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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

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SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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NON-PERTURBATIVE METHODS AND ELECTROWEAK PHYSICS

M. Voloshin
Theoretical Physics Institute
University of Minnesota
MN 55455, Minneapolis
U.S.A

Please note: These are preliminary notes intended for internal distribution only.

1. Standard E.W. Model

- Fundamental fermions:

$$\begin{pmatrix} u_i \\ d_i \end{pmatrix}_L , \quad u_{iR}, \quad d_{iR}, \quad \begin{pmatrix} l_i \\ \nu_i \end{pmatrix}, \quad l_{iR}, \quad \nu_{iR}$$

$$u_i = (u_1, u_2, u_3, c_1, c_2, c_3, t_1, t_2, t_3)$$

$$d_i = (d_1', d_2', d_3', s_1', s_2', s_3', b_1', b_2', b_3')$$

$$d_i' = \underset{\substack{\uparrow \\ \text{KM matrix}}}{V_{ij}}\, d_j \qquad \Big| \qquad l_i = (e, \mu, \tau)$$

- Gauge fields:

$SU(2)_L$: W^a_μ ; $\quad W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\varepsilon^{abc} W^b_\mu W^c_\nu$

$U(1)_Y$: V_μ ; $\quad V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$

$$\frac{g^2}{4\pi} \equiv \alpha_W = \frac{\alpha}{\sin^2\theta_W} \approx \frac{1}{28}, \qquad \frac{g'^2}{4\pi} \equiv \alpha' = \frac{\alpha}{\cos^2\theta_W} \approx \frac{1}{100}$$

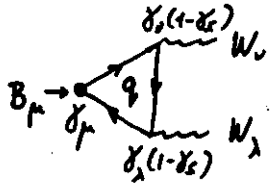
- Higgs doublet

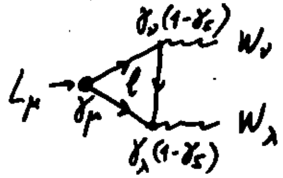
$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \qquad \langle \phi \rangle = v \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$M_W = g v/\sqrt{2} \qquad M_H = 2\sqrt{\lambda}$$

· Baryon and Lepton #'s.

$$B_\mu = \sum_{q=u_i,d_i} \frac{1}{3}(\bar q \gamma_\mu q) \quad , \quad L_\mu = \sum_{\ell=\ell_i,\nu_i} (\bar\ell \gamma_\mu \ell)$$





$$\partial_\mu B_\mu = \partial_\mu L_\mu = N_f \left(\frac{g^2}{32\pi^2} W^a_{\mu\nu} \tilde W^a_{\mu\nu} - \frac{g'^2}{32\pi^2} V_{\mu\nu} \tilde V_{\mu\nu} \right)$$

$\uparrow$
\# of generations
$N_f = 3$

$\tilde F_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\lambda\sigma} F_{\lambda\sigma}$

Limit $\sin^2\theta_W = 0$, $\alpha' = 0 \Rightarrow$ pure $SU(2)_L$ theory.

$$\partial_\mu B_\mu = \partial_\mu L_\mu = N_f \, \partial_\mu K_\mu$$

$$K_\mu = \frac{g^2}{16\pi^2} \varepsilon_{\mu\nu\lambda\sigma} \left(W^a_\nu \partial_\lambda W^a_\sigma + \frac{g}{3} \varepsilon^{abc} W^a_\nu W^b_\lambda W^c_\sigma \right)$$

$$\Delta B = \Delta L = N_f \, \Delta N_{cs} \quad , \quad N_{cs} = \int K_0 \, d^3x$$

$\therefore$ (B+L) violated, (B−L) conserved

To have $\Delta N_{cs} \sim 1$ one encounters fields with amplitude $A \sim \frac{1}{g}$

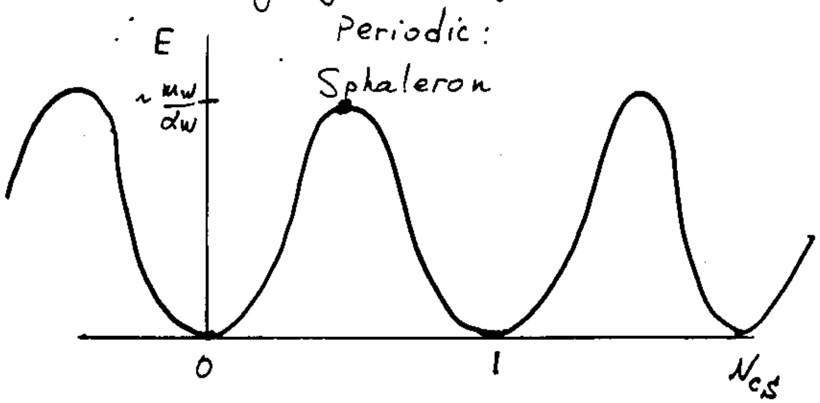
$\Rightarrow$ energy $\sim M_W A^2 \sim \frac{M_W}{\alpha_W}$

} Non-pert field configurations

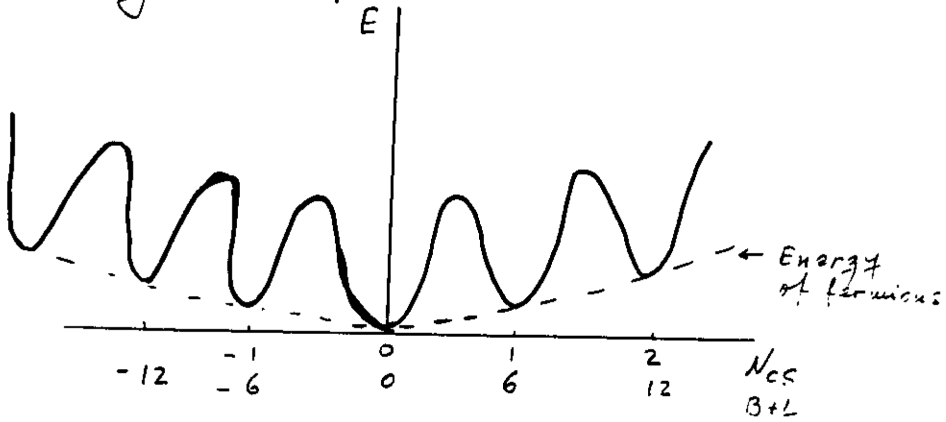
- K_μ - gauge <u>non-invariant</u>

$\Delta N_{cs} = 1$: gauge transformation

$\therefore$ Bosonic gauge theory (without fermions)

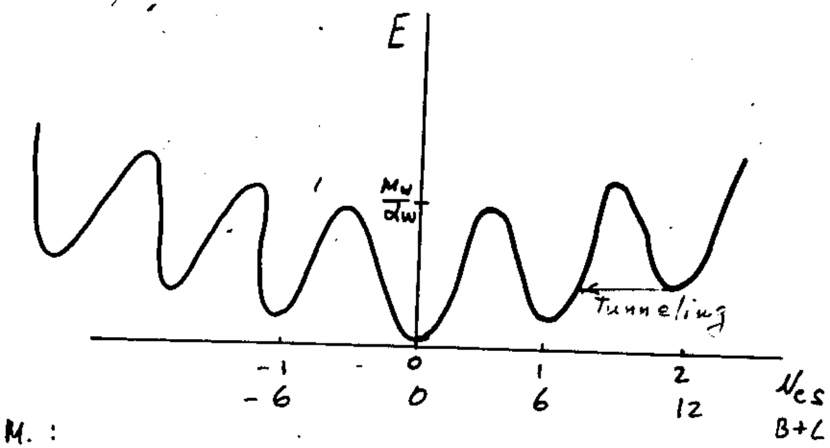


• Theory with fermions (SM)



processes

- Baryon decay / Low energy



S.M.:
$|\Delta B| = 3 \quad |\Delta L| = 3 \Rightarrow$ single nucleon does not decay, but nuclei (e.g. ^3He) can.

In vacuum ΔE_f is very small: in a box with side L $\quad \Delta E_f \sim 1/L \Rightarrow$ non-periodicity of bosonic potential can be neglected.

Tunneling :: instanton.

- Instanton:
Euclidean space

$$W^a_\mu(x) = \frac{2}{g}\,\frac{\rho^2\,\bar\eta_{a\mu\nu}\,x_\nu}{x^2(x^2+\rho^2)}$$

ρ - scale parameter
Shifts, rotations, dilatation
$\bar\eta_{a\mu\nu}$ - 't Hooft symbols

$$\bar\eta_{a\mu\nu} = -\bar\eta_{a\nu\mu}$$
$$\bar\eta_{ai0} = \delta_{ai}$$
$$\bar\eta_{aij} = \epsilon_{aij}$$

$$W^a_{\mu\nu} = \tilde W^a_{\mu\nu}$$

$$S_E = \frac{1}{4}\int W_{\mu\nu}^2\, d^4x = \frac{8\pi^2}{g^2}$$

$$\Delta N_{CS} = \frac{g^2}{32\pi}\int W_{\mu\nu}\tilde W_{\mu\nu}\, d^4x = 1$$

- In Higgs vacuum
$W^{(0)}_\mu(x)$ is valid for $|x| \ll \rho$

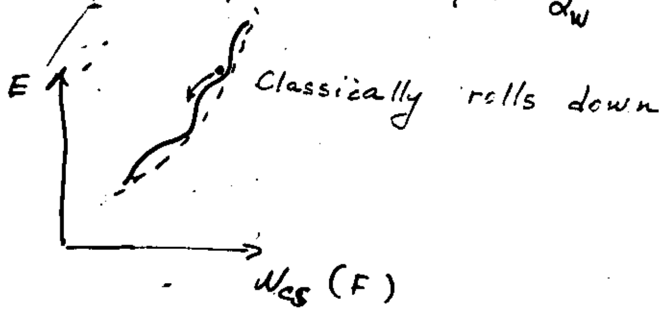
and
$$\phi(x) = \frac{v}{\sqrt2}\,\frac{|x|}{\sqrt{x^2+\rho^2}}\begin{pmatrix}0\\1\end{pmatrix}, \quad |x| \ll \rho$$

$$S_E \approx \frac{8\pi^2}{g^2} + \pi^2 v^2 \rho^2$$

- Tunneling amplitude $\propto e^{-S_E} \sim e^{-\frac{8\pi^2}{g_W^2}} = e^{-\frac{2\pi}{\alpha_W}}$
Probability $\propto e^{-\frac{4\pi}{\alpha_W}} \approx 10^{-170}$ $\quad$ 't Hooft '76
$\Rightarrow$ no possibly observable decays

- High density plasma

Chemical potential $\mu \gtrsim \frac{m_W}{\alpha_W}$ | E per fermion



Fermion plasma with $\mu \gtrsim \frac{m_W}{\alpha_W}$ is absolutely unstable! Rubakov '86

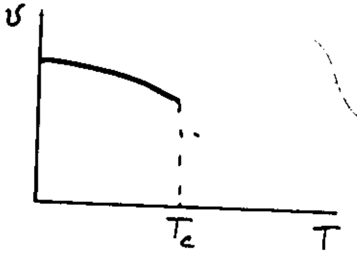
- High temperature | Kuzmin, Rubakov, Shaposhnikov '85

  - $T < E_{SP}$ (top of the barrier)

Thermal fluctuations over the barrier:

$$\Gamma_{B+L} = -\frac{1}{B+L}\frac{d(B+L)}{dt} \propto e^{-E_{SP}/T}$$

$$E_{SP} = E_{SP}(T) \simeq \text{const}\ \frac{m_W(T)}{\alpha_W} = \text{const}\ \frac{v(T)}{g}$$



- Temperature dependence of v | goes back Kirzhnitz & L.

Free energy

$$F(\phi) = V(\phi) + F_{particles}(\phi)$$

$V(\phi)$: T=0 potential incl. rad. corr.; $F_{particles}(\phi)$: free energy of real particles at $T \neq 0$

Lowest order (one-loop: no interaction betw particles)

$$F_{part}(\phi) = F_{boson}(\phi) + F_{fermion}(\phi)$$

$\beta = 1/T$

$$F_{boson}(\phi) = \sum_{d.o.f} \frac{1}{\beta}\int \ln\left(1 - e^{-\sqrt{k^2+m^2}\,\beta}\right)\frac{d^3k}{(2\pi)^3} =$$

$$\sum_{d.o.f.} \frac{1}{2\pi^2\beta^4}\left[\int_{m\beta}^{\infty} \ln(1-e^{-z})\sqrt{z^2-(m\beta)^2}\, z\, dz\right] = \sum_{d.o.f} \frac{1}{2\pi^2\beta^4}\ldots$$

$$F_{fermion}(\phi) = -\sum_{d.o.f.} \frac{1}{\beta}\int \ln\left(1 + e^{-\sqrt{k^2+m^2}\,\beta}\right)\frac{d^3k}{2\pi^3} =$$

$$= \sum_{d.o.f.} \frac{1}{2\pi^2\beta^4}\left[-\int_{m\beta}^{\infty} \ln(1+e^{-z})\sqrt{z^2-(m\beta)^2}\, z\, dz\right] = \sum_{d.o.f} \frac{1}{2\pi^2\beta^4} I$$

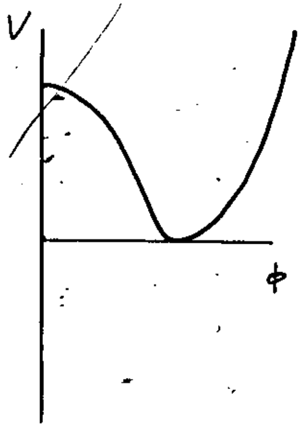
$m = m(\phi)$. E.g. $m_W = g\phi$, $m_q = h_q \cdot \phi$ Yukawa

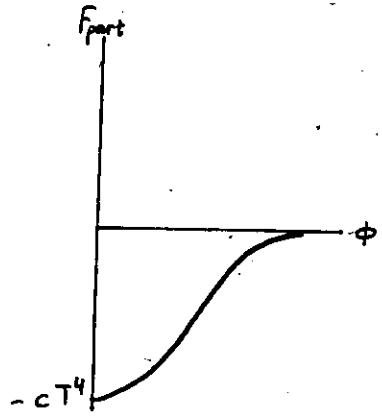
- $T \to \infty$, $\beta \to 0$ ($m\beta \ll 1$)

$$F_b(\phi)/_{d.o.f} = -\frac{\pi^2 T^4}{90} + \frac{m^2T^2}{24}$$

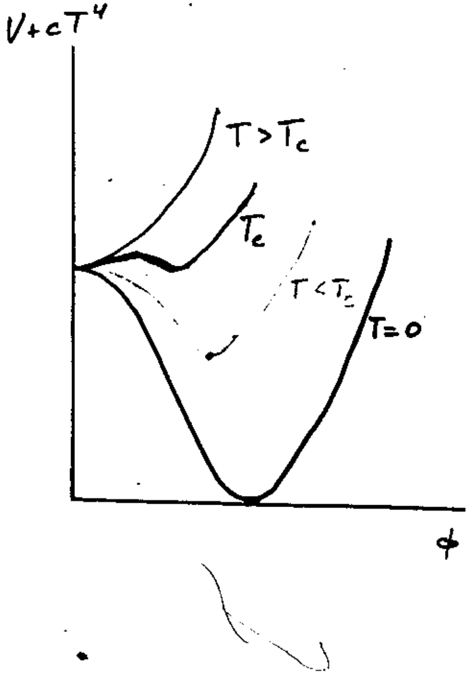
$$F_f(\phi)/_{d.o.f.} = -\frac{7}{8}\frac{\pi^2T^4}{90} + \frac{m^2T^2}{48}$$

$T \to 0$, $\beta \to \infty$ ($m\beta \gg 1$) $\qquad F_b, F_f \sim e^{-m/T}$





adds to



- High temperature. $T > T_c$

$\langle \phi \rangle = 0$, no Higgs-generated masses.

To have $\Delta N_{CS} \sim 1$ at a distance/time scale L, one needs fluctuation with

$$W_\mu \sim \frac{1}{gL}$$

Energy of such fluctuation

$$E \sim (\omega^2, k^2) W^2 L^3 \sim W^2 L \sim \frac{1}{g^2 L}$$

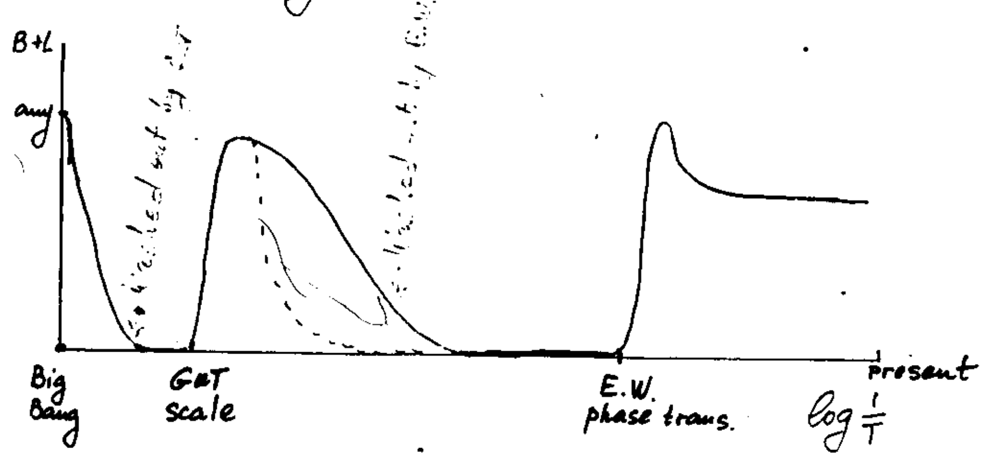
Probability $\sim O(1)$ if $E \sim T \Rightarrow \frac{1}{g^2 L} \sim T$

$$\Rightarrow L \sim \frac{1}{g^2 T} \sim \frac{1}{\alpha_W T}$$

Arnold & McLerran '87

$$\therefore \quad \frac{d\left(\frac{B+L}{V}\right)}{dt} \sim L^{-4} \sim (\alpha_W T)^4$$

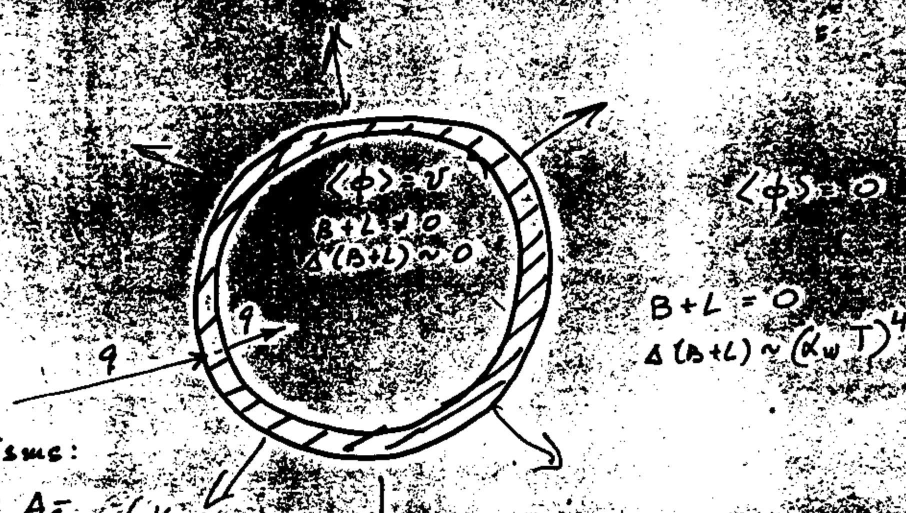
- B+L History of the Universe



If B-L was generated at GUT scale, then B-L conserved by EW. EW makes L = -B (minimal free energy)

If no GUT generated B-L, or if diluted by inflation, B+L generated at EW phase transition.

1-st order phase transition



Mechanisms:

$A_q \neq A_{\bar q}$ (Kaplan & Nelson '91)

Changing phase of ϕ across the wall biases fluctuations with $\Delta N_{CS} = +1$ and $\Delta N_{CS} = -1$. | McLerran, Shaposhnikov, Turok, N. '90

Sakharov's conditions for baryogenesis:

1. $\Delta B \neq 0$
2. $\Delta CP \neq 0$, $\Delta C \neq 0$
3. Non-equillibrium.

|| All can be implemented in S.M. with modest extensions (e.g. 2 Higgs fields).

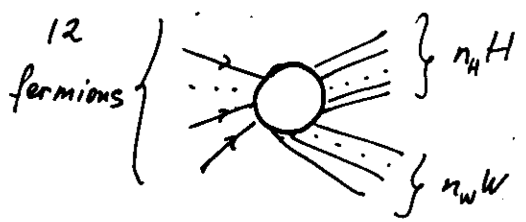
B+L violating scattering (?)

$\Delta(B+L)$ is not suppressed at high density high T $\Rightarrow$ some high energy processes with $\Delta(B+L)$ are not suppressed.

Can $\Delta(B+L)$ be observed in high-energy accelerator experiments?

$$\Delta N_{CS} = 1 \Rightarrow \begin{cases} \Delta L = 3 & (3 \text{ leptons}) \\ \Delta B = 3 & (9 \text{ quarks}) \end{cases}$$
$$\overline{\qquad 12 \text{ fermions} \qquad}$$

Generic process:



Scattering amplitude $\rightarrow$ Multi-point Green f.

$$G(x_1, \ldots, x_{12}, y_1, \ldots, y_{n_H}, w_1, \ldots, w_{n_W}) =$$
$$= \langle T(\psi(x_1) \ldots \psi(x_{12})\, \sigma(y_1) \ldots \sigma(y_{n_H})\, W^{a_1}_{\mu_1}(w_1) \ldots W^{a_{n_W}}_{\mu_{n_W}}$$

$\phi(x) = \frac{v + \sigma(x)}{\sqrt{2}}$

- Euclidean-space path integral calculation of $G(\ldots)$

$$G(\{x\},\{y\},\{w\}) = \int \prod_i \psi(x_i) \cdot \prod_j \sigma(y_j) \cdot \prod_k W(w_k)\, e^{-S[W,\sigma,\psi]} \, \mathcal{D}W \mathcal{D}\sigma \mathcal{D}\psi \mathcal{D}\bar{\psi}$$

Classical configuration with $\Delta N_{CS} = 1$: Instanton

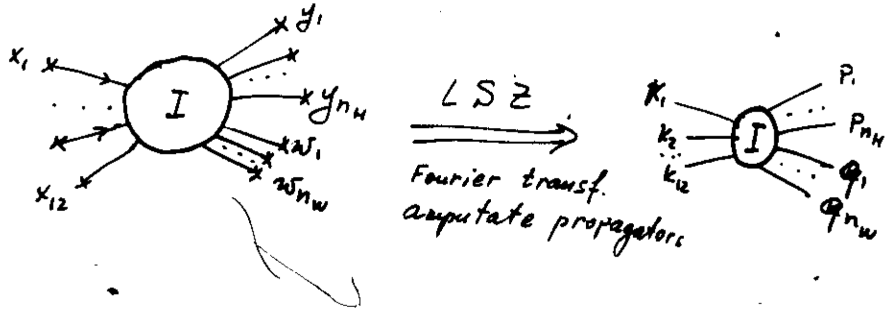
Leading approximation: replace in the path integral $\{\sigma, W\}$ by classical field $\{\sigma_0, W_0\}$, replace $\{\psi\}$ by fermion zero modes | Ringwald '89, Espinosa '89

E.W. Instanton has exactly 12 fermionic zero modes - one for each E.W. doublet.

$$S \to S_{inst} = \frac{8\pi^2}{g^2(\rho)} + \pi^2 v^2 \rho^2$$

$$G = \text{const} \int d\underset{\substack{\uparrow \\ \text{instanton size}}}{\rho}\, D(\rho)\, e^{-S_{inst}} \cdot \int d\underset{\substack{\uparrow \\ SU(2) \text{ group integral}}}{\mu_g} \cdot \int d^4\underset{\substack{\uparrow \\ \text{position of inst}}}{z}\; \psi_0(x_1 - z) \ldots \psi_{12}(x_{12} - z) \cdot$$

$$\sigma(y_1 - z) \ldots \sigma(y_{n_H} - z) \cdot W^{a_1}_{\mu_1}(w_1 - z) \ldots W_{\mu_{n_W}}(w_{n_W} - z)$$



2.
- Fourier transform:

$$\int d^4 z \prod_a f_{\{a\}}(x_a - z) \longrightarrow \Big[\prod_a \underset{\substack{\uparrow \\ \text{individual F.T.}}}{F_{\{a\}}}(p_a)\Big] \underbrace{(2\pi)^4 \delta(\textstyle\sum p)}_{\text{From } \int d^4 z\, e^{iz\sum p_a}}$$

From d^4z int: Conservation l.

- Individual Fourier transforms.

At $|x| \to \infty$ fields fall off as free:

e.g.

$$\sigma = v\left(\frac{x}{\sqrt{x^2+\rho^2}} - 1\right) \to -\frac{v\rho^2}{2x^2} \to \sim -\frac{v\rho^2}{2} e^{-mx}$$

$\Rightarrow$ poles on mass shell (go into amputated propagators)

$\therefore$ Amplitude

$$A \propto \int d\rho\, D(\rho)\, e^{-\frac{8\pi^2}{g^2} - \pi^2 v^2 \rho^2} \cdot \left(2\pi\rho\, F_f(m_f^2)\right)^{12} \cdot$$

$$\cdot \left(-2v\pi^2\rho^2 F_\sigma(m_\sigma^2)\right)^{n_H} \left(\frac{4\pi^2\rho^2}{g} F_W(m_W^2)\right)^{n_W} \times$$

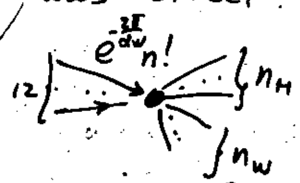
$\cdot$ (Group & kinematical structure)

$F_i(m_i^2)$ - residue of individual F.T. on shell

Formfactor depends on individual $p_i^2 = m_i^2$, not on $(p_i p_j)$ correlations: "point-like" amplitude.

Moreover: $\int d\rho \sim (n_H + n_W)!$ | Ringwald '89, Espinosa (RE)

If RE was correct:

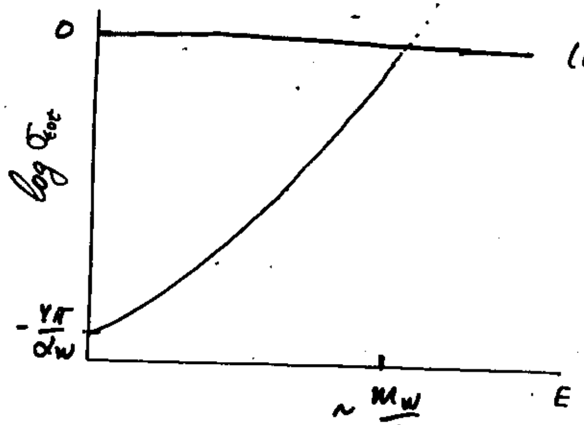
$A_n \neq$ 

$$\sigma_{2\to n} \approx |A_n|^2 \int \frac{d\tau_n}{n_H!\, n_W!} \sim \frac{(n_H+n_W)!^2}{n_H!\, n_W!} \int d\tau_n \cdot e^{-\frac{4\pi}{\alpha_W}}$$

Phase space

$\sigma_{tot}(E) = \sum_{n(E)} \sigma_{2\to n}$ rapidly grows with n

| McLerran, Vainshtein & M. V. '89

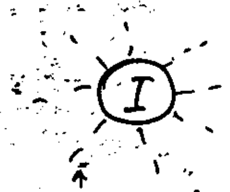


$n_{W,H} \sim \frac{1}{\alpha_W}$.

This makes extremely interesting the question "What is the actual behavior of $\sigma_{\Delta(B+L)}(E)$?"

- RE approximation has very little applicability.

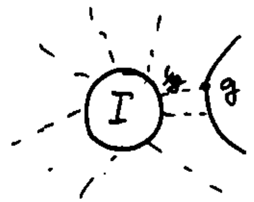
Factors $\psi(x_i)$ $\sigma(y_j)$ $W(w_k)$ were replaced by classical



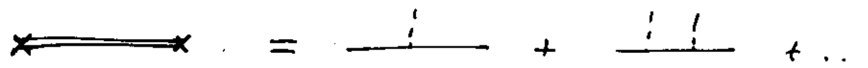
classical fields

In fact $\phi(x) = \phi_0(x) + \phi_q(x)$

generic quantum



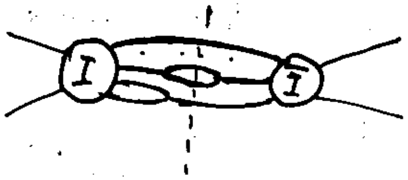
No smallness



Exact propagator in inst bckgrnd

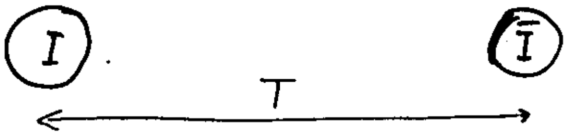
- Total cross section from unitarity (optical theorem)

$$\sigma_{tot} = \text{Im}\, A_{forward}$$



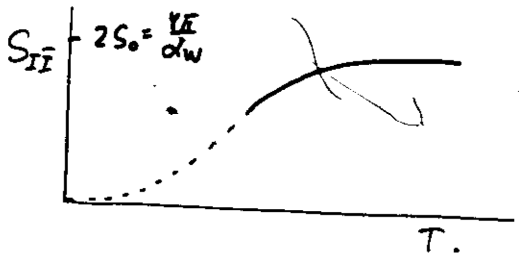
Porratti '9
Zakharov '9

Euclidean-space configuration



$$A_{forw} \sim e^{-S_{I\bar I}(T) + ET}$$ ← Extremize with respect to T.

$I\bar I$-configuration not well defined. However for $T \gg \rho_1, \rho_2$ I and $\bar I$ weakly deform each other. Fix T, minimize $S_{I\bar I}$ with respect to other variables. Slow dependence on T - "valley method".



- "Holy grail" function

Rescale fields & coordinates

$x \to \bar x = g v x$ $\qquad \phi(x) \to v\,\bar\phi(\bar x)$

$W(x) \to v\,\bar W(\bar x)$ $\qquad q(x) \to g^{1/2} v^{3/2}\,\bar q(\bar x)$

$$S = \frac{1}{g^2}\int \bar L\left[\bar W(\bar x), \bar\phi(\bar x), \bar q(\bar x); \frac{\lambda}{g^2}\right] d\bar x$$

$$S_{I\bar I}(T) = \frac{1}{g^2}\,\Phi(gvT)$$

Extremum of $S_{I\bar I}(T) - ET = \frac{1}{g^2}\Phi(gvT;\frac{\lambda}{g^2}) - ET$

at $\frac{v}{g}\Phi'(gvT^*;\frac{\lambda}{g^2}) = E \Rightarrow T^* = \frac{1}{gv} f\left(\frac{Eg}{v};\frac{\lambda}{g^2}\right)$

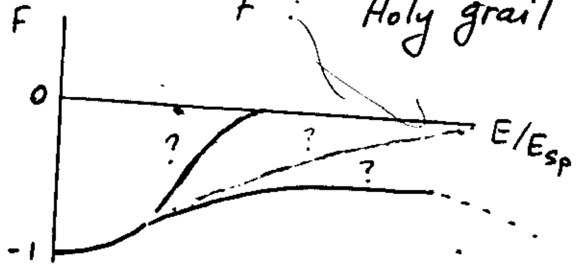
$$\Rightarrow S_{I\bar I}(T^*) - ET^* = \frac{4\pi}{\alpha_W} F\left(\frac{E}{E_{sp}};\frac{\lambda}{g^2}\right)$$

$\left(E_{sp} \approx \frac{M_W}{\alpha_W} \sim \frac{v}{g}\right)$

$$\therefore\ \sigma_{tot} \sim e^{+\frac{4\pi}{\alpha_W} F\left(\frac{E}{E_{sp}};\frac{\lambda}{g^2}\right)}$$

Arnold & Mattis '90
Khlebnikov, Rubakov & Tinyakov '9

F : "Holy grail" function



Presently known terms in expansion:

$$F = -1 + \frac{9}{8}\epsilon^{4/3} - \frac{9}{16}\epsilon^{2} - \frac{3}{32}\left(4 - 3\frac{m_H^2}{m_W^2}\right)\epsilon^{8/3}\ln\epsilon + \dots$$

$$\epsilon = E/E_0 \quad , \qquad E_0 = \sqrt{6}\,\pi\,\frac{m_W}{\alpha_W} \approx 18\ \text{TeV}$$

- Starting from $\epsilon^{10/3}$, F depends on interaction of initial state with final

| Mueller '90
M.V. '91

- Exact Green function in instanton field

$$\langle W_q(p_1)\, W_q(p_2)\rangle \sim W_{cl}(p_1)\, W_{cl}(p_2)\, g^2(p_1 p_2) \ln(p_1\cdot p_2)$$

$$(p_1\cdot p_2) \gg |p_1^2|,\ |p_2^2|$$

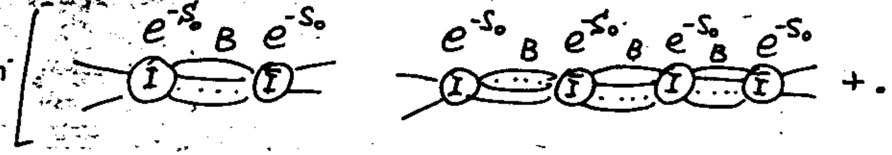
$$\langle W_q W_q\rangle \gg W_{cl} W_{cl}$$

- Knowing expansion in ϵ does not solve the problem of behavior at $\epsilon = O(1)$

• "Premature unitarization": $\sigma_{max} \sim e^{-\frac{2\pi}{\alpha_W}}$ | Zakh… Maggio… & Shif…

Arguments:

$$\sigma = \mathrm{Im}\Big[\ \dots\ +\dots\Big]$$



$$= \mathrm{Im}\left[\frac{B e^{-2S_0}}{1 + \eta B^2 e^{-2S_0}}\right] =$$

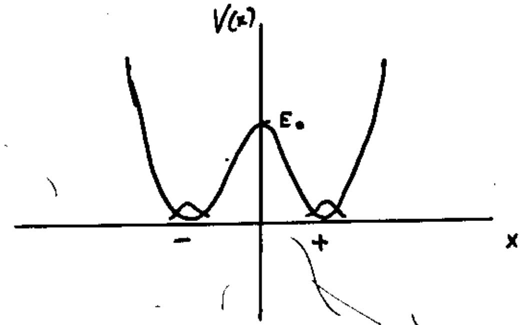
$\eta \sim O(1)$

Im B grows with E

$$= e^{-S_0}\ \mathrm{Im}\left[\frac{B e^{-S_0}}{1 + \eta (B e^{-S_0})^2}\right]$$

$\therefore$ Maximum at $B e^{-S_0} \approx O(1) \Rightarrow \sigma_{max} \sim e^{-S_0}$

• QM example



$$\langle -|\,x\,|+\rangle = \begin{cases} e^{-S_0} & E< \\ e^{-\frac{1}{2}S_0} & E: \end{cases}$$

• However examples of a different behavior exis…

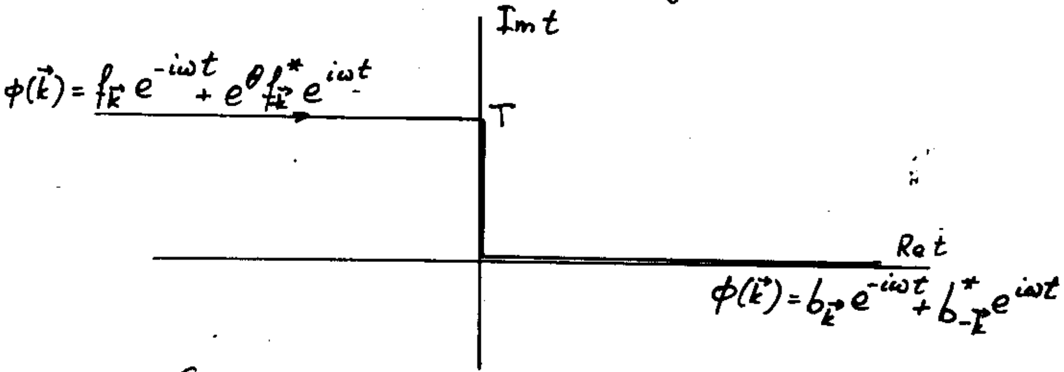
- Rubakov - Tinyakov approach

Instead of $n_{initial} = 2$ consider | Rubakov & Tinyakov '92

$n_{initial} = \nu / g^2$. Finite $\nu \Rightarrow$ WKB for initial state.

The limit $\nu \to 0$ is believed to be smooth.

- Path of largest probability: class. solution



$$E = \int d\vec{k}\, \omega_{\vec{k}}\, e^{\theta} f^*_{\vec{k}} f_{\vec{k}} = \int d\vec{k}\, \omega_{\vec{k}}\, b^*_{\vec{k}} b_{\vec{k}}$$

$$n_i = \int d\vec{k}\, e^{\theta} f^*_k f_k$$

iS = classical action

$$E = 2 \frac{\partial S}{\partial T}, \quad n_i = 2 \frac{\partial S}{\partial \theta}$$

$$\sigma \sim \exp\left(-\frac{1}{g^2} F(E, n_i)\right)$$

$$\frac{1}{g^2} F(E, n) = 2S - ET - n_i \theta$$

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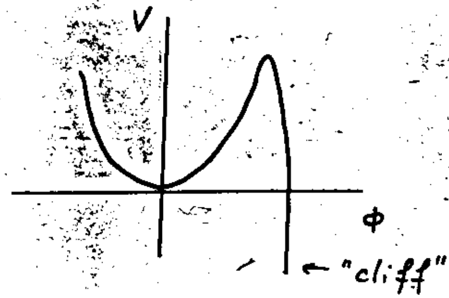
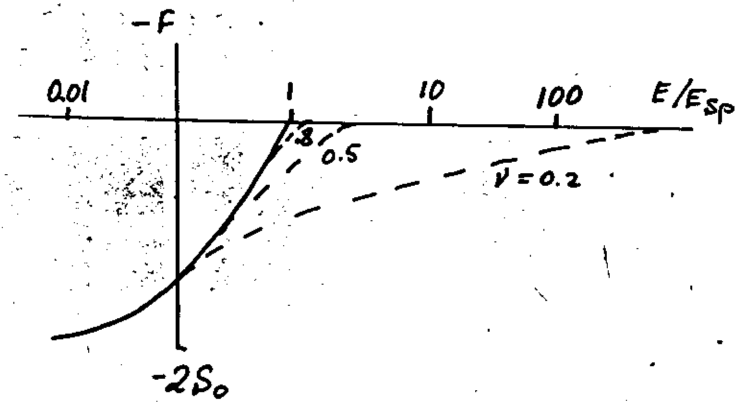
- (1+1) dimensional model | Rubakov & Son '95

$$V(\phi) = \frac{m^2}{2}\phi^2 - \frac{m^2 v^2}{2} \exp\left[2\lambda\left(\frac{\phi}{v} - 1\right)\right]$$

large λ



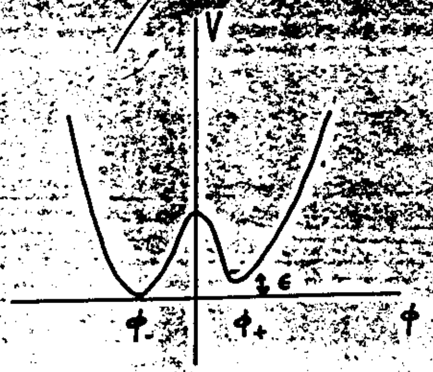


- Multi-instanton contribution is still small when 1-instanton is $O(1)$: no "premature unitarization"
- Also a different from 1/2 factor in Abelian Higgs model ('94 Habib, Mottola, Tinyakov)

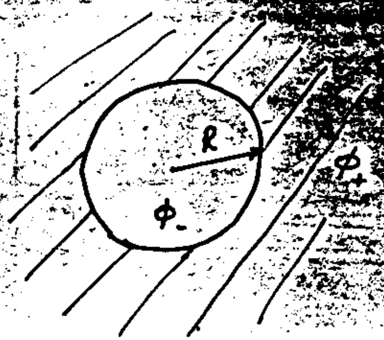
8

2.12 17

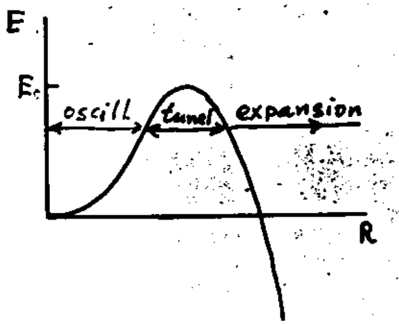
• Another example: induced decay of false vacuum in (3+1) dimensions



Bubble:



$$E(R) = 4\pi\mu R^2 - \frac{4}{3}\pi\epsilon R^3 = \tilde{\mu} R^2 - \tilde{\epsilon} R^3$$



$$A(E) = \langle \text{Bubble}(E) | \phi | 0 \rangle$$

$$A_+ = A_- e^{-S_{tunel}} = \underset{\uparrow \text{ excitation}}{e^{-\frac{\tilde{\mu}^4}{\tilde{\epsilon}^3} c\left(\frac{E}{E_0}\right)}} \; \underset{\uparrow \text{ tunneling}}{e^{-\frac{\tilde{\mu}^4}{\tilde{\epsilon}^3} b\left(\frac{E}{E_0}\right)}}$$

$$\log A_+(E_0) = 0.160 \log A_+(0)$$

M.V. '93

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$$A_+ = \underset{\uparrow \text{ excitation}}{e^{-\frac{\tilde{\mu}^4}{\tilde{\epsilon}^3} c(w)}} \; \underset{\uparrow \text{ tunneling}}{e^{-\frac{\tilde{\mu}^4}{\tilde{\epsilon}^3} b(w)}}$$

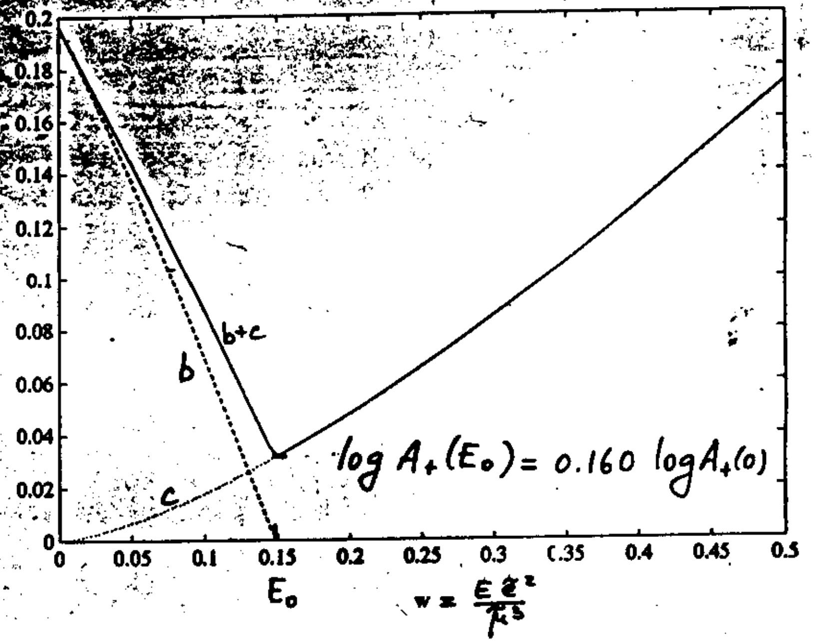
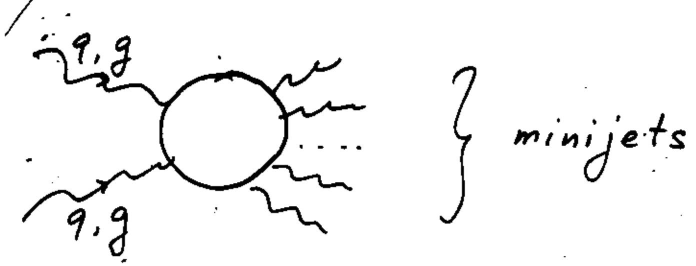


Fig. 3

Even if $\sigma_{max} \sim e^{-S_0} = e^{-\frac{2\pi}{\alpha}}$ the non-perturbative scattering might be observable in QCD



$$n_{jets} \sim \frac{4\pi}{\alpha_s} \sim 40 - 50 \ , \qquad \mu_{jet} \sim 3\text{-}4 \text{ GeV}$$

$$E_{tot} \gtrsim n_{jets} \cdot \mu_{jet} \sim \frac{4\pi}{\alpha_s} \mu_{jet} \sim O(200 \text{ GeV})$$

Zakharov '91

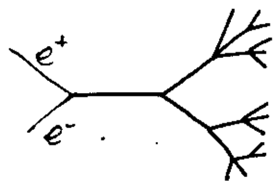
Balitsky & Braun '93-'94

Multi-particle production

Dynamical non-triviality of instanton-induced scattering is associated with multiparticle states. Simplify $\Rightarrow$ consider topologically trivial sector $\Rightarrow$ consider $\lambda\phi^4$ theory.

- Processes with n interacting bosons are described by (at least) $O(n!)$ graphs.

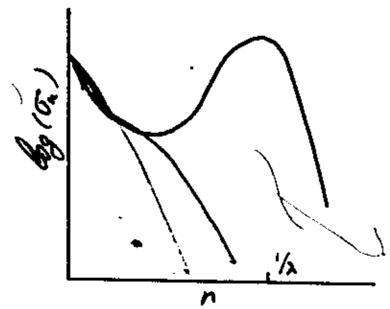
  E.g. $\quad e^+e^- \to nH$



- In s-channel all graphs have same sign

$$A_n \sim n! \, (c\lambda)^{\frac{n}{2}}$$

Cornwall '9
Goldberg '9



- $E \ll \frac{m}{\lambda}$
- $E \sim \frac{m}{\lambda}$
- $E \gg \frac{m}{\lambda}$

Can it be that $\sigma(n \gg 1/\lambda) \gtrsim \sigma(n \sim 1)$??

- Multi-boson amplitudes in $\lambda\phi^4$ theory

- Solvable (hence non-physical) case:

Off-shell n-particle amplitudes at $\varepsilon_i = \vec{p}_i = 0$

$A_n =$ 

$$A_n = \frac{\int \left(\int \sigma(x)\, d^dx\right)^n e^{-S[\phi]} \mathcal{D}\phi}{\int e^{-S[\phi]} \mathcal{D}\phi} \Big|_{\text{connected}}$$

$\sigma(x) = \phi(x) - \langle\phi\rangle$

$$A_n = \left(\frac{d}{dj}\right)^n \ln Z(j) \Big|_{j=0}$$

$Z(j) = \int \exp\left[-S[\phi] + \int j\, \sigma(x)\, d^dx\right] \mathcal{D}\phi\ ;\ j = \text{const}$

$S[\phi] = \int \left(\frac{1}{2}(\partial\phi)^2 + V(\phi)\right) d^dx\ , \quad V(\phi) \to V(\phi) - j\sigma$

$A_n \sim n! \left(\frac{1}{j_c}\right)^n$ where j_c nearest to zero singularity of $\ln Z(j) = -E(j)$ in the complex j plane.

- Unbroken symmetry $(\phi \to -\phi)$

$V(\phi) = \frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4$

Singularity in $E(j) = V(\phi) - j\phi\ \Big|_{\frac{dV}{d\phi} = j}$

<u>Classically</u>: collision of solutions to $\frac{dV}{d\phi} = j$:

$\lambda\phi^3 + m^2\phi = j$

roots collide when also $V''(\phi) = 0$: $3\lambda\phi^2 + m^2 = 0$

$\phi_c = \pm i \frac{m}{\sqrt{3\lambda}} \quad \Rightarrow \quad j_c = \pm i \sqrt{\frac{4}{27}} \frac{m^3}{\sqrt{\lambda}}$

$$\Rightarrow A_{n\,(\text{tree})} \sim \underbrace{n! \left(-\frac{27}{4}\frac{\lambda}{m^2}\right)^{n/2}}$$

Finite $|j_c|$

<u>Loops</u>: shift and modify singularity, but do not eliminate it $\Rightarrow n!\,\lambda^{n/2}$ persists

$E(j)$ from Coleman-Weinberg potential!

• Spontaneously broken symmetry

$$V(\phi) = -\frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4$$

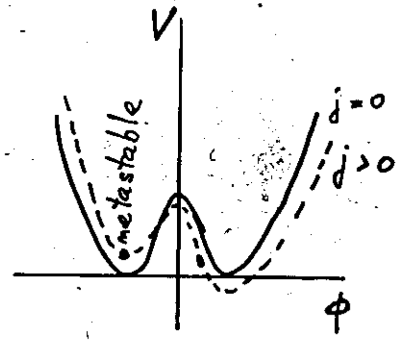
Classically $\langle \phi \rangle = \pm v$, $v = \frac{m}{\sqrt{\lambda}}$

Consider scattering in vacuum with $\langle\phi\rangle = -v$

$\sigma = \phi + v$

$$j_c = \sqrt{\frac{4}{27}}\,\frac{m^3}{\sqrt{\lambda}} \;, \quad A_n \sim n!\left(\frac{27}{4}\frac{\lambda}{m^2}\right)^{\frac{n}{2}}$$

Non-perturbatively



E(j) develops branch point at $j=0$ ("nearest to zero singularity")

$$\text{Decay rate} = 2\,\mathrm{Im}\,E(j) \sim \exp\left[-\frac{2^{\frac{d-1}{2}}}{3^d}(d-1)^{d-1} V_d \,\frac{m^{2d+1}}{\lambda^{\frac{d+1}{2}} j^{d-1}}\right]$$

$\left(V_d = \pi^{d/2}/\Gamma(\tfrac{d}{2}+1)\right)$

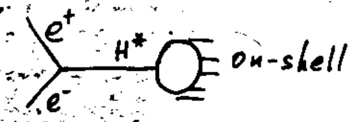
$$\Rightarrow A_n \sim \left(\frac{nd}{d-1}\right)!\,[c(d)]^n \left(\frac{\lambda}{m^2}\right)^{\frac{n}{2}} \left(\frac{\lambda}{m^{4-d}}\right)^{n/d-1}$$

$$c(d) = \left[\frac{3^d}{2^{2d-1}}\,\frac{\Gamma(d/2)}{d^{d-1}\pi^{d/2}}\right]^{\frac{1}{d-1}}$$

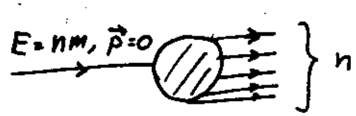
A_n corresponds to contribution of $\frac{n}{d-1}$ loops.

- Production of on-shell multiparticle states

$1^* \to n$ process



• Threshold



$\varepsilon_i = m$
$\vec{p}_i = 0$

• Tree level

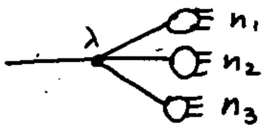
Methods

- Landau WKB method (up to $O(\frac{1}{n^2})$) | M.V. '90
- Recursion equations (exact) | M.V. '92, Argyres, Kleiss & Papadopoulos '92
- Generating field (exact) | L. Brown '92

- Recursion equations

Unbroken $\lambda\phi^4$: $V(\phi) = \frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4$

$$= \sum_{\substack{n_1+n_2+n_3=n \\ (\text{odd } n_1, n_2, n_3)}} \frac{n!}{n_1!\,n_2!\,n_3!}$$



$a(n) = \langle n | \phi(0) | \text{vacuum} \rangle$

$$(n^2-1)\,a(n) = \frac{\lambda}{m^2} \sum \frac{n!}{n_1!\,n_2!\,n_3!}\, a(n_1)\, a(n_2)\, a(n_3)$$

Solution

$$a(n) = n!\left(\frac{\lambda}{8m^2}\right)^{\frac{n-1}{2}}$$

| M.V. '92

Regular method (Füss, 1791):

Generating function

$$f(z) = \sum_{n=0}^{\infty} \frac{z^n}{n!}\, a(n) \quad ; \quad a(n) = \left(\frac{d}{dz}\right)^n f(z)\Big|_{z=0}$$

Rec. eqns. equivalent to

$$z^2 \frac{d^2}{dz^2} f(z) + z\frac{d}{dz} f(z) - f(z) = \frac{\lambda}{m^2} f^3(z)$$

Solution (satisfying $f'(0) = 1$)

$$f(z) = \frac{z}{1 - \frac{\lambda}{8m^2} z^2} \quad \Rightarrow \quad \boxed{a_n = n!\left(\frac{\lambda}{8m^2}\right)^{\frac{n-1}{2}}}$$

$n = 2k+1$

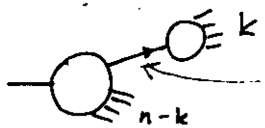
- Broken symmetry

$$U(\phi) = \frac{\lambda}{4}(\phi^2 - v^2)^2$$

$$f(z) = -v\,\frac{1 + z/2v}{1 - z/2v}$$

$$\boxed{a_n = n!\,(2v)^{1-n}}$$

- Tree graphs above threshold



$$\frac{1}{(p_1+\dots p_k)^2 - m^2} = \frac{1}{s_k - m^2}$$

if all energies are $< \omega_0$: $p_i^0 < \omega_0$,

then $s_k < k^2\omega_0^2$

$$\frac{1}{s_k - m^2} > \frac{1}{k^2\omega_0^2 - m^2} > \frac{1}{\frac{9}{8}\omega_0^2 (k^2-1)}$$

$$k_{min} = 3 \qquad \frac{8}{9} = \frac{k_{min}^2 - 1}{k_{min}^2}$$

$$\Rightarrow \quad a(n) > a(n)\Big|_{m^2 \to \frac{9}{8}\omega_0^2}$$

Provides lower bound on cross-section, which grows with energy beyond unitarity lim.

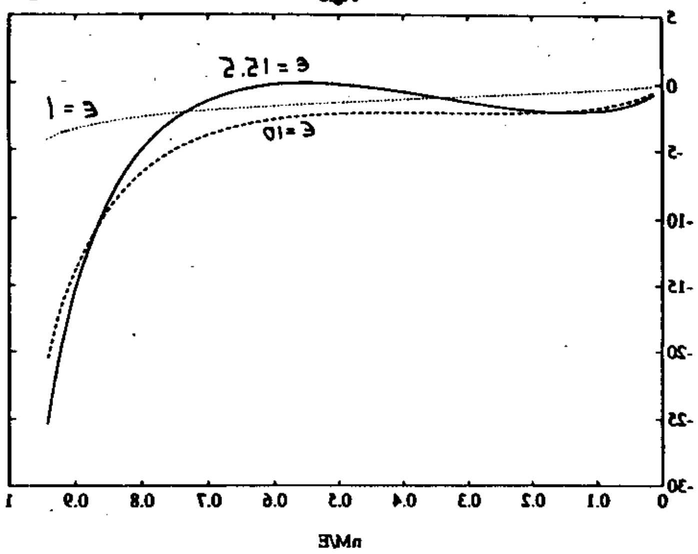
Lower bound on $\sigma_n = \int |A(1\to n)|^2 d\Phi_n$
as given by tree graphs.

$$\sigma_n > \exp\left(\frac{4\pi^2 c}{\lambda} f\left(\frac{E}{E_0}, \frac{nm}{E}\right)\right)$$

$E_0 = \frac{4\pi^2 c\, m}{\lambda}$

$c = 9$ no SSB
$c = \frac{8}{3}$ SSB

Fig.2



In minimal SM:

$$E_0 = \frac{32\pi^2 M_H}{3\lambda} \simeq 65\ \text{TeV}\left(\frac{200\ \text{GeV}}{M_H}\right)$$

$$E_{crit} \simeq 15.5\, E_0 \simeq 1000\ \text{TeV}\left(\frac{200\ \text{GeV}}{M_H}\right)$$

M.V. '92
AKF '92

- Generating field technique | Brown '92

LSZ reduction formula

$$\langle n|\varphi(x)|0\rangle = \left(\prod_{a=1}^{n} \lim_{p_a^2\to m^2} \int d^4x_a\, e^{ip_a x_a}(m^2-p_a^2)\frac{\delta}{\delta\rho(x_a)}\right)\langle 0|\varphi(x)|0\rangle^\rho \Big|_{\rho=0}$$

$$U(\varphi) \to U(\varphi) - \rho(x)\varphi(x)$$

Set $\vec{p}_a = 0$, $\omega_a = \omega$

$$\rho(t) = \rho_0(\omega)\, e^{i\omega t}$$

$$\Rightarrow\ (m^2-p_a^2)\frac{\delta}{\delta\rho(x_a)} \to (m^2-\omega^2)\frac{\delta}{\delta\rho(t)} = \frac{\delta}{\delta z(t)}$$

where $$z(t) = \frac{\rho_0(\omega)e^{i\omega t}}{m^2 - i\epsilon - \omega^2}$$

(coincides with linear response of the field)

Limit $\omega\to m$, $\rho_0(\omega)\to 0$ so that
$z(t)$ - finite: $z(t) \to z_0 e^{imt}$

Tree graphs $\leftarrow$ classical solution for $\langle 0|\varphi(x)|0\rangle^\rho$

Since $\rho\to 0$ the classical equ. is

(unbroken symmetry)

$$\partial^2\phi + m^2\phi + \lambda\phi^3 = 0 \qquad *$$

We need solution, which expands in positive powers of e^{imt}, $\phi(t,\vec{x}) = \phi(t)$

$$\phi_0(t) = \frac{z(t)}{1-\frac{\lambda}{8m^2}z^2(t)} \qquad t \to i\tau$$

$$\langle n|\phi(0)|0\rangle = \left(\frac{\partial}{\partial z}\right)^n \phi_0\Big|_{z=0} = n!\left(\frac{\lambda}{8m^2}\right)^{\frac{n-1}{2}}$$

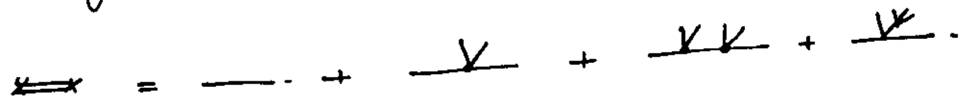
- Equation * is the same as for generating function $f(z)$ if in the latter z is identified as $z = z_0 e^{imt}$

- One-loop

$$\langle 0|\varphi(x)|0\rangle^P = \phi_0(t) + (\text{one loop})$$

$\phi_1 = $ (one-loop) $=$ 

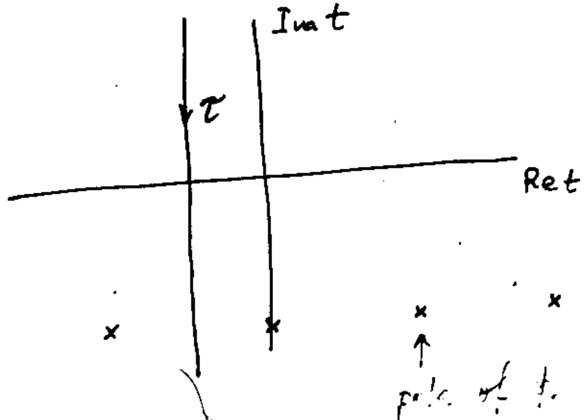
 - exact Green's function in background ϕ_0



$$(\partial^2 + m^2 + 3\lambda\phi_0(x)^2)\, G(x,x') = -i\delta(x-x')$$

$\phi_0(t)$ in complex t plane

$$\phi_0 = \frac{z(t)}{1-\frac{\lambda}{8m^2}z^2(t)}, \quad z(t) = z_0 e^{i}$$



Find G and ϕ_1 on the τ axis,

$\sqrt{\frac{\lambda}{8m^2}}\, z(t) = i e^{m\tau}$. Then $\phi(\tau) = \sum c_n e^{nm\tau}$, $\tau \to$

$$\langle n | \varphi(0) | 0 \rangle = n!\, c_n .$$

In terms of τ: $\quad \phi_0 = \sqrt{\frac{2}{\lambda}}\, \frac{i}{\cosh \tau}$ $\quad$ | m is set as m = 1

$$G(\tau', \vec{x}'; \tau, \vec{x}) = \int G_\omega(\tau'; \tau)\, e^{i\vec{k}(\vec{x}-\vec{x}')} \frac{d^3\vec{k}}{(2\pi)^3}$$

$$\omega = \sqrt{\vec{k}^2 + 1}$$

$$\left(-\frac{d^2}{d\tau^2} + \omega^2 - \frac{6}{(\cosh \tau)^2} \right) G_\omega(\tau, \tau') = \delta(\tau - \tau')$$

Well-known problem of Quantum Mechanics

Results:

$$\phi_{0+1}(t) = \frac{z(t)}{1 - \left(\frac{\bar\lambda}{8\bar m^2}\right) z^2(t)} \left(1 - \frac{3\lambda}{4} F \frac{\left(\frac{\lambda}{8m^2}\right)^2 z^4(t)}{\left(1 - \left(\frac{\lambda}{8m^2}\right) z^2(t)\right)^2} \right)$$

$$\langle 2k+1 | \phi(0) | 0 \rangle = (2k+1)! \left(\frac{\bar\lambda}{8\bar m^2} \right)^k \left(1 - k(k-1) \frac{3^{3/2}\lambda}{16\pi^2} \left(\ln \frac{2+\sqrt3}{2-\sqrt3} - i\pi \right) \right)$$

$\bar\lambda$, $\bar m$ - renormalized λ and m.

Broken symmetry

$$\langle n | \phi(0) | 0 \rangle = n! \left(\frac{-1}{8\bar 8} \right)^{n-1} \left(1 + n(n-1) \frac{\sqrt3\, \lambda}{8\pi} \right)$$

- Nullification

The factor F in 1-loop term corresponds to

$$F = \frac{\sqrt3}{2\pi^2} \left(\ln \frac{2+\sqrt3}{2-\sqrt3} - i\pi \right)$$

Is a consequence of nullification: on-shell amplitudes of $2 \to n$ are zero at threshold for $n > 4$.

For SSB case the one-loop correction is real $\Rightarrow$ $A(2 \to n) = 0$ for $n > 2$.

This is due to the behavior of Green's function in the potential

$$-\frac{N(N+1)}{(\cosh \tau)^2}$$

with N-integer.

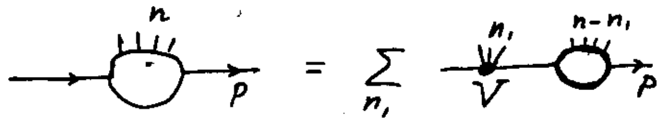
- Deriving nullification

Consider: $V_{int} = \frac{\xi}{2}\phi^2\chi^2$

$D_n(p) =$ 

$q = (m, \vec{0})$

Recursion Equs.:



$V_n = \langle n | V(\phi(0)) | 0 \rangle$, $V(\phi) = \xi\phi^2$

$$D_n(p) = \frac{1}{(p+nq)^2 - m_{0\chi}^2} \sum_{n_1} \frac{n!}{n_1!(n-n_1)!} V_{n_1} D_{n-n_1}(p)$$

$$D_0(p) = \frac{i}{p^2 - m_{0\chi}^2 - V(v)}$$

$$\mathcal{V}(z) = \sum \frac{z^n}{n!} V_n \quad , \quad \mathcal{D}(p;z) = -i \sum_{n=0} \frac{z^n}{n!} D_n(p)$$

$$\mathcal{D}(p;0) = D_0(p)$$

R.E. equivalent to

$$\left(m^2 z^2 \frac{d^2}{dz^2} + (2(p\cdot q) + m^2) z \frac{d}{dz} + p^2 - m_{0\chi}^2 - \mathcal{V}(z)\right)\mathcal{D}(p;z) = 1$$

Unbroken symmetry for ϕ: $\phi(z) = \frac{z}{1 - \frac{\lambda}{8m^2} z^2}$, $\mathcal{V}(z) = \xi\phi^2$

$$\left(m^2 z^2 \frac{d^2}{dz^2} + (2(p\cdot q) + m^2) z \frac{d}{dz} + p^2 - m_\chi^2 - \frac{\xi z^2}{\left[1 - \frac{\lambda}{8m^2} z^2\right]^2}\right)\mathcal{D}(p;z) = 1$$

Introduce dimensionless $\epsilon = \frac{(p\cdot q)}{M^2}$ (= $\frac{\text{energy of outg}}{m}$

$\omega = \frac{\sqrt{\vec{p}^2 + m_{0\chi}^2}}{M}$. $\varepsilon^2 - \omega^2$: on shell .

define $D(p;z) = y^{-\epsilon/2} f(p;y)$ with $y = -\lambda z^2/(8m$

$$\left(4y^2 \frac{d^2}{dy^2} + 4y\frac{d}{dy} - \omega^2 + 8\frac{\xi}{\lambda}\frac{y}{(1+y)^2}\right) f(p,y) = \frac{y^{\epsilon/2}}{m^2}$$

$y = e^{2\tau}$ makes this

$$\left(\frac{d^2}{d\tau^2} - \omega^2 + \frac{N(N+1)}{(\mathrm{ch}\,\tau)^2}\right) f(p;y) = \frac{e^{\epsilon\tau}}{m^2}$$

$N(N+1) = 2\frac{\xi}{\lambda}$.

Solution(s) to homogeneous equ.:

$F(-N, \omega - N, \omega + 1, -y)$ and $(\omega \to -\omega)$

regular at $y = 0$ regular at $y \to \infty$

N-integer $\Rightarrow$ polynomial (Jacobi)

$$F_N(\omega, y) = \frac{\Gamma(1+\omega)}{\Gamma(N+1+\omega)} y^{-\frac{\omega}{2}} (1+y)^{N+1} \frac{d^N}{dy^N}\left(\frac{y^{\omega+N}}{(1+y)^{N+1}}\right)$$

$$\mathcal{D}(p, z(y)) = \frac{y^{-\epsilon/2}}{4\omega m^2}\left[F_N(-\omega, y)\int_0^y u^{\frac{\epsilon}{2}-1} F_N(\omega, u)\,du + F_N(\omega, y)\int_y^\infty u^{\frac{\epsilon}{2}-1} F_N(\omega, u)\,du\right]$$

On-shell amplitude $2\chi \to n\phi \Rightarrow$ residue of $D_n(p)$ at double pole $\epsilon \to -n/2$, $\omega \to -\epsilon = n/2$.

One pole from $F_N(-\omega, y)$, second from integral

$\omega = 1, 2, \dots, N \Rightarrow n \le 2N$

Other known cases

- linear σ-model

$$\frac{\lambda}{4}\left((\sigma^2+\vec{\pi}^2)^2-v^2\right)^2$$

$$A(\pi\pi \to n\sigma) = 0 \quad \text{for } n>1$$

- vector field with Higgs-generated mass

if $\dfrac{4m_V^2}{m_H^2} = N(N+1)$, N-integer >0

then

$A(V_T V_T \to nH) = 0$ for $n > N$ (all possible on-shell process)

$A(V_L V_L \to nH) = 0$ for $n > N+1$ ($n=N+1$ is the only possible on-shell)

- fermions with Higgs-generated mass

if $\dfrac{2m_f}{m_H} = N$

then

$$A(\bar{f}f \to nH) = 0 \quad \text{for } n \geq N$$

- This property is in general lost at loop level, except. $A(2\to 3)=0$ in SSB theory (in all loops).

Approximate integrals of motion?

- In ϕ^4 theories the deep reason / implications for ~~of~~ nullification unknown.

Only the case $N=1$ (e.g. σ model) can be traced to symmetry of a two-oscillator system

$$V(x,y) = \frac{\omega_1^2 x^2}{2} + \frac{\omega_2^2 y^2}{2} + \frac{\lambda}{4}(x^2+y^2)^2$$

$\omega_1 = \omega_2 \Rightarrow O(2)$ symmetry, $Q = \dot{x}y - \dot{y}x$ conserved

$\omega_1 \neq \omega_2$, still conserved "charge":

$$\lambda Q^2 + 4(\omega_1^2-\omega_2^2)\left(\frac{1}{2}\dot{y}^2 + \frac{\omega_2^2}{2}y^2 + \frac{\lambda}{4}y^4 + \frac{\lambda}{4}x^2y^2\right)$$

Libanov, Rubakov & Troitsky '93

Nullification is in general destroyed by loops

Exception: $A_{thr.}(2\to 3) = 0$ in SSB theory in all loops. (Explained below)

- Beyond 1-loop.

SSB theory of one real field ϕ.

- Calculation of $\langle n|\varphi(0)|0\rangle$ at threshold reduces to Euclidean-space problem

$$\Phi(t) = \frac{\int \left(\frac{\int \varphi(t,x)\,dx}{V}\right) e^{-S[\varphi]}\,\mathcal{D}\varphi}{\int e^{-S[\varphi]}\,\mathcal{D}\varphi}$$

$\varphi \to \pm v$ as $t \to \pm\infty$

At $t \to -\infty$

$$\Phi(t) = \sum_{n=0}^{\infty} c_n e^{nmt} \qquad (c_0 = v)$$

$$\langle n|\varphi(0)|0\rangle = n!\, c_n / c_1^n \qquad (c_1 = \langle 1|\varphi(0)|0\rangle \text{ - 1-particle normalization})$$

- Classical:

$$\Phi(t) = v \,\mathrm{th}\, \frac{mt}{2} \qquad (\text{kink})$$

$$\langle n|\varphi(0)|0\rangle = n!\,(-2v)^{1-n}$$

- One-loop

$$\Phi(t) = v\,\mathrm{th}\,\frac{mt}{2}\left(1 + \frac{b(d)\,\lambda}{\mathrm{ch}^2 \frac{mt}{2}}\right)$$

d = # of space dimensions

- General properties

$\Phi(t)$ - real $\Rightarrow$ all c_n real

$\Rightarrow \langle n|\varphi(0)|0\rangle$ - real

$$\mathrm{Im}\, A(1\to n) = A(1\to k) * A(k\to n)$$

1-loop: only $k=2$ contributes

$\Rightarrow A(2\to n)_{tree} = 0$

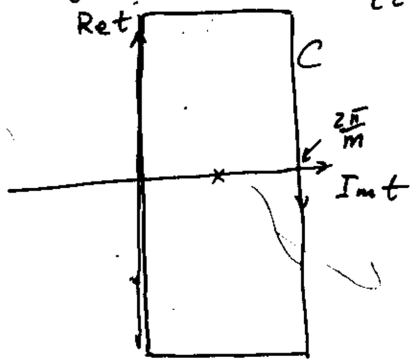
2-loops:

$$A(1\to 2)_{tree} * A(2\to n)_{1\text{-loop}} + A(1\to 3)_{tree} * A(3\to n)_{tree}$$

Special case: $A(1\to 3)_{(\text{all loops})}$

only 2-particle intermediate state contributes to $\mathrm{Im}\, A(1\to 3)$ $\Rightarrow$ $\underline{A(2\to 3)_{(\text{all loops})} = 0}$

- Analytic properties



(t - Euclidean)

$\Phi(t + \frac{2\pi i}{m}) = \Phi(t)$

$\Phi(t) \to \pm v$ as $\mathrm{Re}\, t \to \pm\infty$

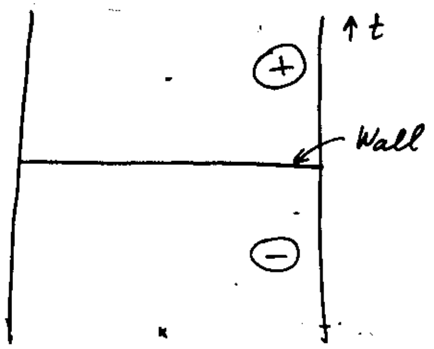
$\Rightarrow N_{poles} - N_{zeros} = 2$ inside

$\Rightarrow |c_n| \sim \left(\frac{1}{R}\right)^n$ - finite radius of convergence

$\Rightarrow n!$ growth is not eliminated by loops

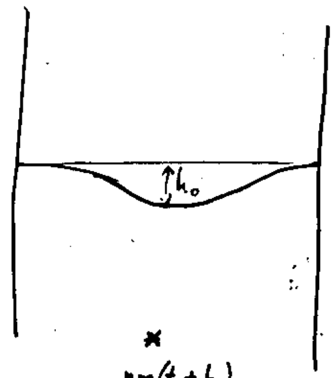
$$C_n = C_n^{(0)} (1 + a n^2 \lambda)$$

$n^2\lambda$ indicates that S.P. for c_n is driven far away from S.P. for $\Phi(t)$ at large n.



e^{nmt}

$C_n^{(0)}$



$$e^{nm(t+h_0)} = e^{nmh_0} e^{nmt}$$

$$C_n \sim C_n^{0} e^{nmh_0} e^{-\mu(A - A_0)}$$

$\mu = \frac{m^3}{3\lambda}$ - surface tension

A - area of the curved wall

$$\therefore \quad C_n \sim C_n^{(0)} e^{Eh_0 - S[h] + S_0}.$$

$E = nm$

$\mathcal{S}[h] = S[h] - Eh_0$: truncated action ($\int p\,dq$) for the wall

$$\mathcal{S}[r] = \int_{-h_0}^{0} \ell_d \mu r^{d-1} \sqrt{1 + \dot{r}^2}\, dt - Eh_0$$

$\ell_d = \frac{2\pi^{d/2}}{\Gamma(\frac{d}{2})}$ - $(d-1)$ dimensional volume of S_d.

$$s[r] = \int p_E\, dr$$

$$p_E = \frac{\ell_d \mu r^{d-1} \dot{r}}{\sqrt{1 + \dot{r}^2}}$$

$$E^2 + p_E^2 = (\ell_d \mu r^{d-1})^2$$

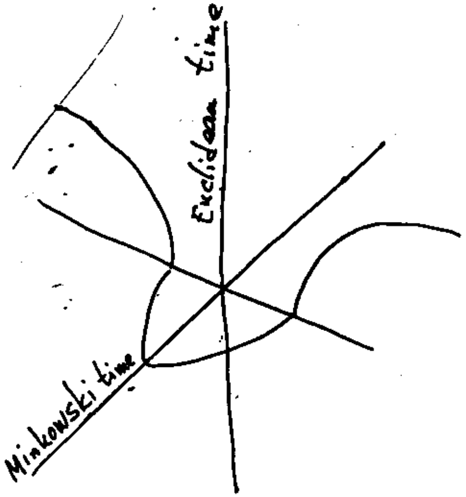
$$\Rightarrow \quad p_E = \sqrt{(\ell_d \mu r^{d-1})^2 - E^2} = \ell_d \mu \sqrt{r^{2d-2} - r_0^{2d-2}}$$

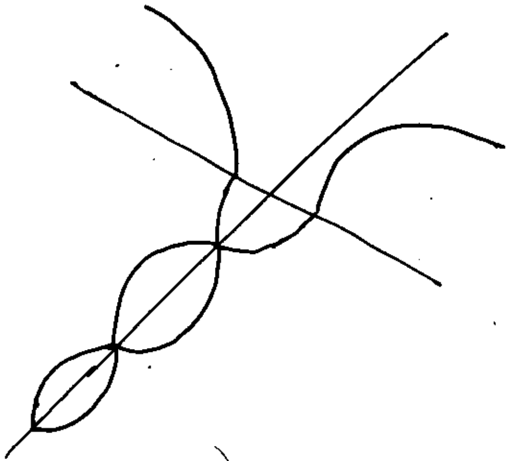
with $\quad r_0 = \left(\frac{E}{\ell_d \mu}\right)^{1/d-1}$

No real solution for p_E at $r < r_0$.

r_0 - classical turning point.

At $r < r_0$ evolution in Minkowski space





$$\langle n|\phi(0)|0\rangle \sim n! \, \frac{\exp(iI(E)/2)}{1-\exp(iI(E))} \, e^{c\cdot E\left(\frac{E}{\mu}\right)^{1/d-1}}$$

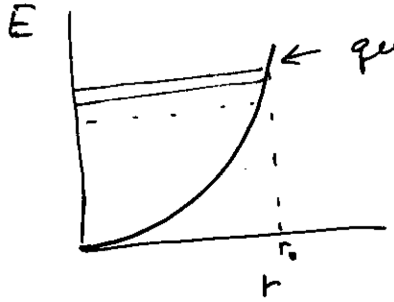
$$\langle n|\phi(0)|0\rangle \sim n! \, \frac{\exp(iI(E)/2)}{1-\exp(iI(E))} \, e^{c\cdot E\left(\frac{E}{\mu}\right)^{1/d-1}}$$

$$I(E) = \oint_{\text{period}} p_M \, dr$$

Bohr-Sommerfeld quantization

$$I(E) = 2\pi N$$

$$E = \frac{4\pi\mu r^2}{\sqrt{1-\dot{r}^2}}$$

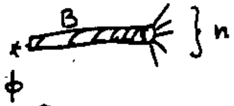


$$E_N = \text{const}\cdot \mu^{\frac{1}{d}} N^{\frac{d-1}{d}}$$

Thin wall applicable when $r_0 \gg \frac{1}{m}$

$E_N \sim \mu r_0^2 \sim \frac{m^3}{\lambda} r_0^2 \Rightarrow E_N \gg \frac{m}{\lambda}$

$\therefore$ Buble = collective resonance of $n \gg 1/\lambda$ quant



$$A(1\to n) = A(1\to B)\cdot A(B\to n) \sim e^{D(E)}$$

$$D(E) \propto E^{3/2}$$

$$\sigma_{tot}(1\to n) \approx \sigma(1\to B)$$

Landau WKB calculation of $\sigma(1\to B)$ yields:

$$\sigma(1\to B) \sim e^{-2D(E)}$$

$\therefore$ Conclusion

$A(1\to n)$ is large: $A(1\to n) \sim n!\, e^{n^{3/2}\lambda^{1/2}}$, for all n being exactly at rest, $\vec{p}_i = 0$.

However the bubles (final state interaction) provide a form factor, which cuts off momenta above the threshold $|p| \gtrsim 1/r_0 \sim 1/E^{1/2}$, so that the total σ is exponentially small.

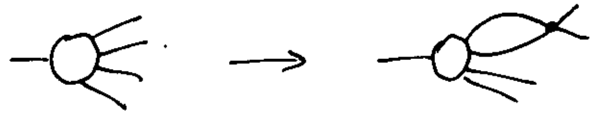
$-d = (2+1)$: infrared behavior at threshold

$\sim \ln \frac{M}{P}$

(In terms of bubble wall : surface tension depends on curvature.)

Standard R.G. techique

|Rubakov, Son '9



Summing in LLA

$$A_n = A_n^{(0)} \Big/ \left(1 + \frac{3\lambda}{8\pi}\log\frac{m}{P}\right)^{\frac{n(n-1)}{2}} \qquad \text{no SSB}$$

$$A_n = A_n^{(0)} \Big/ \left(1 - \frac{3\lambda}{2\pi}\log\frac{m}{P}\right)^{\frac{n(n-1)}{2}} \qquad \text{SSB}$$

- In either case $\log\frac{m}{P}$ dependence is too weak to kill the growth of $\sigma_{tot}(E)$