INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

SMR.858 - 36

II

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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NEUTRINO PHYSICS

A. SMIRNOV
International Centre For Theoretical Physics
Trieste
ITALY

Please note: These are preliminary notes intended for internal distribution only.

LECTURE 2

A. Smirnov

- we have started to discuss the oscillations
- evolution of the neutrino state in vacuum:

$$\nu_e = \cos\theta \cdot \nu_1 + \sin\theta \cdot \nu_2 \, e^{i\Delta\varphi(t)}$$

we have derived the expression for probability:

$$P_{\nu_e \to \nu_\mu} = \sin^2 2\theta \cdot \sin^2 \frac{\pi x}{l_\nu}.$$

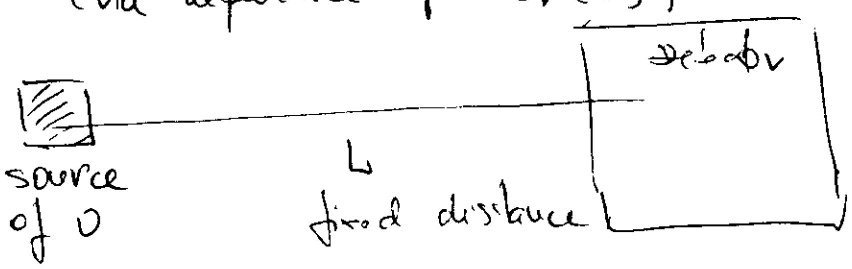
$$P_{\nu_e - \nu_e} = 1 - P_{\nu_\mu \to \nu_e}$$

where

$$l_\nu = \frac{4\pi E}{\Delta m^2}.$$

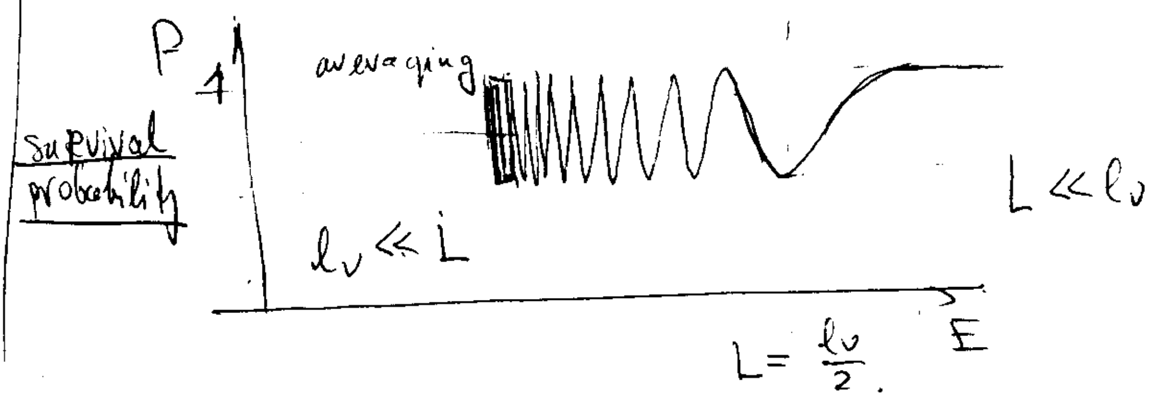
Two points will be important:

(1) Energy dependence of the effect (via dependence of $l_\nu(E)$).



$\sin^2 \frac{x \Delta m^2}{4E}$

in denominator

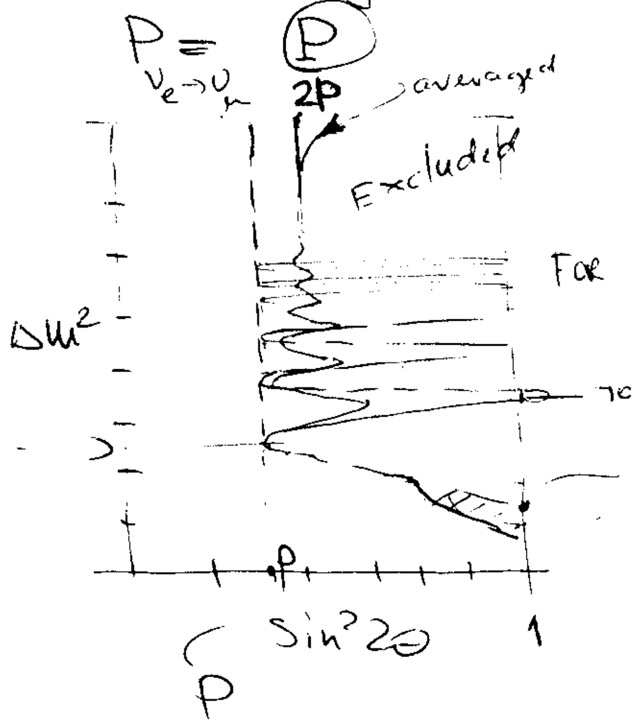


⊛ $\Delta m^2 - \sin^2 2\theta$ plot

Isoprobability line.

$P_{\nu_e \to \nu_\mu} = P$

$$\sin^2 2\theta \cdot \sin^2 \frac{\Delta m^2}{4E} = P$$



small phase:

$$P \simeq \sin^2 2\theta \left(\frac{L \Delta m^2}{4E}\right)^2$$

$$\Delta m^2 = \frac{4 \langle E \rangle}{L} \sqrt{\frac{P}{\sin^2 2\theta}}$$

The region to the left is excluded.

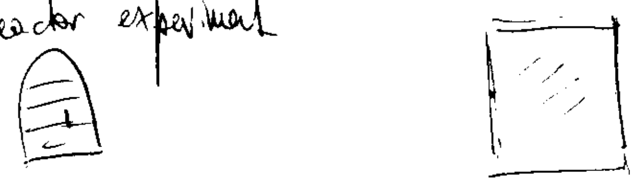
minimum Δm^2.

$$\boxed{\Delta m^2 = \frac{4 \langle E \rangle}{L} \sqrt{P}}$$

○ BUGEY experiment

Reactor experiment

40 m ..



$\Delta m^2 \sim 10^{-2}$

$\sin^2 2\theta \quad 3 \cdot 10^{-3}$

Oscillation in matter.

Flavors of ~~xxx~~ eigenstates:

$$\boxed{\begin{aligned} \nu_e &= c\,\nu_1 + s\,\nu_2 \\ \nu_\mu &= c\,\nu_2 - s\,\nu_1 \end{aligned}}$$

Let us ~~xxxxxx~~ convert these formulas:

$$\nu_1 = c\,\nu_e - s\,\nu_\mu$$
$$\nu_2 = c\,\nu_\mu + s\,\nu_e$$

The ~~x~~ eigenstates ν_1, ν_2 have certain flavour composition: ν_e, ν_μ composition
THE composition is ~~x~~ determined by ~~xxx~~ $\cos\theta$, $\sin\theta$ → by mixing angle.

Vacuum oscillations:

$$\nu_e(t) = \cos\theta\ \nu_1 + \sin\theta\ \nu_2\ e^{i\Delta\varphi}$$

- is phase effect.
- admixtures of ν_1 and ν_2 in $\nu(t)$ are fixed. (by θ.)
- flavor composition of ν_1 and ν_2 is fixed. (also by θ)

$$\boxed{\begin{array}{l} \Delta\varphi = (E_2 - E_1)t \\ l_\nu = \dfrac{2\pi}{E_2 - E_1} \end{array}}$$

Oscillation in uniform medium:

in general $\nu(t) = a(t)\,\nu_{1m} + b(t)\,\nu_{2m}\,e^{i\Delta\varphi_m}$

eigenstates in medium

$\nu_e = \nu_{1m}\cos\theta_m + \nu_{2m}\sin\theta_m$

$$\nu_e(t) = \cos\theta_m(\rho)\ \nu_{1m} + \sin\theta_m(\rho)\ \nu_{2m}\,e^{i\Delta\varphi_m}$$

$\theta_m(n(t))$ the same picture but

$$\boxed{\begin{array}{l} \theta \to \theta_m(E, \Delta m^2, \rho) \\ \Delta\varphi_m \to \Delta\varphi_m(- - - -) \end{array}}$$

- phase effect.
- admixtures of ν_{1m} ν_{2m} in $\nu(t)$ are fixed
- flavor $\overset{(\nu_e\ \nu_\mu)}{\text{composition}}$ of ν_{im} is fixed. although it differs from ~~that~~ in ~~the~~ vacuum case.

In medium with varying density.

- adiabatic case:

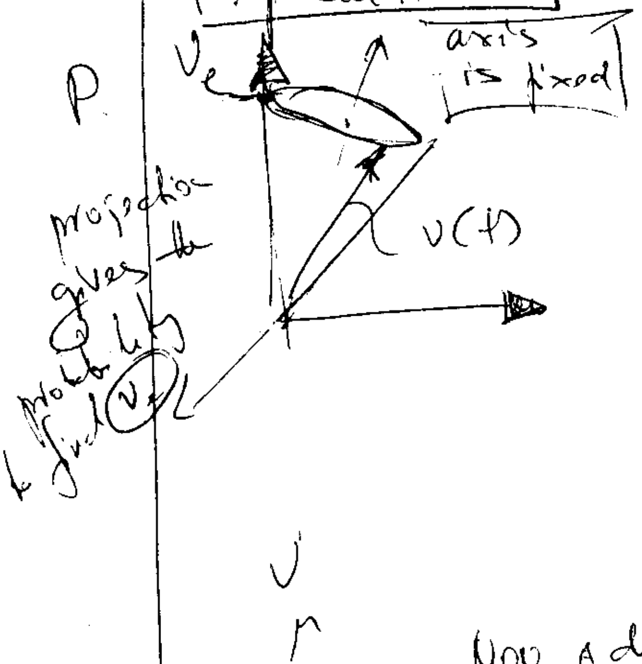
- admixtures of ν_{im} are fixed
- but flavor composition of eigenstate changes with density.

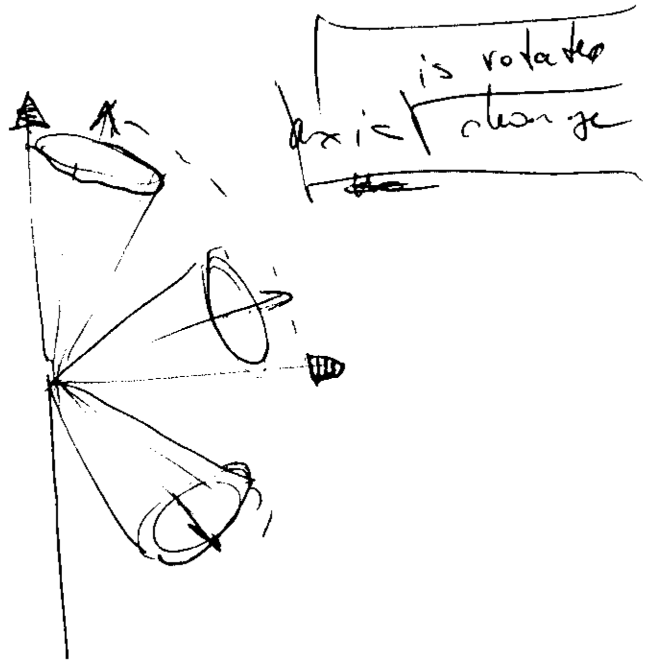
since $\boxed{\theta_m = \theta_m(\rho)}$

this is the effect of resonance conversion

Not a phase effect.

$\boxed{\text{Graphical Representation:}}$





Non adiabatic case.

$\nu_{1m} \leftrightarrow \nu_{2m}$ transitions

What this results come from?

FORMALY from solution of EVOLUTION EQUATION

Let us write the evolution equation.
some scetch:

omitting justifications

1 step:

The evolution equation for one particle with definite mass: m_i

$$\boxed{i\,\frac{d\nu}{dt} = E\,\nu}$$

in the ultrarelativistic limit.

$$\boxed{i\,\frac{d\nu}{dt} \simeq \left(p + \frac{m^2}{2E}\right)\nu} \qquad \begin{matrix}6.1)\\ (*)\end{matrix}$$

2 step:

two flavour neutrinos:

$$\nu_f \equiv \begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$ mixing of these neutrinos

is induced by nondiagonal mass matrix,

M

($\equiv$ if the mass matrix is nondiagonal $\Rightarrow$).

The matrix is diagonalized by: $S(\theta) \equiv \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix}$

$$\nu_f = S \nu_m \qquad \nu_m = \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$ – mass states $\equiv$ eigenstates

$$S^\dagger(\theta) M S(\theta) = \mathrm{diag}(m_1, m_2) \equiv M^{diag}.$$

Let us find evolution equation for flavor states

(a) one can just ~~...~~ generalize (*) equation (6.1)

$$\boxed{i \frac{d\nu_f}{dt} \simeq \left(p \hat{I} + \frac{M^2}{2E} \right) \nu}$$

(b) One can proof ~~this~~ by starting from equation for mass states:

$$i \frac{d \nu_m}{dt} = \left(p \cdot \hat{I} + \frac{M^{diag\,2}}{2E} \right) \nu_m$$

and then ~~make the transforma~~ substitute:

$$\nu_m = S^\dagger \nu_f$$

The solution of this equation will reproduce usual flavor oscillation result.

Once we are interested in evolution of flavor (in flavour space)

$\Rightarrow$ can neglect $p\hat{I}$ - first term.

$$\boxed{i \frac{d\nu_f}{dt} = \frac{M^2}{2E} \nu_f}$$

Hamiltonian like

$$= H \nu$$

s step

$$H \equiv \frac{M^2}{2E}$$

explicitly in terms of mass states and θ:

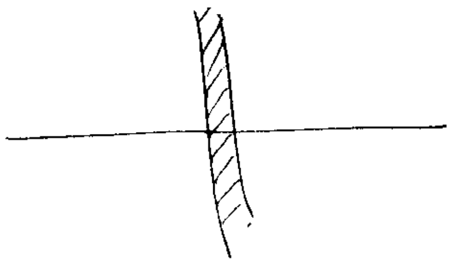
$$i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \frac{\Delta m^2}{4E} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

$\boxed{\text{MATTER EFFECT}}$

- medium is <u>transparent</u> (low energies of neutrinos)

$\Rightarrow$ refraction effect.

$$d\nu = -i\,\nu\cdot(n-1)\cdot p\,dx$$



dx

$$n_R - 1 = \frac{2\pi\cdot f(0)\cdot n}{p^2}$$

p is the momentum

$f(0)$ -- the amplitude of forward scattering

n - density of particles

$$\boxed{d\nu = -i\sqrt{2}\,G_F n\cdot\nu\cdot dx}$$

$$i\frac{\partial\nu}{\partial t} = \sqrt{2}\,G_F n\cdot\nu$$

Equivalently:

- matter effect as potential

$$V = \sqrt{2}\, G_F\, n$$

Fermi constant

can be considered

- Dirac equation

$$\frac{G_F}{\sqrt{2}} \left[\bar{\nu}\gamma^\mu(1+\gamma_5)\nu\right]\left[\underbrace{\bar{e}\gamma^\mu(1+\gamma_5)e}\right]$$

$$\langle\Psi| \downarrow |\Psi\rangle$$

$$\downarrow$$

$$\langle\Psi|\bar{e}\gamma^0 e|\Psi\rangle = \sqrt{2}\, G_F\, n_e$$

for scattering on electrons

- correction to self energy:

Green function in medium

~~for flavour oscillations~~.

F.

Properties:

(1) in the SM: V is diagonal:

$$V = \begin{pmatrix} V_e & 0 \\ 0 & V_x \end{pmatrix}$$

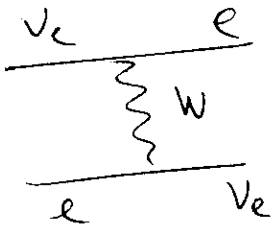
(2) ~~for flavour oscillations~~

only difference is important:

$$V_e - V_x$$

(3) for flavour oscillations: $\nu_e - \nu_\mu$, $\nu_e - \nu_\tau$:

$V_e - V_x$ due to scattering of on electrons due to CC:



Evolution equation in matter:

$$\boxed{i \frac{d\nu_f}{dt} = \left(\frac{M^2}{2E} + V\right)\nu}$$

or explicitly for flavour case:

$$H = \begin{bmatrix} \sqrt{2}\, G_F N_e - c_2 & s_2 \\ s_2 & c_2 \end{bmatrix}$$

Wolfenstein equation

$$c_2 = \frac{\Delta m^2}{4E} \cos 2\Theta, \quad s_2 = \frac{\Delta m^2}{4E} \sin 2\Theta$$

How to solve the equation?

- find the eigenstates of A
  $\equiv$ neutrino eigenstates ν_{im}.
  and Eigenvalues E_{im}

~~or [illegible]~~

(12.1) $$\boxed{\vec{\nu}_f = S(\Theta_m) \cdot \vec{\nu}_m}$$

analogy with vacuum.

mixing matrix in matter.

Θ_m is mixing angle in matter

- find the ~~equation~~ evolution equation for ν_m.
  (substituting (12.1) in the equation for ν_f

- solve the equation for ν_{im}

- f using (12.1)
  write the solution for $\underline{\nu_f}$
  (w.f).

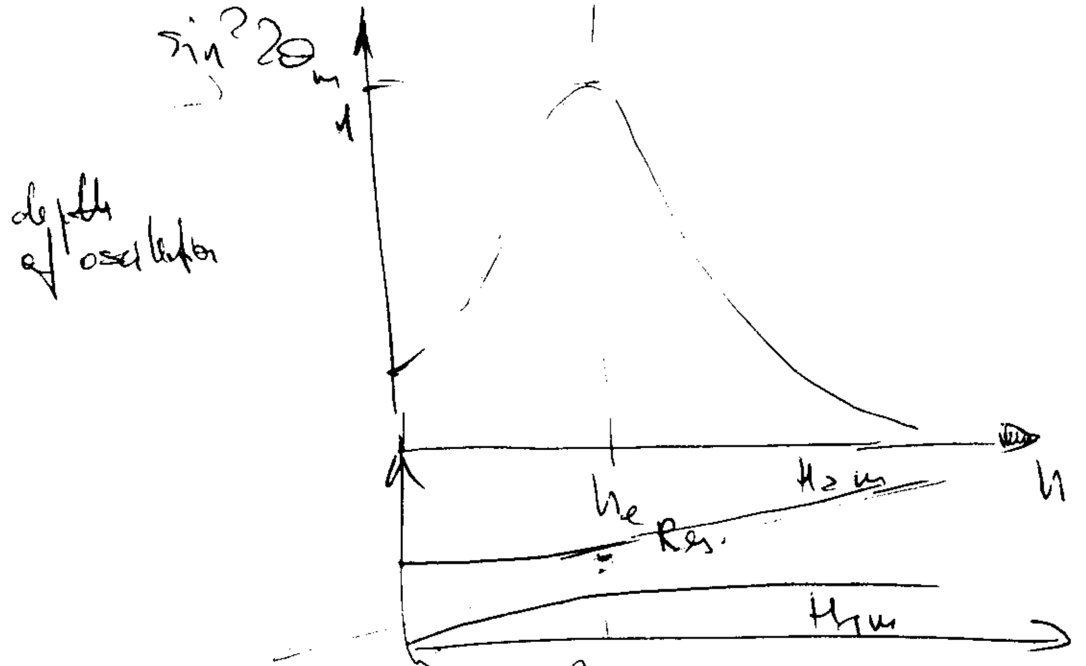
Thus one can reproduce the results discussed Before

Remarks:

(1) ~~[illegible]~~ $S(\Theta_m)$ is determined by diagonalization of $H(n_e, \dots)$

$\Rightarrow$ gives Θ_m:

$$\boxed{\tan 2\Theta_m = \frac{\sin 2\Theta}{\cos 2\Theta - \sqrt{2}\, G_F\, n_e \frac{2E}{\Delta m^2}}}$$



$$n^R_{Res} = \frac{\Delta m^2}{2E} \frac{\cos 2\Theta}{\sqrt{2}\, G_F}$$

$\boxed{\text{Resonance density}}$

Eigenvalues H_{1m}, H_{2m}

$$H_{1m} - H_{2m} = \sqrt{}$$

Equation for eigen-states: (substituting (12.1) into eq. for ν_f)

$$\boxed{i\frac{d}{dt}\vec{\nu}_m = \begin{pmatrix} H_{1m} & i\dot{\theta}_m \\ -i\dot{\theta}_m & H_{2m} \end{pmatrix}\vec{\nu}_m} \quad (14.1)$$

allow to analyse

$$\dot{\theta}_m \equiv \frac{d\theta_m}{dx}$$

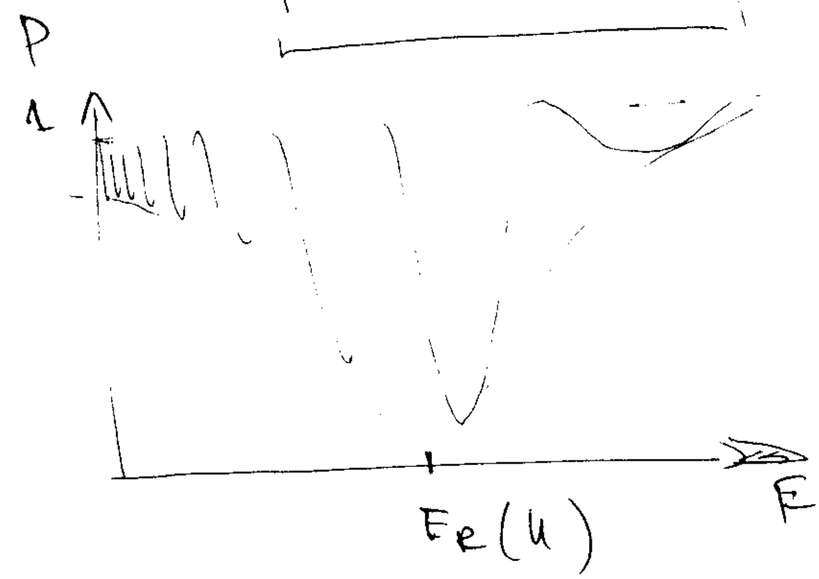
Constant density:

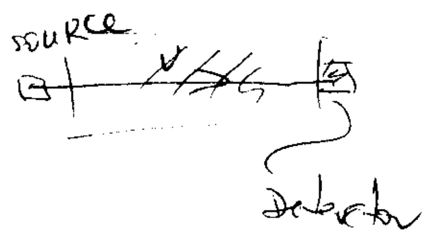
- $\dot{\theta}_m = 0$
- θ_m = constant
- equations for ν_{im} decouple

as in vacuum case but parameters of oscillations are ~~different~~ differ from vacuum ones.

depth:

$$\boxed{\begin{aligned} A &= \sin^2 2\theta_m \\ l_m &= \frac{2\pi}{H_{2m} - H_{1m}} \end{aligned}}$$





Resonance enhancement of ν oscillation

$\boxed{\text{Variable density.}}$

$\dot{\theta}_m \neq 0$

If $\boxed{|\dot{\theta}_m| \ll |H_{2m} - H_{1m}|}$ adiabaticity condition.

$\dot{\theta}$ can be neglected in eq. (14.1).

$\equiv$ $\nu_{1m} \leftrightarrow \nu_{2m}$ transitions can be neglected.

$\Rightarrow$ as in $n = \text{const}$ case

However NOW $\theta_m = \theta_m(t)$

$\Rightarrow$ θ_m determine flavour composition of ~~ν~~ ν_{im} (eigenstates).

$\nu_{1m} = \cos\theta_m \, \nu_e - \sin\theta_m \, \nu_\mu$

$\Rightarrow$ $\boxed{\text{FLAVOUR COMPOSITION CHANGES DURING THE PROPAGATION}}$

$\hookrightarrow$ Resonance conversion

ν_{2m} :

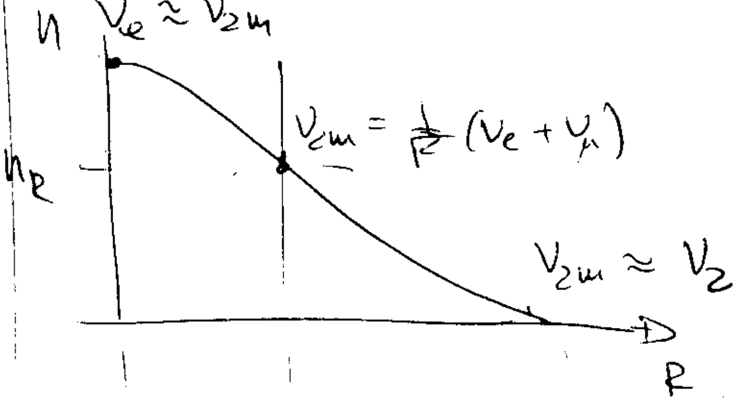
$n \gg n_R$ $\quad \theta_m \approx \frac{\pi}{2}$ $\quad \nu_{2m} \approx \nu_e$

$n = n_R$ $\quad \theta_m = \frac{\pi}{4}$ $\quad \nu_{2m} = \frac{\nu_e + \nu_\mu}{\sqrt{2}}$

$n \ll n_R$ $\quad \theta_m \approx \theta$ $\quad$ and if θ is small ; $\nu_{2m} \approx \nu_\mu$

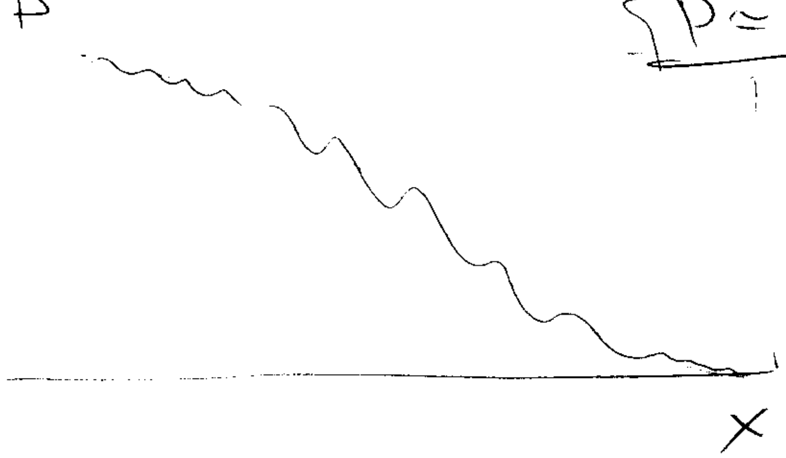
if ν_e is produced at $n \gg n_R$:

$$\nu(0) \approx \nu_e \approx \nu_{2m}$$



probability to find ν_e in final state is

$$\boxed{P \simeq \sin^2\theta}$$



$\boxed{\text{If density is changed fastly:}}$

$\dot{\theta}_m$ is big:

transitions : $\nu_{1m} \Leftrightarrow \nu_{2m}$.

~~[illegible]~~ important parameter :

$$\varkappa_R = \frac{H_1 - H_2}{\dot{\theta}_m}$$

$$\boxed{P_{12} \sim \exp\left[-\frac{\pi}{2}\varkappa_R\right]}$$

If $P_{12} \sim 1$ No conversion effect,

("double" change)

(1) flavour composition is changed completely

(2) $\nu_{2m} \to \nu_{1m}$.

as the result flavour of state is not changed!

in the resonance:

$$\boxed{\varkappa_R = \frac{\Delta m^2}{2E} \frac{\sin^2 2\theta}{\cos 2\theta \cdot \frac{1}{n_e}\frac{dn_e}{dx}}} \propto \frac{1}{E}$$

The bigger E,
and the smaller Θ
the adiabaticity is
<u>worse</u>