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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

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SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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PROTON DECAY, MAGNETIC MONOPOLES & ALL THAT

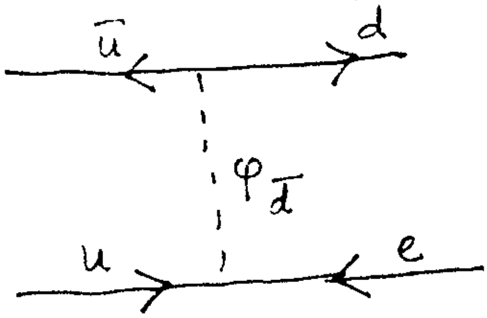
Q. SHAFI
Bartol Research Institute
University of Delaware
217 Sharp Laboratory
Delaware, 19716 Newark
U.S.A.

Please note: These are preliminary notes intended for internal distribution only.

Proton Decay, Magnetic Monopoles & All That

Dimension four p decay in MSSM

$d^c d^c u^c$; $q d^c L$



← Unacceptable

Simplest Solution

Eliminate one or more of the trilinear vertices.

Introduce Z_2 matter parity such that 'matter' superfields change sign under it

⇒ proton is stable in MSSM

⇒ lightest sparticle is stable

Note: Trilinear couplings involving 'higgs superfield' is present so that quarks & leptons can acquire mass ⇒ Z_2 unbroken

∵ higgs → + higgs

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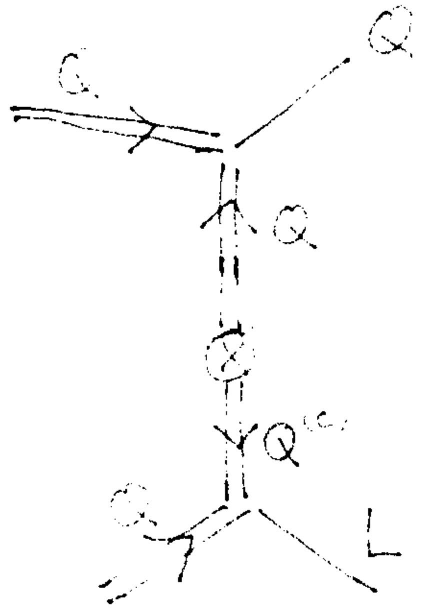
Dimension Five Proton Decay

Need $q\,u^c H_u$, $q\,d^c H_d$, $L e^c H_d$ } for masses — <u>allowed</u> by Z_2 matter parity

$\Rightarrow$ new diagrams which mediate p decay when MSSM is embedded in GUTS



$\leftarrow$ Hard to eliminate this!

In SU(5) this leads to $\tau_{p \to K^+ \bar{\nu}} \sim 10^{28}$ yrs

By assuming 'extreme' values for a no. of parameters you find that τ_p can be increased to $\sim 10^{33}$ yrs

Supersymmetric Trinification

- Motivation

- Minimal Model $\longleftrightarrow$ Gauge Hierarchy

- Proton Stability
- Monopoles
- Higgs Doublets as "Pseudo – Nambu Goldstone" bosons

- Unification of $SU(3)$'s
- Origin of M_{GUT}?
- Comparison with $SU(5)$

Motivation

Gauge Hierarchy Problem

Try resolving this in $SU(5)$

(need $\underset{\sim}{75}$, $\underset{\sim}{50}+\underset{\sim}{\overline{50}}$, ...?)

In one $SO(10)$ scheme using the D-W mechanism you need 2 (or 3) $\underset{\sim}{45}$, 2 $\underset{\sim}{10}$, $\underset{\sim}{54}$, $126+\widetilde{126}$

- Proton Decay

  Why dont we see it? (Minimal $SU(5)$)

- Not clear how $SU(5)$ or $SO(10)$ arise from superstrings

Chiral Superfields

$$\lambda \doteq \lambda_s + \theta \lambda_f + \theta\theta F_\lambda$$

Superpotential W is constructed out of chiral superfields.

From W we can derive the Yukawa couplings as well as higgs couplings. W (at renormalizable level) contains terms that are at most trilinear in the superfields

Note: Often people use the same symbol to denote the superfields & its scalar comp[illegible]

SUSY protects the higgs doublet from acquiring huge radiative corrections

In ordinary GUTS (SU(5) say) the doublet is accompanied by a color triplet

$\left(\underset{\sim}{5} \to (3,1) + (1,2) \right)$

Want the color triplet to be superheavy, especially since it mediates p decay

$\Rightarrow$ doublet - triplet splitting problem

$W \supset \bar{5} \; 24 \; 5$

$$\Rightarrow \begin{pmatrix} \bar{\underset{\sim}{3}} & \bar{\underset{\sim}{2}} \end{pmatrix} \begin{pmatrix} a & & & & \\ & a & & & \\ & & a & & \\ & & & b & \\ & & & & b \end{pmatrix} \begin{pmatrix} \underset{\sim}{3} \\ \underset{\sim}{2} \end{pmatrix}$$

$\Rightarrow \quad m_{\underset{\sim}{3}} \sim a \, , \quad m_{\underset{\sim}{2}} \sim b$

with $b = -\frac{3}{2} a$ (since $\mathrm{tr}\, \underset{\sim}{24} = 0$)

D-W (1981):

$$W \supset \underset{\sim}{10}_1 \quad \underset{\sim}{45} \quad \underset{\sim}{10}_2$$

$$\underset{\sim}{45} = \begin{pmatrix} u & & & & \\ & a & & & \\ & & a & & \\ & & & b & \\ & & & & b \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\underset{\sim}{2} \sim b \quad , \quad \underset{\sim}{3} \sim a$$

Now $a \neq 0$, $b = 0$ possible

since the trace of $U(5)$ generators

is nonvanishing.

Supersymmetric Trinification

Gauge Group $G \equiv SU(3)_c \times \underbrace{SU(3)_L \times SU(3)_R}_{\substack{\text{contains} \\ SU(2)_L \times U(1)}}$

Matter superfields :

$$\lambda_i = \begin{pmatrix} H^u & H^d & L \\ e^c & \nu^c & N \end{pmatrix}_i \sim (1, \bar{3}, 3)$$

$\uparrow SU(3)_L \downarrow$

$\longleftarrow SU(3)_R \longrightarrow$

$$Q_i = \begin{pmatrix} u \\ d \\ g \end{pmatrix}_i \sim (3, 3, 1)$$

$$Q_i^{(c)} = (u^c \; d^c \; g^c) \sim (\bar{3}, 1, \bar{3})$$

Need Higgs superfields for gauge symmetry breaking ([illegible]). Two super-multiplets needed to break G to $SU(3)_c \times SU(2)_L \times U(1)$. These transform as λ_i and denoted by λ ($\bar{\lambda}$) and λ' ($\bar{\lambda}'$) [Note the simplicity. Cf: $SU(5)$, $SO(10)$]

$$|\lambda| = |\bar{\lambda}^*| = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & N \end{pmatrix} \longrightarrow SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$\longrightarrow$ [illegible] $U(1)_Y$

$$|\lambda'| = |\bar{\lambda}'^*| = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & v^{c\prime} & 0 \end{pmatrix}$$

In analogy with MSSM

introduce $\mathbb{Z}_2$ matter parity

such that

$$\left.\begin{array}{l} \text{'higgs'} \rightarrow \lambda, \lambda' \text{ are invariant} \\ \text{'matter'} \rightarrow \lambda_i, Q_i, Q_i^{(c)} \text{ change sign} \end{array}\right\}$$

Two important consequences

i) Add'l particles g_i, H_i^u, etc. acquire masses via $Q_i Q_j^{(c)} \lambda, \ldots$

ii) Dimension 5 p decay allowed by G (& mediated by g_i) is eliminated.

(2 higgs supermultiplets)

Question

Can we extract the EW doublets from λ? (say)

(without fine tuning!)

Consider the following G invariant coupling

$$\epsilon^{\alpha\beta\gamma}\,\epsilon_{ABC}\,\lambda_\alpha^A\,\lambda_\beta^B\,\lambda_\gamma^C \quad (\equiv \lambda^3)$$

It contains the term $H^u H^d \langle N \rangle \sim M_{GUT} H^u H^d$

'μ problem'

$\Rightarrow$ Gauge Hierarchy problem

Simplest Solution : Eliminate this term either by imposing some add'l discrete symmetry, say Z_2, or an R-symmetry.

(you really need $Z_4 \supset Z_2^m$)

matter parity; unbroken

Bottom Line

With discrete $Z_3 \times Z_4$ one can suppress the λ^3 term to an adequate level

$$\lambda^3 \, (\lambda' \bar{\lambda}')^3 / M^6$$

$$\Rightarrow \sim \mathcal{O}(\text{TeV}) \, H^u H^d$$

Under R: $\quad \lambda \rightarrow e^{-\frac{iR}{3}} \lambda$

$$W \rightarrow e^{iR} W$$

Of course, we dont want too many 'light' particles so require

$$\bar{\lambda} \rightarrow e^{\frac{iR}{3}} \bar{\lambda}$$

$$\lambda', \bar{\lambda}' \rightarrow e^{\frac{iR}{3}} (\lambda', \bar{\lambda}')$$

Let's look at the 'μ term'. The leading contribution comes from

$$\lambda^3 \, (\lambda' \bar{\lambda}')^3 / M^6$$

$\sim$ Planck scale

which gives rise to $\mu \sim$ TeV

to break $G \rightarrow 3\text{-}2\text{-}1$

So with a minimal number (2) of higgs superfields you can solve this hierarchy (μ) problem (Try doing this with $SU(5)$ or $SO(10)$!)

To summarize the 'minimal higgs'

Scenario :

- 'Resolution' of the hierarchy problem
- Stable proton
- 'Low energy' spectrum consists of
  MSSM + $(\bar{\underset{\sim}{5}} + \underset{\sim}{5})$ of $SU(5)$
- $\tan\beta \approx m_t/m_b$ (H^u, H^d reside in λ)
- Stable LSP
- Massive neutrinos
- 'Triply' charged monopoles

Note : To achieve MSSM & preserve the rest include an add'l higgs supermultiplet.

Electroweak Doublets as "Pseudo Goldstones"

(cf: pions) Inoue et al, Anselm.., Berezhiani, Barbieri, Dvali, ..., [illegible], Randall ⋮

Need at least two higgs λ, λ' to break

$G \equiv (SU(3))^3$ to $SU(3)_c \times SU(2) \times U(1)$

$$\lambda = \begin{pmatrix} \left.\begin{matrix} 0 \\ 0 \end{matrix}\right\} H_1 & \left.\begin{matrix} 0 \\ 0 \end{matrix}\right\} H_2 & \left.\begin{matrix} 0 \\ 0 \end{matrix}\right\} L \\ 0 & 0 & N \end{pmatrix}, \quad \lambda' = \begin{pmatrix} \left.\begin{matrix} 0 \\ 0 \end{matrix}\right\} H_1' & \left.\begin{matrix} 0 \\ 0 \end{matrix}\right\} H_2' & \left.\begin{matrix} 0 \\ 0 \end{matrix}\right\} L' \\ 0 & \nu^{c\prime} & 0 \end{pmatrix}$$

With λ, λ' decoupled in the renormalizable superpotential (employ discrete sym) we have as global symmetry

$G_{gl} = G_{(\lambda)} \times G_{\lambda'}$. This gives rise

to a pair of $SU(2)$ doublets 'pseudogoldstones'

With SUSY unbroken:

$$P \propto L \langle \nu^{c\prime} \rangle - H_2' \langle N \rangle$$

$$\bar{P} \propto \bar{L} \langle \bar{\nu}^{c\prime} \rangle - \bar{H}_2' \langle \bar{N} \rangle$$

(2)(iii) Quantum nos. of $P \sim H_{(down)}$

" " " $\overline{P} \sim H_{(up)}$

Alas, $\overline{P}$ does not couple to the t quark at renormalizable level.

One simple resolution is to introduce an add'l 'higgs' superfield λ'' such that a linear combination of $\overline{P}$ and H_1'' (from λ'') happens to be the light state.

For instance, introduce

$$M \lambda'' \overline{\lambda}'' + f \underbrace{\overline{\lambda}''}_{\text{breaks } G_{(gl)}} \overline{\lambda}' \overline{\lambda}$$

Note: $\langle \overline{\lambda}'' \rangle = 0$

In SUSY limit, get

$$\begin{array}{c|cc} & H_1'' & \bar{P} \\ \hline \bar{H}_1'' & M & z \end{array}$$

$\Rightarrow$ $\cos\alpha\,\bar{P} - \sin\alpha\,H_1''$ is light with

$$\sin\alpha = \frac{z}{(M^2+z^2)^{1/2}}, \quad z = f\sqrt{N^2+\nu_c^2}$$

This scenario is realized by introducing appropriate discrete symmetries (e.g. $Z_3 \times Z_4$) which contain the Z_2 matter parity.

Application to Radiative Electroweak Breaking

One finds the boundary conditions:

$$\mu_1^2 = \mu_2^2 = (b^2 \cos^2\alpha + 1)$$

$$\mu_3^2 = (b^2 + 1)\cos\alpha$$

where $b = -1$ in minimal SUGRA.

If you require the parameter $r \equiv \sqrt{\mu_1^2/\mu_3^2}$ to be close to unity (note $r = 1$ for $\alpha = 0$), you find that $\tan\beta$ is also close to unity, say $\lesssim 2$.

Unification of Couplings in

$SU(3)_c \times SU(3)_L \times SU(3)_R$

Introduce :

$$C_{cR} : (\lambda, Q, Q_c) \longrightarrow (Q, \lambda, Q_c)$$

$$C_{cL} : (\lambda, Q, Q_c) \longrightarrow (Q_c, Q, \lambda)$$

'Permutation' symmetry has 6 elements and forms semidirect product with $(SU(3))^3$.

$$\lambda^a \leftrightarrow Q^a \longleftrightarrow Q^a_{(c)} \quad . \quad a = 1, 2, 3$$

$$\lambda - \bar{\lambda} \leftrightarrow Q - \bar{Q} \leftrightarrow Q_{(c)} - \bar{Q}_{(c)}$$

$$\lambda' - \bar{\lambda}' \leftrightarrow Q' - \bar{Q}' \leftrightarrow Q'_{(c)} - \bar{Q}'_{(c)}$$

Ensure that the add'l 'colored' fields are superheavy.

Proton Decay

Without C it is straightforward to eliminate dim 5 p decay. But with C present things are more interesting.

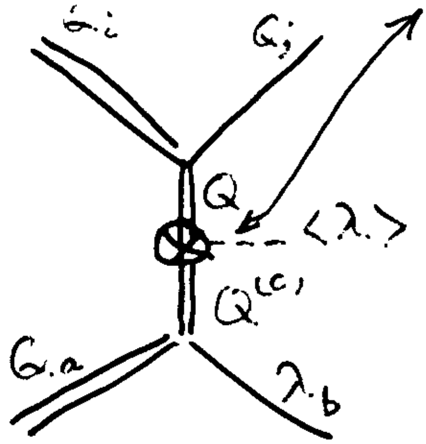
~~First~~ Approach

new fields required by the unification requirement

$$\lambda_i \lambda_j \lambda \to G_i G_j G \; ; \; \lambda^3 \to Q\,Q^{(c)}\lambda \; ; \; Q_a Q_b^{(c)} \lambda \to Q_a Q^{(c)} \lambda_b$$

needed for lepton masses

suppressed



Q, $Q^{(c)}$ without subscripts are the 'colored partners' of 'higgs' λ

Is dim 5 back??

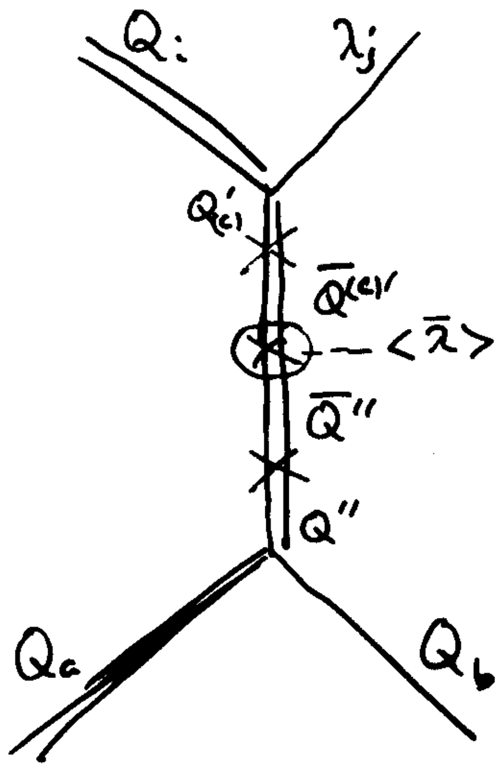
Remarkable feature:

dim 5 suppressed by M_W/M_G (gauge hierarchy! factor)

$\Rightarrow$ effectively dim 6 $\Rightarrow$ p 'stable'

p decay in the PGB model

Dimension five is present in this case. However, compared to $SU(5)$, some mixing angles from the superheavy sector are present in the amplitude.



$\overline{\lambda}\,\overline{\lambda}'\,\overline{\lambda}'' \Rightarrow \overline{\lambda}\,\overline{Q}'^{(c)}\,\overline{Q}''$

but coeffs. not related by C.

Bottom Line : p decay expected, somewhat suppressed relative to $SU(5)$

Origin of GUT scale

Suggestion:

Spontaneous breaking of R-parity (& SUSY) in the hidden sector induce R-parity noninvariant terms in the observable sector which help generate the GUT scale.

Consider for instance the coupling

$$\frac{R\,(\lambda'\bar{\lambda}')^2}{M_P^2}$$

which can stabilize the $\lambda'-\bar{\lambda}'$ vev ($\underset{\sim M_{SUSY}^{(hidden)}}{\langle R\rangle} \neq 0$)

Similar terms arise in the $\lambda-\bar{\lambda}$ sector.

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With $\langle R \rangle \sim (m_{3/2} M_P)^{1/2}$, induce a term in the potential

$$\frac{m_{3/2} \, |\lambda'|^6}{M_P^3}$$

which leads to $|\langle \lambda' \rangle| \sim (m_{3/2}/M_P)^{1/4} M_P$

$$\sim M_{GUT} \; (\ll M_P)$$

Similarly for $(\lambda - \bar{\lambda})$ vevs.

Magnetic Monopoles

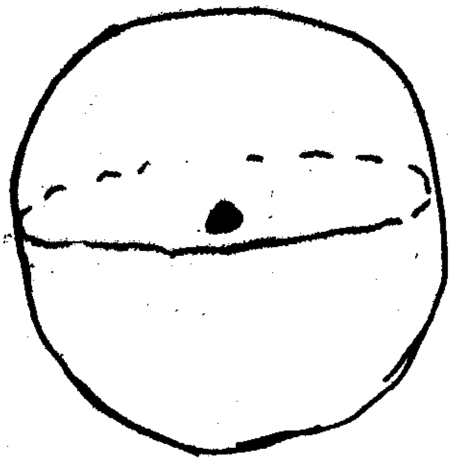
- SU(5) predicts superheavy ($\sim 10^{17}$ GeV) monopoles which carry one unit of 'Dirac charge' ($eg = \frac{n}{2}$)

- In $(SU(3))^3$ the lightest ($\sim 10^{17}$ GeV) monopole carries three units of 'Dirac charge'.

The difference has to do with the 'global structure' of the standard model embedding in these two GUTS.

Monopoles & Topology

S^1 is the equatorial ⊙



$$A_\varphi^N = g(1-\cos\theta) \quad , 0 \le \theta \le \pi/2$$

$$A_\varphi^S = g(1+\cos\theta) \quad , \pi/2 \le \theta \le \pi$$

$$A_\varphi^N(\theta=\pi/2) - A_\varphi^S(\theta=\pi/2) = 2g = \frac{1}{ie}(\partial_\varphi \Omega)\Omega^{-1}$$

$$\Omega(\varphi) = \exp(i2eg\varphi)$$

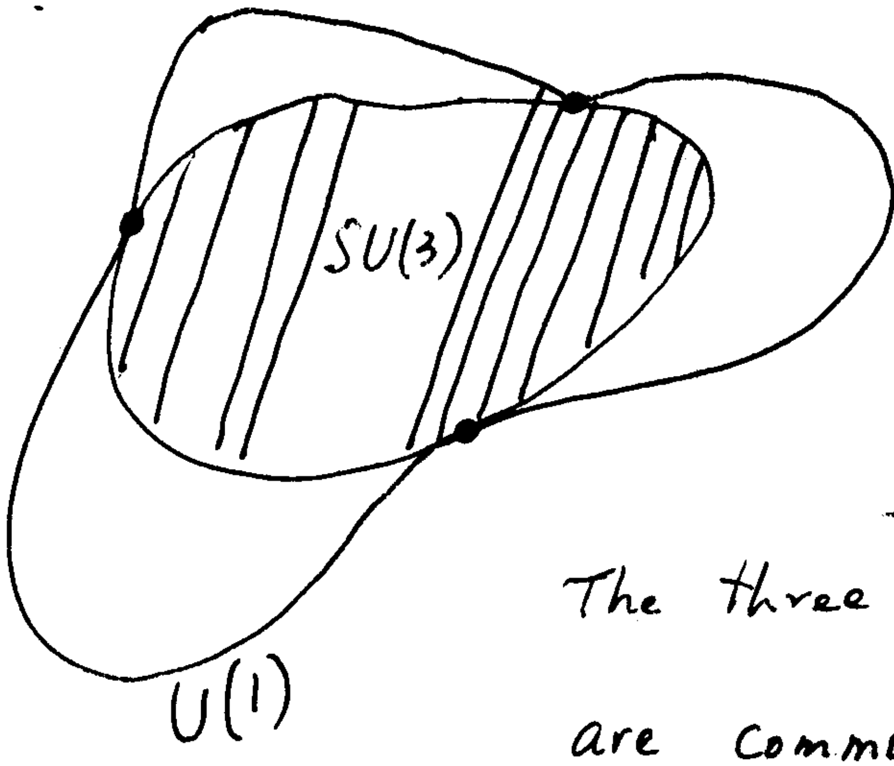
$$\Rightarrow \quad eg = n/2$$

More generally, for $G \to H$, need to compute $\pi_2(G/H) \sim \pi_1(H)$ if G is simply connected

$G = SU(2)$, $H = U(1)$

$\Rightarrow \pi_1(U(1)) \sim \pi_1(S^1) \sim \mathbb{Z}$

$SU(5) \supset SU(3)_c \times U(1)_{em}$



The three 'red points' are common & correspon[d] to the center of $SU(3)_c$. So we need not 'wrap' around the full U(1) circle.

In $(SU(3))^3$ you have to go around the full U(1) circle: $SU(3)_c \times U(1)_{em}$

Polyakov – t'– Hooft Magnetic Monopoles $\left(e\,g = \frac{n}{2} \right)$

Dirac charge $g_D = \frac{1}{2e}$ (defined so that the total magnetic flux from g is $4\pi g$; Electric flux from e is e)

Core size $R \sim M_G^{-1} \sim 10^{-30}$ cm (monopole mass 'concentrated' here)

Mass $m \sim \frac{M_G}{\alpha_G} \sim 24 \times 10^{16} \ldots \sim 10^{17}$ GeV $(\sim 10^{-5} g)$

$(M_G \sim 2 \times 10^{16}$ GeV$)$

<u>Note</u>: $R \sim \frac{1}{\alpha} m^{-1} \Rightarrow R \gg$ Compton wavelength

Almost 'classical' object

Core radius is chosen to minimize the sum of the energy stored in the magnetic field outside the core & the energy due to the scalar field gradient inside the core

$$\left.\begin{aligned} E_{mag} &\sim \frac{4\pi g^2}{R_c} \sim \frac{\pi}{e^2 R_c} \\ E_{core} &\sim 4\pi R_c v^2 \sim 4\pi \frac{M_x^2}{\alpha_G} R_c \end{aligned}\right\} \Rightarrow \boxed{R_c \sim M_x^{-1}}$$

Parker Limit

Gravitational fields $\Rightarrow$ typical galactic infall velocity of order $10^{-3}c$
(indep of mass)

Galactic magnetic field strength $B \sim 3\times 10^{-6}$ gauss

Coherence length $L \sim 10^{21}$ cm

Monopole crossing one coherence length is accelerated to

$$\boxed{v \sim \left[\frac{2 g_D B L}{m}\right]^{1/2} \sim 10^{-3} c \left(\frac{10^{17}\,\text{GeV}}{m}\right)^{1/2}}$$

Parker Bound

Mag Field of our galaxy accelerates monopoles

Energy density in field is $U \sim B^2/8\pi$

$\frac{dU}{dt} \sim \langle g n \vec{v}\cdot\vec{B} \rangle$, n = monopole density

Require that field energy not substantially depleted in time $\sim 10^8$

$$\Rightarrow \quad F = \frac{nv}{4\pi} \lesssim \frac{B}{32\pi^2 g \tau} \sim 10^{-16}\,\text{cm}^{-2}\,\text{sec}^{-1}\,\text{sr}^{-1}\left(\frac{B}{3\times10^{-6}\,g}\right)\left(\frac{10^8\,\text{yr}}{\tau}\right)$$

{ Assumes that gravitational effects on the trajectory are negligible ; not true for $m \gtrsim 10^{17}$ GeV }

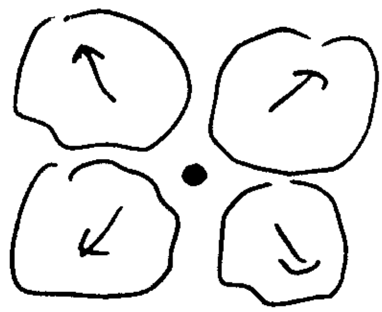
Monopoles versus Inflation

In standard cosmology and with the 'usual' higgs potential, the monopole number density far exceeds what is acceptable.

Typically

$$r_{in} \equiv n/T^3 \sim \frac{1}{\xi^3 T^3} \sim \mathcal{O}(10^{-2}) \text{ or so}$$

size of fluctuation region of higgs $\sim T^{-1}$



$\Rightarrow$ monopole number density far too large!

($r_f \sim r_{in}$ since expansion prevents m-$\bar{m}$ annihilation)

$\Rightarrow$ INVOKE INFLATION

Question : Can superheavy monopoles survive inflation ?

Note: It's possible to write down models in which lighter (~ 10^{12} – 10^{13} GeV) monopoles survive at an observable level

SUSY GUTs suggest intriguing new possibility for superheavy monopoles

$$\begin{tikzcd}
\text{Flat directions} \arrow[rr, leftrightarrow] \arrow[dr, leftrightarrow] & & \text{Origin of GUT scale} \arrow[dl, leftrightarrow] \\
& \text{Monopole suppression} &
\end{tikzcd}$$

Consider $(SU(3))^3$ with $\lambda\ (\bar{\lambda})$ fields

W contains $\lambda^3 \ \left(\equiv \epsilon^{\alpha\beta\gamma} \epsilon_{ABC} \lambda_\alpha^A \lambda_\beta^B \lambda_\gamma^C \right.$

$$\langle \lambda \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \langle N \rangle \end{pmatrix} \quad (= \langle \bar{\lambda} \rangle)$$

$\uparrow$ so SUSY is unbroken

Note that λ^3 vanishes along $\langle N \rangle$

Indeed terms $\lambda^3 \dfrac{(\lambda \bar{\lambda})^n}{M_p^{2n}}$ vanish along $\langle N \rangle$

Also derivates of these terms vanish.

$\Rightarrow$ W and V vanish (along $\langle N \rangle = \langle \bar{N} \rangle$)

$\Rightarrow$ flat dir^n

It is expected that SUSY breaking will 'lift' this degeneracy such that finally,

$$|\langle \lambda \rangle| = |\langle \bar{\lambda} \rangle| \sim 10^{16}\ \mathrm{GeV}$$

As an example consider

$$V(\phi) = -\underbrace{M_s^2}_{\sim \mathrm{TeV}} \phi^2 + \frac{K^2 \phi^{n+4}}{M_p^n} \quad (n \geq 2)$$

Note that for $T \neq 0$ there are add'l contributions $\propto \sigma T^2 \phi^2$

$\Rightarrow T > T_c (\sim M_s)$, $\langle \phi \rangle_T = 0$ and the gauge symmetry is restored!

In other words,

Superheavy monopoles first appear when the temperature drops below M_S. Moreover, at this stage the universe is dominated by the energy density in the ϕ field ($\rho_\phi^{in} \sim M_S^2 M_G^2 \gg M_S^4$)

The decay of ϕ leads to a large amount of entropy production which dilutes the 'initially large' monopole density.

The 'entropy increase' factor Δ is of order $f^{-1} M_s^{5/2} M_P^{-1/2} \langle\phi\rangle^3$

$$\left(\Gamma_\phi = f^2 M_s^3 / \langle \phi \rangle^2 \right)$$

so that:

$$r_{final} \equiv n_{in} / T_f^3$$

$$\sim \left(\frac{f}{1}\right)\left(\frac{r_{in}}{10^{-2}}\right)\left(\frac{M_s}{10^3}\right)^{5/2}\left(\frac{10^{16}}{\langle\phi\rangle}\right)^3 10^{-31}$$

$r \sim 10^{-30}$

$\Rightarrow 10^{-16}\ cm^{-2}\,s^{-1}\,sr^{-1}$

close to the 'Parker bound'

Conclusion

Superheavy monopoles 'survive' inflation in certain SUSY GUTS

	$SU(5)$	$(SU(3))^3$ (A)	$(SU(3))^3$ (B)
$\sin^2\theta_W(M_Z)$	✓	✓	✓
Proton Decay	Should see it (have seen?)	Stable	dim5 Suppressed relative to $SU(5)$
Gauge Hierarchy	?	'Easy' with discrete sym	Higgs are 'pseudogoldstones'
$\tan\beta$	?	$\approx m_t/m_b$	Of order unity when pseudosymmetry slightly broken
L.S.F.	✓	✓	✓
Neutrinos	?	massive	massive
String [illegible]	?	'Promising'	'Promising'

Challenges:

- Connection of $(SU(3))^3$ to superstrings?
- SUSY breaking mechanism?
- Fermion masses? (Exploit M_{GUT}/M_P ?)