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III

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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CP VIOLATION

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INT, Seattle CP Violation

Theme: What is ~~CP~~?
How does it come about in the SM?
How can we test the SM picture of ~~CP~~?

[List of Topics
Focus on Beauty system.]

CP Violation

Boris Kayser

The plan

What is $\not{CP}$, and why do we care about it?

The discovery of small $\not{CP}$ effects in K decay.

Where does this $\not{CP}$ come from?
How can it be produced by the known weak interaction as described in the SM?

The SM prediction of large $\not{CP}$ effects providing clean information on the underlying CKM phases, in B decay.

$\not{CP}$ in the B system —
What $\not{CP}$ effects will one look for?
How will the generic underlying parameters be cleanly extracted?
What will it take to do the experiments?
What are the independent underlying parameters?

Further testing the SM of $\not{CP}$ in other places —
Direct $\not{CP}$ in K decay
A clean test of the SM in rare K decay
Electric dipole moments of fermions.

C, P, and CP

Charge conjugation C —
For every particle there is a corresponding antiparticle
Takes particle into antiparticle:

$$C|f\rangle = \eta_c|\bar{f}\rangle .$$

[2]

Parity P —
Inverts the coordinate system:

$$\vec{r} \to -\vec{r} .$$

$$\vec{p} = m\frac{d\vec{r}}{dt} \to -\vec{p}, \quad \vec{L} = \vec{r}\times\vec{p} \to +\vec{L} .$$

$$P|f(\vec{p}, s_z)\rangle = \eta_p|f(-\vec{p}, s_z)\rangle .$$

CP = C*P —

$$CP|f(\vec{p}, s_z)\rangle = \eta_{CP}|\bar{f}(-\vec{p}, s_z)\rangle .$$

[2]

$\cancel{CP}$ in the neutral K system.

Consider neutral kaons at rest.
K^0, $\overline{K^0}$, of opposite strangeness. Spinless, antiparticles.
We choose the phase convention so that

$$CP|K^0\rangle = +1|\overline{K^0}\rangle.$$

Then (simple field theory)

$$CP|\overline{K^0}\rangle = +1|K^0\rangle.$$

Weak interactions can change S so they mix K^0 and $\overline{K^0}$.

The time evolution of a neutral kaon is described by

$$i\frac{\partial}{\partial t}\begin{bmatrix} a(t) \\ \overline{a}(t) \end{bmatrix} = \mathcal{M}\begin{bmatrix} a(t) \\ \overline{a}(t) \end{bmatrix}$$

amp. for K^0 — $a(t)$

amp. for $\overline{K^0}$ — $\overline{a}(t)$

A particle and its antiparticle do not always behave the same way.

$$\nu \longleftarrow \overset{\pi^+}{\bullet} \longrightarrow \mu^+$$

but

$$\overline{\nu} \longleftarrow \overset{\pi^-}{\bullet} \longrightarrow \mu^-$$

∴ Nature is not matter-antimatter symmetric in the sense of being invariant under C.

$$\Gamma\left[\nu \longleftarrow \overset{\pi^+}{\bullet} \longrightarrow \mu^+\right]$$

$$> \Gamma\left[\nu \longleftarrow \overset{\pi^+}{\bullet} \longrightarrow \mu^-\right] \longleftarrow \text{Rate may vanish.}$$

Is nature matter-antimatter symmetric in the sense of being invariant under CP?

$$CP\left[\nu \longleftarrow \overset{\pi^+}{\bullet} \longrightarrow \mu^+\right]$$

$$= \left[\mu^- \longleftarrow \overset{\pi^-}{\bullet} \longrightarrow \overline{\nu}\right]$$

and they do have equal rates. However—

$$\mathcal{M} = \begin{bmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} \\ \mathcal{M}_{21} & \mathcal{M}_{22} \end{bmatrix}$$, a non-Hermitean

effective Hamiltonian, is known as the neutral K mass matrix.

We assume CPT invariance.
This implies that

(Disc. classes)

$$\mathcal{M}_{11} \equiv \langle K^0 | \mathcal{M} | K^0 \rangle = \langle \overline{K^0} | \mathcal{M} | \overline{K^0} \rangle \equiv \mathcal{M}_{22}.$$

Writing $\mathcal{M}_{11} \equiv M - i\frac{\Gamma}{2}$, $\mathcal{M}_{22} \equiv \overline{M} - i\frac{\overline{\Gamma}}{2}$,

$M = \overline{M}$ Particle and antiparticle have same mass

$\Gamma = \overline{\Gamma}$ They also have same total width.

If CP invariance holds,

$$\mathcal{M}_{12} \equiv \langle K^0 | \mathcal{M} | \overline{K^0} \rangle = \langle K^0 | (CP)^\dagger \mathcal{M} (CP) | \overline{K^0} \rangle$$
$$= \langle (CP) K^0 | \mathcal{M} | (CP) \overline{K^0} \rangle = \langle \overline{K^0} | \mathcal{M} | K^0 \rangle \equiv \mathcal{M}_{21}.$$

Then $\mathcal{M}$ has the form

$$\mathcal{M} = \begin{bmatrix} X & Y \\ Y & X \end{bmatrix},$$

so that the mass eigenstates are $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

That is, they are the CP eigenstates

$$|K_+\rangle \equiv \frac{|K^0\rangle + |\overline{K^0}\rangle}{\sqrt{2}}, \quad |K_-\rangle = \frac{|K^0\rangle - |\overline{K^0}\rangle}{\sqrt{2}}$$

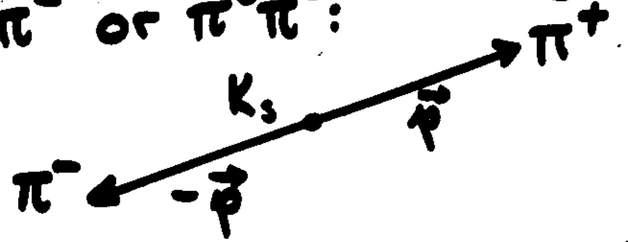
CP: $+1$ $\qquad$ -1

Experimentally, the two mass eigenstates are—

K_S $\quad (\tau_S = 0.89 \times 10^{-10}$ sec)

K_L $\quad (\tau_L = 5.19 \times 10^{-8}$ sec)

Essentially all K_S decays are to $\pi^+\pi^-$ or $\pi^0\pi^0$:



$$CP|\pi^+(\vec{p})\pi^-(-\vec{p})\rangle = +|\pi^-(-\vec{p})\pi^+(\vec{p})\rangle$$

$\therefore$ If CP is conserved,

$$CP(K_S) = +1, \quad CP(K_L) = -1.$$

Then $K_L \to \pi^+\pi^-$ is forbidden.

But $K_L \to \pi^+\pi^-$ does occur.

(Christenson, Cronin, Fitch, Turlay)

$$BR(K_L \to \pi^+\pi^-) = 2.0 \times 10^{-3}$$

$\therefore$ CP is not always conserved.

To be sure, $\dfrac{\Gamma(K_L \to \pi^+\pi^-)}{\Gamma(K_S \to \pi^+\pi^-)} = (2.269 \times 10^{-3})^2 = 5 \times 10^{-6}$;

so $\not{CP}$ is here small.

It is also observed that

$$\frac{\Gamma(K_L \to \pi^-\ell^+\nu) - \Gamma(K_L \to \pi^+\ell^-\bar{\nu})}{\text{"} \quad + \quad \text{"}} = 3.27 \times 10^{-3} \neq 0.$$

CP-noninvariance.

<u>What is the origin of $\not{CP}$?</u>

All $\not{CP}$ - effects seen so far occur in K decays.

K decays are due to the weak interaction.

Perhaps $\not{CP}$ comes from the weak interaction.

This is the possibility we will emphasize.

How can the weak interaction cause ~~CP~~? [3,4]

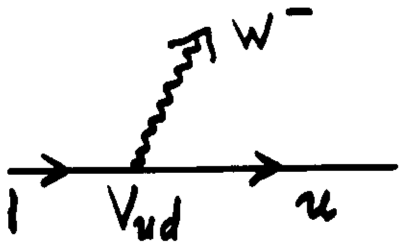
According to the S(tandard) M(odel), the weak couplings of the quarks are described by $\mathcal{H} = \mathcal{H}_{cc} + \mathcal{H}_{NC}$, with

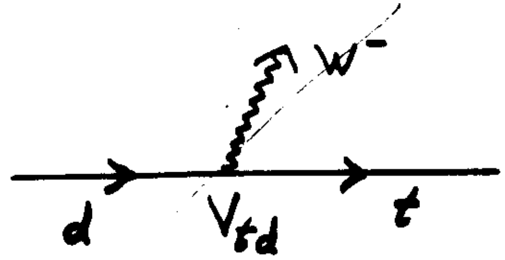
$$\mathcal{H}_{cc} = \frac{g}{\sqrt{2}} W^{\mu} \sum_{\substack{\alpha = u,c,t \\ i = d,s,b}} V_{\alpha i}\, \bar{\alpha}_L \gamma_\mu i_L$$

$$+ \frac{g}{\sqrt{2}} W^{\mu\dagger} \sum_{\substack{\alpha \\ i}} V^*_{\alpha i}\, \bar{i}_L \gamma_\mu \alpha_L .$$

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

is the C(abibbo)–K(obayashi)–M(askawa) quark mixing matrix. [4]





$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}$$

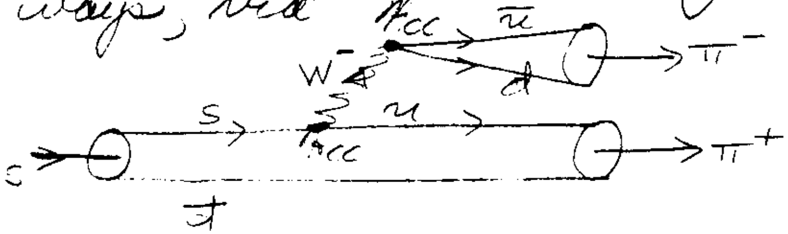
The weak interaction & the SM

It is well-established, as you know, hadrons are made of quarks. The SM describes things in terms of quarks.

$|K_S\rangle \simeq \frac{|s\bar d\rangle + |d\bar s\rangle}{\sqrt 2}$, $|\pi^+\rangle = |u\bar d\rangle$, $|\pi^-\rangle = |d\bar u\rangle$

CC

$K_S \to \pi^+\pi^-$ proceeds, among other ways, via



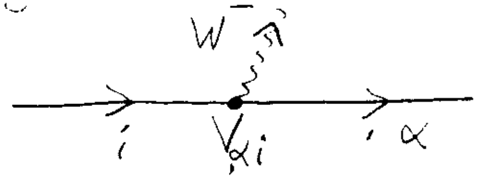
As you know, we have 6 quarks

Q				Notation
$+2/3$	u	c	t	α
$-1/3$	d	s	b	i

$\xrightarrow{\hspace{6em}}$ Mass

In a weak decay, a heavy quark decays into lighter ones.

In



any $-$ve quark i (d, s, or b) can turn into any $+$ve one α (u, c, or t).

$$\mathcal{L}_{NC} = \frac{g}{\cos\theta_W} Z^\mu \sum_{\substack{q=u,c,t,\\ d,s,b}} \left\{ \left[I_3(q_L) - Q(q)\sin^2\theta_W\right]\bar q_L \gamma_\mu q_L - Q(q)\sin^2\theta_W\, \bar q_R \gamma_\mu q_R \right\}$$

$$(CP)\, W^\mu V_{\alpha i}\, \bar\alpha_L \gamma_\mu i_L\, (CP)^{-1} =$$

$$= \underbrace{[\eta(W)\eta(\alpha)\eta^*(i)]}_{\text{Phases which we can choose } = 1} W^{\mu\dagger} V_{\alpha i}\, \bar i_L \gamma_\mu \alpha_L .$$

$$\mathcal{H}_{cc} = \frac{g}{\sqrt2} W^\mu \sum_{\alpha,i} V_{\alpha i}\, \bar\alpha_L \gamma_\mu i_L + \frac{g}{\sqrt2} W^{\mu\dagger} \sum_{\alpha,i} V^*_{\alpha i}\, \bar i_L \gamma_\mu \alpha_L$$

$$\to \frac{g}{\sqrt2} W^\mu \sum_{\alpha,i} V^*_{\alpha i}\, \bar\alpha_L \gamma_\mu i_L + \frac{g}{\sqrt2} W^{\mu\dagger} \sum_{\alpha,i} V_{\alpha i}\, \bar i_L \gamma_\mu \alpha_L$$

$\mathcal{H}_{cc}$ is CP-invariant if and only if V is real, or can be made real by changing the phase convention.

$\mathcal{H}_{NC}$ is necessarily CP-invariant.

In the SM, $\not{CP}$ arises from complex phases in V.

How these phases can produce physical $\not{CP}$ effects will be explained shortly.

The CKM matrix

The SM requires that the CKM matrix V be unitary.

Apart from this unitarity, V is unpredicted, so its elements must be determined experimentally.

How many parameters are needed to fully specify V?

Rephasing invariance

The physics does not change when, say,

$$d \to e^{i\theta} d .$$

Under this rephasing, for $\alpha = u, c, t$,

$$V_{\alpha d} \bar{\alpha}_L \gamma_\mu d_L \to V_{\alpha d} \bar{\alpha}_L \gamma_\mu (e^{i\theta} d_L)$$

$$= (e^{i\theta} V_{\alpha d}) \bar{\alpha}_L \gamma_\mu d_L .$$

$\therefore$ The phase of any one $V_{\alpha i}$ depends on phase conventions.

One may go from one phase convention to another by multiplying any column, or any row, of V by a phase factor, or by carrying out several such operations.

However, the only phases in V one can remove this way are those that correspond to the relative phases of the quark fields. Multiplying all up quarks α and down quarks i by the same phase factor $e^{i\theta}$ doesn't change any of the $V_{\alpha i}$:

$$V_{\alpha i}\,\overline{\alpha_L}\,\gamma_\mu\, i_L \rightarrow V_{\alpha i}\,(\overline{e^{i\theta}\alpha_L})\,\gamma_\mu\,(e^{i\theta} i_L)$$

$$= V_{\alpha i}\,\overline{\alpha_L}\,\gamma_\mu\, i_L\,.$$

Only changes in the relative phases of the quark fields correspond to changes of the phases of the $V_{\alpha i}$.

Suppose there are N quark generations

$$\underbrace{\begin{pmatrix}u\\d\end{pmatrix}, \begin{pmatrix}c\\s\end{pmatrix}, \begin{pmatrix}t\\b\end{pmatrix}, \;\cdots,\; \begin{pmatrix}\cdots\\\cdots\end{pmatrix}}_{N}.$$

Then V is $N \times N$.

This unitary V contains —

$$2N^2 - N - 2\,\frac{N(N-1)}{2} = N^2$$

real parameters.

From these, we can remove $2N-1$ unphysical phases by multiplying columns and rows by phase factors.

$$\cancel{\left[\begin{matrix} & e^{i\varphi} \\ & e^{i\varphi} \\ & \vdots \\ & e^{i\varphi}\end{matrix}\right]}$$

~~Phases in last column can be removed indirectly~~

Number of physical parameters =

$$= N^2 - (2N-1) = (N-1)^2$$

$$= \begin{cases} 0 & N=1 \\ 1 & N=2 \\ 4 & N=3 \end{cases}$$

If we choose the parameters as mixing angles and phases, how many of them are phases?

If V were real, it would contain

$$N^2 - N - \frac{N(N-1)}{2} = \frac{1}{2}N(N-1) \text{ mixing angles.}$$

The number of phases in a complex V is then —

$$(N-1)^2 - \frac{1}{2}N(N-1) = \frac{1}{2}(N-1)(N-2) = \begin{cases} 0 & N=1 \\ 0 & N=2 \\ 1 & N=3 \end{cases}$$

(Kobayashi, Maskawa)

The 2X2 case as problem 6, in class. [6]

If N is only 2, the weak interaction as described in the SM cannot violate CP.

Processes that "know" about only 2 generations won't violate CP.

(Wolfenstein)

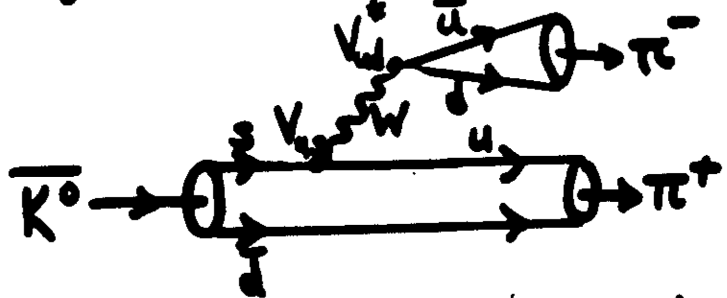
The quarks:

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix} \longrightarrow$$

Mass

A K

~~Typical weak~~ decay:



K decay: $s \rightarrow u + \bar{u} + d$

generation: 2 1 1 1

$\cancel{CP}$ will be small, as observed, in K decay.

To see large ($\sim 100\%$) $\cancel{CP}$ effects, we must observe processes that involve all three generations.

$\therefore$ Let us study the decays of the Beauty mesons B.

$$B^+ = [\bar{b}u],\ B_d = [\bar{b}d],\ B_s = [\bar{b}s]$$

b is in generation 3.

In addition, $\cancel{CP}$ in B decays can yield $\boxed{\text{clean}}$ information on the phases in V.

Thus, we can cleanly test whether these phases are the origin of $\cancel{CP}$.

$\cancel{CP}$ in the B system

What $\cancel{CP}$ effects do we look for?

$$\langle f|T|i\rangle \underset{CP}{=} \langle CP[f]|T|CP[i]\rangle$$

$$\text{Asym} \equiv \frac{\Gamma(B^+\to f) - \Gamma(B^-\to \bar{f})}{\text{"} + \text{"}} \neq 0 \Rightarrow \cancel{CP}$$

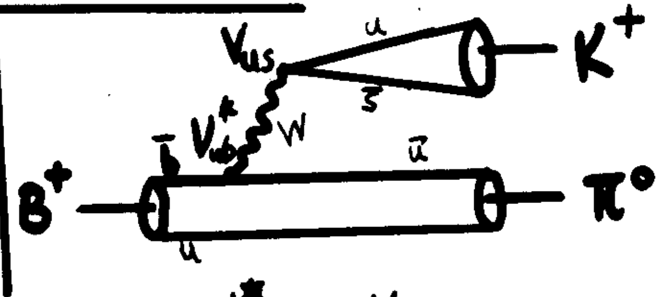
We look for partial decay rate asymmetries of this kind.

Note: Assuming CPT-invariance, $\Gamma_{tot}(B^+) = \Gamma_{tot}(B^-)$, so there must be compensating asymmetries.

Complex phases in V lead to physical $\cancel{CP}$ effects, including rate asymmetries, through interference between amplitudes.

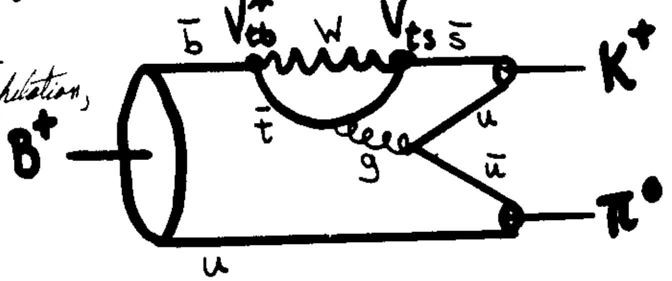
Example: $B^\pm \to \pi^0 K^\pm$

$\Gamma(B^+ \to \pi^0 K^+) = \Big|$



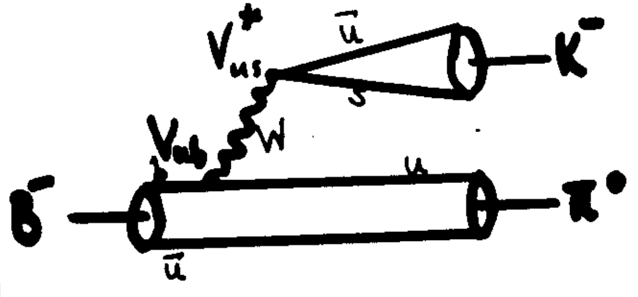
Among others, diagrams (color-suppressed, $\bar{b}u$ annihilation, perhaps others; $\bar{c}$, $\bar{u}$ [illegible]).

$+$



$\Big|^2$

$\Gamma(B^- \to \pi^0 K^-) = \Big|$



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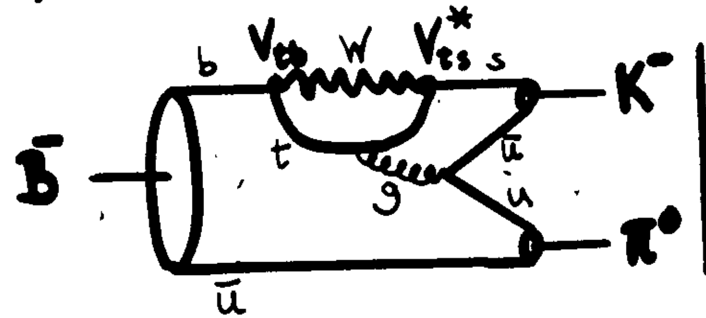


$\Big|^2$

Generalizing, suppose $B^+ \to f$ receives contributions from two diagrams:

$$\Gamma(B^+ \to f) = \left| M e^{i\delta^f_{CKM}} e^{i\alpha_s} + M' e^{i\delta'^f_{CKM}} e^{i\alpha'_s} \right|^2$$

$$\Gamma(B^- \to \bar{f}) = \left| M e^{-i\delta^f_{CKM}} e^{i\alpha_s} + M' e^{-i\delta'^f_{CKM}} e^{i\alpha'_s} \right|^2$$

In going from a process to its CP conjugate the CKM phases reverse, but nothing else changes.

The interfering amplitudes have different relative phase in $B^+ \to f$ and $B^- \to \bar{f}$.

If both CKM and strong phases are present,

$$\Gamma(B^+ \to f) \neq \Gamma(B^- \to \bar{f}). \qquad \cancel{CP}$$

$$\Gamma(B^+ \to f) = M^2 + M'^2 + 2MM'\cos(\varphi + \varphi_S),$$

$$\Gamma(B^- \to \bar{f}) = M^2 + M'^2 + 2MM'\cos(-\varphi + \varphi_S),$$

where

$$\varphi \equiv \delta^f_{CKM} - \delta'^f_{CKM}, \quad \varphi_S \equiv \alpha_S - \alpha'_S.$$

To test the SM of $\not{CP}$, one would like to determine φ.

$\Gamma(B^+ \to f)$ and $\Gamma(B^- \to \bar{f})$ do not determine φ.

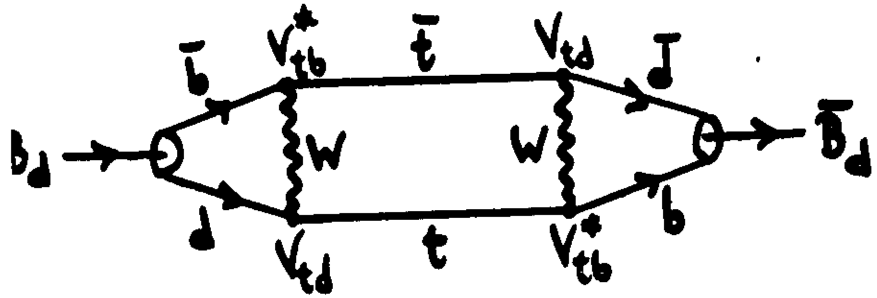
<u>In general</u>, in $B^\pm$ decays a clean test of the SM is not possible.

<u>Decays of Neutral B Mesons</u>

With few exceptions, it is ~~from~~ <u>neutral</u> B decays that can yield clean KM phase information. ~~can be obtained~~.

We treat $B_d(\bar{b}d)$. $B_s(\bar{b}s)$ is similar.

The $B_d(\bar{b}d)$ and $\bar{B}_d(b\bar{d})$ mix via



$$\arg[A(B_d \to \bar{B}_d)] = \arg[(V_{td}V^*_{tb})^2] \equiv -2\,\delta^m_{CKM}.$$

As for the neutral kaons, so for the $B_d, \overline{B_d}$ system there is a mass matrix

$$\mathcal{M} = \begin{bmatrix} X & \mathcal{M}_{12} \\ \mathcal{M}_{21} & X \end{bmatrix} \quad \left(\begin{matrix} \mathcal{M}_{11} = \mathcal{M}_{22} \equiv X \\ \text{by CPT} \end{matrix} \right)$$

in which $\mathcal{M}_{21}$ = The $\bar{t}t$ box diagram.

The neutral B mass eigenvalues are

$$\lambda_\pm = X \pm \sqrt{\mathcal{M}_{12}\mathcal{M}_{21}} \equiv m_\pm - i\frac{\Gamma_\pm}{2}.$$

Since $\mathcal{M}_{12} = \mathcal{M}_{21}(V \to V^*) = \mathcal{M}_{21}^*$,

$$\Gamma_+ = \Gamma_- \equiv \Gamma.$$

This is true even if the $\bar{t}t$ box isn't.

(Neither B mass eigenstate has a special decay mode with a big BR, and the two eigenstates have many modes in common.)

The mass eigenstates are

$$|B_\pm\rangle = \frac{|B^0\rangle \pm \rho |\overline{B^0}\rangle}{\sqrt{1+|\rho|^2}},$$

where

$$\rho \equiv \sqrt{\frac{\mathcal{M}_{21}}{\mathcal{M}_{12}}} = \sqrt{\frac{(V_{td}V_{tb}^*)^2}{(V_{td}^*V_{tb})^2}} = e^{-2i\delta^m_{CKM}}.$$

From—

amp. for B_d

$$i\frac{\partial}{\partial t}\begin{bmatrix} a(t) \\ \bar{a}(t) \end{bmatrix} = \mathcal{M}\begin{bmatrix} a(t) \\ \bar{a}(t) \end{bmatrix},$$

amp. for $\overline{B}_d$

$$|B_d\rangle \longrightarrow |B_d(t)\rangle = e^{-i(m - i\frac{\Gamma}{2})t} \times$$

$$\times \left\{ c|B_d\rangle - i e^{-2i\delta^m_{CKM}} s |\overline{B}_d\rangle \right\}$$

Here, $m \equiv \frac{m_+ + m_-}{2}$, and with $\Delta m = m_+ - m_-$,

$$c \equiv \cos\left(\frac{\Delta m}{2}t\right), \quad s \equiv \sin\left(\frac{\Delta m}{2}t\right).$$

0.76 ± 0.06

$$\left.\frac{\Delta m}{\Gamma}\right)_{B_d} \equiv X_d = \begin{cases} \cancel{0.68 \pm 0.10} \\ \cancel{0.79 \pm 0.10} \end{cases}$$

ARGUS & CLEO & LEP (t-depend)

(check Glasgow)

The $B_d - \overline{B}_d$ oscillation is as fast as the decay.

If a $|B\rangle$ was $|B_d\rangle$ at some time $t=0$, it oscillates between $|B_d\rangle$ and $|\overline{B}_d\rangle$ subsequently, until decay.

Suppose f is a final state accessible to both B_d and $\overline{B}_d$.

Examples: $\rho^+\pi^-$, $D^0 K_s$, $\pi^+\pi^-$, ΨK_s.

Then, for $B_d(t) \to f$,

$$\Gamma_f(t) = |\langle f|T|B_d(t)\rangle|^2 = e^{-\Gamma t} \times$$

$$\times \left| c \underbrace{\langle f|T|B_d\rangle}_{M e^{i\delta^f_{CKM}} e^{i\alpha_s}} - i e^{-2i\delta^m_{CKM}} s \underbrace{\langle f|T|\overline{B}_d\rangle}_{\overline{M} e^{-i\overline{\delta}^f_{CKM}} e^{i\overline{\alpha}_s}} \right|^2$$

Assume one diagram dominates

This is the only real assumption

$$(t) = e^{-\Gamma t}\{c^2 M^2 + s^2 \overline{M}^2 - 2cs M\overline{M} \sin(\varphi + \varphi_s)\}.$$

For the CP-mirror image decay, $\overline{B}_d(t) \to \overline{f}$,

$$(t) = e^{-\Gamma t}\{c^2 M^2 + s^2 \overline{M}^2 - 2cs M\overline{M} \sin(-\varphi + \varphi_s)\}.$$

Here

$$\varphi = 2\delta^m_{CKM} + \delta^f_{CKM} + \overline{\delta}^f_{CKM}, \quad \varphi_s = \alpha_s - \overline{\alpha}_s$$

↑ Weak CKM phase of interest

↑ Unknown strong phase

φ occurs in an interference term.

CP invariance $\Rightarrow \Gamma_f(t) = \bar{\Gamma}_{\bar{f}}(t)$.

$\therefore\ \varphi \neq 0 \Rightarrow \cancel{CP}$

With Γ and Δm known, $\Gamma_f(t)$ and $\bar{\Gamma}_{\bar{f}}(t)$ determine $M, \bar{M}$, $s_+ \equiv \sin(\varphi + \varphi_S)$, and $s_- \equiv \sin(-\varphi + \varphi_S)$.

Then

$$\sin^2\varphi = \frac{1}{2}\left[1 - s_+ s_- \pm \sqrt{(1-s_+^2)(1-s_-^2)}\right]$$

Then $\sin^2\varphi$ can be compared to the prediction from the CKM matrix to test the SM of $\cancel{CP}$.

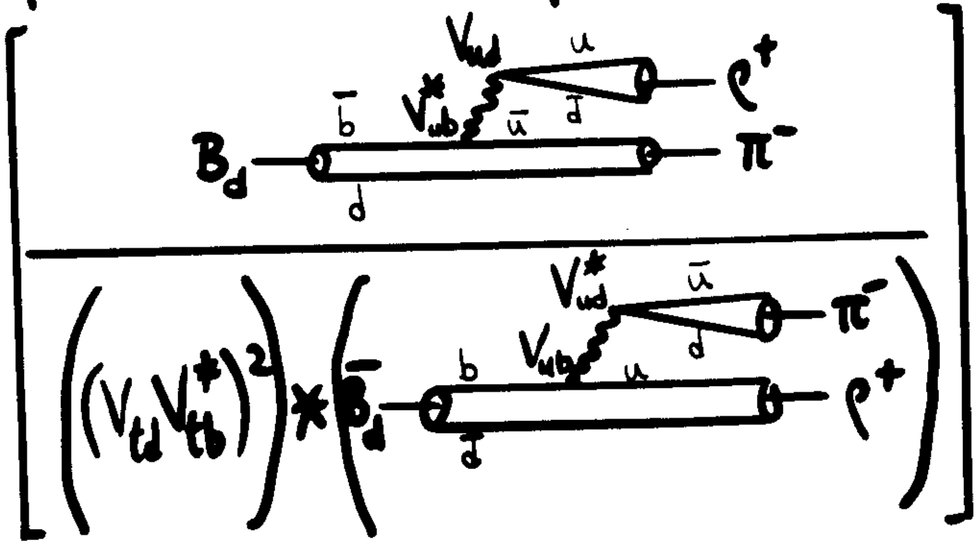
(Aleksan, Dunietz, B.K., Le Diberder)

Note the determination of φ is theoretically clean.

The φ that is probed in a given decay $B_d(t) \to f$ is

$$\varphi = \text{CKM Phase}\left[\frac{A(B_d \to f)}{A(B_d \to \bar{B}_d) * A(\bar{B}_d \to f)}\right].$$

Example: In $B_d(t) \to \rho^+\pi^-$, the φ we probe is the CKM phase of



That is, in this decay

$$\varphi = 2\arg[V_{ud}V_{ub}^*V_{tb}V_{td}^*]$$

The CKM phase φ probed in any B decay is

arg[Some Product of CKM Elements].

Special case of the formalism:

f is a CP eigenstate

$CP|f\rangle = \eta_f |f\rangle$ $\quad (\pi^+\pi^-, \psi K_S, \dots)$

$\equiv |\bar f\rangle$

If

$$\langle f|T|B_d\rangle = M e^{i\delta^f_{CKM}} e^{i\alpha_s},$$

then

$$\langle f|T|\bar B_d\rangle = \eta_f \langle \bar f|T|\bar B_d\rangle = \eta_f M e^{-i\delta^f_{CKM}} e^{i\alpha_s} \equiv \bar M e^{-i\bar\delta^f_{CKM}} e^{i\bar\alpha_s}.$$

$$\bar M = M,\quad \bar\delta^f_{CKM} = \delta^f_{CKM},\quad \varphi_s \equiv \alpha_s - \bar\alpha_s = \begin{cases} 0, & \eta_f = + \\ \pi, & \eta_f = - \end{cases}$$

$$\Gamma_f(t) = M^2 e^{-\Gamma t}\{1 \underset{(+)}{-} \sin(\Delta m t)\,\eta_f \sin\varphi\},$$

with $\varphi = 2(\delta^m_{CKM} + \delta^f_{CKM})$.

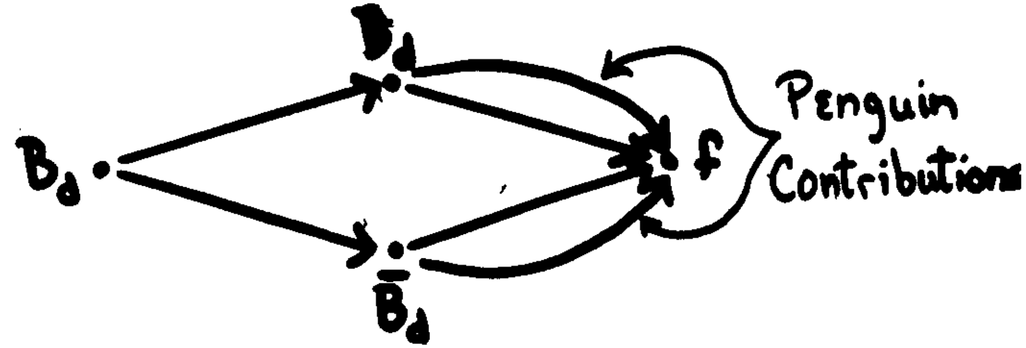
(Bigi, Carter, Sanda, Dunietz, Rosner)

The asymmetry

$$\frac{\Gamma_f(t) - \bar\Gamma_f(t)}{\text{''} + \text{''}} = -\sin(\Delta m t)\,\eta_f \sin\varphi$$

clearly determines $\sin\varphi$.

Key assumption: $A(B_d \to f)$ and $A(\bar B_d \to f)$ are each dominated by one diagram.



How will the experiments be done?

$\not{CP}$ in B decays will be studied at both hadron accelerators and e^+e^- colliders. These studies will complement each other.

How many B's are needed?

Example: $\overset{(-)}{B}_d(t) \to \psi K_s$.

Suppose —

$$\text{Asym} = \frac{\Gamma[B_d(t)\to\psi K_s] - \Gamma[\bar{B}_d(t)\to\psi K_s]}{\text{''} \quad + \quad \text{''}} \sim 10\%$$

To see this Asym above $\sqrt{N}$ statistical fluctuations, you need $\sim 10^3$ detected $B_d(t)\to\psi K_s$ decays.

The ψ is detected via $\psi\to\mu^+\mu^-$ or e^+e^-.

$$BR(\psi \to \underbrace{\mu^+\mu^- \text{ or } e^+e^-}_{\text{together}}) \cong 12\%.$$

$\therefore 10^4$ $B_d(t)\to\psi K_s$ decays are needed.

$$BR(B_d \to \psi K_s) \cong \overset{4}{\cancel{3}} \times 10^{-4}.$$

$\therefore \sim 10^8$ B_d's are needed.

For other, typical, decay modes of interest, the number of B's needed is similar.

How easily can one get $10^8 - 10^9$ B's?

The e^+e^- colliders as an example:

$$\underset{\sigma = 1\,\text{nb}}{e^+e^- \longrightarrow} \Upsilon(4s) \longrightarrow \underset{\text{BR: } 50\%}{B_d\bar{B}_d} \text{ or } \underset{50\%}{B^+B^-}$$

To get 10^8 $B_d\bar{B}_d$ pairs in two years, we need a luminosity of —

$$\mathcal{L} = 10^8/[(0.5\,\text{nb})(\underbrace{6\times 10^7\,\text{sec}}_{\text{full time}})] = \underbrace{3\times 10^{33}/\text{cm}^2\text{-sec}}_{\mathcal{L}\text{ design for SLAC B}}.$$

The present peak luminosity at CESR is

$$\mathcal{L}_{present} = 3 \times 10^{32} / \text{cm}^2 \cdot \text{sec}$$

$\therefore$ Producing enough B's will be a challenge.

We need B factories!

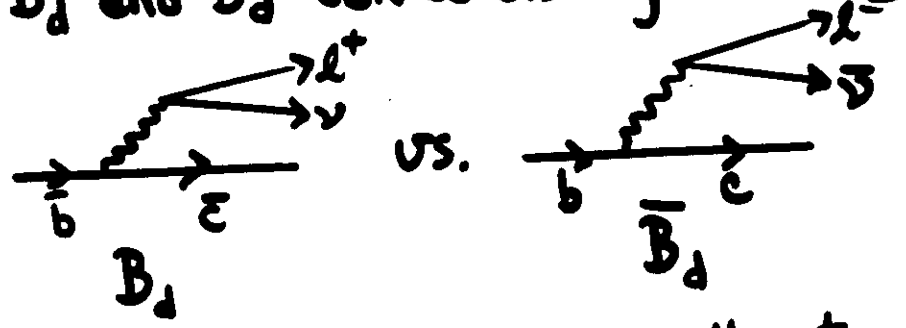
Being built at SLAC and KEK.

Tagging

To compare $\Gamma[B_d(t) \to \psi K_S]$ with $\Gamma[\overline{B}_d(t) \to \psi K_S]$, we must be able to tag the parent of each decay as a $B_d(t)$ or a $\overline{B}_d(t)$.

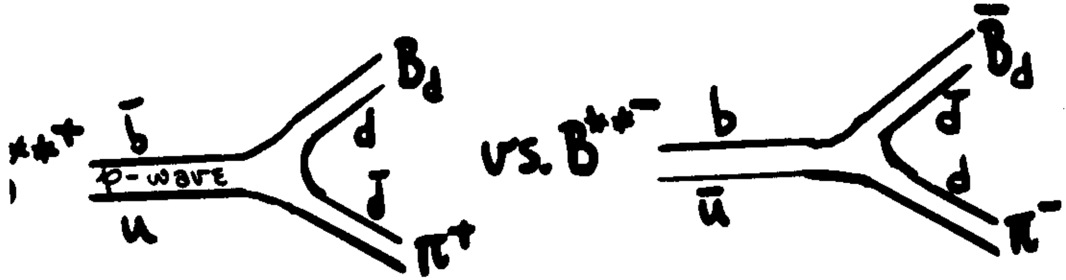
One way: $e^+e^- \to \Upsilon(4s) \to \underset{\hookrightarrow \psi K_S}{B_d} + \underset{\hookrightarrow \ell^- + \ldots}{\overline{B}_d}$

B_d and $\overline{B}_d$ can be distinguished by



The remaining B in the pair is then tagged.

Another way: $\bar{p}p \to B^{**(\pm)} + \ldots$, $B^{**(\pm)} \hookrightarrow \pi^{(\pm)} + \overset{(-)}{B}_d$



"Self-tagging."

(Grónau, Nippe, Rosner)

One or more B^{**} resonances have been seen at $M_{B^{**}} \sim 5.7$ GeV $\{M_B \sim 5.3$ GeV$\}$ at LEP in, for example,

$$e^-e^+ \to Z^0 \to b\bar{b}$$
$$\bar{b} \to B^{**0} \longrightarrow B^{(*)+}\pi^-$$

The LEP [illegible] suggest (but don't prove) that [illegible] of B_d mesons in hadronic [illegible] are produced via a B^{**+} decay. This would [illegible].

Quantum Mechanics of ~~CP~~ Experiments at B Factories

$\boxed{\text{Tagging via the EPR effect}}$ (B.K., Stodolsky)

Produce the B mesons via

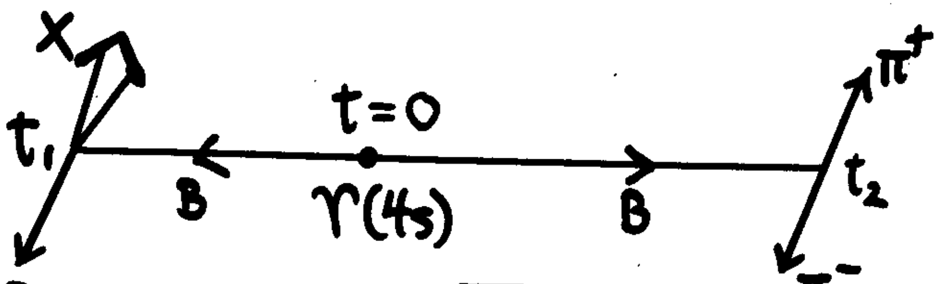
$$e^+e^- \xrightarrow{\text{Big }\sigma} \underbrace{\Upsilon(4s)}_{J=1} \xrightarrow{BR\sim 100\%} \underbrace{BB}_{p\text{ wave}}$$

Neutral B pair: Each B oscillates between B_d and $\overline{B_d}$. However, at any given time, if one B is found to be a $\overline{B_d}$, the other must be a B_d.

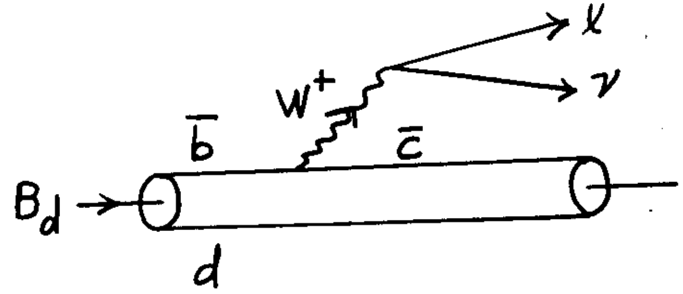
(Einstein, Podolsky, Rosen)

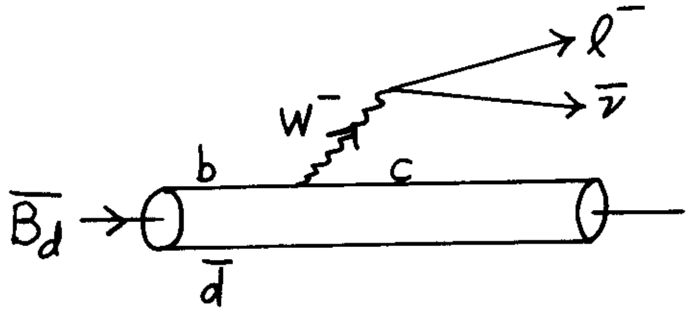
Traditional description: Rudaz

2) Consider, then,



ℓ^- (tags parent as $\overline{B_d}$)





Suppose that $t_1 < t_2$.

The decay $B \to \ell^- X$ on the left tells us that, <u>at time t_1</u>, the B on the right is a B_d.

This remaining B will now mix.

Then, assuming the B's are nonrelativistic and neglecting their motion,

$$\Gamma[\text{One } B \to \ell^- X \text{ at } t_1 ; \text{ Other } B \to \pi^+\pi^- \text{ at } t_2]$$

$$\propto \underbrace{e^{-\Gamma t_1} e^{-\Gamma t_1}}_{\substack{\text{Prob. that} \\ \text{both } B\text{'s} \\ \text{lived to } t_1}} \underbrace{e^{-\Gamma(t_2-t_1)}\{1-(\sin\varphi)\sin[\Delta m(t_2-t_1)]\}}_{\text{Evolution after time } t_1}$$

$$= e^{-\Gamma(t_2+t_1)}\{1-(\sin\varphi)\sin[\Delta m(t_2-t_1)]\}$$

Suppose that $t_2 < t_1$.

The decay $B \to \pi^+\pi^-$ on the right tells us that, at time t_2, the B on the left is the one that cannot decay to $\pi^+\pi^-$.

(Lipkin)

One obtains the same expression for the joint probability as when $t_1 < t_2$.

(Blankenbecler)

When we do not make the non-relativistic approximation, we must ask —

in which Lorentz frame does the decay of one B fix the state of the other at the same time?

And other questions.

To deal with the non-localization of [illegible] QM [illegible] [illegible] relevant, and to avoid [illegible] the same [illegible] then [illegible] in which we [illegible] a [illegible] B [illegible], but [illegible] the amplitude for [illegible] sequence of events.

(B. K., Stodolsky)

The final expressions are unchanged except that $t(\Upsilon[4S]$ rest frame$) \to \tau(B$ frames$)$

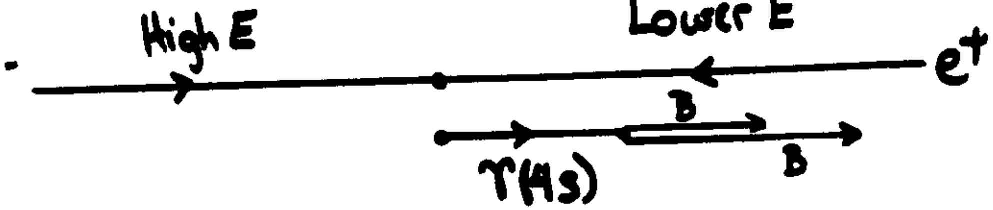
If decay times cannot be measured, the ~~CP~~ sine wave contributes

$$c \int_0^\infty dt_1 \int_0^\infty dt_2\, e^{-\Gamma(t_2+t_1)} \sin[\Delta m(t_2-t_1)] = 0.$$

$\therefore$ Timing information is crucial.

If $\Upsilon(4S)$ is at rest in lab., slow B does not go far enough for its pathlength to be measured.

$\therefore$ Build an asymmetric collider.



[$39\frac{1}{2}$ pp to here]

What independent CKM phases α exist?

What is there to measure?

Recall that

$\alpha = \arg$ [Some Product of CKM Elements].

So, the question is —

What are the independent phases of CKM products?

(Aleksan, Kayser, London)

What Are the Independent Phases of CKM Products?

(Aleksan, B.K., London)

The SM requires that V be unitary.

[Δ's]

If

$$\varphi = \text{Arg}[\text{Some Convention-Independent Product of CKM Elements}],$$

then

$$\varphi = n_\alpha \alpha + n_\beta \beta + n_\epsilon \epsilon + n_{\epsilon'} \epsilon' \;;$$

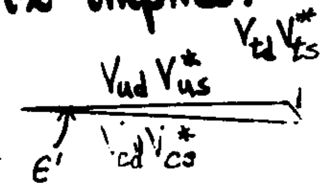
$$n_{\alpha,\beta,\epsilon,\epsilon'} = \text{integers}.$$

The angles $\alpha, \beta, \epsilon, \epsilon'$ are the independent phases of products of CKM elements.

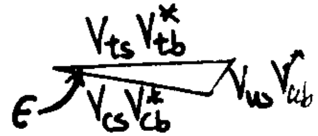
Studies of CP in B decay are probes of these four angles, or an equivalent set of 4.

Unitarity of the CKM matrix implies:

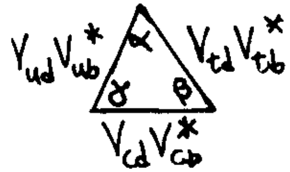
ds $\quad \underset{\lambda}{V_{ud}V_{us}^*} + \underset{\lambda}{V_{cd}V_{cs}^*} + \underset{\lambda^5}{V_{td}V_{ts}^*} = 0$



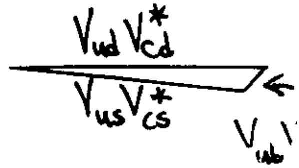
sb $\quad \underset{\lambda^4}{V_{us}V_{ub}^*} + \underset{\lambda^2}{V_{cs}V_{cb}^*} + \underset{\lambda^2}{V_{ts}V_{tb}^*} = 0$



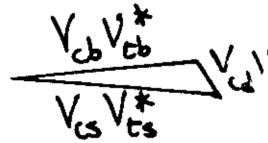
db $\quad \underset{\lambda^3}{V_{ud}V_{ub}^*} + \underset{\lambda^3}{V_{cd}V_{cb}^*} + \underset{\lambda^3}{V_{td}V_{tb}^*} = 0$



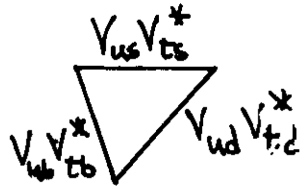
uc $\quad \underset{\lambda}{V_{ud}V_{cd}^*} + \underset{\lambda}{V_{us}V_{cs}^*} + \underset{\lambda^5}{V_{ub}V_{cb}^*} = 0$



ct $\quad \underset{\lambda^4}{V_{cd}V_{td}^*} + \underset{\lambda^2}{V_{cs}V_{ts}^*} + \underset{\lambda^2}{V_{cb}V_{tb}^*} = 0$



ut $\quad \underset{\lambda^3}{V_{ud}V_{td}^*} + \underset{\lambda^3}{V_{us}V_{ts}^*} + \underset{\lambda^3}{V_{ub}V_{tb}^*} = 0$



(Bjorken; Jarlskog

The θ(milliradian) angle ϵ' may be too small to measure.

Experiments aimed at α, β, and ϵ are being contemplated.

Wolfenstein has introduced a very good ($\sim 3\%$) approximation to V_{CKM} in which $\epsilon = \epsilon' = 0$.* The only remaining independent angles are then α and β in the db Δ.

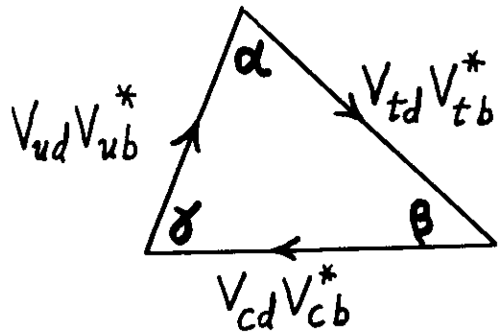
⇒ If there is time, show that with

$$V_{CKM} \sim \begin{bmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{bmatrix},$$

there is a phase convention in which all elements are $\simeq$ real except V_{ub}, V_{td}. Otherwise, assign it as a problem.

.5

We focus, then, on the db Δ.



The Goal

Measure the 4 independent angles of the unitarity triangles, or at least 3 of them. Focus first on α, β.

Beyond that, overconstrain the system as much as possible:

See if different probes of the same angle yield the same result.

See if the angles in the db triangle add up to π.

See if the angles implied by the lengths of the sides agree with those inferred from ~~CP~~ effects.

See if you get a consistent picture.

Probes of the SM Angles

Decay Mode	φ	Where?
$B_d(t) \to \pi^+\pi^-, \pi^+\rho^-, \pi^+a_1^-$	2α	e^+e^-, Had(?)
$B_d(t) \to \psi K_S, \psi K^{*0}$, $K^{*0} \to K_S\pi^0$	2β	e^+e^-, Had
$B_d(t) \to D^{(*)+} D^{(*)-}$	2β	?
$B_s(t) \to D_s^+ K^-$	γ	Had
$B^+ \to D^0_{CP} K^+$, $D^0_{CP} \to \pi^+\pi^-, K^+K^-, \ldots$	γ	Had, e^+e^-(?)
$B_s(t) \to \psi\varphi$	2ϵ	Had

*Theorist's impression

Experiments at e^+e^- B factories and hadron facilities will be complementary.

Testing/overconstraining the SM of ~~CP~~ outside the B system

Search for direct ~~CP~~ in the K system

The ~~CP~~ effects definitely observed so far are

$$\left|\frac{A[K_L\to\pi^+\pi^-]}{A[K_S\to\pi^+\pi^-]}\right| \equiv |\eta_{+-}| = (2.3\times10^{-3}) \neq 0$$

$$\{\, |A[K_L\to\pi^0\pi^0]/A[K_S\to\pi^0\pi^0]| \equiv |\eta_{00}| \simeq |\eta_{+-}|.$$

and

$$\frac{\Gamma[K_L\to\pi^-\ell^+\nu] - \Gamma[K_L\to\pi^+\ell^-\bar\nu]}{\text{''} \quad + \quad \text{''}} \equiv \delta$$

$$= 3.3\times10^{-3} \neq 0.$$

Both of these effects can be explained by supposing that K_L and K_S are admixtures of [illegible] eigenstates K_+ and K_- (~~CP~~ in K^0-$\bar K^0$ mixing; indirect ~~CP~~). It is not necessary to suppose that, in addition, K_- decays to $\pi\pi$, or that the K^0 and $\bar{K}^0$ decay amplitudes for decay to leptons differ (~~CP~~ in decay amplitudes; direct ~~CP~~).

$\Rightarrow$ a ~~CP~~ wiggle in $\Gamma[K^0(t)\to\pi^+\pi^-]$ a la that in $\Gamma[B_d(t)\xrightarrow{CP}\text{hadrons}]$. Seen, for example, at CPLEAR in $\bar{p}p\,(\text{at rest})\longrightarrow K^-\pi^+K^0$ (or $K^+\pi^-\bar K^0$).

In superweak models (Wolfenstein) essentially the only ~~CP~~ in the K^0 system is ~~CP~~ in K^0-$\bar K^0$ mixing, induced by [illegible] at some high mass scale.

However, in the SM, one expects both direct and indirect ~~CP~~, both in the K^0 and the B system.

~~CP~~ in K^0-$\bar K^0$ mixing:

Allowing for ~~C~~ but not ~~CPT~~ in [illegible], we have

$$\mathcal{M} = \begin{bmatrix} X & \mathcal{M}_{12} \\ \mathcal{M}_{21} & X \end{bmatrix} \qquad (S.2.1)$$

This leads to (problem)

$$|K_S\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}\left[|K_+\rangle + \epsilon|K_-\rangle\right]$$

$$|K_L\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}\left[|K_-\rangle + \epsilon|K_+\rangle\right], \qquad (S.2.2)$$

where

$$|K_\pm\rangle \equiv \frac{|K^0\rangle \pm |\overline{K^0}\rangle}{\sqrt{2}}\,, \quad CP|K_\pm\rangle = \pm |K_\pm\rangle\,, \qquad (S.3.1)$$

and

$$\epsilon = \frac{\sqrt{M_{12}} - \sqrt{M_{21}}}{\sqrt{M_{12}} + \sqrt{M_{21}}}\,. \qquad (S.3.2)$$

If neutral $K \to \pi\pi$ only via K_+ piece,

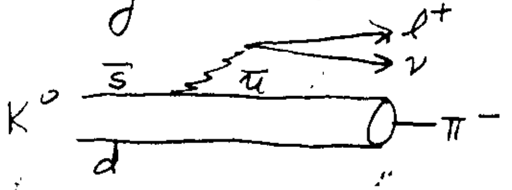
$$\eta_{+-} \equiv \frac{\langle \pi^+\pi^- |T| K_L\rangle}{\langle \pi^+\pi^- |T| K_S\rangle} = \frac{\epsilon \langle \pi^+\pi^- |T| K_+\rangle}{\langle \pi^+\pi^- |T| K_+\rangle} = \epsilon = \eta_{00} \qquad (S.3.3)$$

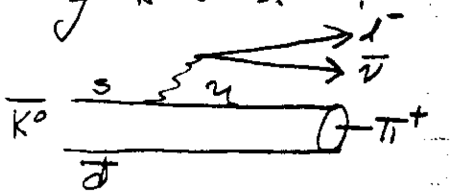
Then, one can search for direct CP by trying to show that $\eta_{+-} \neq \eta_{00}$.

Turning to the semileptonic asymmetry δ, we note from (S.2.2) that

$$|K_{S(L)}\rangle = \frac{1}{\sqrt{2(1+|\epsilon|^2)}}\left[(1+\epsilon)|K^0\rangle \ (\pm)\ (1-\epsilon)|\overline{K^0}\rangle\right] \qquad (S.3.4)$$

∴ Only $K^0 \to l^+$ and only $\overline{K^0} \to l^-$:





Now, if there is no direct CP, we have

$$\langle \pi^- l^+ \nu |T| K^0\rangle = \langle \pi^+ l^- \bar{\nu} |T| \overline{K^0}\rangle\,.$$

↑ we certainly expect this in the SM, since each side of the Eq. has only one diagram

Then

$$\langle \pi^- l^+ \nu |T| K_L\rangle \propto 1+\epsilon$$

$$\langle \pi^+ l^- \bar{\nu} |T| K_L\rangle \propto 1-\epsilon$$

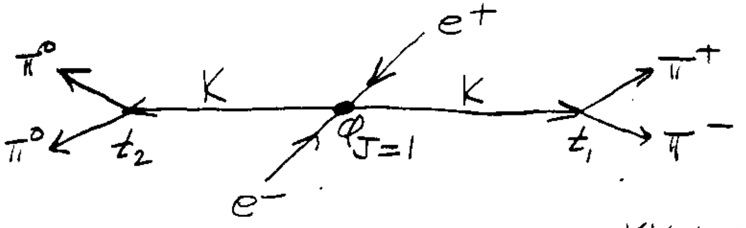
$$\delta = \frac{|1+\epsilon|^2 - |1-\epsilon|^2}{+} \cong 2\,\mathrm{Re}\,\epsilon \quad \text{for small } \epsilon.$$

For consistency, we must have

$$\delta = 3.3\times 10^{-3} \leq 2|\eta_{\overset{+-}{00}}| = 4.6\times 10^{-3} \ ✓$$

The several ~~attempts to demonstrate that~~ $\eta_{+-} \neq \eta_{00}$ (to show that "$\mathrm{Re}\,\frac{\epsilon'}{\epsilon}$" $= \frac{1}{6}\left[1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|^2\right] \neq 0$) include the plans at the ϕ factory DAPHNE:

(Lipkin; Dunietz, Hauser, Rosner)



KK : p wave

The rate will be studied as a function of the difference between the two decay times. To see that this probes direct CP, consider equal decay times:

As for $\phi \to K\bar{K}$, so here, the decay $K \to \pi^+\pi^-$ at the same time puts the other K in the state which $\to \pi^+\pi^-$. There is no CP-violating amplitude, that state is $|K_->$. But $|K_->$ doesn't go to $\pi^0\pi^0$ either (when there is no direct CP). Using the amplitude approach, one easily shows that Prob $[t_2 = t_1] \propto |\eta_{+-} - \eta_{00}|^2$. Optimal procedure will require measurements at $t_2 \neq t_1$, too.

The existing results on ϵ'/ϵ are

$$\mathrm{Re}(\epsilon'/\epsilon) = \begin{cases} (23 \pm 3.5 \pm 6) \times 10^{-4} & \text{NA31/CERN} \\ (7.4 \pm 5.2 \pm 2.9) \times 10^{-4} & \text{E731/Fermilab} \end{cases}$$

$$\Rightarrow (14 \pm 8) \times 10^{-14} \quad \text{Combined}$$

Comparison with SM predictions

Unlike the asymmetries in certain B decays, ϵ_K and ϵ'_K are not simply related to CKM elements with no theoretical uncertainties in the relations. For example, SM calculations generally predict that $(\epsilon'/\epsilon)_K$ is nonzero and positive, but its magnitude could be anywhere from 3×10^{-3} on down to zero or even -ve, although this last is unlikely.

(Buchalla, Buras, ...)

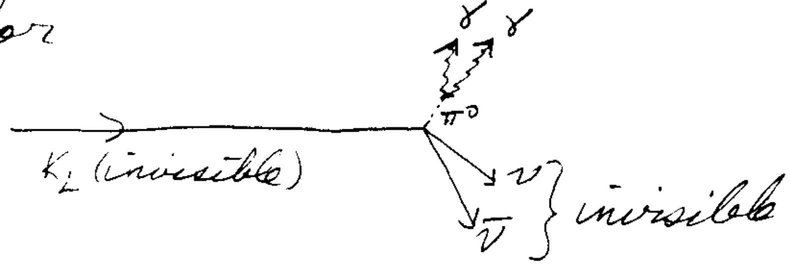
Measurement of $(\epsilon'/\epsilon)_K$, particularly to show that it is nonzero, is very important (direct CP, as we already expected from ...), but theoretical uncertainties prevent us (at present) from using the precise size of $(\epsilon'/\epsilon)_K$ as a quantitative test of the SM of CP.

$K_L \to \pi^0 \nu \bar{\nu}$

This is potentially a theoretically clean test of the SM of CP, complementing the tests in B decay. Experimentally, it is very hard: the SM prediction is that

$$BR[K_L \to \pi^0 \nu\bar{\nu}] \sim 2 \times 10^{-11}.$$

Look for

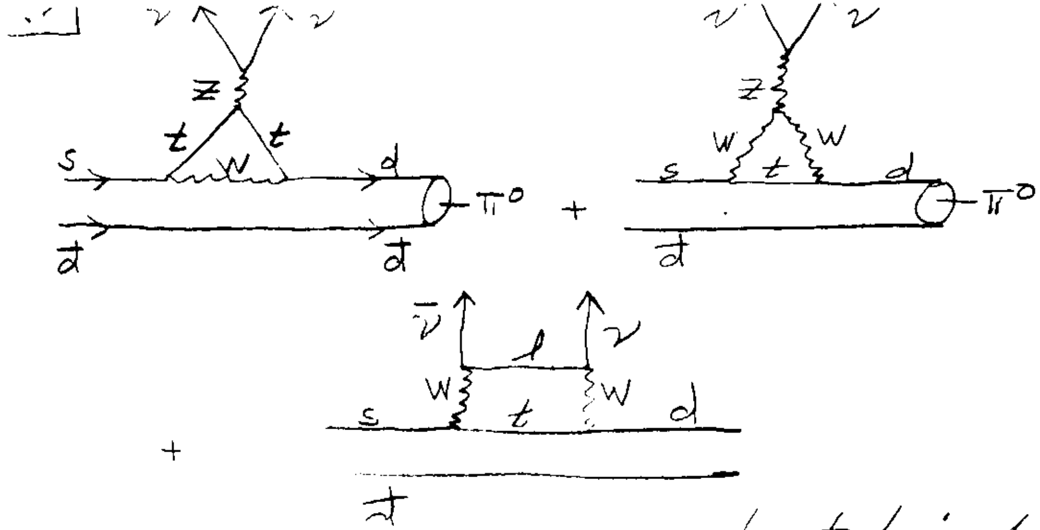


Must make sure you don't have $K_L \to \pi^0\pi^0$ [$BR \sim 10^{-3}$] and fail to see the 2nd π^0.

What has $K_L \to \pi^0 \nu\bar{\nu}$ got to do with CP?

The decay need not violate CP (unlike $K_L \to \pi\pi$), but in the SM it is strongly dominated by a CP-violating amplitude.

The SM contributing diagrams are (for the $s\bar{d}$ piece of K_L) →



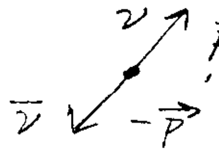
All are $\propto V_{ts} V_{td}^*$. (t dominates because of its heaviness.)

When the heavy degrees of freedom are integrated out one obtains an $\mathcal{L}_{eff}$ for $K_L \to \pi^0 \nu\bar{\nu}$ of the form

$$\mathcal{L}_{eff} = C\left[V_{ts} V_{td}^* (\bar{d}s)_{V-A} + V_{ts}^* V_{td} (\bar{s}d)_{V-A}\right][\bar{\nu}\nu]_{V-A}$$

Accurately calculated since the hard [illegible] (Buchalla + Buras) [illegible] short-distance physics. (This is for given m_t)

Now, in the $\nu\bar{\nu}$ c.m. frame



M. E.	$\bar\nu\nu$ State Produced
$\langle\bar\nu\nu\vert\bar\nu\gamma(1+\gamma_5)\nu\vert 0\rangle$	$J=1$, $CP=+$
$\langle\bar\nu\nu\vert\bar\nu\gamma_4(1+\gamma_5)\nu\vert 0\rangle$	$J=0$, $CP=-$ (Amp $\propto m_\nu/E_\nu$)

Then, in the K_L rest frame,

$$\pi^0 \longleftarrow \underset{K_L}{\bullet} \longrightarrow (\bar\nu\nu)$$

$$CP[\text{Final state}] = CP(\bar\nu\nu)\underbrace{CP(\pi^0)}_{-1}(-1)^L = +1$$

But, if CP were conserved in the K system, CP[K_L] would be -1.

$\therefore$ $K_L\to\pi^0\nu\bar\nu$ does not conserve CP in the SM.

The SM hadronic M.E. in this decay is, from our $\mathcal{H}_{eff}$,

$$\equiv \langle\pi^0\vert V_{ts}V_{td}^*(\bar d s)_{V-A} + V_{ts}^*V_{td}(\bar s d)_{V-A}\vert K_L\rangle$$

$$= \langle\pi^0\vert V_{ts}^*V_{td}(\bar s d)_{V-A}\vert K^0\rangle\langle K^0\vert K_L\rangle$$

$$+\langle\pi^0\vert V_{ts}V_{td}^*(\bar d s)_{V-A}\vert\bar K^0\rangle\langle\bar K^0\vert K_L\rangle$$

Using CP to relate

$\langle\pi^0\vert(\bar d s)_{V-A}\vert\bar K^0\rangle$ to $\langle\pi^0\vert(\bar s d)_{V-A}\vert K^0\rangle$,

we have in our phase conventions (cf. (5.3.4))

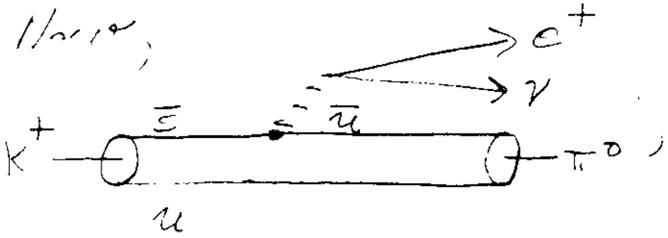
$$\lambda \propto \langle\pi^0\vert(\bar s d)_{V-A}\vert K^0\rangle\left[V_{ts}^*V_{td}(1+\epsilon) - V_{ts}V_{td}^*(1-\epsilon)\right]$$

$$\simeq \quad " \quad [2i\,\mathrm{Im}(V_{ts}^*V_{td})]$$

Neglecting $[\epsilon\,\mathrm{Re}(V_{ts}^*V_{td})]/[\mathrm{Im}(V_{ts}^*V_{td})]$

$K_L \sim K_- + \epsilon K_+$, so a is dominantly direct ~~CP~~: $K_-\to\pi^0\nu\bar\nu$.

Now,



, which is measured,

is $\propto \langle\pi^0\vert(\bar s u)_{V-A}\vert K^+\rangle$. By an isospin rotation, this gives $\langle\pi^0\vert(\bar s d)_{V-A}\vert K^0\rangle$.

Thus, given an accurate m_t and $BR(K^+\to\pi^0 e^+\nu)$, $BR(K_L\to\pi^0\nu\bar\nu)$ gives clean CKM information, which must be consistent with that from other sources. The accuracy of the predicted $BR(K_L\to\pi^0\nu\bar\nu)$ for infinitely precise input params. is 1%. (Buras)

Electric dipole moments of fermions

In the literature, it is proved that if a quantum system with definite spin $\vec{S}$ has an electric dipole moment, $\vec{\mu}_{El} = g_{El}\,\vec{S}$, then $\underline{T}$ is violated.

Proof: Interaction energy of the system in a static external $\vec{E}$ field is

$$\mathcal{E} = -\vec{\mu}_{El}\cdot\vec{E} = -g_{El}\,\vec{S}\cdot\vec{E}$$

under T, $\vec{E}\to\vec{E}$, but $\vec{S}\to-\vec{S}$.

Thus, $\mathcal{E}\to-\mathcal{E}$.

If nature is CPT-invariant, as it is if it is described by a local, relativistically-invariant quantum field theory, such as the SM, then any $\vec{\mu}_{El}$ on a system of definite $\vec{S}$ violates CP as well.

Let us try to prove more ~~directly that~~ for a spin-1/2 fermion f ($n, e, t, \ldots$) a μ_{El} violates CP.

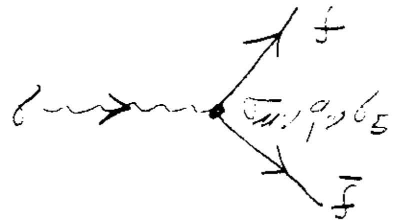
An electric dipole moment for $s=1/2$ is the static limit of the coupling

$$\langle f(p_2,s_2)|J_\mu^{EM}|f(p_1,s_1)\rangle = iE(q^2)\,\bar{u}_2\sigma_{\mu\nu}q_\nu\gamma_5 u_1$$

$$q\equiv p_1-p_2$$

$$E(q^2=0)=\mu_{El}\,.$$

Let us view this in the vacuum channel



One can show that

$$\langle f\bar{f}|J_\mu^{EM}|0\rangle \underbrace{\eta_\mu}_{\gamma\text{ polarization}} = iE(q^2)\,\bar{u}_f\sigma_{\mu\nu}q_\nu\gamma_5 v_{\bar{f}}\,\eta_\mu$$

creates $f\bar{f}$ in a state which in the non-relativistic limit is P_1. Now,

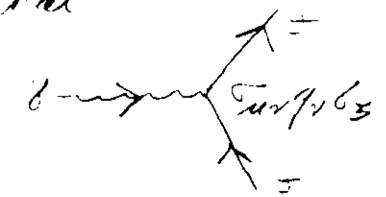
$CP[f\bar{f}; L, S] = (-1)^{S+1} = -1$ for $^1P_1$.

But $CP[\gamma] = +1$.

Thus, 

does not conserve CP.

Have we proved that, for $s = \frac{1}{2}$, an EDM violates CP even if CPT invariance does not hold? Actually, no! We assumed crossing - the assumption that



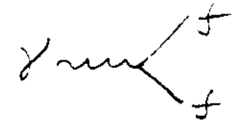
$\neq 0$

$\Rightarrow$ 

$\neq 0$,

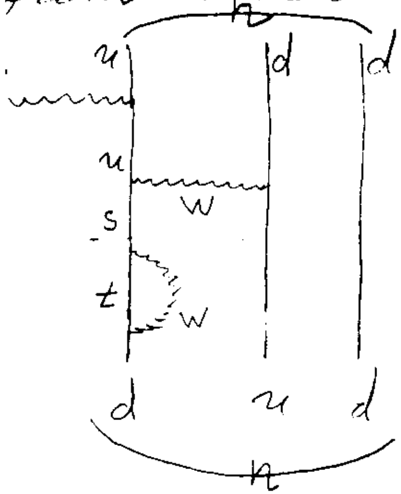
so that if the latter violates CP, so does the former. But the only way I know to prove such a crossing relation is to assume field theory, so that CPT holds.

The SM is a field theory, so we can look at



in the cross channel $\gamma \to f\bar{f}$.

There, an electric-dipole coupling requires $\not{CP}$, and the SM can generate such a coupling through CP violation coming from such diagrams as

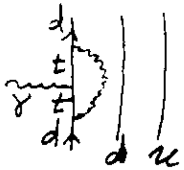


$\propto V_{td} V_{ts}^* V_{us} V_{ud}^*$ A $\not{CP}$ phase can enter

From contribs. like this,

$\mu_{El}(n) \sim e \times 10^{-30}$ cm

(Gavela et al)

The lowest-order processes like 

$\propto V_{td} V_{td}^*$ have no phase, so don't contribute to this flavor-conserving effect.

From various possible SM contributions, $\mu_{El}(n)|_{SM} \lesssim e \times 10^{-30}$ cm.

(Review: Barr & Marciano)

Natural size for $\mu_{El}(n) = e \times$ size of $n \sim e \times 10^{-13}$ cm.

Present bound: 1.1×10^{-25} e cm

A $\mu_{El}(n)$ at $\sim 10^{-26}$ e cm would be inconsistent with the SEWM of CP, but one could perhaps explain it in terms of a small θ_{QCD}.

(Wolfenstein)

$\mu_{El}(e)$ would not have this problem.

Presently, $\mu_{El}(e) \lesssim 10^{-26}$ e cm (PDG), and the SM $\Rightarrow$ $\ll 10^{-50}$ e cm owing to the small masses of the neutrinos.

Prove that for massless neutrinos, the SM $\Rightarrow$ $\mu_{El}(e) = 0$.