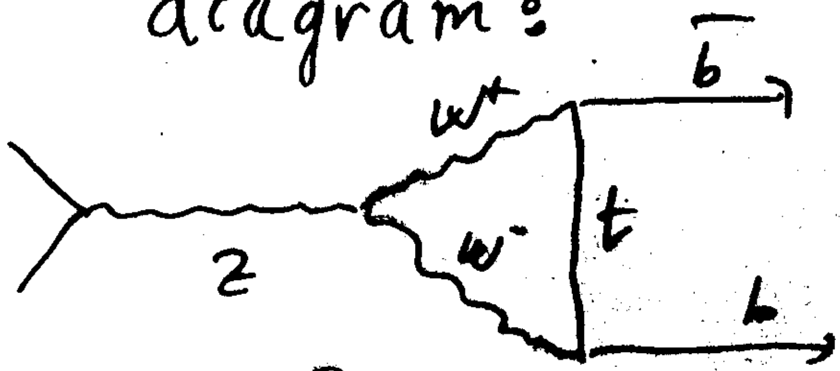
(better than nothing, if H_0 not discovered...)

What (anything) "else"?

<u>One</u> (possible) small surviving disagreement between SM and LEP1 measurements:

$$R_b \equiv \frac{\Gamma_b}{\Gamma_h} \equiv \frac{\Gamma_{Z \to b\bar{b}}}{\Gamma_{Z \to had}} \quad \text{(Figure)}.$$

Relevant diagram:

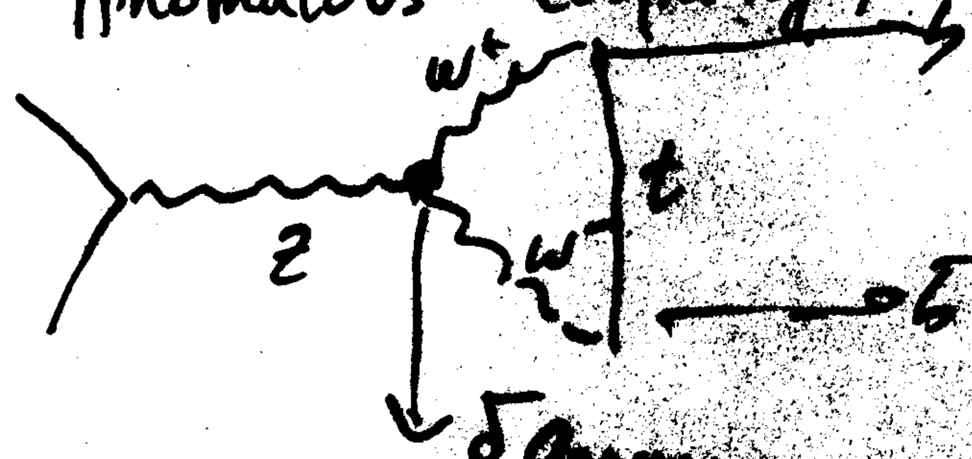
~ 

Suggestions ??

I) Add the "Sparticles" vertex

(could do the job...)

II) Modify the ZWW coupling
("Anomalous" coupling)



(could do the job)

Very speculative (?)

(would be "much" better than (g-2))

Conclusions

Until now, the MSM is do[illegible]
its best .....

LEP1, SLC have tried [illegible]
to (dis?) prove its val[illegible]...

Great expectation for the
soon coming series of
"tests" (LEP2, LHC .....).
But, whatever will happe[illegible]
HONOUR TO THE S[illegible]

5.2 The Effective Electroweak Mixing Angle $\sin^2\theta_{eff}^{lept}$

The asymmetry measurements from LEP can be combined into a single observable, the effective electroweak mixing angle, $\sin^2\theta_{eff}^{lept}$, defined as:

$$\sin^2\theta_{eff}^{lept} \equiv \frac{1}{4}(1 - g_{V\ell}/g_{A\ell}), \tag{13}$$

without making any strong model-specific assumptions.

For a combined average of $\sin^2\theta_{eff}^{lept}$ from $A_{FB}^{0,\ell}$, $\mathcal{A}_\tau$ and $\mathcal{A}_e$ only the assumption of lepton universality, already inherent in the definition of $\sin^2\theta_{eff}^{lept}$, is needed. In practice no further assumption is involved if the quark forward-backward asymmetries, $A_{FB}^{0,b}$ and $A_{FB}^{0,c}$, are included in this average, as these asymmetries have a reduced sensitivity to corrections particular to the hadronic vertex. The results of these determinations of $\sin^2\theta_{eff}^{lept}$ and their combination are shown in Table 16. Also the comparison with the measurement of the left-right asymmetry, A_{LR}, at SLD [16] is given. It should be noted that A_{LR} measures the same quantity as $\mathcal{A}_e$ from τ polarisation, with minimal model dependence. Both measurements have small systematic errors.

	$\sin^2\theta_{eff}^{lept}$	Average by Group of Observations	Cumulative Average	χ^2/d.o.f.
$A_{FB}^{0,\ell}$	0.23096 ± 0.00073			
$\mathcal{A}_\tau$	0.2324 ± 0.0010			
$\mathcal{A}_e$	0.2328 ± 0.0011	0.2318 ± 0.0005	0.2318 ± 0.0005	2.5/2
$A_{FB}^{0,b}$	0.23182 ± 0.00064			
$A_{FB}^{0,c}$	0.2310 ± 0.0021	0.2318 ± 0.0006	0.2318 ± 0.0004	2.6/4
$\langle Q_{FB}\rangle^{(a)}$	0.2320 ± 0.0016	0.2320 ± 0.0016	0.2318 ± 0.0004	2.6/5
A_{LR} (SLD)	0.2294 ± 0.0010	~~[illegible]~~	~~[illegible]~~	~~[illegible]~~

→ 0.23[illegible] ?

Table 16: Comparison of several determinations of $\sin^2\theta_{eff}^{lept}$ from asymmetries. Averages are obtained as weighted averages assuming no correlations. The second column lists the $\sin^2\theta_{eff}^{lept}$ values derived from the quantities listed in the first column. The third column contains the averages of these numbers by groups of observations, where the groups are indicated by the horizontal lines. The last column shows the cumulative averages. The χ^2 per degree of freedom for the cumulative averages also is given. $^{(a)}$The LEP results for the hadronic charge asymmetry [52–55], $\langle Q_{FB}\rangle$, and their method of combination are unchanged with respect to Reference 23.

5.3 Number of Neutrinos

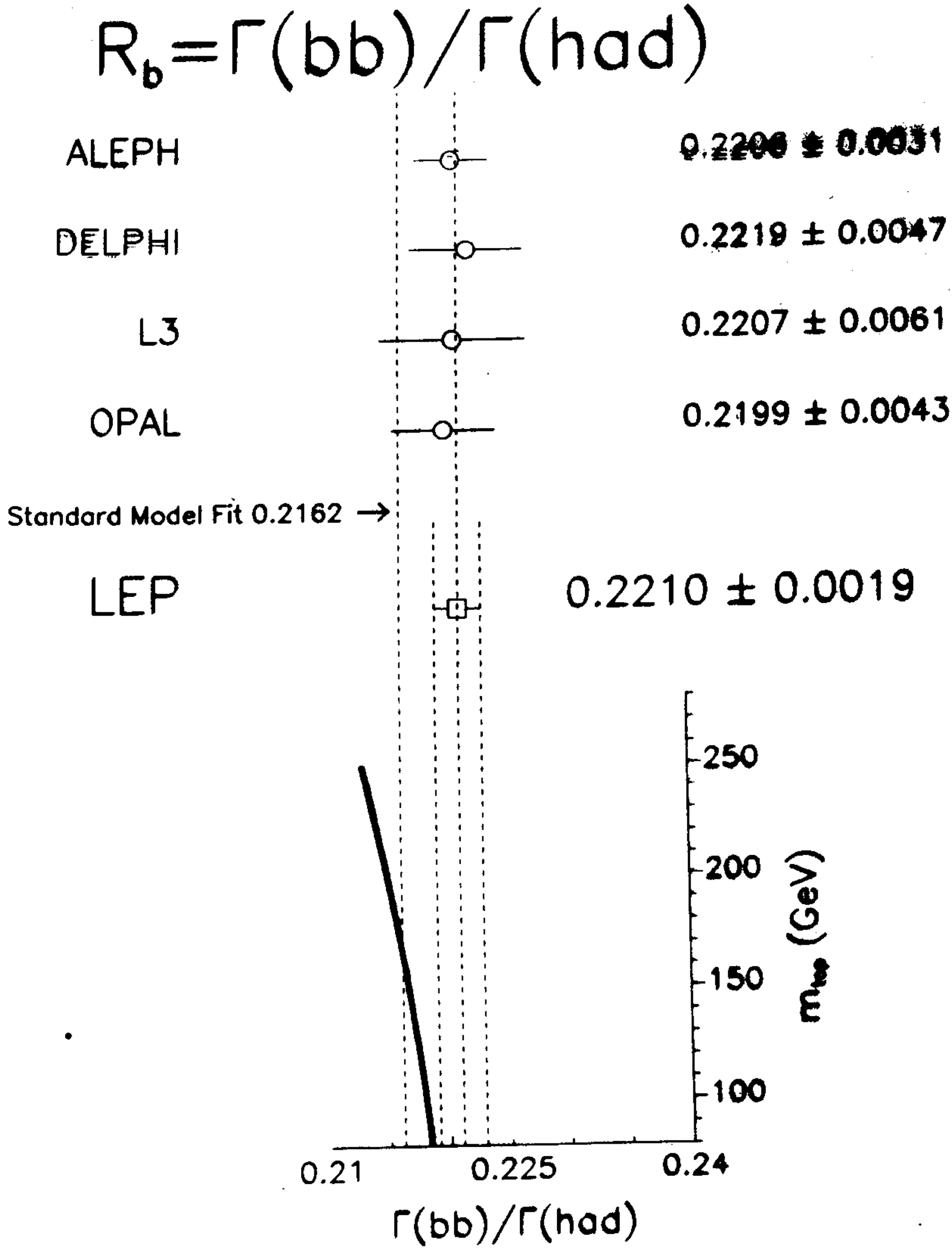
An important aspect of our measurement concerns the information related to Z decays into invisible channels. Using the results of Tables 7 and 8 the ratio of the Z decay width into invisible particles and the leptonic decay width is determined:

$$\Gamma_{inv}/\Gamma_{\ell\ell} = 5.951 \pm 0.032 \, .$$

	Measurement	Standard Model Fit	Pull
a) LEP			
line-shape and lepton asymmetries:			
m_Z [GeV]	91.1887 ± 0.0022	91.1885	0.0
Γ_Z [GeV]	2.4971 ± 0.0032	2.4979	−0.3
σ_h^0 [nb]	41.492 ± 0.081	41.441	[illegible]
R_ℓ	20.800 ± 0.035	20.783	[illegible]
$A_{FB}^{0,\ell}$	0.0172 ± 0.0013	0.0157	1.1
+ correlation matrix Table 8			
τ polarisation:			
$\mathcal{A}_\tau$	[illegible]	0.145	−0.8
$\mathcal{A}_e$	[illegible]	0.145	−0.9
b and c quark results:			
R_b	0.2204 ± 0.0020	0.2157	2.4
R_c	0.1606 ± 0.0095	0.172	−1.2
$A_{FB}^{0,b}$	0.1015 ± 0.0036	0.1015	0.0
$A_{FB}^{0,c}$	0.0760 ± 0.0089	0.0724	0.4
+ correlation matrix Table 14			
$q\bar{q}$ charge asymmetry:			
$\sin^2\theta_{eff}^{lept}$ ($\langle Q_{FB} \rangle$)	0.2320 ± 0.0016	0.2318	0.1
b) $p\bar{p}$ and νN			
m_W [GeV] ($p\bar{p}$ [57])	80.26 ± 0.16	80.34	−0.6
$1 - m_W^2/m_Z^2$ (νN [11–13])	0.2253 ± 0.0047	0.2238	0.4
c) SLC			
$\sin^2\theta_{eff}^{lept}$ (A_{LR} [18])	[illegible] $\pm$ 0.0010	[illegible]	−2.4

Table 17: Summary of measurements included in the combined analysis of Standard Model parameters. Section a) summarises LEP averages, section b) electroweak precision tests from $p\bar{p}$ colliders and νN scattering, section c) gives the result for $\sin^2\theta_{eff}^{lept}$ from the measurement of the left-right polarisation asymmetry at SLD. The Standard Model fit results in column 3 and the pulls (difference to measurement in units of the measurement error) in column 4 are derived from the fit including all data (Table 18, column 4) for a fixed value of m_H = 300 GeV.

Detailed studies of the theoretical uncertainties in the Standard Model predictions are carried out in the working group on 'Precision Calculations at LEP-I' [50]. The theoretical uncertainties are dominated by the uncertainty in the value of $\alpha(m_Z^2)$ due to the contribution of light quarks to the photon vacuum-polarisation. Recently there have been several preprints on reevaluations of $\alpha(m_Z^2)$ [4–6]. The span of results ranges from $\alpha(m_Z^2) = 1/(128.$[illegible]$ \pm $ [illegible]$)$ [4] to $\alpha(m_Z^2) = 1/(128.896 \pm 0.090)$ [6]. For the results presented in this section, a value of $\alpha(m_Z^2) = 1/(128.896 \pm 0.090)$ [6] is used, where the error is propagated in the fits. This uncertainty causes an error of 0.00023 on the Standard Model prediction of $\sin^2\theta_{eff}^{lept}$ and an error of 4 GeV on m_t. The effect on the Standard Model prediction for $\Gamma_{\ell\ell}$ is negligible. Using instead $\alpha(m_Z^2) = 1/(128.$[illegible]$ \pm 0.10)$ [4] results in a shift of the


$R_b = \Gamma(bb)/\Gamma(had)$
ALEPH
0.2206 ± 0.0031
DELPHI
0.2219 ± 0.0047
L3
0.2207 ± 0.0061
OPAL
0.2199 ± 0.0043
Standard Model Fit 0.2162 →
LEP
0.2210 ± 0.0019
250
200
150
100
m_{top} (GeV)
0.21
0.225
0.24
Γ(bb)/Γ(had)