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SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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SUPERSYMMETRIC GUTS

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Minimal SUSY SU(5)

In minimal (non SUSY) SU(5) the matter fermions (quarks/leptons) are in the $10 + \bar{5}$ representations (per family)

$$\bar{5}^i = (d_c^1, d_c^2, d_c^3, e, \nu)$$

$$10_{[iK]} = \begin{bmatrix} 0 & u_c & u_c & u & d \\ & 0 & u_c & u & d \\ & & 0 & u & d \\ & & & 0 & e_c \\ & & & & 0 \end{bmatrix}$$

Electroweak Higgs doublet H becomes a member of $\bar{5}$-plet.

$$5_H = \begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ H^- \\ H^0 \end{pmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \text{colored triplet} \\ \left.\begin{matrix} \\ \\ \end{matrix}\right\} \text{Higgs doublet} \end{matrix}$$

The first stage of GUT symmetry breaking is induced by the VEV of 24-plet

$$\langle \Sigma_i^K \rangle = \begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix} \Sigma$$

In the SUSY case all the fields become superfields

Higgs and matter fields become a <u>chiral</u> superfields

$$[\text{Higgs}, \text{matter}](x_\mu) \Longrightarrow S(x_\mu, \theta_+) \qquad \theta_\pm = \frac{1 \pm i\gamma_5}{2}\theta$$

and gain superpartners (Higgsinos, squarks / sleptons)

Gauge fields become <u>Vector</u> supermultiple

$$G_\mu \to V(x_\mu, \theta).$$

Since our fields $\Sigma_i^{\ K}$, $\bar{5}_{Hi}$, $10_{[iK]}$, $\bar{5}^i$ now become chiral superfields, the $\Sigma_i^{\ K}$ scalar becomes a complex

$$\Sigma_j^{\ K} = A_j^{\ K} + i B_j^{\ K} \quad \left(A^* = A\ , \quad B^+ = -B^+ \right).$$

Also we have to double Higgs 5-plets. $5_H + \bar{5}_H$ because: (1) with just one $5_H (\bar{5}_H)$ we can not give masses to down (up) quarks, since 5_H^* is superfield of antichirality $5_H^*(x_\mu\ \theta_-)$;

(2). Anomaly cancelation.

Superpotential $W(s_i)$ must be a gauge invariant polinomial (up to a trilinear in S_i for renormalizability)

But in supergravity the renormalizability is not a necessary requirement.

Scalar potential — $\sum_a \left| \frac{\partial W}{\partial S_a} \right|^2 \Big|_{S_a \equiv \text{scalar}}$

Youkawa couplings — $\frac{\partial^2 W}{\partial S_a \partial S_b} \Big|_{S_a \equiv \text{scalar}} \bar{\tilde{S}}_a \tilde{S}_b$

Consider following superpotential:

$$W = \frac{M}{2} Tr\Sigma^2 + \frac{h}{3} Tr\Sigma^3 + \lambda \bar{5}_H \Sigma 5_H + M' \bar{5}_H 5_H + g^u_{\alpha\beta} 5_H 10^\alpha 10^\beta + g^d_{\alpha\beta} \bar{5}_H 10^\alpha \bar{5}^\beta \qquad (1)$$

$M, M' \sim M_{GUT}$.

This superpotential <u>is not</u> the most general $SU(5)$-invariant one; I have excluded terms like:

$$M''_a 5_H \bar{5}^a + C_{\alpha\beta\gamma} \bar{5}^\alpha 10^\beta \bar{5}^\gamma$$

which are $SU(5)$-invariants. The reason is that these terms can lead to a very rapid proton decay (for not very small $C_{\alpha\beta\gamma}$)

For example, $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ decomposition gives

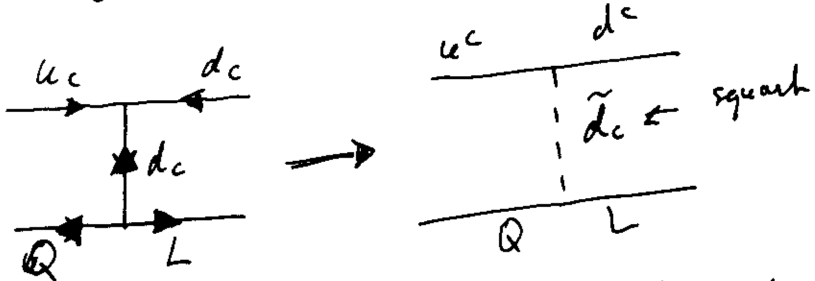
$$\bar{5}^i 10_{ik} \bar{5}^k = \left\{ \begin{array}{l} \text{Let us denote} \\ i,k \equiv a,b \ \text{ for } i,k = 1,2,3 \\ i,k \equiv A,B \ \text{ for } i,k = 4,5 \end{array} \right\} =$$

$$= \bar{5}^a 10_{ab} \bar{5}^b + \bar{5}^a 10_{aA} \bar{5}^A + \bar{5}^A 10_{AB} \bar{5}^B =$$

$$= d^c u^c d^c + d^c Q L + L L e^c$$

(generation index is suppressed)

This couplings lead to the proton decay via $d=6$ operator trough the exchange of scalar quarks.



this mechanism of the proton decay has to do nothing with grand unification, but with supersymmetry, since the couplings

$$d^c u^c d^c + d^c Q L + \cdots$$

can be presented already at the level of the minimal SUSY standard model. So we have to exclude them by certain symmetry e.g. matter parity (Z_2):

$$(\bar{5}^\alpha, 10^\alpha) \to -(\bar{5}^\alpha, 10^\alpha)$$

$$(\text{Higgs fields}) \to (\text{Higgs fields}).$$

Now, the superpotential (1) is a most general under $SU(5) \otimes Z_2$.

GUT symmetry breaking

Let us see if the system has a "right" minimum in which $SU(5)$ is broken down to $G_W = SU(3) \otimes SU(2) \otimes U(1)$ by large VEV ($\sim M_G$).

Let us put $10^\alpha = \bar 5^\alpha = 5_H = \bar 5_H = 0$

Then the vacuum condition are:

(D) $[\Sigma^*, \Sigma]_i^k = 0$

(F) $M\Sigma_i^k + h\left(\Sigma_i^{2k} - \delta_i^k \frac{t_2\Sigma^2}{5}\right) = 0$

The condition (D) is equivalent to $[A,B]=0$ and thus, Σ can be taken diagonal.

Let us take

$$\Sigma = \mathrm{diag}(\Sigma_1\ \Sigma_2 \cdots \Sigma_5)$$

Then (D) is satisfied automatically and we are left with

$$M\Sigma_i + h\left(\Sigma_i^2 - \frac{t_2\Sigma^2}{5}\right) = 0$$

So the only possible forms are:

$$\Sigma = \mathrm{diag}[\underbrace{\Sigma'}_{n}\ \underbrace{\Sigma''}_{5-n}]$$

$T_2\Sigma = 0 \Rightarrow \quad n\Sigma' + (5-n)\Sigma'' = 0$

and the possible solutions are

① $$\Sigma = \begin{pmatrix} 2 &&&& \\ & 2 &&& \\ && 2 && \\ &&& -3 & \\ &&&& -3 \end{pmatrix}\frac{M}{h}$$ G_W-symmetric

② $$\Sigma = \begin{pmatrix} 1 &&&& \\ & 1 &&& \\ && 1 && \\ &&& 1 & \\ &&&& -4 \end{pmatrix}\frac{M}{3h}$$ $SU(4)\otimes U(1)$-symmetric

③ $\Sigma = 0$ $\quad SU(5)$-symmetric.

Vacuum (1) is a "right" one, but in SUSY limit all three are <u>exactly</u> degenerated. In fact the degeneracy of the vacuum is larger, because we have derived ①-(3) under the condition $5_H = \bar 5_H = 10^\alpha = \bar 5^\alpha = 0$. If we relax this condition we will find many other vacua (some of which are <u>continuously</u> degenerated) with $5_H, \bar 5_H$ and even $\bar 5^\alpha, 10^\alpha$ having large VEVs.

Degeneracy will be lifted by the supersymmetry breaking.

Let us come back to the vacuum (1), and find the mass of the Higgs doublet $H(\bar{H})$ and colored triplets $T(\bar{T})$.

This masses are:

$$M_T = 2\frac{M}{h}\lambda + M'$$

$$M_H = -3\frac{M}{h}\lambda + M'$$

In general unless we impose some symmetry, $M' \sim M \sim M_{GUT}$ and $\lambda \sim h \sim 1$.

So, to obtain $M_H \sim 100\,\text{GeV}$ we have to <u>fine tune</u> the parameters in such a way that

$$-3\frac{M}{h}\lambda + M' = 0$$

and thus

$$M_T = 5\frac{M}{h}\lambda \Rightarrow \text{Triplet is superheavy.}$$

is this fine tuning necessity?

For example, one can introduce some extra symmetry (say Z_2):

$$5_H \to -5_H, \quad \overline{5_H} \to \overline{5_H}$$

Then both $M = \lambda = 0$ and doublet is light automatically(!). But, then triplet is <u>also</u> light and this can be a disaster, since triplet can mediate the proton decay via:

(a) <u>$d=6$ operators</u> induced by scalar Higgs triplet exchange and are presented already at the <u>nonSUSY</u> GUT level

$$\bar{5}_H^i 10_{ik} \bar{5}^k = \left\{ \begin{array}{l} \text{again} \\ i,k = 1,2,3 \equiv a,b \\ i,k = 4,5 \equiv A,B \end{array} \right\} \Rightarrow \text{couplings of } \bar{T} =$$

$$= \bar{5}_H^a 10_{ab} \bar{5}^b + \bar{5}_H^a 10_{aA} \bar{5}^A =$$

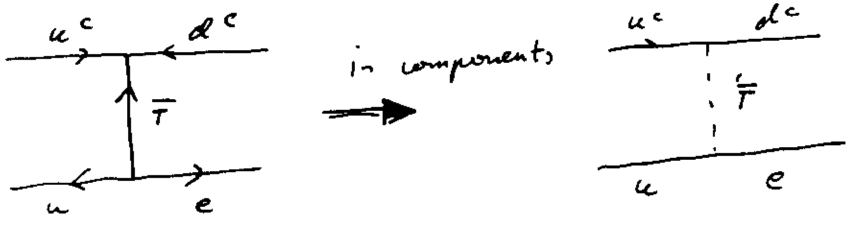
$$= \underbrace{\bar{T} u^c d^c + \bar{T} Q L}$$

$$5_i^H 10_{km} 10_{ne} \epsilon^{ikmne} = 5_a^H 10_{bc} 10_{AB} \epsilon^{abcAB} +$$

$$+ 5_a^H 10_{bA} 10_{cB} \epsilon^{abAcB} + \dots =$$

$$= T u^c e^c + T Q Q$$

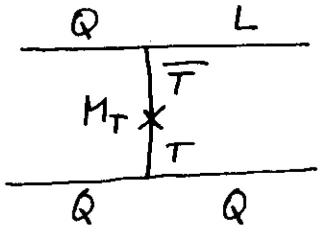
For example

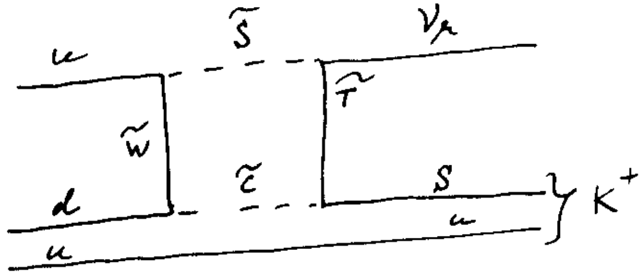


So, the triplet has to be superheavy (in minimal SUSY SU(5)).

(8) But even if triplet is superheavy, there still can be $d=5$ operators leading to the proton decay, which are suppressed just by one power of M_G^{-1}. This are operators mediated by the colored Higgsino exchange. In superoperators they have a form of F-term.

$$\frac{1}{M}[qqql]_F =$$





$$p \to K^+ \nu_\mu$$

How we can suppress $d=5$ operators?

$\boxed{1}$ eliminate chirality flip SUSY mass term $\bar T T M_T$.

For this it is enough to introduce two pairs of $\bar 5_H, 5_H$ and $\bar 5_H', 5_H'$ and Z_2' symmetry under which

$$(\bar 5_H, 5_H') \to -(\bar 5_H, 5_H')$$

and all other Higgs fields are invariant.

then terms $\Sigma\, 5_H \bar 5_H + M' \bar 5_H 5_H$ are forbidden and there is no $M_T T \bar T$ term.

$\boxed{2}$ Second possibility is to suppress the couplings of T ($\bar{T}$) with ~~m~~ matter superfields. This can be done if there is some new physics at M_G which induces ~~the~~ effective couplings of the form.

$$\frac{1}{M_G} \bar{5}_H^K \Sigma_K^m 10_{mi} \bar{5}^i + (\text{"old" couplings } g \bar{5}_H^m 10_{mi} \bar{5}^m)$$

the effective couplings for $\bar{T}$ and $\bar{H}$-doublet (after GUT symmetry breaking by $\langle\Sigma\rangle = (2\,2\,2\,-3\,-3)\langle\Sigma\rangle$) become

$$g_{\bar{T}} = \frac{2\Sigma}{M_G} + g$$

$$g_H = -\frac{3\Sigma}{M} + g$$

Now we can fine tune $\frac{2\Sigma}{M_G} + g = 0$. and triplet $\bar{T}$ gets decoupled.

In SU(5) this ~~no~~ leads to another fine tuning, but in SO(10) can be done naturally.

Let us ask the following question.

assume that the susy breaking scale in the low energy sector is $M_S \sim 100$ GeV (meaning that this is a mass splitting in the light matter and Higgs doublet supermultiplets). Is the ~~two~~ tree level hierarchy allowing stable in this ~~[illegible]~~ situation?

Answer is „no" and this is related with the existance of light ~~[illegible]~~ G_W - singlets in the theory.

Assume that we want to make $M_H = 0$ dynamically without a fine tuning.. ~~[illegible]~~ For this reason let us introduce a singlet S and consider superpotential.

$$W = \frac{M}{2}\Sigma^2 + \frac{h}{3}\Sigma^3 + \lambda \Sigma \bar{5}_H 5_H + g S \bar{5}_H 5_H$$

$$+ (\text{„Yukawa" couplings}).$$

Since S enters in combination with $5_H \bar{5}_H$, it is undetermined in the vacuum with $5_H = \bar{5}_H = 0$.

Singlet is „sliding" and may have whatever VEV. In G_W-symmetric vacuum the masses of $T(\bar{T})$ and $H(\bar{H})$ are

$$M_T = 2\frac{M}{h}\lambda + gS$$

$$M_H = -3\frac{M}{h}\lambda + gS$$

Now assume that somehow SUSY is broken and the doublet gets a negative $[\text{mass}]^2 \sim [100\ \text{GeV}]^2$. Then S will adjust

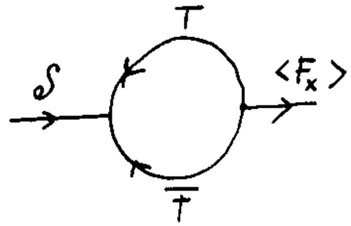
$$S = 3\frac{\lambda}{g}\frac{M}{h} \quad \text{and} \quad M_T = 5\frac{M}{h}\lambda$$

and triplet is superheavy (!).

But this mechanism does not works. First, the SUSY breaking will also induce a (positive) $[\text{mass}]^2$ for the S and

S can never be precisely $= \frac{3\lambda}{g}\frac{M}{h}$ (unless it's mass is very small).

More importantly, there are radiative corrections of the form



$+ \; \dots$

$\langle F_X \rangle$ is an insertion that breaks SUSY and in realistic cases is $\sim M_G m_s$. So effectively this induces the term

$$F_S F_X^* (\text{loop factor})$$

some constant

$$|F_S|^2 + g F_S 5_H \bar{5}_H + F_S F_X^* c' + \dots$$

So the doublet gets large ($\sim m_s M_G$) mass/VEV. The hierarchy is destabilized.

So the light singlets coupled to both light and heavy superfields (provided the mass splitting in the latter is large) are disaster.

Solutions to the hierarchy / D-T splitting problem

① Missing partner / Missing VEV

② Higgs doublets as pseudogoldstones

① Missing partner

Let us introduce representation that contains triplet states $T(\bar{T})$ and no doublets $H(\bar{H})$. Such is a $50 + \overline{50}$.

Now, the Higgs triplets from $5_H \bar{5}_H$ can get masses from the coupling.

$$5_H 75 \overline{50} + \bar{5}_H 75\, 50 + M\, 50 \overline{50}$$

(where $\langle 75 \rangle \equiv G_W$-singlet). But doublets are massless because they have no partners in $50 + \overline{50}$.

② Higgs doublets as pseudogoldstones.

There is at least one example when scalar mass can be protected by the symmetry. This is the case if scalar is a Goldstone particle.

But if the global symmetry is explicitly broken, the ~~doublet~~ scalar can get a small mass measured by the amount of explicit breaking.

We can implement this idea in the SUSY GUTs to solve the hierarchy problem.

In this approach doublets can be identified with the pseudo Goldstones of accidental global symmetry of the Higgs (super) potential.

Consider e.g. $SU(6)$-gauge theory. To break $SU(6) \to G_W$ we need two Higgs superfields: Σ_i^k adjoint (35-plet) and $6_H, \bar{6}_H$ fundamental.

If the crossing coupling $\bar{6}_H \Sigma 6_H$ is absent, the (renormalizable) superpotential of Higgs sector has a larger $SU(6)_\Sigma \otimes SU(6)_{6_H}$ global symmetry.

$$W(\text{Higgs}) = W(\Sigma) + W(6_H, \bar{6}_H)$$

In the "right" G_W-symmetric vacuum.

$$\Sigma = \begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & -2 & \\ & & & & & -2 \end{bmatrix} \langle\Sigma\rangle \quad \langle 6_H \rangle = \langle \bar{6}_H \rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \langle 6 \rangle$$

the global pseudosymmetry is broken

$$\left[SU(6)_\Sigma \otimes SU(6)_{6_H}\right] \to \left[SU(4) \otimes SU(2) \otimes U(1)\right]_\Sigma \otimes \left[SU(5)\right]_{6_H}$$

We can count number of massless modes and find that there is a one pair

$$H = \langle 6 \rangle D_\Sigma + \langle \Sigma \rangle D_6$$

$$\bar{H} = \langle 6 \rangle \bar{D}_\Sigma + \langle \Sigma \rangle \bar{D}_{\bar{6}}$$

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which is not eaten up by gauge supermultiplet and thus is a physical massless (in SUSY limit) field to be identified with electroweak Higgs doublet.

$H, \bar{H}$ will get a small $\sim m_S$ mass after SUSY breaking.