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SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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CP VIOLATION AND BEAUTY

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CP Violation and Beauty

CP Violation and Beauty

$\not{CP}$ in K decays.

The possibility that $\not{CP}$ comes from the known weak interaction, hence from phase factors in the quark mixing matrix V.

Then we expect large $\not{CP}$ effects in B (beauty meson) decays.

How phase factors in V lead to $\not{CP}$.

How decays of neutral B mesons can cleanly test whether $\not{CP}$ is due to phases in V.

What phases there are to measure, and which B decays measure them.

How a knowledge of the phases in V determines all of V.

C, P, and CP

Charge conjugation C –
Takes particle into antiparticle:

$$C|f\rangle = \eta_c|\bar{f}\rangle .$$

Parity P –
Inverts the coordinate system:

$$\vec{r} \to -\vec{r} .$$

$$\vec{p} = m\frac{d\vec{r}}{dt} \to -\vec{p}, \quad \vec{L} = \vec{r}\times\vec{p} \to +\vec{L} .$$

$$P|f(\vec{p},s_z)\rangle = \eta_p|f(-\vec{p},s_z)\rangle .$$

CP = C * P –

$$CP|f(\vec{p},s_z)\rangle = \eta_{cp}|\bar{f}(-\vec{p},s_z)\rangle .$$

$\not{C}\not{P}$ in the neutral K system

Consider neutral kaons at rest.

We choose the phase convention so that

$$CP|K^0\rangle = +1|\overline{K^0}\rangle.$$

Then (simple field theory)

$$CP|\overline{K^0}\rangle = +1|K^0\rangle.$$

The time evolution of a neutral kaon is described by

amp. for K^0

$$i\frac{\partial}{\partial t}\begin{bmatrix} a(t) \\ \bar{a}(t) \end{bmatrix} = \mathcal{M}\begin{bmatrix} a(t) \\ \bar{a}(t) \end{bmatrix}$$

amp. for $\overline{K^0}$

$$\mathcal{M} = \begin{bmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} \\ \mathcal{M}_{21} & \mathcal{M}_{22} \end{bmatrix}$$, a non-Hermitean

effective Hamiltonian, is known as the neutral K mass matrix.

We assume CPT invariance.
This implies that

$$\mathcal{M}_{11} \equiv \langle K^0 | \mathcal{M} | K^0 \rangle = \langle \overline{K^0} | \mathcal{M} | \overline{K^0} \rangle \equiv \mathcal{M}_{22}.$$

Writing $\mathcal{M}_{11} \equiv M - i\frac{\Gamma}{2}$, $\mathcal{M}_{22} \equiv \overline{M} - i\frac{\overline{\Gamma}}{2}$,

$M = \overline{M}$ Particle and antiparticle have same mass

$\Gamma = \overline{\Gamma}$ They also have same total width.

If CP invariance holds,

$$\mathcal{M}_{12} \equiv \langle K^0 | \mathcal{M} | \overline{K^0} \rangle = \langle K^0 | (CP)^\dagger \mathcal{M} (CP) | \overline{K^0} \rangle$$

$$= \langle (CP) K^0 | \mathcal{M} | (CP) \overline{K^0} \rangle = \langle \overline{K^0} | \mathcal{M} | K^0 \rangle \equiv \mathcal{M}_{21}.$$

Then $\mathcal{M}$ has the form

$$\mathcal{M} = \begin{bmatrix} X & Y \\ Y & X \end{bmatrix},$$

so that the mass eigenstates are $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

That is, they are the CP eigenstates

$$|K_+\rangle \equiv \frac{|K^0\rangle + |\bar{K}^0\rangle}{\sqrt{2}}, \quad |K_-\rangle = \frac{|K^0\rangle - |\bar{K}^0\rangle}{\sqrt{2}}$$

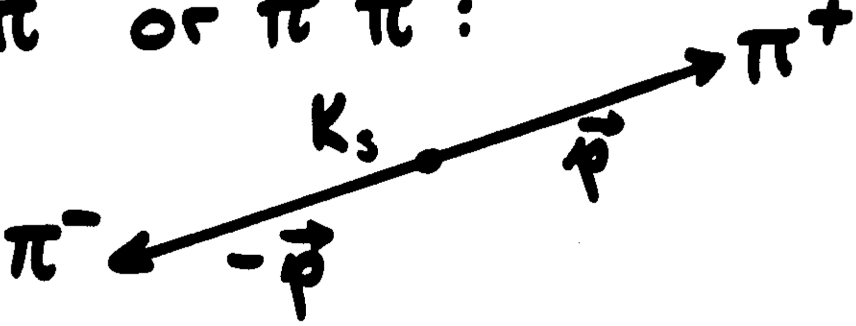
P: $\quad +1 \qquad\qquad -1$

Experimentally, the two mass eigenstates are—

$K_S \quad (\tau_S = 0.89 \times 10^{-10}$ sec$)$

$K_L \quad (\tau_L = 5.17 \times 10^{-8}$ sec$)$

Essentially all K_S decays are to $\pi^+\pi^-$ or $\pi^0\pi^0$:



$$CP|\pi^+(\vec{p})\pi^-(-\vec{p})\rangle = +|\pi^-(-\vec{p})\pi^+(\vec{p})\rangle$$

$\therefore$ If CP is conserved,

$$CP(K_S) = +1, \quad CP(K_L) = -1.$$

Then $K_L \to \pi^+\pi^-$ is forbidden.

But $K_L \to \pi^+\pi^-$ does occur.

(Christenson, Cronin, Fitch, Turley

$$BR(K_L \to \pi^+\pi^-) = 2.0 \times 10^{-3}$$

$\therefore$ CP is not always conserved.

It is also observed that

$$\frac{\Gamma(K_L \to \pi^- \ell^+ \nu) - \Gamma(K_L \to \pi^+ \ell^- \bar{\nu})}{\text{"} \quad + \quad \text{"}}$$

$$= 3.27 \times 10^{-3} \neq 0.$$

CP-noninvariance.

<u>What is the origin of $\cancel{CP}$?</u>

All $\cancel{CP}$ - effects seen sofar occur in K decays.

K decays are due to the weak interaction.

Perhaps $\cancel{CP}$ comes from the weak interaction.

This is the possibility we will emphasize.

How can the weak interaction cause $\not{CP}$?

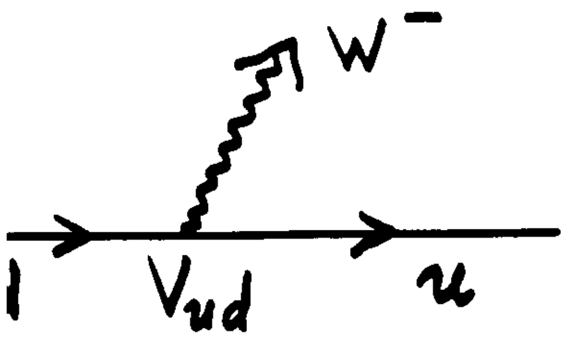
According to the S(tandard) M(odel), the weak couplings of the quarks are described by $\mathcal{H} = \mathcal{H}_{CC} + \mathcal{H}_{NC}$, with

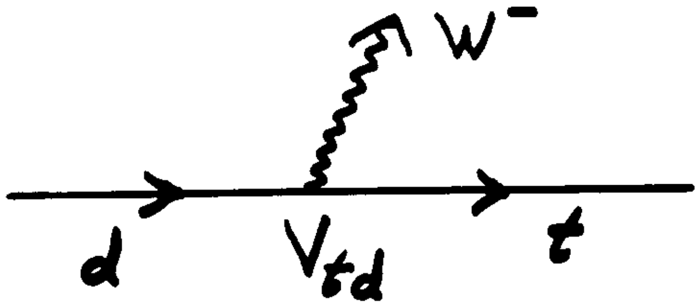
$$\mathcal{H}_{CC} = \frac{g}{\sqrt{2}} W^{\mu} \sum_{\substack{\alpha = u,c,t \\ i = d,s,b}} V_{\alpha i} \bar{\alpha}_L \gamma_\mu i_L$$

$$+ \frac{g}{\sqrt{2}} W^{\mu\dagger} \sum_{\substack{\alpha \\ i}} V^*_{\alpha i} \bar{i}_L \gamma_\mu \alpha_L .$$

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

is the C(abibbo) - K(obayashi) - M(askawa) quark mixing matrix.





$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}$$

$$\mathcal{H}_{NC} = \frac{g}{\cos\theta_W} Z^\mu \sum_{\substack{q=u,c,t,\\ d,s,b}} \left\{ \left[I_3(q_L) - Q(q)\sin^2\theta_W \right] \bar{q}_L \gamma_\mu q_L - Q(q)\sin^2\theta_W \, \bar{q}_R \gamma_\mu q_R \right\}$$

$$(CP) W^\mu V_{\alpha i} \bar{\alpha}_L \gamma_\mu i_L (CP)^{-1} =$$

$$= \underbrace{[\eta(W)\eta(\alpha)\eta(i)]}_{\text{Phases which we can choose } = 1} W^{\mu\dagger} V_{\alpha i} \bar{i}_L \gamma_\mu \alpha_L .$$

$$\mathcal{H}_{cc} = \frac{g}{\sqrt{2}} W^\mu \sum_{\alpha,i} V_{\alpha i} \bar{\alpha}_L \gamma_\mu i_L + \frac{g}{\sqrt{2}} W^{\mu\dagger} \sum_{\alpha,i} V_{\alpha i}^* \bar{i}_L \gamma_\mu \alpha_L$$

$$\rightarrow \frac{g}{\sqrt{2}} W^\mu \sum_{\alpha,i} V_{\alpha i}^* \bar{\alpha}_L \gamma_\mu i_L + \frac{g}{\sqrt{2}} W^{\mu\dagger} \sum_{\alpha,i} V_{\alpha i} \bar{i}_L \gamma_\mu \alpha_L$$

$\mathcal{H}_{cc}$ is CP-invariant if and only if V is real, or can be made real by changing the phase convention.

$\mathcal{H}_{NC}$ is necessarily CP-invariant.

In the SM, $\cancel{CP}$ arises from complex phases in V.

How these phases can produce physical $\cancel{CP}$ effects will be explained shortly.

The CKM matrix

The SM requires that the CKM matrix V be unitary.

Apart from this unitarity, V is unpredicted, so its elements must be determined experimentally.

How many parameters are needed to fully specify V?

Rephasing invariance

The physics does not change when, say,

$$d \to e^{i\theta} d .$$

Under this rephasing, for $\alpha = u, c, t$,

$$V_{\alpha d}\, \bar{\alpha}_L \gamma_\mu d_L \to V_{\alpha d}\, \bar{\alpha}_L \gamma_\mu (e^{i\theta} d_L)$$

$$= (e^{i\theta} V_{\alpha d})\, \bar{\alpha}_L \gamma_\mu d_L .$$

$\therefore$ The phase of any one $V_{\alpha i}$ depends on phase conventions.

One may go from one phase convention to another by multiplying any column, or any row, of V by a phase factor, or by carrying out several such operations.

Suppose there are N quark generations

$$\underbrace{\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}, \dots, \begin{pmatrix} \cdots \\ \cdots \end{pmatrix}}_{N}.$$

Then V is $N \times N$.

This unitary V contains –

$$2N^2 - N - 2\,\frac{N(N-1)}{2} = N^2$$

real parameters.

From these, we can remove $2N-1$ unphysical phases by multiplying columns and rows by phase factors.

$$\begin{bmatrix} & e^{i\varphi} \\ & e^{i\varphi} \\ & \vdots \\ & e^{i\varphi} \end{bmatrix}$$

Phases in last column can be removed indirectly

Number of physical parameters =

$$= N^2 - (2N-1) = (N-1)^2$$

$$= \begin{cases} 0 & N=1 \\ 1 & N=2 \\ 4 & N=3 \end{cases} .$$

If we choose the parameters as mixing angles and phases, how many of them are phases?

If V were real, it would contain $N^2 - N - \frac{N(N-1)}{2} = \frac{1}{2}N(N-1)$ mixing angles.

The number of phases in a complex V is then –

$$(N-1)^2 - \frac{1}{2}N(N-1) = \frac{1}{2}(N-1)(N-2) = \begin{cases} 0 & N=1 \\ 0 & N=2 \\ 1 & N=3 \end{cases}$$

(Kobayashi, Maskawa)

If N is only 2, the weak interaction as described in the SM cannot violate CP.

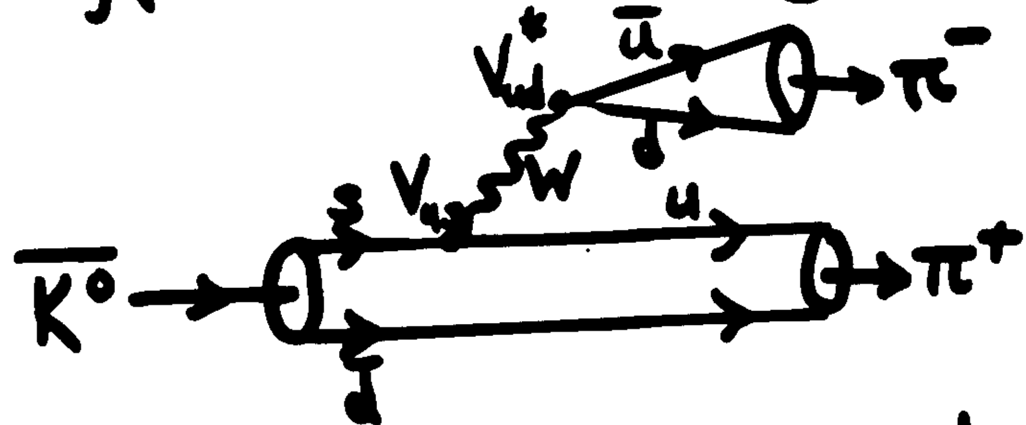
Processes that "know" about only 2 generations won't violate CP.

The quarks:

$$\begin{pmatrix}u\\d\end{pmatrix} \quad \begin{pmatrix}c\\s\end{pmatrix} \quad \begin{pmatrix}t\\b\end{pmatrix}$$

$$\xrightarrow[\text{Mass}]{\hspace{6cm}}$$

Typical weak decay:



K decay: $s \rightarrow u + \bar{u} + d$

generation: 2 1 1 1

$\cancel{CP}$ will be small, as observed, in K decay.

To see large ($\sim 100\%$) $\cancel{CP}$ effects, we must observe processes that involve all three generations.

$\therefore$ Let us study the decays of the Beauty mesons B.

$$B^+ = [\bar{b}u], \quad B_d = [\bar{b}d], \quad B_s = [\bar{b}s]$$

b is in generation 3.

In addition, $\cancel{CP}$ in B decays can yield clean information on the phases in V.

Thus, we can cleanly test whether these phases are the origin of $\cancel{CP}$.

$\not{CP}$ in the B system

What $\not{CP}$ effects do we look for?

$$\langle f|T|i\rangle \underset{CP}{=\!=} \langle CP[f]|T|CP[i]\rangle$$

$$\text{Asym} \equiv \frac{\Gamma(B^+\to f)-\Gamma(B^-\to \bar{f})}{\text{"} \quad + \quad \text{"}} \neq 0 \Rightarrow \not{CP}$$

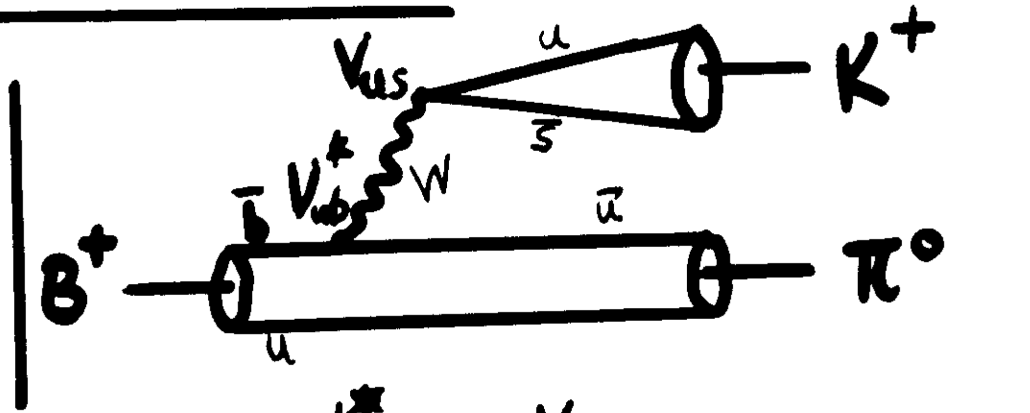
We look for partial decay rate asymmetries of this kind.

Note: Assuming CPT-invariance $\Gamma_{tot}(B^+)=\Gamma_{tot}(B^-)$, so there must be compensating asymmetries.

Complex phases in V lead to physical $\not{CP}$ effects, including rate asymmetries, through interference between amplitudes.

Example: $B^\pm \to \pi^0 K^\pm$

$(B^+ \to \pi^0 K^+) =$



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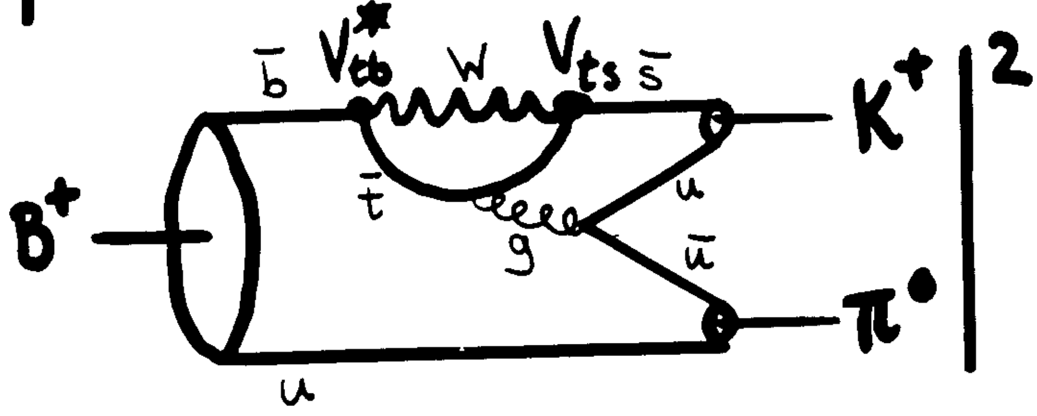
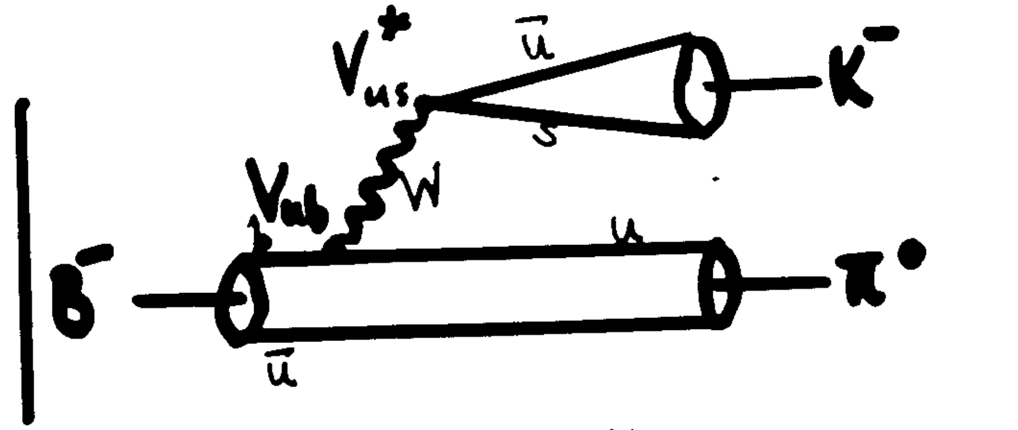


$(B^- \to \pi^0 K^-) =$



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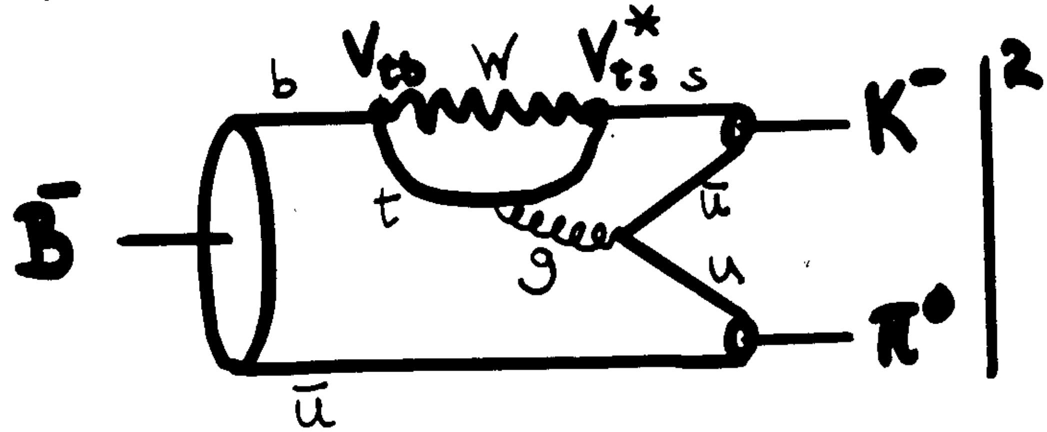


Generalizing, suppose $B^+ \to f$ receives contributions from two diagrams:

$$\Gamma(B^+ \to f) = |M e^{i\delta^f_{CKM}} e^{i\alpha_s} + M' e^{i\delta'^f_{CKM}} e^{i\alpha'_s}|^2$$

$$\Gamma(B^- \to \bar{f}) = |M e^{-i\delta^f_{CKM}} e^{i\alpha_s} + M' e^{-i\delta'^f_{CKM}} e^{i\alpha'_s}|^2$$

In going from a process to its CP conjugate, the CKM phases reverse, but nothing else changes.

The interfering amplitudes have different relative phase in $B^+ \to f$ and $B^- \to \bar{f}$.

If both CKM and strong phases are present,

$$\Gamma(B^+ \to f) \neq \Gamma(B^- \to \bar{f}). \qquad \cancel{CP}$$

$$\Gamma(B^+ \to f) = M^2 + M'^2 + 2MM' \cos(\varphi + \varphi_S),$$

$$\Gamma(B^- \to \bar{f}) = M^2 + M'^2 + 2MM' \cos(-\varphi + \varphi_S),$$

where

$$\varphi \equiv \delta^f_{CKM} - \delta'^f_{CKM}, \quad \varphi_S \equiv \alpha_S - \alpha'_S .$$

To test the SM of $\not{CP}$, one would like to determine φ.

$\Gamma(B^+ \to f)$ and $\Gamma(B^- \to \bar{f})$ do not determine φ.

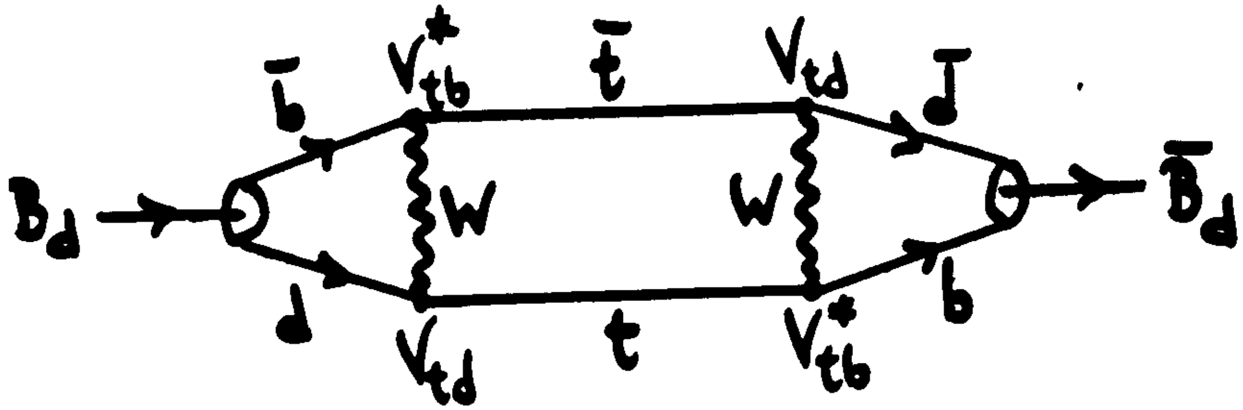
<u>In general</u>, in $B^\pm$ decays a clean test of the SM is not possible.

Decays of Neutral B Mesons

It is from neutral B decays that clean CKM phase information can be obtained.

We treat $B_d(\bar{b}d)$. $B_s(\bar{b}s)$ is similar.

The $B_d(\bar{b}d)$ and $\bar{B}_d(b\bar{d})$ mix via



$$\arg[A(B_d \to \bar{B}_d)] = \arg[(V_{td}V^*_{tb})^2] \equiv -2\,\delta^m_{CKM}.$$

As for the neutral kaons, so for the $B_d, \overline{B_d}$ system there is a mass matrix

$$\mathcal{M} = \begin{bmatrix} X & \mathcal{M}_{12} \\ \mathcal{M}_{21} & X \end{bmatrix} \quad \begin{pmatrix} \mathcal{M}_{11} = \mathcal{M}_{22} \equiv X \\ \text{by CPT} \end{pmatrix}$$

in which $\mathcal{M}_{21}$ = The $\bar{t}t$ box diagram.

The neutral B mass eigenvalues are

$$\lambda_\pm = X \pm \sqrt{\mathcal{M}_{12}\mathcal{M}_{21}} \equiv m_\pm - i\frac{\Gamma_\pm}{2}.$$

Since $\mathcal{M}_{12} = \mathcal{M}_{21}(V \to V^*) = \mathcal{M}_{21}^*$,

$$\Gamma_+ = \Gamma_- \equiv \Gamma.$$

(Neither B mass eigenstate has a special decay mode, and the two eigenstates have many modes in common.)

The mass eigenstates are

$$|B_\pm\rangle = \frac{|B^0\rangle \pm \rho|\bar{B}^0\rangle}{\sqrt{1+|\rho|^2}},$$

where

$$\rho \equiv \sqrt{\frac{\mathcal{M}_{21}}{\mathcal{M}_{12}}} = \sqrt{\frac{(V_{td}V_{tb}^*)^2}{(V_{td}^*V_{tb})^2}} = e^{-2i\delta^m_{CKM}}.$$

rom—

p. for B_d

$$i\frac{\partial}{\partial t}\begin{bmatrix} a(t) \\ \bar{a}(t)\end{bmatrix} = \mathcal{M}\begin{bmatrix} a(t) \\ \bar{a}(t)\end{bmatrix},$$

mp. for $\bar{B}_d$

$$|B_d\rangle \longrightarrow |B_d(t)\rangle = e^{-i(m-i\frac{\Gamma}{2})t} \times$$

$$\times\left\{c|B_d\rangle - ie^{-2i\delta^m_{CKM}} s|\bar{B}_d\rangle\right\}$$

lere, $m \equiv \frac{m_+ + m_-}{2}$, and with $\Delta m = m_+ - m_-$,

$$c \equiv \cos\left(\frac{\Delta m}{2}t\right), \quad s \equiv \sin\left(\frac{\Delta m}{2}t\right).$$

$$\left.\frac{\Delta m}{\Gamma}\right)_{B_d} \equiv x_d = \begin{cases} 0.68 \pm 0.10 & \text{ARGUS \& CLEO} \\ 0.79 \pm 0.10 & \text{LEP} \end{cases}$$

The $B_d - \bar{B}_d$ oscillation is as fast as the decay.

Suppose f is a final state accessible to both B_d and $\bar{B}_d$.

Examples: $\rho^+\pi^-$, $D^0 K_s$, $\pi^+\pi^-$, ψK_s.

Then, for $B_d(t) \rightarrow f$,

$$\Gamma_f(t) = |\langle f | T | B_d(t) \rangle|^2 = e^{-\Gamma t} \times$$

$$\times \left| c \underbrace{\langle f | T | B_d \rangle}_{M e^{i\delta^f_{CKM}} e^{i\alpha_s}} - i e^{-2i\delta^m_{CKM}} s \underbrace{\langle f | T | \bar{B}_d \rangle}_{\bar{M} e^{-i\bar{\delta}^f_{CKM}} e^{i\bar{\alpha}_s}} \right|^2$$

Assume one diagram dominates

$$(t) = e^{-\Gamma t}\{c^2 M^2 + s^2 \bar{M}^2 - 2cs M\bar{M} \sin(\varphi + \varphi_s)\}.$$

For the CP-mirror image decay, $\bar{B}_d(t) \rightarrow \bar{f}$,

$$(t) = e^{-\Gamma t}\{c^2 M^2 + s^2 \bar{M}^2 - 2cs M\bar{M} \sin(-\varphi + \varphi_s)\}.$$

Here

$$\varphi = 2\delta^m_{CKM} + \delta^f_{CKM} + \bar{\delta}^f_{CKM}, \quad \varphi_s = \alpha_s - \bar{\alpha}_s$$

Weak CKM phase of interest

Unknown strong phase

φ occurs in an interference term.

CP invariance $\Rightarrow \Gamma_f(t) = \bar{\Gamma}_{\bar{f}}(t)$.

$\therefore \varphi \neq 0 \Rightarrow \cancel{CP}$

With Γ and Δm known, $\Gamma_f(t)$ and $\bar{\Gamma}_{\bar{f}}(t)$ determine $M, \bar{M}$, $S_+ \equiv \sin(\varphi + \varphi_s)$, and $S_- \equiv \sin(-\varphi + \varphi_s)$.

Then

$$\sin^2\varphi = \frac{1}{2}\left[1 - S_+S_- \pm \sqrt{(1-S_+^2)(1-S_-^2)}\right]$$

Then $\sin^2\varphi$ can be compared to the prediction from the CKM matrix to test the SM of $\cancel{CP}$.

(Aleksan, Dunietz, B.K., Le Diberder)

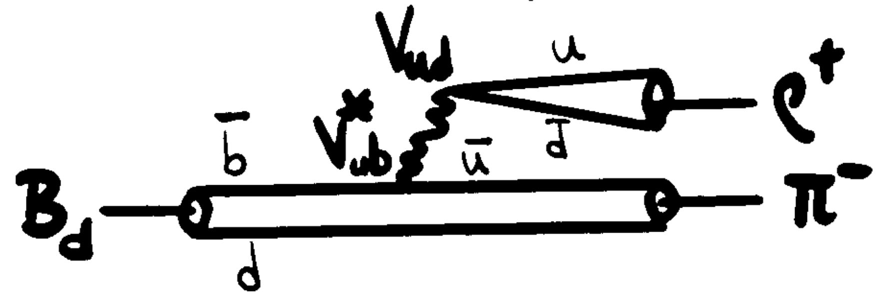
Note the determination of φ is theoretically clean.

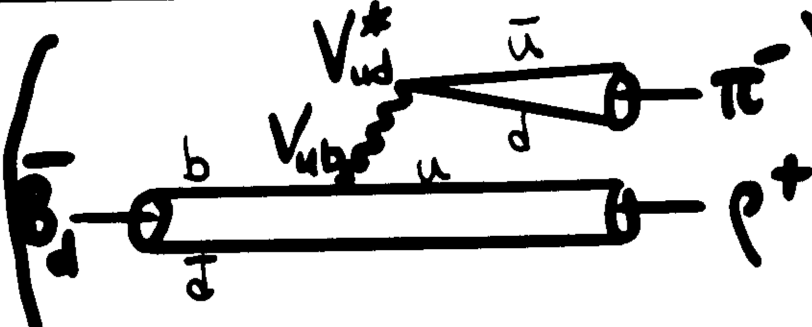
The φ that is probed in a given decay $B_d(t) \to f$ is

$$\varphi = \text{CKM Phase}\left[\frac{A(B_d \to f)}{A(B_d \to \overline{B}_d) * A(\overline{B}_d \to f)}\right].$$

Example: In $B_d(t) \to \rho^+\pi^-$, the φ we probe is the CKM phase of

$$\left[\frac{\text{(diagram 1)}}{\left((V_{td}V_{tb}^*)^2\right) * \left(\text{(diagram 2)}\right)}\right]$$





That is, in this decay

$$\varphi = 2\arg[V_{ud}V_{ub}^*V_{tb}V_{td}^*].$$

The CKM phase φ probed in any B decay is

arg [Some Product of CKM Elements].

Special case of the formalism:
f is a CP eigenstate

$$CP|f\rangle = \eta_f |f\rangle \qquad (\pi^+\pi^-, \Psi K_S, \dots)$$

If

$$\langle f|T|B_d\rangle = M e^{i\delta^f_{CKM}} e^{i\alpha_s},$$

then

$$\langle f|T|\bar{B}_d\rangle = \eta_f \langle \bar{f}|T|\bar{B}_d\rangle$$
$$= \eta_f M e^{-i\delta^f_{CKM}} e^{i\alpha_s}$$
$$\equiv \bar{M} e^{-i\bar{\delta}^f_{CKM}} e^{i\bar{\alpha}_s}.$$

$$\bar{M} = M,\ \bar{\delta}^f_{CKM} = \delta^f_{CKM},\ \varphi_s \equiv \alpha_s - \bar{\alpha}_s = \begin{cases} 0, & \eta_f = + \\ \pi, & \eta_f = - \end{cases}$$

$$\overset{(-)}{\Gamma}_f(t) = M^2 e^{-\Gamma t}\left\{1 \overset{(-)}{+} \sin(\Delta m\, t)\, \eta_f \sin\varphi \right\},$$

with $\varphi = 2(\delta^m_{CKM} + \delta^f_{CKM})$.

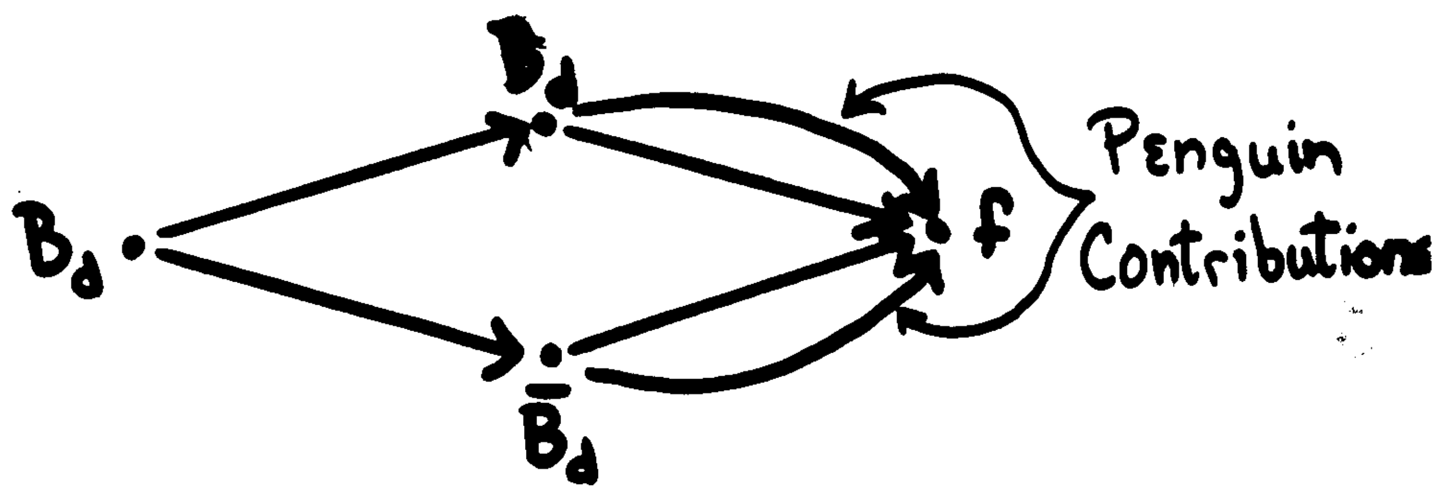
(Bigi, Carter, Sanda, Dunietz, Rosner)

The asymmetry

$$\frac{\Gamma_f(t) - \bar{\Gamma}_f(t)}{\text{''} \;+\; \text{''}} = -\sin(\Delta m\, t)\, \eta_f \sin\varphi$$

cleanly determines $\sin\varphi$.

Key assumption: $A(B_d \to f)$ and $A(\bar{B}_d \to f)$ are each dominated by one diagram.



In charged B decays, $\Gamma^+ \neq \Gamma^-$ requires both CKM and strong phases.

In neutral B decays, $\Gamma_f(t) \neq \overline{\Gamma}_{\bar{f}}(t)$ requires no strong phases.

Why?

When we go from $\Gamma_f(t)$ to $\overline{\Gamma}_{\bar{f}}(t)$, all CKM phases reverse. Thus, if $\Gamma_f(t)$ and $\overline{\Gamma}_{\bar{f}}(t)$ are to differ, some phase must not reverse. What is *that* phase?

$$\Gamma_f(t) = e^{-\Gamma t} \left| c \langle f|T|B_d\rangle - is(\overset{\exp(-2i\delta^m_{CKM})}{\rho} \langle f|T|\overline{B_d}\rangle) \right|^2$$

$$\overline{\Gamma}_{\bar{f}}(t) = e^{-\Gamma t} \left| c \langle f|T|B_d\rangle^{*CKM} - is(\rho \langle f|T|\overline{B_d}\rangle)^{*CKM} \right|^2$$

Makes strong phase unnecessary

$*CKM$: CKM phases reverse, and nothing else changes

(3-1)

1 How will the experiments be done?

~~CP~~ in B decays will be studied at both hadron accelerators and e^+e^- colliders. These studies will complement each other.

How many B's are needed?

Example: $\overset{(\leftrightarrow)}{B_d}(t) \to \Psi K_s$.

Suppose —

$$\text{Asym} = \frac{\Gamma[B_d(t)\to\Psi K_s] - \Gamma[\bar{B}_d(t)\to\Psi K_s]}{\text{''} \quad + \quad \text{''}} \sim 10\%.$$

To see this Asym above $\sqrt{N}$ statistical fluctuations, you need $\sim 10^3$ detected $B_d(t)\to\Psi K_s$ decays.

The Ψ is detected via $\Psi\to\mu^+\mu^-$ or e^+e^-.

$BR(\Psi\to\mu^+\mu^- \text{ or } e^+e^-) \cong 12\%$.

$\therefore 10^4$ $B_d(t) \to \Psi K_s$ decays are needed.

$BR(B_d \to \Psi K_s) = 3 \times 10^{-4}$.

$\therefore \sim 10^8$ B_d's are needed.

For other, typical, decay modes of interest, the number of B's needed is similar.

How easily can one get $10^8 - 10^9$ B's?

The e^+e^- colliders as an example:

$$e^+e^- \xrightarrow{} \Upsilon(4s) \longrightarrow B_d \bar{B}_d \quad \text{or} \quad B^+B^-$$

$\sigma = 1\,\text{nb}$ BR: 50% 50%

To get 10^8 $B_d\bar{B}_d$ pairs in two years, we need a luminosity of —

$$\mathcal{L} = 10^8 / [(0.5\,\text{nb})(6 \times 10^7\,\text{sec})] = 3 \times 10^{33} / \text{cm}^2\text{-sec.}$$

The present peak luminosity at CESR is

$$\mathcal{L}_{present} = 3 \times 10^{32} / cm^2 \cdot sec$$

∴ Producing enough B's will be a challenge.

We need B factories!

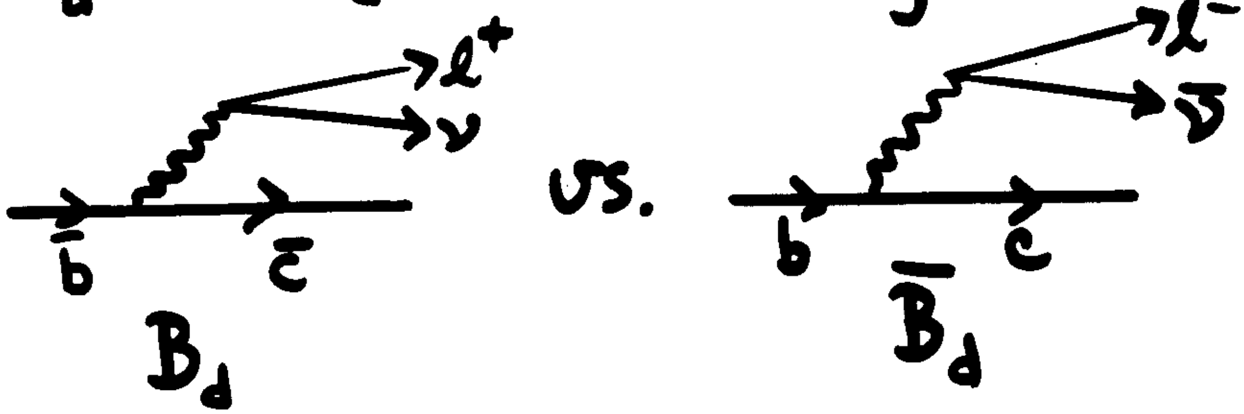
Being built at SLAC and KEK.

Tagging

To compare $\Gamma[B_d(t) \to \psi K_s]$ with $\Gamma[\overline{B}_d(t) \to \psi K_s]$, we must be able to tag the parent of each decay as a $B_d(t)$ or a $\overline{B}_d(t)$.

One way: $e^+e^- \to \Upsilon(4s) \to \underset{\hookrightarrow \psi K_s}{B_d} + \underset{\hookrightarrow \ell^- + \ldots}{\overline{B}_d}$

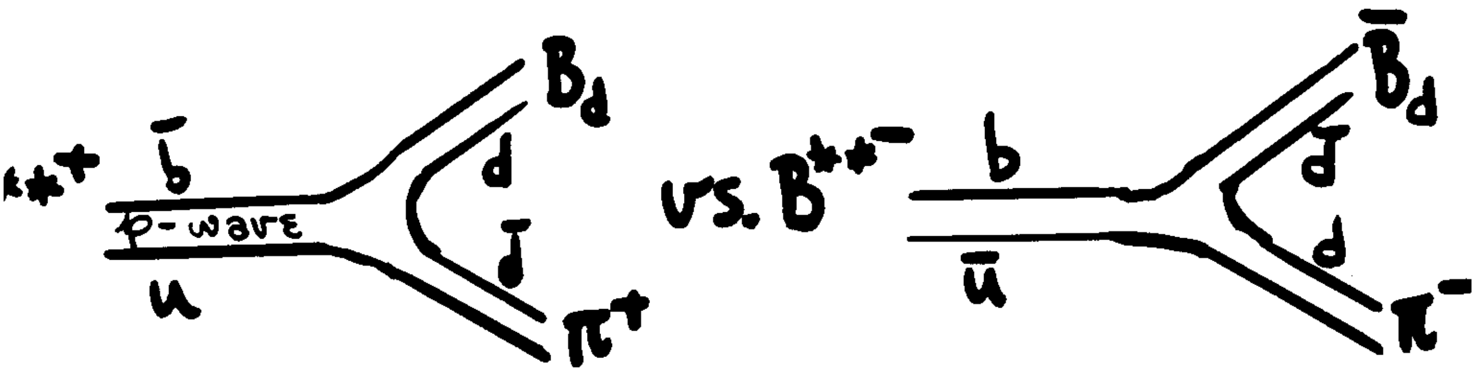
· B_d and $\bar{B}_d$ can be distinguished by



The remaining B in the pair is then tagged.

Another way: $\bar{p}p \to B^{**(\pm)} + \cdots$

$\quad \hookrightarrow \pi^{(\pm)} + \overset{(-)}{B_d}$



"Self-tagging."

(Gronau, Nippe, Rosner)

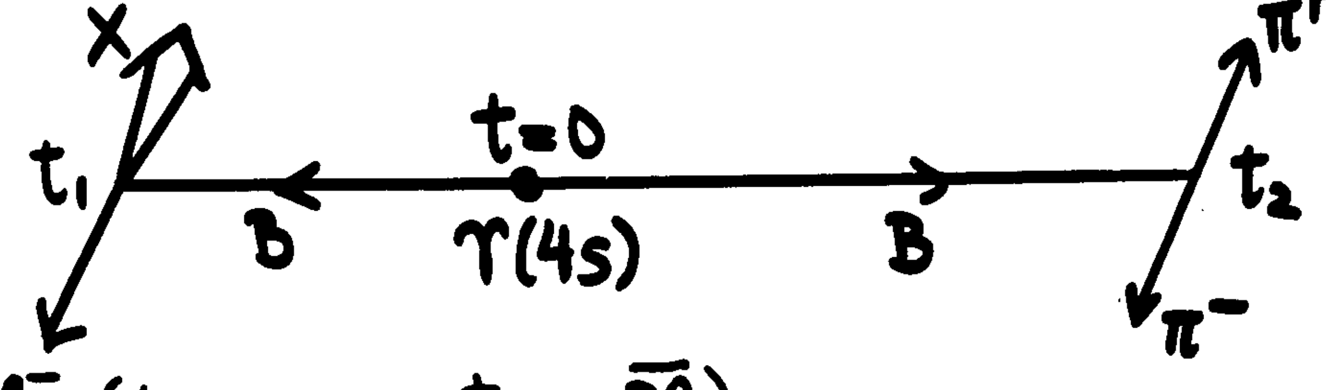
Why an Asymmetric Collider?

Answer: Quantum mechanics!

Produce the B mesons via

$$\underbrace{e^+e^- \xrightarrow{\text{Big }\sigma} \Upsilon(4s)}_{J=1} \xrightarrow{BR\sim 100\%} \underbrace{B^0\overline{B^0}}_{p\text{ wave}} \quad (\text{or } B^+B^-)$$

Neutral B pair: Until one decays, at any given time, we have one B^0 and one $\overline{B^0}$, despite mixing.



ℓ^- (tags parent as $\overline{B^0}$)

Suppose at time t_1, 1^{st} $B \to \ell^- X$.
Then at t_1, this B was a $\overline{B^0}$.
Then at t_1, other B was a B^0!

(Einstein, Podolsky, Rosen)

The remaining B will now mix.

$$\Gamma\left[1^{st} B \to \ell^- X \text{ at } t_1\,;\ 2^{nd} B \to \pi^+\pi^- \text{ at } t_2\right]$$

$$\underbrace{e^{-\Gamma t_1}\, e^{-\Gamma t_1}}_{\substack{\text{Prob. that}\\ \text{both B's}\\ \text{lived to } t_1}}\ \underbrace{e^{-\Gamma(t_2-t_1)}\left\{1+(\sin\varphi)\sin[\Delta m(t_2-t_1)]\right\}}_{\text{Evolution after time } t_1}$$

$$e^{-\Gamma(t_2+t_1)}\left\{1+(\sin\varphi)\sin[\Delta m(t_2-t_1)]\right\}$$

This holds even if $t_2 < t_1$. (Blankenbecler)

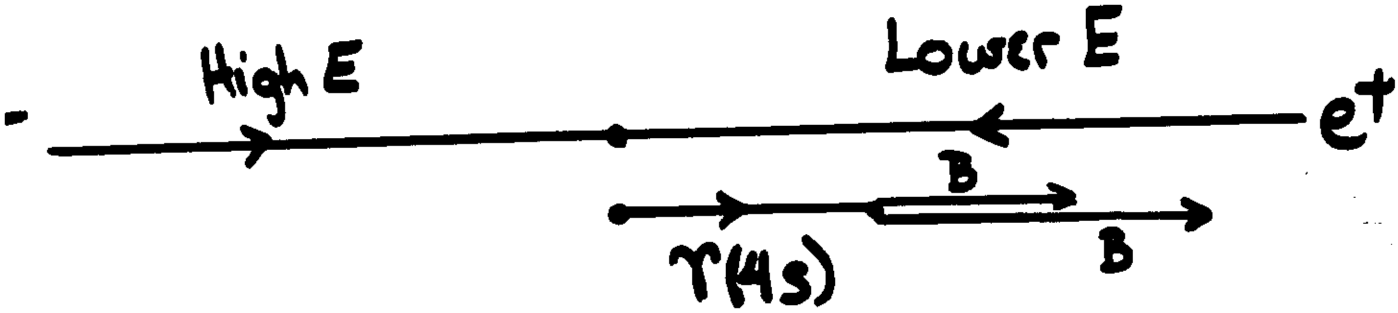
If decay times cannot be measured, the ~~CP~~ sine wave contributes

$$\propto \int_0^\infty dt_1 \int_0^\infty dt_2 \, e^{-\Gamma(t_2+t_1)} \sin[\Delta m(t_2-t_1)] = 0.$$

$\therefore$ Timing information is crucial.

If $\Upsilon(4s)$ is at rest in lab., slow B does not go far enough for its pathlength to be measured.

$\therefore$ Build an asymmetric collider.



Usefulness of non-CP-eigenstates

Originally, the test of the SM of ~~CP~~ was to be based on B→ CP eigenstates.

The challenge: To gather statistics.

A typical B decay mode has a small BR.)

To maximize the impact of a given luminosity, study not just B→CP eigenstates, but also B→ other final states.

If a CP eigenstate and a non-CP-eigenstate are both produced by the same quark-level process, they both probe the same φ. One can improve the statistics on this φ by considering both of these final states.

Attention is still focussed on two-body final states.

A two-body final state f can fail to be a CP-eigenstate for two reasons:

- f is not a CP-self-conjugate collection of particles (e.g. $\rho^+\pi^-$, $D^{*+}D^-$, $K^+\pi^-$)

  Here one can use the technique for $f \neq \bar{f}$.

- f is a CP-self-conjugate collection of particles, but is not a CP eigenstate because it is a mixture of partial waves with different CP parities (e.g. $\psi\rho^0$, $D^{*+}D^{*-}$)

  Here one can isolate a CP-eigenstate component of f by angular measurements.

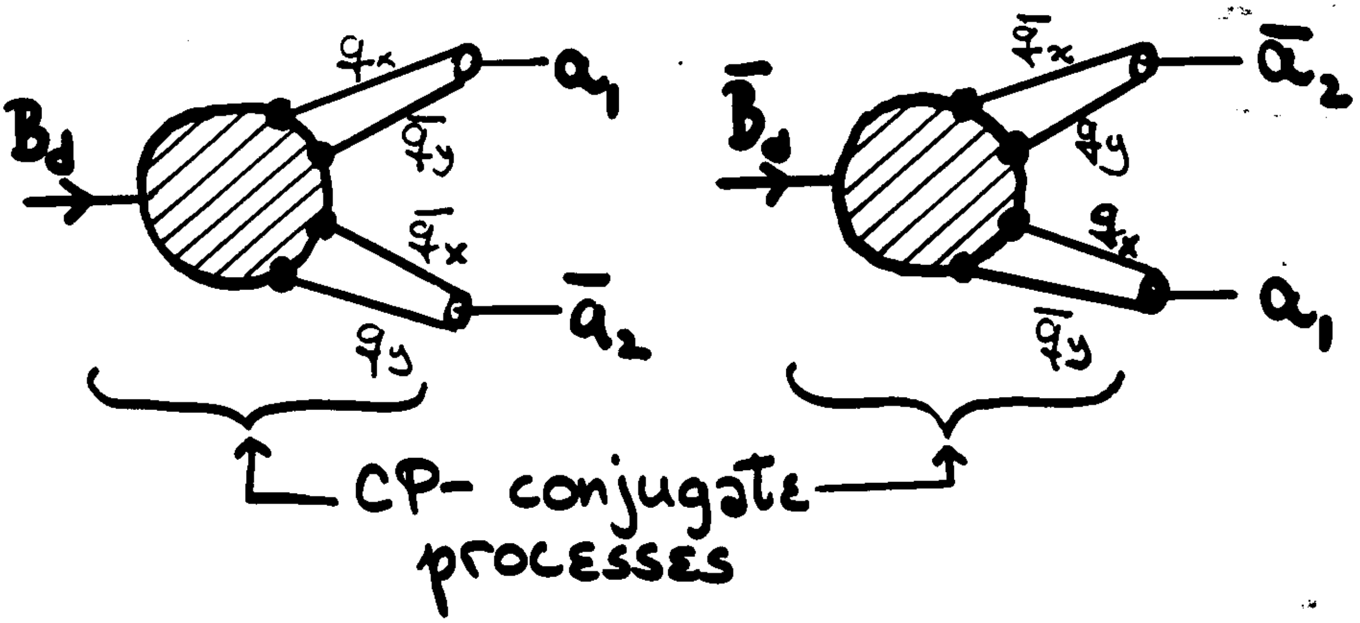
I. Non-CP-Self-Conjugate Collections of Particles

A useful class —
Final state $f = a_1 \bar{a}_2$, where a_1 and a_2 are different mesons but have the same quark content $q_x \bar{q}_y$. Then $a_1 \bar{a}_2 = (q_x \bar{q}_y)(\bar{q}_x q_y)$ is a CP-self-conjugate collection of quarks.

Example —

$$f = \rho^+ \pi^- = (u\bar{d})(\bar{u}d)$$

When $a_1\bar{a}_2 = (q_x\bar{q}_y)(\bar{q}_x q_y)$, each diagram in $B_d \rightarrow a_1\bar{a}_2$ has a simply-related analog in $\bar{B}_d \rightarrow a_1\bar{a}_2$.

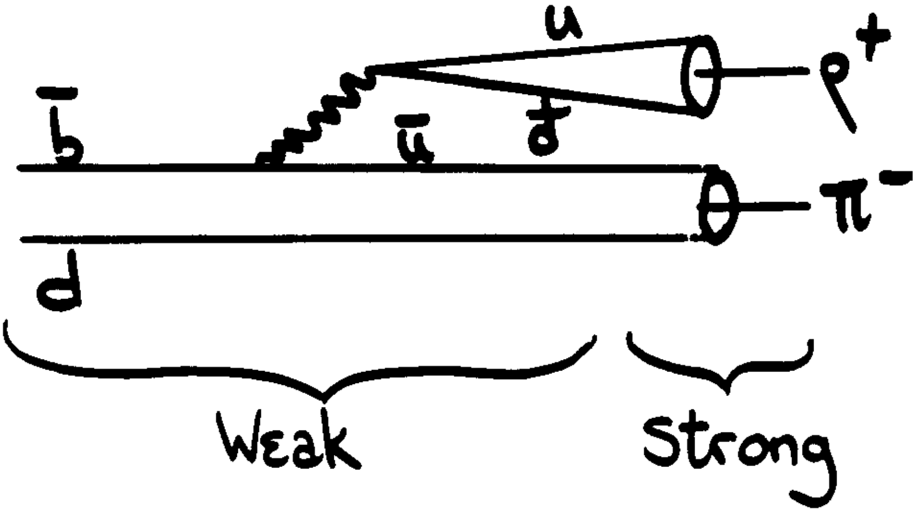


Only the (strong) binding of the final state quarks to make $a_1\bar{a}_2$ breaks the CP-conjugacy of the two diagrams

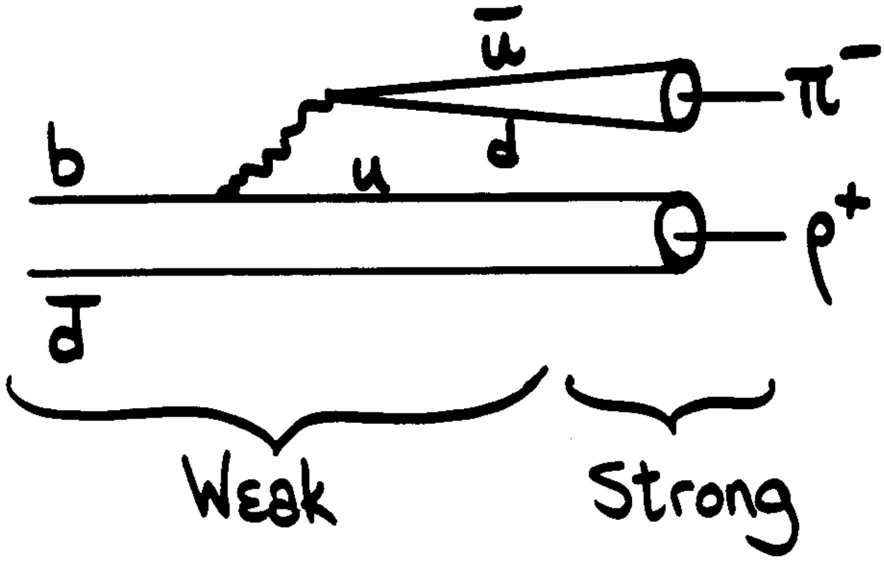
Therefore, the two diagrams should have comparable size.

(Aleksan, Dunietz, B.K., Le Diberder)

$B_d \to \rho^+ \pi^-$



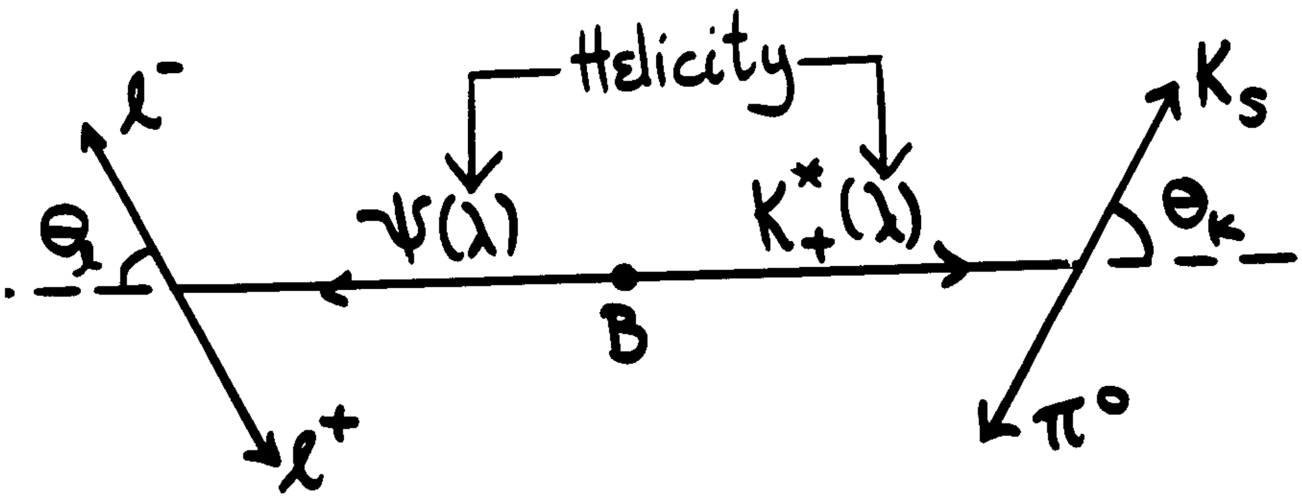
$\overline{B}_d \to \rho^+ \pi^-$



We expect these interfering amplitudes to be of comparable size.
∴ $\not{CP}$ can be large.

I. CP-Self-Conjugate Collection of Particles with Spin

Example –



Only $K^*_+ \equiv \frac{K^{*0} + \overline{K^{*0}}}{\sqrt{2}}$ contributes.

$$\text{Helicity} \xrightarrow{CP} -\text{Helicity}$$

$\therefore$ ψK^*_+ state is very likely not a CP eigenstate.

However,

$$\frac{d\Gamma}{d(\cos\theta_K)} = \underbrace{a\cos^2\theta_K}_{\text{From } \lambda=0} + \underbrace{b\sin^2\theta_K}_{\text{From } \lambda=\pm 1}$$

The $\cos^2\theta_K$ in the K angular distribution selects

$$B \longrightarrow \underbrace{\psi(\lambda=0) + K^*_+(\lambda=0)}_{\text{CP eigenstate}}$$

(B.K., Kuroda, Peccei, Sanda)

This decay probes the same φ as does $B \to \psi K_S$: φ_0.

Accuracy in $B \to \psi K^*$ is improved by using all available angles.

Then, for a given number of decays,

$$\text{Error}(\sin\varphi_0)\Big|_{\psi K^*} = (1 - 2.5) * \text{Error}(\sin\varphi_0)\Big|_{\psi K_s}$$

(Dunietz, Quinn, Snyder, Toki, Lipkin)

If nature is really kind, all $B \to \psi K^*$ yield $\lambda = 0$.

ARGUS: $\dfrac{\Gamma_{\lambda=0}}{\Gamma_{\text{All }\lambda}} > 0.78$ @ 95% CL

CLEO: $\dfrac{\Gamma_{\lambda=0}}{\Gamma_{\text{All }\lambda}} = 0.78 \pm 0.10 \pm 0.10$

What independent CKM phases φ exist?

Recall that

$\varphi = \arg$[Some Product of CKM Elements].

What independent phases of products of CKM elements exist?

The SM requires that V be unitary.

From experimental data, the rough magnitudes of the $V_{\alpha i}$ are summarized (Wolfenstein) by $(\lambda = 0.22)$ —

$$V_{CKM} \sim \begin{bmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{bmatrix}$$

By rephasing and unitarity:

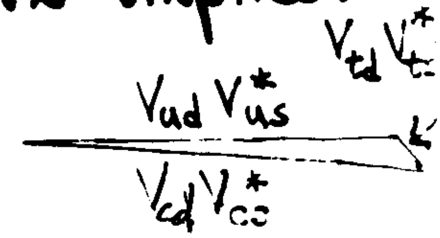
- • real
- ⊙ ≈ real
- • complex

The independent phases are $\arg(V_{ub})$ and $\arg(V_{td})$.

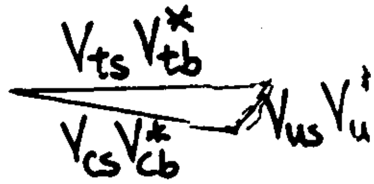
The phase-convention-independent picture: [Δ's].

Unitarity of the CKM matrix implies:

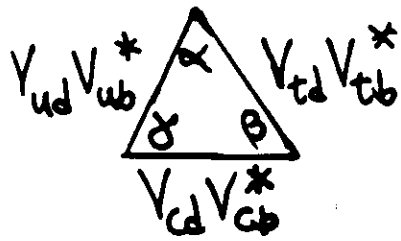
s $$\underset{\lambda}{V_{ud}V_{us}^*} + \underset{\lambda}{V_{cd}V_{cs}^*} + \underset{\lambda^5}{V_{td}V_{ts}^*} = 0$$



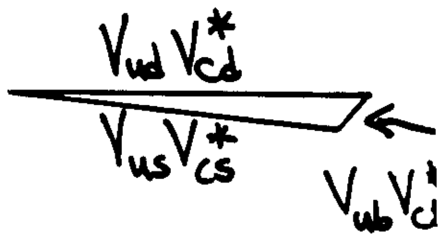
b $$\underset{\lambda^4}{V_{us}V_{ub}^*} + \underset{\lambda^2}{V_{cs}V_{cb}^*} + \underset{\lambda^2}{V_{ts}V_{tb}^*} = 0$$



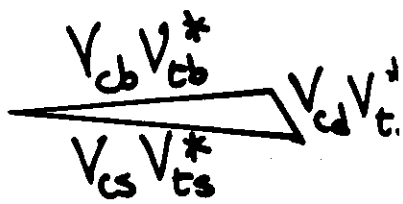
b $$\underset{\lambda^3}{V_{ud}V_{ub}^*} + \underset{\lambda^3}{V_{cd}V_{cb}^*} + \underset{\lambda^3}{V_{td}V_{tb}^*} = 0$$



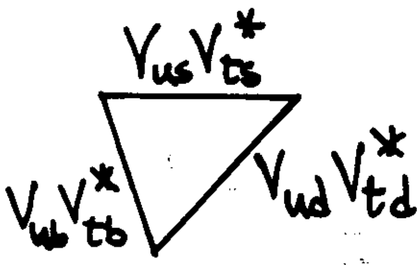
c $$\underset{\lambda}{V_{ud}V_{cd}^*} + \underset{\lambda}{V_{us}V_{cs}^*} + \underset{\lambda^5}{V_{ub}V_{cb}^*} = 0$$



t $$\underset{\lambda^4}{V_{cd}V_{td}^*} + \underset{\lambda^2}{V_{cs}V_{ts}^*} + \underset{\lambda^2}{V_{cb}V_{tb}^*} = 0$$



t $$\underset{\lambda^3}{V_{ud}V_{td}^*} + \underset{\lambda^3}{V_{us}V_{ts}^*} + \underset{\lambda^3}{V_{ub}V_{tb}^*} = 0$$



1)

$$V = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$

$$\begin{bmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix}$$

with

$$\lambda = 0.2205 \pm 0.0018$$

$$A = 0.84 \pm 0.12$$

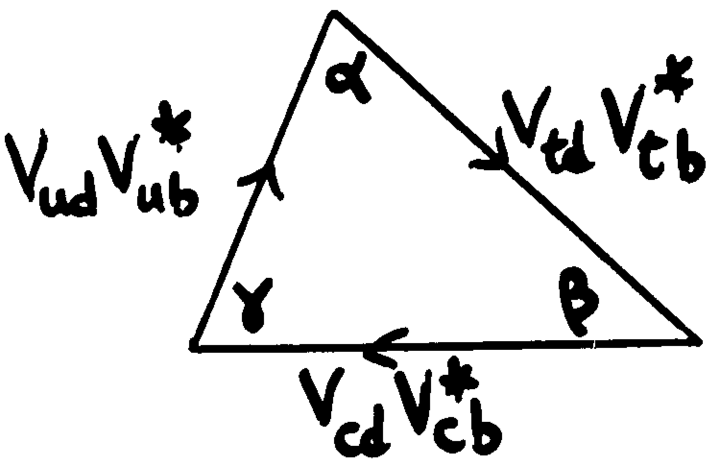
$$\sqrt{\rho^2+\eta^2} = 0.36 \pm 0.09$$

$$\rho, \eta = ?$$

(Ali + London)

In the Wolfenstein approximation, all Δ's collapse except the db and ut Δ's, which become equivalent.

We focus, then, on the db Δ:



Each angle in any of the unitarity Δ's is essentially the phase of some product of CKM elements.

For example,

$$\gamma = \pi - \left| \arg\left(V_{ud}V_{ub}^* / V_{cd}V_{cb}^* \right) \right|.$$

The angles in the unitarity Δ's are convention-independent.

In the Wolfenstein approximation, the independent phases φ are

γ [$\simeq -\arg(V_{ub})$ in our phase convention]

and

β [$\simeq -\arg(V_{td})$ in our phase convention].

As a check of the SM of $\cancel{CP}$, we can also measure α and see if

$$\alpha = \pi - \beta - \gamma .$$

We would like to find B decays in which the measured φ is α, β, or γ.

<u>Which B decay modes probe each angle?</u>

In $B_q(t) \to f$, $q = d$ or s, the φ probed is

$$\varphi = \text{CKM Phase}\left[\frac{A(B_q \to f)}{A(B_q \to \bar{B}_q) * A(\bar{B}_q \to f)}\right].$$

Now,

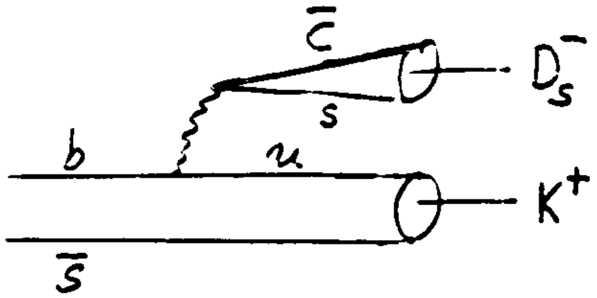
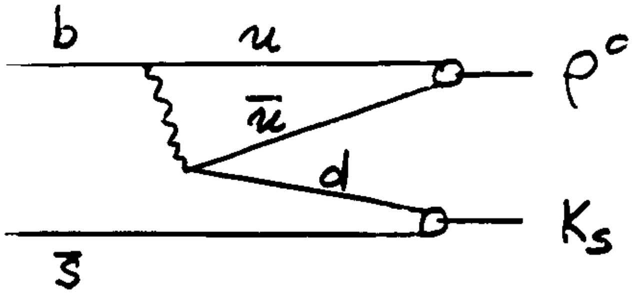
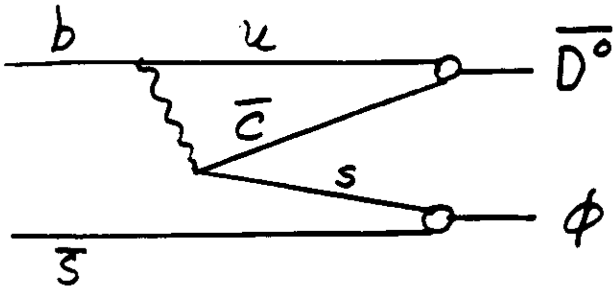
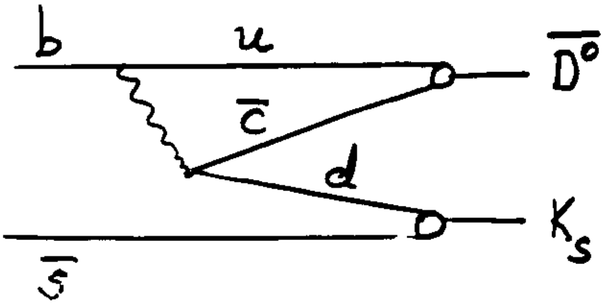
$$\arg[A(B_d \to \bar{B}_d)] = \arg[(V_{td}V_{tb}^*)^2] \simeq -2\beta,$$

$$\arg[A(B_s \to \bar{B}_s)] = \arg[(V_{ts}V_{tb}^*)^2] \simeq 0.$$

Therefore —

To Probe	Study Decays of	Involving
γ	B_s	$b \to u + \begin{cases} \bar{c}s \\ \bar{c}d \\ \bar{u}s \\ \bar{u}d \end{cases}$
β	B_d	$b \to c + \begin{cases} \bar{c}s \\ \bar{c}d \\ \bar{u}s \\ \bar{u}d \end{cases}$
α	B_d	$b \to u + \begin{cases} \bar{c}s \\ \bar{c}d \\ \bar{u}s \\ \bar{u}d \end{cases}$

Possibly useful modes for γ:

Mode	"BR"	
	2×10^{-4}	
	5×10^{-7}	Penguins
	2×10^{-5}	
	10^{-4}	Tiny Interference

Additional probes of γ use –

Mode	"BR"	
$B^{\pm} \rightarrow D^{0}_{CP} K^{\pm}$	2×10^{-4}	(Gronau-Wyler)
$\overset{(-)}{B}_{d} \rightarrow D^{0}_{CP} \overset{(-)}{K}{}^{*0}$	2×10^{-5}	(Dunietz)
$B^{\pm} \rightarrow \pi^{0} K^{\pm}$	$\sim 10^{-5}$	(Gronau Hernández London Rosner)

An interesting approach: $B^{\pm} \rightarrow D + K^{\pm}$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad \hookrightarrow K^+K^-$

(Gronau & Wyler)

his approach illustrates that sometimes lean CKM phase information can be xtracted even from charged B decays.

$$\langle (K^+K^-)_D K^+ | T | B^+ \rangle =$$

$$= \underbrace{\langle K^+K^- | T | D^0 \rangle}_{\parallel\,(CP)} \underbrace{\langle D^0 K^+ | T | B^+ \rangle}_{D}$$

$$+ \overbrace{\langle K^+K^- | T | \bar{D}^0 \rangle} \overbrace{\langle \bar{D}^0 K^+ | T | B^+ \rangle}^{\bar{D}}$$

$$\propto D + \bar{D}.$$

Suppose D and $\bar{D}$ have relative CKM phase δ_{CKM} and relative strong phase α_{ST}

4] Then

$$\Gamma^+ \equiv \Gamma[B^+ \to (K^+K^-)_D K^+] = |D+\bar{D}|^2$$
$$= |D|^2 + |\bar{D}|^2 + 2|D||\bar{D}| \cos(\delta_{CKM} + \alpha_{ST})$$

$$\Gamma^- \equiv \Gamma[B^- \to (K^+K^-)_D K^-]$$
$$= |D|^2 + |\bar{D}|^2 + 2|D||\bar{D}| \cos(-\delta_{CKM} + \alpha_{ST})$$

Measure $|D|$ in $B^+ \to D^0 + K^+$, $D^0 \hookrightarrow K^-\pi^+$

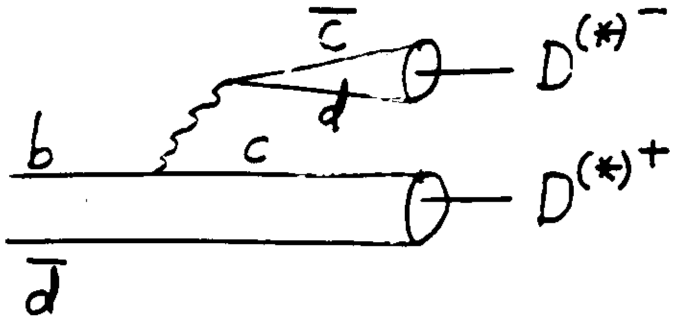
Measure $|\bar{D}|$ in $B^+ \to \bar{D}^0 + K^+$, $\bar{D}^0 \hookrightarrow K^+\pi^-$

Then Γ^+ determines $\cos(\delta_{CKM} + \alpha_{ST}) \equiv C_+$,
and Γ^- " $\cos(-\delta_{CKM} + \alpha_{ST}) \equiv C_-$.

Then $\sin^2\delta_{CKM} = \frac{1}{2}\left[1 - C_+C_- \pm \sqrt{(1-C_+^2)(1-C_-^2)}\right]$.
From the diagrams, $\delta_{CKM} = \gamma - \pi$.
<u>Perhaps</u> feasible at a symmetric collider. (Stone)

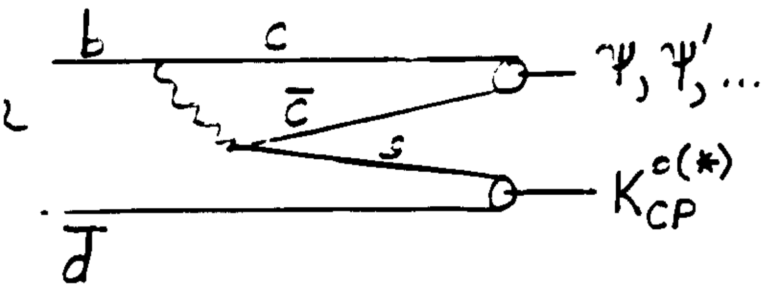
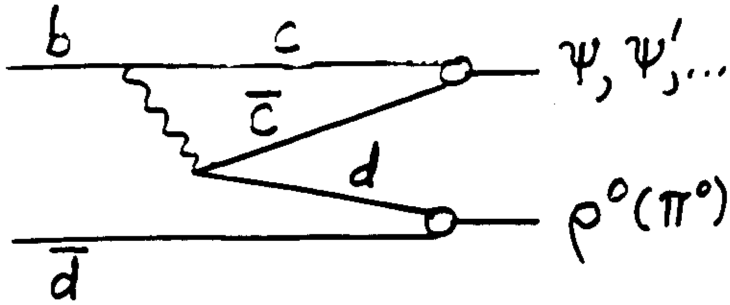
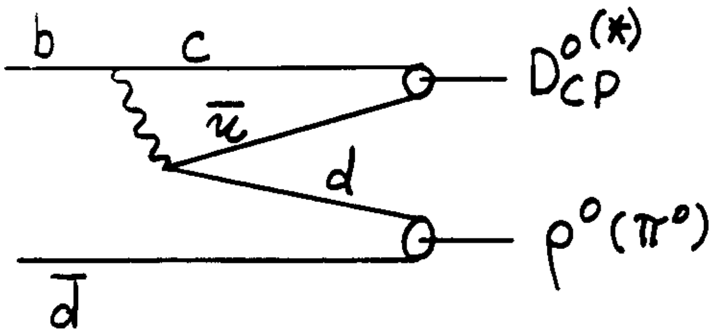
Color allowed

Probes of β

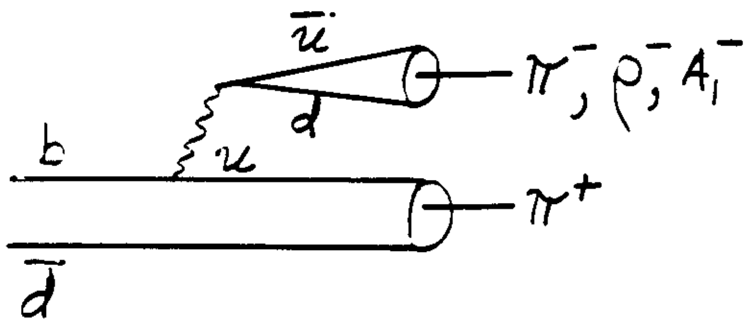
Mode	Rough BR	Penguins Worrisome?
	6×10^{-4}	Maybe Like $B_d \to \pi^+\pi^-$

Color suppressed

Probes of β

Mode	Rough BR	Penguins Worrisome
	3×10^{-4}	No
	2×10^{-5}	Maybe
	10^{-4}	No

Probes of α

	Mode	Rough BR	Penguins Worrisome?
3		2×10^{-5}	Maybe

To test the SM of $\not{CP}$, study decays which involve different quark processes but allegedly yield the same CKM phase.

To improve statistics, study several decays which involve the same quark process.

Let us now go beyond the Wolfenstein approximation, treat V_{CKM} exactly, and ask again —

What Independent Phases φ Exist?

Frequently, V is parametrized as

$$V = V(\underbrace{\theta_1, \theta_2, \theta_3}_{\text{Mixing angles}}, \delta) \quad \leftarrow \text{Complex phase}$$

$$\delta = 0 \Rightarrow \cancel{CP} = 0$$

But $\cancel{CP}$ in a B decay does not measure δ in any direct way. It measures

$$\varphi = \text{Arg}[\text{Some Product of CKM Elements}]$$

$$= \text{Complicated Function}(\theta_1, \theta_2, \theta_3, \delta). \quad [V_{CKM}]$$

$\underline{V}$ In one popular parametrization,

$$= \begin{bmatrix} c_1 c_3 & s_1 c_3 & s_3 e^{-i\delta} \\ -s_1 c_2 - c_1 s_2 s_3 e^{i\delta} & c_1 c_2 - s_1 s_2 s_3 e^{i\delta} & s_2 c_3 \\ s_1 s_2 - c_1 c_2 s_3 e^{i\delta} & -c_1 s_2 - s_1 c_2 s_3 e^{i\delta} & c_2 c_3 \end{bmatrix}$$

where $c_i = \cos\theta_i$, $s_i = \sin\theta_i$.

∴ In studies of $\cancel{CP}$ in B decays, take the basic parameters to be the phases φ of products of CKM elements.

What independent phases of products of CKM elements exist?

The SM requires that V be unitary [Δ's]

The 18 angles in the unitarity triangles are phases of products of CKM elements.

At most, 4 of these angles are independent.

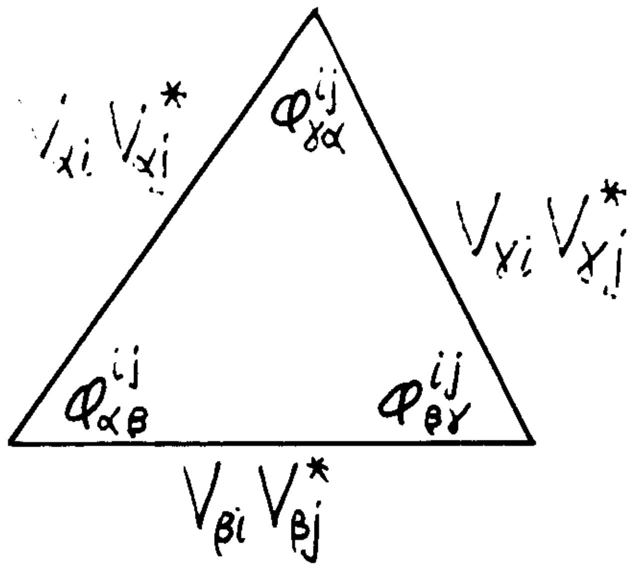
Simple direct proof:

Notation: $V_{\alpha i}$

Greek: u, c, t $\longrightarrow \alpha$; $i \longleftarrow$ Roman: d, s, b

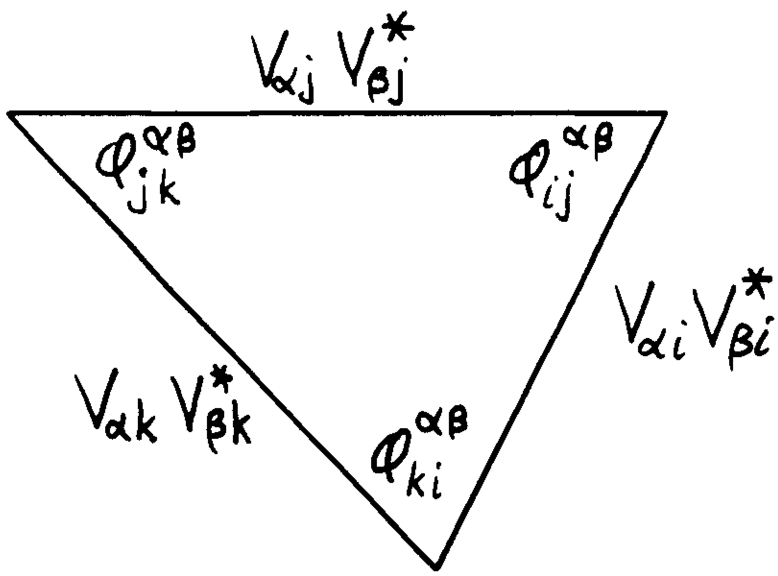
The Δ stating that columns i and j are $\perp$.

he ij "Δ":



The Δ stating that rows α and β are $\perp$:

he $\alpha\beta$ "Δ":



We write the angles as $\varphi^{\text{triangle}}_{\text{adjacent lines}}$.

roof that only 4 angles are independent:

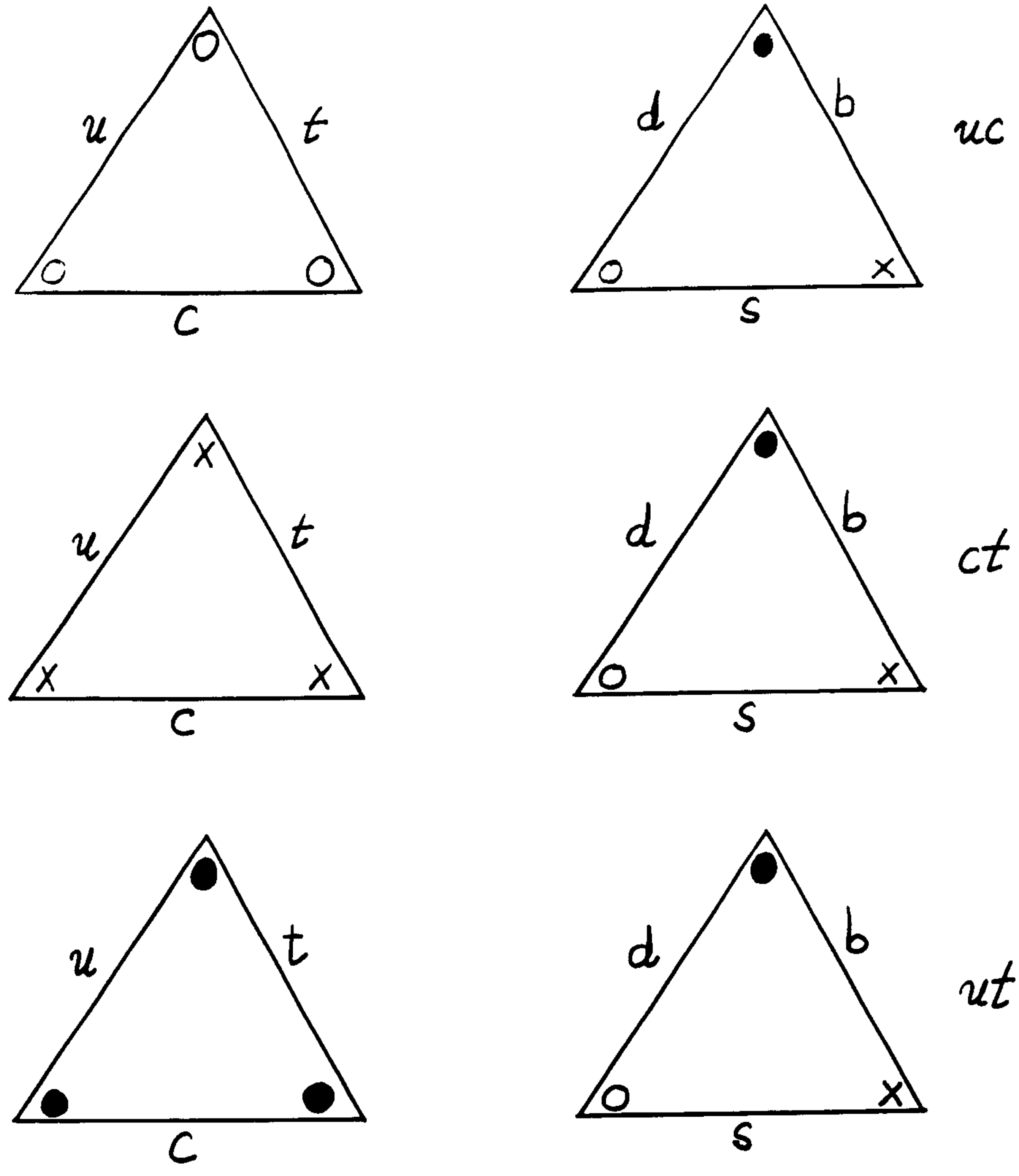
'ote that —

$$\varphi_{ds}^{\overset{\text{Row}}{uc}} = \arg\left[\frac{V_{ud}V_{cd}^{*}}{V_{us}V_{cs}^{*}}\right] = \arg\left[\frac{V_{ud}V_{us}^{*}}{V_{cd}V_{cs}^{*}}\right] = \varphi_{uc}^{\overset{\text{Column}}{ds}} .$$

:very angle in a "row triangle" is also zn angle in a "column triangle".

Ne focus, then, on the "column triangles".

Column Δ's Row Δ's



The angles of each color add up to π.
We can then eliminate one of the column Δ's.
The remaining two contain 4 independent angles.

Call the independent angles

$$\varphi_q \; ; \; q = 1, 2, 3, 4.$$

If

$$\varphi = \text{Arg}\left[\text{Some Rephasing-Invariant Product of CKM Elements}\right],$$

then

$$\varphi = \sum_{q=1}^{4} n_q \varphi_q \; ; \; n_q = \text{integer}.$$

The φ_q are the independent phases of products of CKM elements.

Studies of $\cancel{CP}$ in B decay are probes of the 4 independent phases φ_q.

To show that $\varphi = \sum_{q=1}^{4} n_q \varphi_q$ —

Choose the phase convention by multiplying rows and columns by phases until

$$V_{CKM} = \begin{bmatrix} * & * & r \\ * & * & r \\ r & r & r \end{bmatrix}$$

$r =$ real element ≥ 0
$* =$ complex element

In this phase convention,

$$\varphi_{ut}^{db} = \arg(V_{ud} V_{ub}^* V_{tb} V_{td}^*) = \arg(V_{ud})$$

$$\varphi_{ut}^{sb} = \arg(V_{us} V_{ub}^* V_{tb} V_{ts}^*) = \arg(V_{us})$$

$$\varphi_{ct}^{db} = \arg(V_{cd} V_{cb}^* V_{tb} V_{td}^*) = \arg(V_{cd})$$

$$\varphi_{ct}^{sb} = \arg(V_{cs} V_{cb}^* V_{tb} V_{ts}^*) = \arg(V_{cs}) .$$

In our phase convention,

$$\varphi \equiv \arg[\text{Some rephasing-invariant product of CKM elements}] =$$

$$= n_{ud} \arg(V_{ud}) + n_{us} \arg(V_{us})$$

integers

$$+ n_{cd} \arg(V_{cd}) + n_{cs} \arg(V_{cs})$$

$$= n_{ud}\, \varphi_{ut}^{db} + n_{us}\, \varphi_{ut}^{sb} + n_{cd}\, \varphi_{ct}^{db} + n_{cs}\, \varphi_{ct}^{sb}$$

In short,

$$\varphi = \sum_{q=1}^{4} n_q \varphi_q \,.$$

Since φ and the φ_q are rephasing-invariant, so is this relation.

Suppose that B decay experiments determine the φ_q.

Once the 4 independent φ_q are known, the entire CKM matrix follows from them!

Proof:

In the previously-adopted phase convention, the phases of the $V_{\alpha i}$ that are not real and positive are determined directly by the φ_q.

How can the phases φ_q determine the magnitudes $|V_{\alpha i}|$?

Unitarity!

The 4 independent $\varphi_q \Rightarrow$ All 18 φ_q

$\Rightarrow$ Shapes of all the triangles $\Rightarrow$

Ratios of lengths of sides in each triangle

Then, for example,

$$\left|\frac{V_{cd}V_{cs}^*}{V_{ud}V_{us}^*}\right| \left|\frac{V_{cd}V_{cb}^*}{V_{ud}V_{ub}^*}\right| \left|\frac{V_{us}V_{ub}^*}{V_{cs}V_{cb}^*}\right| = \frac{|V_{cd}|^2}{|V_{ud}|^2} \,.$$

We can get $|V_{td}|^2/|V_{ud}|^2$ similarly.

Unitarity: $|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 1$.

Then $|V_{ud}|^2, |V_{cd}|^2, |V_{td}|^2$ are all determined.

four magnitudes (e.g., $|V_{ud}|$, $|V_{us}|$, $|V_{cd}|$, $|V_{cs}|$), plus one sign, plus no phases, fully determine V.

(Jarlskog)

What we have shown here is that four phases, plus no magnitudes, also fully determine V.

Is it practical to determine the 4 independent φ_q from $\not{CP}$ effects in B decay?

An illuminating choice of $\varphi_1, \ldots, \varphi_4$ is:

Angle	Choice	Expected Size
φ_1	$\arg(V_{ud} V_{ub}^* V_{td}^* V_{tb}) \equiv \alpha$	~ 1
φ_2	$\arg(V_{cd} V_{cb}^* V_{td}^* V_{tb}) \equiv \beta$	~ 1
φ_3	$\arg(V_{cs} V_{cb}^* V_{ts}^* V_{tb}) \equiv \epsilon$	~ 0.05
φ_4	$\arg(V_{ud} V_{us}^* V_{cd}^* V_{cs}) \equiv \epsilon'$	~ 0.002

To determine ϵ or ϵ', one must measure a small $\not{CP}$ effect, or the small difference between two large $\not{CP}$ effects.

An illustrative complete set of B decay experiments

Decay Mode	Quantity Measured
$\overset{(-)}{B}_d \to \pi^+\pi^-$	$\sin 2\alpha$
$\overset{(-)}{B}_d \to \psi K_s$	$\sin 2\beta$
$B^\pm \to D K^\pm$, $D \to K^+K^-$	$\sin^2(\gamma - \epsilon')$
$\overset{(-)}{B}_s \to \psi\phi$	ϵ
$\overset{(-)}{B}_s \to D_s^\pm K^\mp$	$\sin^2(\gamma - 2\epsilon + \epsilon')$
or	
$\overset{(-)}{B}_d \to \rho\pi$	$\cos 2\alpha$

All discrete ambiguities can be resolved.

Gronau&Wyler, [2] Dunietz, [3] Aleksan, Dunietz, B.K.
Snyder&Quinn

Suppose only α, β and ϵ can be measured.

To a good approximation,

$$\left|\frac{V_{ub}}{V_{cb}}\right|^2 \cong \left|\frac{\sin\beta \ \sin\epsilon}{\sin\alpha \ \sin\gamma}\right|$$

nd

$$\left|\frac{V_{td}}{V_{ts}}\right|^2 \cong \left|\frac{\sin\gamma \ \sin\epsilon}{\sin\alpha \ \sin\beta}\right| .$$

These are interesting CKM parameters which are difficult to obtain in other ways.

Summary

If the known weak interaction is behind $\not{CP}$, then $\not{CP}$ comes from phases in the CKM matrix V.

B decays will cleanly test whether phases in V are the source of $\not{CP}$.

If the answer is "yes", we will see some very pretty physics.

If the answer is "no", we will have evidence of physics beyond the SM.