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GAUGE YUKAWA UNIFICATION AND THE MASS OF THE TOP QUARK

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Please note: These are preliminary notes intended for internal distribution only.

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G. Zoupanos

Gauge - Yukawa Unification

Standard Model

Very successful ⇝ highly non-trivial part of a (more fundamental theory of E.P.P.

BUT

no. of free parameters (~20)

Renormalization ⇝ free parameter.

Traditional way of reducing number of parameters

SYMMETRY

Celebrated example : GUTs

e.g. minimal $SU_5 \rightsquigarrow$ testable $\sin^2\theta_w$

LEP Data $\rightsquigarrow$ $N=1$ SU_5

However more SYMMETRY
(e.g. SO_{10}, E_6, E_7, E_8) does not
necessarily lead to more predictions
for the S.M. parameters.
(due to new degrees of freedom,
various ways and channels of
breaking e.t.c)

Extreme case : Superstring Ths

GUTs can also relate Yukawa couplings among themselves and might lead to predictions

e.g. in SU_5 ⇝ successful m_τ / m_b

In SO_{10} all elementary particle of each family (both chiralities + ν_R) are in a common rep 16-plet.

Natural gradual extension: attempt to relate the couplings of the two sectors

⇝ Gauge-Yukawa Unification

Searching for a symmetry is needed one that relates fields with different spins

⇝ Supersymmetry
BUT $N=2$

Fayet

In $SU(5)$

Yukawa couplings

$$f_1\, 10_f\, 10_f\, 5_H + f_2\, 10_f\, \bar{5}_f\, \bar{5}_H$$

$$\langle 5_H \rangle = (0, 0, 0, 0, v)$$

$$\longrightarrow v f_1\, \bar{u} u + v f_2\, (\bar{d} d + \bar{e} e)$$

i.e. $\text{"}m_{e,\mu,\tau}\text{"} = \text{"}m_{d,s,b}\text{"}$ Boundary condition at the unification scale.

The $SU(5)$ symmetric relations are obtaining significant renormalisation corrections.

$$m(\mu) = m - m g_n^2\, b_m^{(n)} \ln\left(\frac{\Lambda}{\mu}\right) \longleftrightarrow$$

$$\longrightarrow \frac{d \ln m(\mu)}{d \ln \mu} = b_m^{(n)}\, g_n^2(\mu)$$

where g_n^2 solutions of

$$\frac{d g_n^2(\mu)}{d \ln \mu} = -2 b_n g_n^4(\mu)$$

with $b_n = (11n - 2N_F)/48\pi^2$, $n \geq 2$

$b_1 = - N_F/24\pi^2$

and $b_m^{(n)} = -\frac{3}{8\pi^2} \underbrace{\sum_a (T^a T^a)_{ij}}_{\underset{C_2(R)}{\equiv}}$

$$C_2(R) = \frac{\dim(adj)}{\dim(R)} \ell(R_i) \quad \text{for } SU(n), n \geq 2$$

$$(T_1^0)^2 = \frac{3}{5}\left(\frac{Y}{2}\right)^2$$

$$\rightarrow \frac{m(\mu)}{m(\mu_0)} = \left[\frac{g_n(\mu)}{g_n(\mu_0)}\right]^{-\frac{b_m^{(n)}}{b_n}}$$

$\rightarrow \frac{m_b}{m_\tau} \simeq 3$ at $\mu = \mu_{th} \sim 10$ GeV

also $\underset{\simeq 200}{\frac{m_\mu}{m_e}} = \underset{\simeq 20}{\frac{m_s}{m_d}}$

GYU - functional relationship derived in some principle.

in
- Superstrings
- Composite models

in principle such relations exist.
in practice both have more problems than the S. M.

Attempts to relate gauge and Yukawa couplings:

- Requiring absence of quadratic divergencies (Decker, Pestieau, Veltman)

$$m_e^2 + m_\mu^2 + m_\tau^2 + 3\left(m_u^2 + m_d^2 + m_c^2 + m_s^2 + m_t^2 + m_b^2\right) = \frac{3}{2} m_W^2 + \frac{3}{4} m_Z^2 + \frac{3}{4} m_H^2$$

- Spontaneous breaking of susy via F-terms

$$\sum_J (-1)^{2J}(2J+1)\, m_J^2 = 0$$

Veltman '81

Requiring absence of quadratic divergences found:

$$m_e^2 + m_\mu^2 + m_\tau^2 + 3(m_u^2 + m_d^2 + m_c^2 + m_s^2 + m_t^2 + m_b^2) = \frac{3}{2} m_W^2 + \frac{3}{4} m_Z^2 + \frac{3}{4} m_H^2$$

For $m_H^2 \ll m_W^2 \rightarrow m_t = 69\,\mathrm{GeV}$

" $m_H^2 = m_W^2 \rightarrow m_t = 77.5\,\mathrm{GeV}$

⋮

" $m_H^2 \simeq (300\,\mathrm{GeV})^2 \rightarrow m_t = 180\,\mathrm{GeV}$

Ferrara, Girardello and Palumbo considering the spont. breaking of a susy theory found

$$\sum_J (-1)^{2J} (2J+1)\, m_J^2 = 0$$

Veltman

Dim. reg. : quadratic div. manifest themselves as pole singularities in $d=2$.

To make them vanish one impose the condition:

$$\frac{1}{4} f(d)\left[m_e^2 + m_\mu^2 + m_\tau^2 + 3\left(m_u^2+m_d^2+m_c^2+m_s^2+m_t^2+m_b^2\right)\right]$$

$$= \frac{d-1}{2}\, m_W^2 + \frac{d-1}{4}\, m_Z^2 + \frac{3}{4}\, m_H^2$$

where $f(d) = \mathrm{Tr}[1]$

Veltman chose $f(d)=4$ and using susy arguments he put $d=4$

Osland + Wu using point splitting reg. found same relation

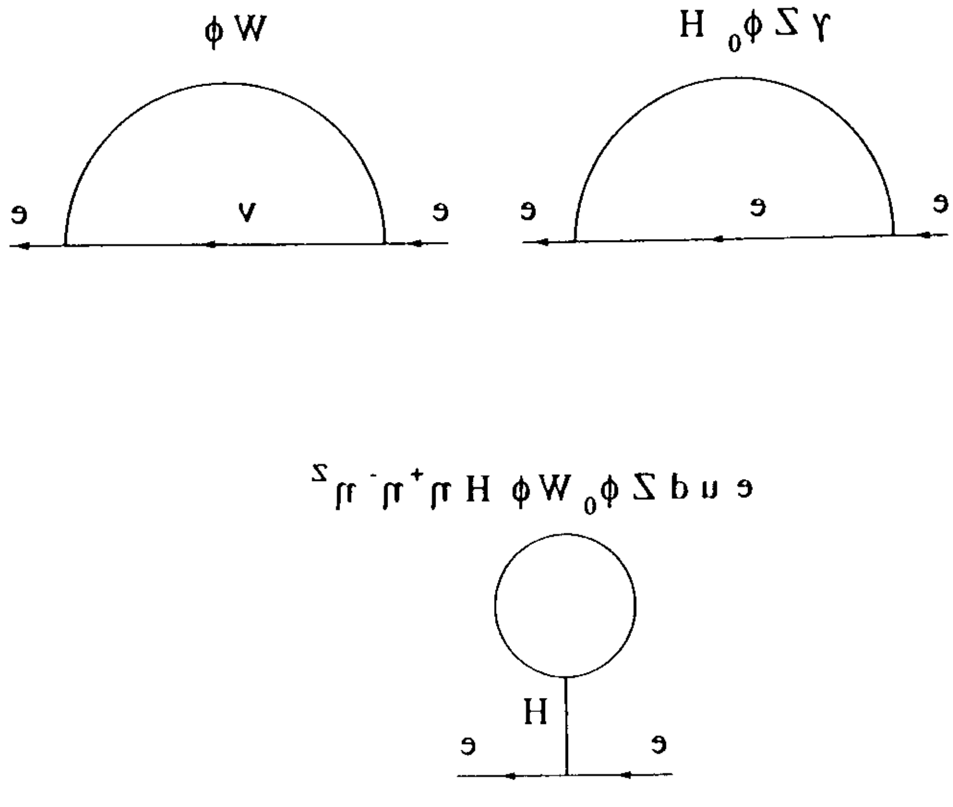


Figure 1

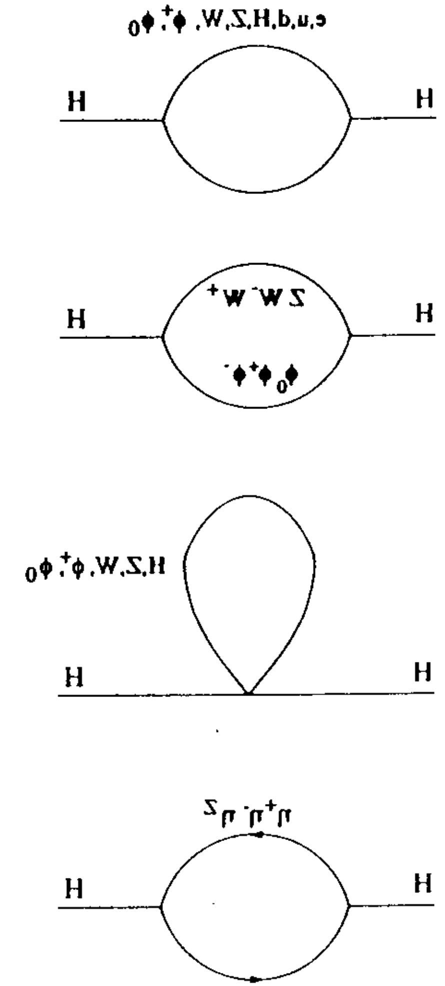


Figure 2

- Pendleton - Ross infrared fixed point

$$\frac{d}{dt}\left(Y_t/\alpha_3\right) = 0 \rightsquigarrow m_t \sim 100 \text{ GeV}$$

(Divergent! in 2-loops, Zimmermann)

- Infrared quasi-fixed point (Hill)

$$\frac{d}{dt}\left(Y_t\right) = 0 \rightsquigarrow m_t \sim 280 \text{ GeV}$$

$$Y_t \equiv \frac{h_t^2}{4\pi}$$

- Susy Pendleton - Ross

$$\rightsquigarrow m_t \simeq 140 \text{ GeV} \cdot \sin\beta$$

$$\tan\beta = \frac{v_U}{v_D}$$

- Susy Quasi-fixed point (Barger et. al., Carena et. al.)

$$\rightsquigarrow m_t \simeq 200 \text{ GeV} \cdot \sin\beta$$

Standard Model

- Pendleton - Ross infrared fixed point:

For strong α_3, i.e. $\alpha_1 = \alpha_2 = 0$

$$\frac{d\alpha_3}{dt} = -7\,\alpha_3^2$$

$$\frac{dY_t}{dt} = -\frac{Y_t}{4\pi}\left(8\,\alpha_3 - \frac{9}{2}\,Y_t\right) \; ; \; Y_t = \frac{h_t^2}{4\pi}$$

$$\text{P-R} : \frac{d}{dt}\left(Y_t/\alpha_3\right) = 0 \;\; \rightsquigarrow \;\; Y_t = \frac{2}{9}\,\alpha_3$$

$$\rightsquigarrow \;\; m_t^{P-R} = \sqrt{\frac{8\pi}{9}\,\alpha_3} \cdot v \sim 100 \text{ GeV}$$

In the reduction scheme same result is obtained from the requirement that the system is described by a single coupling thy with a renormalized power series expansion in α_3.

•• Infrared Quasi-fixed point:

Vanishing β-function for Y_t

$$\Rightarrow \frac{9}{2} Y_t^{Q-f} = 8\,\alpha_3$$

$$\Rightarrow m_t^{Q-f} = \sqrt{8}\, m_t^{P-R} \sim 280 \text{ GeV}$$

* Quasi-fixed point would also become an exact fixed point if $b_3 = 0$.

Susy S.M.

- Pendleton - Ross:

$$\frac{d\alpha_3}{dt} = -3\,\alpha_3^2$$

$$\frac{dY_t}{dt} = Y_t\left(\frac{16}{3}\frac{\alpha_3}{4\pi} - 6\,Y_t\right)$$

$$\frac{d}{dt}\left(Y_t/\alpha_3\right) = 0 \rightsquigarrow Y_t^{susy\,P-R} = \frac{7}{18}\alpha_3$$

$$\rightsquigarrow m_t^{susy\,P-R} = \sqrt{\frac{7}{18}\,4\pi\,\alpha_3}\cdot v\cdot \sin\beta$$

$$\sim 140\ \text{GeV}\cdot\sin\beta$$

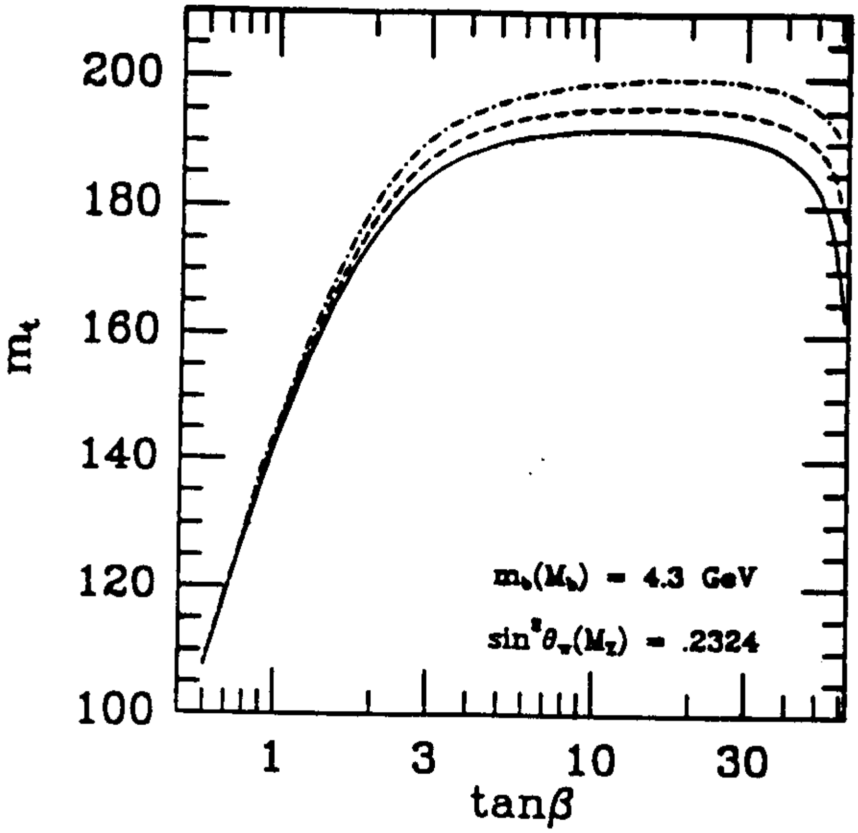
$$\tan\beta = \frac{v_U}{v_D}$$

- • Quasi-fixed point:

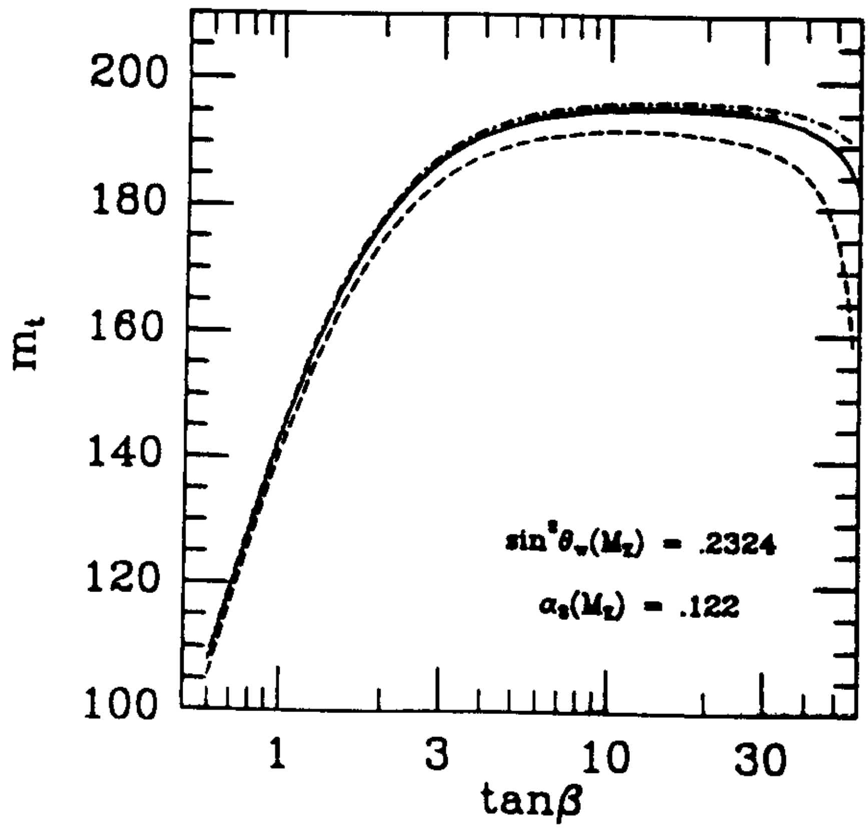
$$Y_t^{susy\,Q-f} = \frac{16}{18}\frac{\alpha_3}{4\pi}$$

$$\rightsquigarrow m_t^{susy\,Q-f} \sim 200\ \text{GeV}\cdot\sin\beta$$

Quasi-fixed point is reached if h_t becomes strong at scales $\mu = 10^{14} - 10^{19}$ GeV



Carena
Pokorski
Wagner



Search for renormalization group invariant relations among Gauge and Yukawa couplings in Unification schemes before symmetry breaking.

- Finite SU_5

  Kapetanakis, Mondragon, Z '92

  " " '93

- Minimal Finite SU_5

  Kubo, Mondragon, Z '94

- [illegible] Pati Salam

  $SU_4 \times SU_{2L} \times SU_{2R}$

  Kubo, Mondragon, Tracas, Z '94

- G-Y Unif. and t-b hierarchy '95

  Kubo, Mondragon, Olechowski, Z

in a GUT with

g - gauge coupling

g_i - other couplings (Yukawas, self-couplings)

Any R.G.I. relation among the couplings can be expressed as

$$\Phi(g, g_1, \cdots) = \text{const}$$

$$\Rightarrow \frac{d}{dt}\Phi = 0 \quad , \quad t = \ln\mu$$

$$\frac{d\Phi}{dt} = \frac{\partial\Phi}{\partial g}\underbrace{\frac{dg}{dt}}_{\beta_g} + \sum_i \frac{\partial\Phi}{\partial g_i}\underbrace{\frac{dg_i}{dt}}_{\beta_i} = 0$$

This is math. equivalent to

$$\frac{dg}{\beta_g} = \frac{dg_1}{\beta_1} = \frac{dg_2}{\beta_2} = \cdots \quad \text{characteristic system}$$

$$\Rightarrow \beta_g \frac{dg_i}{dg} = \beta_i \quad \text{// Reduction eqs}$$

The strongest requirement is to demand the formal power series solutions to the REs

$$g_i = \sum_{n=0} \rho_i^{(n+1)} g^{2n+1}$$

Remarkably, uniqueness of these solution can be decided already at 1-loop!

Assume

$$\beta_i = \frac{1}{16\pi^2}\left[\sum_{j,k,l \neq g} \beta_i^{(1)} g_j\, g_k\, g_l + \sum_{j \neq g} \beta_i^{(1)j} g_j\, g^2\right] + \cdots$$

$$\beta_g = \frac{1}{16\pi^2} \beta_g^{(1)} g^3 + \cdots$$

Assume $\rho_i^{(n)}$, $n \leq r$ have been uniquely determined

To obtain $\rho_i^{(r+1)}$ insert g_i in REs and collect terms of $O(g^{2r+3})$

$$\rightsquigarrow \sum_{l\neq g} M(r)_i^l \rho_l^{(r+1)} = \text{Lower order terms known by assumption}$$

$$M(r)_i^l = 3\sum_{j,k\neq g} b_i^{(1)} \rho_j^{(1)} \rho_k^{(1)} + b_i^{(1)l} - (2r+1) b_g^{(1)} \delta_i^l$$

$$0 = \sum_{j,k,l\neq g} b_i^{(1)jkl} \rho_j^{(1)} \rho_k^{(1)} \rho_l^{(1)} + \sum_{l\neq g} b_i^{(1)l} \rho_l^{(1)} - b_g^{(1)} \rho_i^{(1)}$$

$\rightsquigarrow$ $\rho_i^{(n)}$ for all $n > 1$ for a given set of $\rho_i^{(1)}$ can be uniquely determined if

$$\det M(n)_i^l \neq 0 \text{ for all } n \geq 0$$

$\mathcal{N}=1$ gauge theories

Consider a chiral, anomaly free, $N=1$ globally supersymmetric gauge th. based on a group G with gauge coupling g.

Superpotential

$$W = \frac{1}{2} m_{ij} \phi^i \phi^j + \frac{1}{6} C_{ijk} \phi^i \phi^j \phi^k$$

m_{ij}, C_{ijk} gauge invariant tensors

ϕ^i - matter fields transforming as an irr. rep. R_i of G.

Renormalization constants associated with W

$$\phi^{0i} = (Z^i_j)^{1/2}\phi^j \,, \quad m^0_{ij} = Z^{i'j'}_{ij}\, m_{i'j'} \,, \quad C^0_{ijk} = Z^{i'j'k'}_{ijk} C_{i'j'k}$$

$N=1$ non-renormalization thm ensures absence of mass and cubic-int-term infinities

$$Z^{ijk}_{i'j'k'}\, Z^{1/2\, i'}_{i''}\, Z^{1/2\, j'}_{j''}\, Z^{1/2\, k'}_{k''} = \delta^i_{(i''}\, \delta^j_{j''}\, \delta^k_{k'')}$$

$$Z^{ij}_{i'j'}\, Z^{1/2\, i'}_{i''}\, Z^{1/2\, j'}_{j''} = \delta^i_{(i''}\, \delta^j_{j'')}$$

(in the background field method)

$$Z_g Z_V^{1/2} = 1$$

$\Rightarrow$ Only surviving infinities are Z^i_{j}, (Z_V)

i.e. one infinity for each field.

The 1-loop β-function of the gauge coupling is

$$\beta_g^{(1)} = \frac{dg}{dt} = \frac{g^3}{16\pi^2}\left[\sum_i \ell(R_i) - 3C_2(G)\right]$$

$\ell(R_i)$ - Dynkin index of R_i

$C_2(G)$ - quadratic Casimir of the adjoint rep

β-functions of C_{ijk}, by virtue of the non renormalization thm, are related with the anomalous dim matrix γ_{ij} of ϕ^i

$$\beta_{ijk} = \frac{dC_{ijk}}{dt} = C_{ijl}\gamma^l_k + C_{ikl}\gamma^l_j + C_{jkl}\gamma^l_i$$

$$\gamma_i^j = Z^{-\frac{1}{2}}{}_i^k \frac{d}{dt} Z^{\frac{1}{2}}{}_k^j$$

$$= \frac{1}{32\pi^2}\left[C^{jkl}C_{ikl} - 2g^2 C_2(R_i)\delta_i^j\right]$$

$C_2(R_i)$ – quadratic Casimir of R_i

$C^{ijk} = C^*_{ijk}$

Finite Unification

Old days...

... divergences are "hidden under the carpet" (Dirac, Lects on Q.F.T., '64)

Recent years ...

... divergences reflect existence of a higher scale where new degrees of freedom are excited. Not just artifacts of pert. th.

However the presence of quadratic divergences means that physics at one scale are very sensitive to unknown physics at higher scales.

$\rightsquigarrow$ SUSY ths which are free of quadratic divergences in spite of any experimental evidence...

$\rightsquigarrow$ Natural to expect that beyond unification scale the theory should be completely finite.

- $N=4 \rightsquigarrow$ finite to all orders in pert.
- $N=2 \rightsquigarrow$ only 1-loop contributions to β-function. Possible to arrange the spectrum so that theory is finite.

Multiplicities for massless irreducible reps with maximal helicity 1

Spin \ N	1	1	2	2	4
1	—	1	—	1	1
1/2	1	1	2	2	4
0	2	—	4	2	6

$$N=2 \quad : \quad \beta(g) = \frac{2g^3}{(4\pi)^2}\left(\sum_i T(R_i) - C_2(G)\right)$$

e.g. $SU(N)$ with $2N$ fundamental reps $\rightsquigarrow \beta(g) = 0$

$SU(5)$: $p(5+\bar{5})$; $q(10+\overline{10})$; $r(15+\overline{15})$

with $p+3q+7r = 10$

$SO(10)$: $p(10+\overline{10})$; $q(16+\overline{16})$

with $p+2q = 8$

E_6 : $4(27+\overline{27})$

Finite Unified Theories

$N=1$

- Complete classification of chiral $N=1$ susy models with a simple gauge group satisfying 1-loop finiteness has been done.

  Hamidi, Patera, Schwarz; Jiang, Zhou

- • 1-loop finiteness $\rightsquigarrow$ 2-loop finiteness

  Parkes, West. Jones, Mezinescu

- • • There exist simple criteria that guarantee all loop finiteness in the sense of $\beta^{4}=0$

  Lucchesi, Piguet Sibold Kazakov Ermushev Tarasov

- We examined promising 1(2) loop models;
  make them all-loop finite;
  $\Longrightarrow$ top quark mass ~ 189 GeV

Kapetanakis, Mondragon, Z '92

Chiral, anomaly free, globally $N=1$ SUSY gauge theory with group G has superpotential:

$$W = a_i \phi_i + \frac{1}{2} m_{ij} \phi_i \phi_j + \frac{1}{6} C_{ijk} \phi_i \phi_j \phi_k$$

$\left.\begin{array}{l} a_i \\ m_{ij} \\ C_{ijk} \end{array}\right\}$ gauge inv tensors

$C_{ijk} \rightarrow$ Yukawa couplings

ϕ_i – matter fields in irreducible reps of G.

Necessary and sufficient conditions for $N=1$ 1-loop finiteness:

- Vanishing of β_g implies:

$$\sum_i \ell(R_i) = 3 C_2(G)$$

$\ell(R_i)$: Dynkin index of irrep R_i

$C_2(G)$: Quadratic Casimir of G (adjoint)

$\rightsquigarrow$ Selection of the field content (representations) of the theory.

CHIRAL TWO-LOOP-FINITE SUPERSYMMETRIC THEORIES ☆

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Any globally supersymmetric theory in four dimensions that is one-loop finite is automatically (at least) two-loop finite. We classify all such theories that are chiral and have a simple gauge group.

One of the less satisfying aspects of GUTs ("grand-unified theories") and super GUTs is that they contain many arbitrary parameters. One principle that could serve to limit the number of parameters is the requirement of finiteness. This possibility has been raised with the discovery that there are large classes of supersymmetric gauge theories that are free from ultraviolet divergences at all orders of perturbation theory [#1]. The theories for which this has been established are all $N = 4$ and some [2] $N = 2$ super Yang–Mills theories. Finiteness allows some parameters to be introduced via mass and soft supersymmetry-breaking terms, but it relates all the dimensionless couplings. Unfortunately, all $N = 2$ or $N = 4$ theories are nonchiral ("vector-like") and do not appear suited to the construction of a realistic model. Recently, the possibility has been raised that certain $N = 1$ theories could also be finite [3,4]. A theory containing Yang–Mills and chiral superfields is ultraviolet finite at loop provided that certain conditions (described below) restricting the representations and couplings of the chiral superfields are satisfied. It has been proved by direct calculation [3] and by considerations involving the chiral anomaly [4] that these conditions ensure two-loop finiteness as well, without any additional restrictions. It is an open question whether any of the $N = 1$ theories of this class are finite beyond two loops. The purpose of this letter is to list all chiral solutions of the one-loop conditions that are based on a simple gauge group.

Consider a globally supersymmetric $N = 1$ theory in four dimensions with a simple Yang–Mills group G. In addition to the gauge superfield, it can contain chiral superfields in an arbitrary representation R of G with irreducible components R_i:

$$\mathrm{R} = \bigoplus_i \mathrm{R}_i \,. \tag{1}$$

Our task is to find the possible choices of R and associated couplings that ensure one-loop finiteness. We only consider chiral theories ($\mathrm{R} \neq \bar{\mathrm{R}}$). This restricts G to those groups that have complex representations, namely SU(n) with $n \geqslant 3$, SO($4k + 2$) with $k \geqslant 2$, and E_6. Cancellation of the gauge-current anomaly

$$A(\mathrm{R}) = \sum_i A(\mathrm{R}_i) = 0 \tag{2}$$

is also imposed, since it is a necessary requirement for a consistent quantum theory. The anomaly condition is nontrivial only for SU(n).

There are two additional conditions required by one-loop finiteness [3,4]. The first is the one-loop finiteness of the gauge-field self energy. The condition is

$$l(\mathrm{R}) = \sum_i l(\mathrm{R}_i) = 3C_2(\mathrm{G}) \,, \tag{3}$$

where $l(\mathrm{R}_i)$ is the "index" of R_i [5] and $C_2(\mathrm{G})$ is

☆ Work supported in part by the U.S. Department of Energy under contract DEAC 03-81-ER40050.

[1] On leave from Centre de recherches de mathématiques appliquées, Université de Montréal, Montréal, Québec, Canada.

[#1] For a review see ref. [1].

the eigenvalue of the second-order Casimir operator (which coincides with the index of the adjoint representation). Since indices are always positive (except for singlets, which are excluded), eq. (3) already limits R to a finite number of possibilities for a given group G.

The second condition is the one-loop finiteness of the chiral superfield self-energy. In terms of coefficients d describing the cubic self-coupling of the chiral superfields in the superpotential, the condition is

$$\sum_{\substack{b,c\\ j,k}} d^{ijk}_{abc}\,\bar{d}^{\,i'jk}_{a'bc} = 2g^2\delta_{aa'}\delta_{ii'}C_2(\mathrm{R}_i)\,. \tag{4}$$

The subscripts a, b, c label components of the representations $\mathrm{R}_i, R_j, \mathrm{R}_k$.

The only irreducible representations that can occur in R are ones whose indices do not exceed $3C_2(G)$. Singlets (with nonzero couplings) are excluded by (4). All relevant representations of the groups with complex representations are listed in table 1. It also gives the indices and anomalies, normalized to be unity for the fundamental representation. Complex-conjugate representations, which have the same index and opposite anomaly, are not shown.

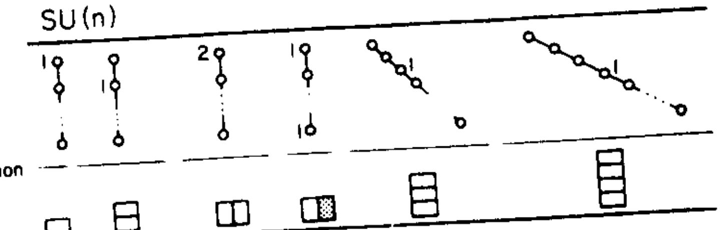
In seeking solutions to eqs. (2)–(4) it is convenient to consider first the trace of (4) given by summing over $a = a'$ and the m_α values of $i = i'$ for which $\mathrm{R}_i = \mathrm{R}_k$. This results in conditions of the form

$$\sum_{\beta\gamma} |C_{\alpha\beta\gamma}|^2 = m_\alpha I(\mathrm{R}_\alpha)\,. \tag{5}$$

Eq. (5) is weaker than (4), but it is useful for quickly eliminating many candidates from the list of admissible R's. Detailed examination of (4) then eliminates

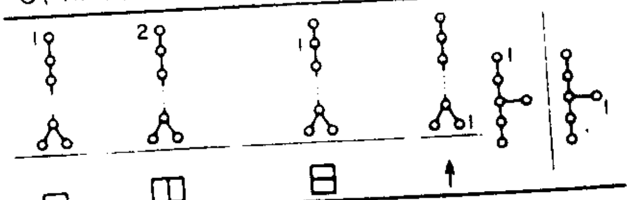
Table 1
Properties of relevant irreducible representations

SU(n)



Representation						
Dimension	n	n(n-1)/2	n(n+1)/2	n^2-1	n(n-1)(n-2)/6	n(n-1)(n-2)(n-3)/24
Index	1	n-2	n+2	2n	(n-2)(n-3)/2	(n-2)(n-3)(n-4)/6
Anomaly	1	n-4	n+4	0	(n-3)(n-6)/2	(n-3)(n-4)(n-8)/6

O(4k+2) E(6)



Representation						
Dimension	4k + 2	2(k+1)(4k+1)	(4k+2)(4k+1)/2	2^{2k}	27	78
Index	1	4k + 4	4k	2^{2k-3}	1	4

Table 2
Multiplicities m_α of the irreducible components R_α of R for all the solutions.

Irrep	27	$\overline{27}$	Comments
E(6)	n	$12-n$	$7 \leqslant n \leqslant 12$

Irrep	10	54	45	16	$\overline{16}$	Comments
SO(10)	8	0	0	n	$8-n$	$5 \leqslant n \leqslant 8$
	2	1	1	1	0	
	$12-2m-2n$	1	0	n	m	$n+m \leqslant 4$, $n > m$

Irrep	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	Adj
SU(n)	$2n-4$	$2n+4$	0	1	1	0	0
$n \geqslant 7$	$n-4$	$n+4$	0	1	1	0	1

Irrep	8	$\bar{8}$	28	70	63
SU(8)	1	5	1	1	1

Irrep	3	$\bar{3}$	6	8
SU(3)	3	10	1	0
	0	7	1	1

Irrep	4	$\bar{4}$	15	6	10
SU(4)	0	8	1	1	1
	4	12	0	1	1

Irrep	5	$\bar{5}$	10	$\overline{10}$	15	$\overline{15}$	24
SU(5)	3	14	2	0	1	0	0
	6	14	0	1	1	0	0
	5	7	4	2	0	0	0
	5	10	5	0	0	0	0
I →	6	9	4	1	0	0	0
	7	8	3	2	0	0	0
	8	10	3	1	0	0	0
	1	2	1	0	1	1	1
	1	9	0	1	1	0	1
	2	3	3	2	0	0	1
	3	5	3	1	0	0	1
II →	4	7	3	0	0	0	1
	5	6	2	1	0	0	1
	6	8	2	0	0	0	1
	8	9	1	0	0	0	1
	3	4	1	0	0	0	2

Irrep	6	$\bar{6}$	15	$\overline{15}$	21	$\overline{21}$	20	35
SU(6)	0	16	3	0	1	0	0	0
Sib.ld et. al. →	8	16	0	1	1	0	0	0
	0	4	5	3	0	0	0	0
	3	5	4	3	0	0	0	0
	0	12	6	0	0	0	0	0
	2	10	5	1	0	0	0	0
	4	8	4	2	0	0	0	0
	8	12	3	1	0	0	0	0
	0	2	1	0	0	0	3	1
	0	4	2	0	0	0	2	1
	3	5	1	0	0	0	2	1
	0	6	3	0	0	0	1	1
	2	4	2	1	0	0	1	1
	3	7	2	0	0	0	1	1
	6	8	1	0	0	0	1	1
	1	3	1	0	1	1	0	1
	2	10	0	1	1	0	0	1
	1	3	3	2	0	0	0	1
	0	8	4	0	0	0	0	1
	2	6	3	1	0	0	0	1
	3	9	3	0	0	0	0	1
	5	7	2	1	0	0	0	1
	6	10	2	0	0	0	0	1
	9	11	1	0	0	0	0	1
	0	4	2	0	0	0	0	2
	3	5	1	0	0	0	0	2
	0	2	1	0	0	0	1	2

some [1 for SU(5) and 11 for SU(6)] of the class allowed by (2), (3), and (5). The complete list of complex representations satisfying (2), (3) and (4) is given in table 2. The conjugate representations, which are also solutions, are not tabulated. In most cases the couplings are uniquely determined (up to a change of basis), but in a few cases there are free parameters or discrete alternatives. Note that there are no solutions for SO(4k + 2) with $k > 2$.

Scanning the tables for potentially realistic schemes,

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•• Vanishing of $\gamma^{(1)i}_{j}$ implies:

$$C^{ik\ell} C_{jk\ell} = 2 \delta^i_j g^2 C_2(R_i)$$

g: gauge coupling constant
$C_2(R_i)$: quadratic Casimir of R_i
$C^{ijk} = (C_{ijk})^*$

$\rightsquigarrow$ Yukawa couplings are related to the gauge coupling const.

• 1-loop finiteness condts do not restrict the form of a_i, m_{ij}

•• Existence of a $U(1)$ is incopatible with the first condt.

•• The second condt forbids the presence of singlets with nonzero coupling

$$\ell(R_i)\,\delta_{ab} = Tr\,(T_a T_b)$$

$$C_2(R_i)\,\delta_{ij} = \sum_a (T^a T^a)_{ij}$$

where $T_a\,[a = 1, \dots, \dim(adj)]$ is the a-th generator in the R_i rep.

$$C_2(R_i) = \frac{\dim(adj)}{\dim(R_i)}\,\ell(R_i) \qquad //$$

e.g. in $SU(N)$

N has $\ell = 1 \rightsquigarrow C_2(N) = \frac{N^2-1}{N}$

N^2-1 " $\ell = 2N \rightsquigarrow C_2(N^2-1) = 2N$

$\rightsquigarrow$ SUSY can be broken by adding G-invariant soft terms (subject to mild constraints)

* 1-loop finiteness condts are necessary and sufficient to guarantee 2-loop finiteness. { Parkes, West, Jones, Mezincescu, Machacek, Vaughn

$$\beta_g^{(1)} = \mu \frac{\partial}{\partial \mu} g = \frac{g^3}{16\pi^2}\left[\sum_i \ell(R_i) - 3\, C_2(G)\right]$$

$$\beta_{ijk}^{(1)} = \mu \frac{\partial}{\partial \mu} C_{ijk} = \gamma^{(1)\ell}_{(i}\, C_{jk)\ell}$$ due to non-renormalization thm

$$\gamma^{(1)\,i}_{j} = \frac{1}{16\pi^2}\frac{1}{2}\left[C^{imn} C_{jmn} - 2\,\delta^i_j\, g^2 C_2(R_i)\right]$$

$$\beta^{(2)} = \frac{1}{(16\pi^2)^2}\, 2g^5\left[\sum_i \ell(R_i) - 3\, C_2(G)\right]$$

$$- \frac{1}{(16\pi^2)^2}\,\frac{g^3}{r}\, C_2(R_i)\left[C^{imn} C_{jmn} - 2\delta^i_j g^2 C_2(R_i)\right]$$

r : $\mathrm{tr}\,\delta^{ab}$

$$\gamma^{(2)i}_{\ \ j} = \frac{1}{(16\pi^2)^2}\, 2g^4 C_2(R_i)\left[\sum_i \ell(R_i) - 3C_2(G)\right]$$

$$- \frac{1}{(16\pi^2)^2}\,\frac{1}{2}\left[C^{ik\ell} C_{jkm} + 2g^2 (R^a)^i_{\ m} (R^a)^\ell_{\ j}\right]$$

$$\cdot \left[C^{mpq} C_{\ell pq} - 2\delta^m_\ell g^2 C_2(R_i)\right]$$

- 1-loop finiteness condts ensure that $\beta^{(3)}_{(g)}$ in 3-loops vanishes but in general the chiral superfield $\gamma^{(3)}$ does not (Grisaru, Milewski, Zanon; Parkes, West)

What happens in higher loops?
So far 1-loop finiteness condts (on γs) are telling us:

$$C_{ijk} = C_{ijk}(g)$$

$$\beta^{(1)}_{ijk} = 0$$

** Necessary + sufficient condts for $\beta(g)$ and β_{ijk} to vanish to all orders:

Lucchesi
Piguet
Sibold

1. $\beta^{(1)}(g) = 0$
2. $\gamma^{(1)i}_{j} = 0$
3. $\beta_{ijk} = \beta(g) \dfrac{d C_{ijk}}{d g}$ admit a power series solution which in lowest order is a solution of condt 2.

3. → 3'. There exist solutions to $\gamma^{(1)} = 0$ of the form

$C_{ijk} = \rho_{ijk}\, g$, ρ_{ijk} - complex

→ 4. These solutions are isolated and non-degenerate

⟹ last condt introduces new constraints

Condts 1 + 2 $\rightsquigarrow$ 2-loop finiteness

3. All-loops finiteness:

Find a unique solution of the system of eqs!

Recall

- a_i, m_{ij} (linear, mass) terms unaffected

•• breaking of SUSY

$\cancel{F}$ since singlets are excluded ($\gamma^{(1)}=0$)

$\cancel{D}$ " $U(1)$ " ($\beta^{(1)}=0$)

$\rightsquigarrow$ soft terms + more condts

Model I

$G = SU(5)$; 3 fermion families;
no Higgs in the adjoint

Content: fermion super-mults   scalar super-mults

$$\underset{\bar{5}_i \;\; 10_i}{3(\bar{5}+10)} + \underset{H_a \;\; \bar{H}_a}{6(5+\bar{5})} + \underset{N}{(10+\overline{10})}$$

Finiteness condts:

1. $\beta(g)^{(1)} = 0$ is automatically satisfied

2. Consider most general $SU(5)$-inv, $N=1$-inv, cubic superpotential:

$$W = \frac{1}{2} g_{ija} 10_i 10_j H_a + g_{ia} 10_i N H_a + \bar{g}_{ija} 10_i 5_j \bar{H}_a + \frac{1}{2} g'_{ijk} 10_i \bar{5}_j \bar{5}_k + \frac{1}{2} f_{ab} N \bar{H}_a \bar{H}_b + \frac{1}{2} f_{ab} \bar{N} H_a H_b + \frac{1}{2} h_a N N H_a + \frac{1}{2} \bar{h}_a \bar{N} \bar{N} \bar{H}_a + \frac{1}{2} g_{iab} 10_i \bar{H}_a \bar{H}_b + p_{ia} N \bar{5}_i \bar{H}_a + \frac{1}{2} t_{ij} N \bar{5}_i \bar{5}_j$$

$i, j, k = 1, 2, 3$; $a, b = 1, \dots, 6$

Condt ② i.e. $\gamma^{(1)} = 0$ impose relations among Yukawa and gauge cpls

$$H: 3 g^{ija} g_{ijb} + 6 g^{ia} g_{ib} + 4 f^{ca} f_{cb} + 3 h^a h_b = \delta^a_b \frac{24}{5} g^2$$

$$\bar{5}: 4 \bar{g}^{ila} \bar{g}_{ima} + 4 g'^{ilk} g'_{imk} + 4 t^{lj} t_{mj} + p^{la} p_{ma} = \delta^l_m \frac{24}{5} g^2$$

$$\bar{H}: 4 \bar{g}^{ija} \bar{g}_{ijb} + 4 f^{ca} f_{cb} + 3 \bar{h}^a \bar{h}_b + 4 q^{ica} q_{icb} + 4 p^{ia} p_{ib} = \delta^a_b \frac{24}{5} g^2$$

$$N: 3 g^{ia} g_{ia} + f^{cb} f_{cb} + 3 h^a h_a + 2 p^{ia} p_{ia} + t^{ij} t_{ij} = \frac{36}{5} g^2$$

$$\bar{N}: \bar{f}^{ab} \bar{f}_{ab} + 3 \bar{h}^a \bar{h}_a = \frac{36}{5} g^2$$

$$10: 3 g^{lki} g_{mki} + 2 \bar{g}^{lki} \bar{g}_{mki} + 3 g^{la} g_{ma} + g'^{ljk} g'_{mjk} + q^{lab} q_{mab} = \delta^l_m \frac{36}{5} g^2$$

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— To find unique solution impose additional discrete syms.

Q-parity: 10_i, $\bar{5}_i$ odd
all the rest even

and a $Z_3 \times Z_7$

	10_1	10_2	10_3	$\bar{5}_1$	$\bar{5}_2$	$\bar{5}_3$	N
Z_7	1	2	4	4	1	2	0
Z_3	1	2	0	0	0	0	0

	H_1	H_2	H_3	H_4	H_5	H_6
Z_7	5	3	6	0	0	0
Z_3	1	2	0	0	1	2

Then only few terms survive leading to the unique solution:

$$f_{44}^2 = \bar{f}_{44}^2 = 0 \qquad h_4^2 = \bar{h}_4^2 = \frac{8}{5} g^2$$

$$f_{56}^2 = f_{65}^2 = \bar{f}_{56}^2 = \bar{f}_{65}^2 = \frac{6}{5} g^2$$

$$g_{iii}^2 = \frac{8}{5} g^2 \qquad \bar{g}_{iii} = \frac{6}{5} g^2$$

$\rightsquigarrow$ All - loop finite

- This model could result from a string theory:

  CY manifold with non standard embedding

  Witten
  Diestler
  Greene
  Kirlin
  Miron

  $$E_8 \times E_8' \longrightarrow SU(5) \times E_8'$$

Content : $m(10) + n(\bar{5}) + \delta(10+\overline{10}) + \epsilon(5+\bar{5})$

m, n, δ, ϵ topological #s of the manifold.

In our case: $m = n = 3$

$$\delta = 1$$
$$\epsilon = 6$$

Then break gauge symmetry via Hosotani (Wilson flux mechanism):

CY should admit freely acting discrete group F

Model II

$G = SU(5)$; 3 fermion families and 24

Hamidi, Schwarz
Jones, Raby
Leon et. al.
Kazakov

Content

$$3(\bar{5}+10) + 4(\overset{H_a}{5}+\overset{\bar{H}_a}{\bar{5}}) + 24$$

ferm. super-mults

scalar sup-mults

Finiteness condts

1. $\beta^{(1)}(g) = 0$ is automatically satisfied

2. Consider most general $SU(5)$-inv, $N=1$, cubic superpotential:

$$W = \frac{1}{2} g_{ija} 10_i 10_j H_a + \bar{g}_{ija} 10_i \bar{5}_j H_a + \frac{1}{2} g'_{ijk} 10_i \bar{5}_j \bar{5}_k$$

$$+ \frac{1}{2} g_{iab} 10_i \bar{H}_a \bar{H}_b + f_{ab} \bar{H}_a 24 H_b + p(24)^3 + h_{ia} \bar{5} 24 \ldots$$

$i, j, k = 1, 2, 3$; $a, b = 1, \ldots, 4$

$\gamma^{(1)} = 0 \rightsquigarrow$ relations among Yukawa and gauge couplings.

$$\overline{H}: \quad 4\,\bar{g}_{ija}\,\bar{g}^{ibb} + \frac{24}{5} f_{ac} f^{bc} + 4 g_{iac} g^{ibc} = \frac{24}{5} g^2 \delta_a^b$$

$$H: \quad 3\, g_{ija}\, g^{ijb} + \frac{24}{5} f_{ca} f^{cb} + \frac{24}{5} h_{ia} h^{ib} = \frac{24}{5} g^2 \delta_a^b$$

$$5: \quad 4\,\bar{g}_{kia}\,\bar{g}^{kja} + \frac{24}{5} h_{ia} h^{ja} + 4 g'_{ikl} g'^{jkl} = \frac{24}{5} g^2 \delta_i^j$$

$$10: \quad 2\,\bar{g}_{ika}\,\bar{g}^{jka} + 3\, g_{ika}\, g^{jka} + g_{iab}\, g^{jab} + g'_{kli}\, g'^{klj} = \frac{36}{5} g^2 \delta_i^j$$

$$24: \quad f_{ab} f^{ab} + \frac{21}{5} p p^* + h_{ia} h^{ia} = 10 g^2$$

- Imposing the same discrete symmetries as before we find the unique solution:

$$g_{iii}^2 = \frac{8}{5} g^2, \quad \bar{g}_{iii}^2 = \frac{6}{5} g^2$$

$$f_{ii}^2 = 0, \quad f_{44}^2 = g^2, \quad p^2 = \frac{15}{7} g^2$$

All loop finite model

- SU(5) breaks down to the standard model due to $\langle 24 \rangle$

- Use the freedom in mass parameters to obtain only a pair of Higgs fields light, acquiring v.e.v.

- Get rid of unwanted triplets rotating the Higgs sector (after a fine tuning)

  see Leon et. al.

- Adding soft terms we can achieve SUSY breaking.

- Using R.G.Es (gauge couplings,

$$\sin^2\theta_W(M_Z) = 0.233$$

$$M_X = 2\cdot 10^{16}$$

$$\alpha_{em}^{-1}(M_Z) = 127.9$$

$$\alpha_s(M_Z) = 0.120$$

$$\alpha_X = 0.0425$$

Then using above and initial values at M_X:

$$g_t^2 = \frac{8}{5}(4\pi\alpha_X) \quad ||$$

$$\bar{g}_b^2 = \bar{g}_\tau^2 = \frac{6}{5}(4\pi\alpha_X) \quad ||$$

we obtain

$$m_b \simeq 3.5 \text{ GeV} \qquad (3.1 - 4.1 \text{ GeV})$$

$$m_\tau \simeq 1.78 \text{ GeV}$$

$$m_t \simeq 189 \text{ GeV}$$

1) Gauge Couplings Unification

$\sin^2\theta_W$, $\alpha_{em} \rightsquigarrow \alpha_3(M_Z)$ — Amaldi et. al.

2) Bottom - Tau Yukawa Unif.

SU_5 - type

$\rightsquigarrow m_t \sim 100 - 200$ GeV — Barger et. al, Carena et. al

*3) Top - Bottom - Tau Yuk. Uni.

$h_t^2 = \frac{4}{3} h_{b,\tau}^2$ in SU_5 - FUT

Similar to SO_{10} — Shafi et. al, Barger et. al

$\rightsquigarrow m_t \sim 160 - 200$ GeV — Carena et. al

*4) Gauge - Top - Bottom - Tau Uni.

FUT SU_5 : $h_t^2 = \frac{8}{5} g_U^2$; $h_b^2 = \frac{6}{5} g_U^2$

Min. Susy SU_5 : $h_t^2 = 0.97\, g_U^2$; $h_b^2 = .57\, g_U^2$

Conclusions

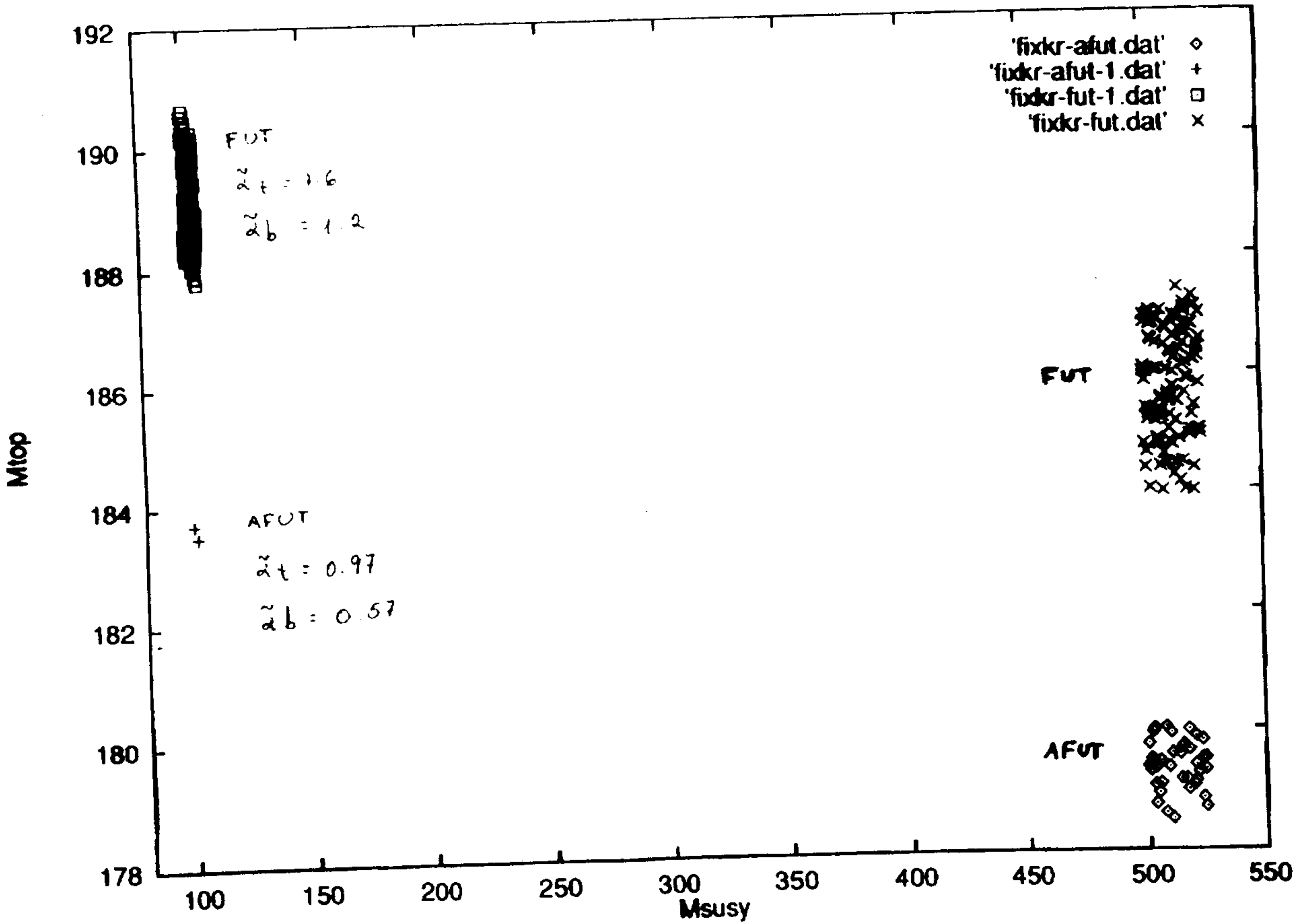
- Examined 3-families, $SU(5)$, $N=1$ which are 1(2)-loop finite and can be made all-loop finite imposing discrete symmetries.

- The values of the Yukawas at the unification point are determined by the theory.

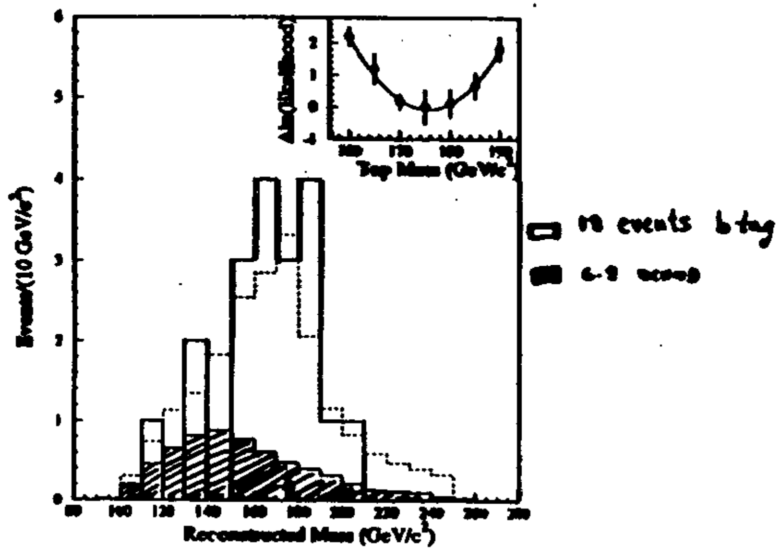
- High value of the top mass:
  185 – 200 GeV

- Consistent gauge-Yukawa Uni. in minimal $N=1$, SU_5:
  180 – 190 GeV

- Consistent gauge-Yukawa Uni. in minimal $N=1$, $SU_4 \times SU_{2L} \times SU_{2R}$
  195 – 240 GeV


'fixkr-afut.dat'
'fixkr-afut-1.dat'
'fixkr-fut-1.dat'
'fixkr-fut.dat'
FUT
$\tilde{\alpha}_t = 1.6$
$\tilde{\alpha}_b = 1.2$
AFUT
$\tilde{\alpha}_t = 0.97$
$\tilde{\alpha}_b = 0.57$
FUT
AFUT
Mtop
Msusy
192
190
188
186
184
182
180
178
100
150
200
250
300
350
400
450
500
550

Mass Fit to the Tagged Events



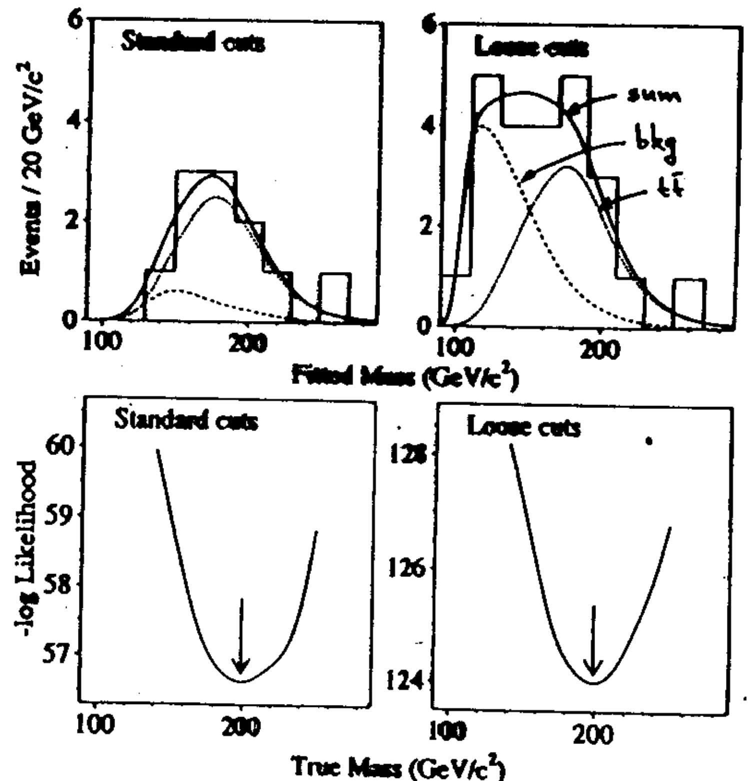
$M_t = 176 +/- 8 +/- 10$ GeV

Significance of mass peak (KS test): 0.02

DØ

Mass Analysis Results

- Standard cuts - 11 of 14 events fitted
- Loose cuts - 24 of 27 events fitted
  * Results insensitive to constraining the background normalization



$M_t = 199^{+19}_{-21} \pm 22$ GeV

SUMMARY OF TOP SECTION

1. Top quark is observed
   at mass of $m_t = 180 \pm 12$ GeV

2. Production cross section
   $\sigma = 6.6 {+2.1 \atop -1.6}$ pb
   agrees with expectation for $t\bar{t}$

3. Decay properties and kinematics
   of signal are consistent with top