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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
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SMR.858 - 44

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SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

12 June - 28 July 1995

4-DIMENSIONAL SUPERSTRING MODELS

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Please note: These are preliminary notes intended for internal distribution only.

4-Dimensional
Superstring
Models

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Introduction and Outline

Superstrings are a unified description of
- gravity
- gauge bosons
- chiral fermions
- scalars

which is free of ultraviolet infinities and is perturbatively exactly solvable.

Superstring Phenomenology is a young field

- No quantitative predictions at low energies (yet)
- Many important qualitative predictions and insights:

1. Weakly coupled scalar/pseudoscalar (dilaton/axion) + other scalars (moduli).
2. Gauge coupling unification $\lesssim 10^{18}$ GeV without GUT unification.
3. Global continuous symmetries are "accidental"
4. SUSY breaking scenarios
5. 3 light generations are "special" and N_{gen} is related to other properties of SM. (or MSSM)

+ GUT's, neutrinos, fermion mass hierarchies, cosmological constant, ...

Outline

Lecture #1
- What is string theory?

Lecture #2
- Supersymmetry and currents on the worldsheet
- The heterotic superstring in 10D and 4D
- Quasi-realistic 4D heterotic superstring vacua
- Examples

Lecture #3
- Gauge coupling unification
- SUSY breaking
- Global symmetries, proton decay, light fermion masses.

Lecture #4
- Free fermionic construction of 4D heterotic superstring vacua

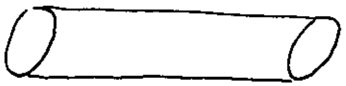
Lecture #5
- Quasi-realistic free fermionic models

1. Flipped SU(5) GUT
2. SU(5) GUT with SU(3) horizontal symmetry

What is String Theory?

You cannot understand the new predictions of string theory unless you understand how string theory is different from spacetime qft.

- Unlike e.g. gauge theories, there is **no** adequate nonperturbative formulation of superstrings
- String theory, as we use it now, is a 1st-quantized (perturbative) description of dynamics on a 2-dim. world-sheet



$X^\mu(\sigma^0, \sigma^1)$ $\mu = 0, 1, \dots D-1$

worldsheet proper-time, string coord.

X^μ maps worldsheet into D-dim spacetime (target space)

closed strings: $X^\mu(\sigma^0, 0) = X^\mu(\sigma^0, \pi)$

1st quantized. $X^\mu(\sigma^0) \longrightarrow X^\mu(\sigma^0, \sigma^1)$

2-dimensional general coordinate invariance $\Rightarrow$ world-sheet action for X^μ:

$$S = \frac{1}{4\pi\alpha'} \int_{\Lambda_g} d^2\sigma \sqrt{-\det g_{\alpha\beta}}\, g^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu} \qquad (1)$$

$\alpha, \beta = 0, 1$

$\mu, \nu = 0, \dots D-1$

$\eta_{\mu\nu}$ = Minkowski 4D metric

$g_{\alpha\beta}$ = world-sheet metric

Λ_g = 2-D surface (cylinder, torus, ...)

$\frac{1}{2\pi\alpha'}$ = string tension $\alpha' = (\text{length})^2$

- No dynamics for $g_{\alpha\beta}$:

$$\int d^2\sigma \sqrt{-g}\, R = 4\pi \chi \quad \text{Euler characteristic}$$

Integrate out $g_{\alpha\beta}$ to get Nambu-Goto action:

$$S = -\frac{1}{2\pi\alpha'} (\text{area of world-sheet})$$

- Another local symmetry

$$g_{\alpha\beta}(\sigma_0,\sigma_1) \to e^{f(\sigma_0,\sigma_1)} g_{\alpha\beta}(\sigma^0,\sigma^1)$$

= Weyl invariance

- Gauge-fix 3 local symms
  $\Rightarrow$ fixes 3 indep. components of $g_{\alpha\beta}$

e.g. in complex coordinates

$$z = \sigma_0 + i\sigma_1$$
$$\bar{z} = \sigma_0 - i\sigma_1$$

$$g_{zz} = g_{\bar{z}\bar{z}} = 0$$

$$g_{z\bar{z}} = g_{\bar{z}z} = \frac{1}{2} e^{\phi(z,\bar{z})}$$

and Liouville mode $\phi(z,\bar{z})$ drops out of the action, classically.

- Residual coord. invariance
  = conformal transformations
  (2D conformal mappings)

$$z \to f(z) \qquad \bar{z} \to f(\bar{z})$$

- So "flat-gauge" action is a 2D conformal field theory of D free bosons

$$S = \frac{1}{4\pi\alpha'} \int_{\Lambda_g} dz\, d\bar{z}\, \partial_z X^\mu \partial_{\bar{z}} X_\mu$$

The eqns of motion

$$\partial_z \partial_{\bar{z}} X^\mu(z,\bar{z}) = 0$$

$\Rightarrow$ $\partial_z X^\mu$ is an analytic function of z
$\partial_{\bar{z}} X^\mu$ is an antianalytic function
i.e. a fn. of $\bar{z}$

We say that $X^\mu(z,\bar{z})$ splits into $\left\{\begin{array}{l}\text{left-moving}\\ \text{right-moving}\end{array}\right.$ pieces

Define normal mode operators α_n^μ, $\tilde{\alpha}_n^\mu$

$$\partial_z X^\mu \sim \sum_n z^{-n-1} \alpha_n^\mu$$

$$\partial_{\bar{z}} X^\mu \sim \sum_n (\bar{z})^{-n-1} \tilde{\alpha}_n^\mu$$

defines a Hilbert space of states.

Questions:

- string theory seems trivial (!?)
- what is relation to real physics?

Problems:

- we have not yet quantized the theory
- we have ignored other world-sheet fields besides X^μ
- we have a 1st-quantized S-matrix formalism, not a 2nd-quantized string field theory $\Phi(X^\mu(\sigma_0,\sigma_1))$

Quantization:

- The price of gauge-fixing is either
  - Fadeev-Popov ghost fields on world-sheet
  - constraints (analog of Gauss Law) on Hibert space of states built up from $\alpha_n^\mu, \tilde{\alpha}_n^\mu$.

Let's quantize using constraints (canonical quantization)

Constraints: 3 indep. components of 2D energy-momentum tensor T_{ab} must annihilate physical states.

- Since $T_{z\bar{z}} = 0$ by eqn. of motion, there a 2 constraint operators:

$$T(z) \equiv T_{zz} = -\tfrac{1}{2}\partial_z X^\mu \partial_z X_\mu$$

$$\bar{T}(\bar{z}) \equiv T_{\bar{z}\bar{z}} = -\tfrac{1}{2}\partial_{\bar{z}} X^\mu \partial_{\bar{z}} X_\mu$$

- Expand these in "Virasoro" modes:

$$T(z) = \sum_n z^{-n-2} L_n$$

- After canonical quantization, <u>a suprise</u>
  $[T(z), T(z')]$ has a Schwinger term
  $\Rightarrow$ an anomaly

In modes: Virasoro algebra

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12} n(n^2-1)\,\delta_{n,-m}$$

$c = D$ is "central charge"

$c \neq 0 \Rightarrow$ an anomaly

Constraints on states:

$$L_n |phys\rangle = 0 \quad n>0$$
$$(L_0 - a)|phys\rangle = 0$$

constant "a" because of normal-ordering ambiguity

- To construct full spectrum of physical states:
  - fix residual conformal symm. by "light-cone gauge" condition $X^+(\sigma^0,\sigma^1) \propto \sigma^0$ (X^+, X^- are then gone)
  - use transverse mode ops $\alpha_n^i, \tilde{\alpha}_n^i$ to construct all states satisfying constraints

- What about the apparent anomaly?
  - Weyl anomaly: Liouville mode still coupled
  - If anomaly $\Rightarrow$ nonunitary spectrum ie. physical states with negative norm
  - For $D \neq 26$ we find nonunitary behavior but for $D=26$ anomaly is <u>absent</u>
  - In covariant path-integral quantization, world-sheet Fadeev-Popov ghosts contribute $c = -26$
  - For $D=26$ we also find "a" $= 1$ so $(L_0 - 1)|phys\rangle = 0$

$D=26$ is called "critical dimension" of bosonic string theory

<u>But</u>

- we assumed X^μ the only world-sheet fields (+ ghosts)
- we assumed only local symmetries on world-sheet are 2D coord inv. + Weyl
- we assumed that cancelling Weyl anomaly is sufficient to obtain unitary theory (in general, false)

Mass and spin of physical states:

A detail we skipped is

$$\alpha_0^\mu = \tilde{\alpha}_0^\mu = \sqrt{\tfrac{\alpha'}{2}}\, p^\mu \qquad \text{center of mass momentum of string}$$

$\Rightarrow$ an additional constraint which ties together left and right movers:

$$(L_0 - \bar{L}_0)|phys\rangle = 0$$

And from p^μ dependence we see that

$$(L_0 - 1)|phys\rangle = 0$$
$$(\bar{L}_0 - 1)|phys\rangle = 0$$

are <u>mass-shell conditions</u>

$$\alpha' M^2 |phys\rangle \equiv -\alpha' p^\mu p_\mu |phys\rangle$$
$$= \left[2(N-1) + 2(\tilde{N}-1)\right]|phys\rangle$$

where $\begin{cases} N = \sum_{n=1}^{\infty} \alpha_{-n}^i \alpha_n^i \\ \tilde{N} \end{cases}$ are occupation #'s

$$(N - \tilde{N})|phys\rangle = 0$$

Simplest physical state: $N = \tilde{N} = 0$

call it $|0, p^\mu\rangle$

$$\alpha' M^2 |0, p^\mu\rangle = -4|0, p^\mu\rangle$$

$\Rightarrow$ an on-shell scalar tachyon!

Note $|0, p^\mu\rangle$ is created from the cft vacuum by a "vertex operator"

$$|0, p^\mu\rangle = \underbrace{:e^{ip_\mu X^\mu(0,0)}:}_{\text{A local op. which creates a physical state at a point on the world-sheet}}|0\rangle$$

— Next simplest physical state: $N = \tilde{N} = 1$

$$|\Omega^{ij}, p^\mu\rangle = \alpha_{-1}^i \tilde{\alpha}_{-1}^j |0, p^\mu\rangle$$

$$\alpha' M^2 |\Omega^{ij}, p^\mu\rangle = 0$$

- symmetric part is a graviton
- trace part is a massless scalar "dilaton"
- antisymmetric part if 4D would be $\equiv$ an axion

— So this is gravity + matter in D-dims!

fixes $\alpha' \sim \frac{1}{M_{Pl}^2}$ $\quad M_{Pl} \sim 10^{19}$ GeV

Green Functions in String Theory:

Use vertex ops to create on-shell external states at points on closed 2D surfaces, e.g.

contribution to 4-pnt function

$g_{\alpha\beta}$ gauged away $\Rightarrow$ only need topological + other global parameters to define distinct 2D surfaces

topology: Riemann surfaces of genus g

N-pnt function is defined to be the simple sum over all distinct world-sheet surfaces

$$\sum_{\text{genus}} \underbrace{e^{(n+2g-2)\phi_0}} \int [dx^\mu dg]\, e^{-S} \prod_{i=1}^{n} \int d^2 z_i\, V_i(z_i)$$

this factor because $e^{-\frac{\phi_0}{4\pi}\int d^2 z \sqrt{g} R} = e^{-\chi\phi_0}$

and $\chi = 2-2g-n$

ϕ_0 is a parameter $\equiv$ vev of dilaton field

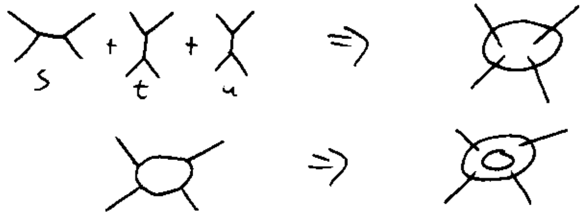
Note $\sum_{\text{genus}}$ = perturbation series with effective coupling constant

$$\frac{g_s^2}{4\pi} \equiv e^{2\phi_0}$$

– Note the dilaton vev is not determined in string perturbation theory

$\Rightarrow$ $\boxed{\text{the value of } g_s \text{ is unknown}}$

– We see roughly why string theory is free of ultraviolet divergences



– There are more string consistency conditions besides Virasoro conditions + anomaly cancellation

For $g>0$ there are additional global parameters that we must integrate over

e.g. torus complex parameter τ

2D coord inv. of global parameters $\Rightarrow$ "modular invariance of Green fns.

– There is also tree-level "modular invariance"

$\equiv$ s-t-u duality of 4-pnt. fns.

$\equiv$ vertex ops. form a mutually local closed associative algebra

$\boxed{\text{All of this needed for a unitary finite theory}}$

Next Lecture:

- We will add more local symmetry to the world-sheet (ie. supersymmetry)
- get rid of tachyon, start doing real physics
- add more world-sheet degrees of freedom to get ∞ number of string "models"

  $\equiv$ string theory perturbed around ∞ number of different backgrounds or "string vacua"

Questions I will not answer

- What is correct nonperturbative formulation of string theory?
- What is true vacuum of string theory?
- If string coupling is not small, is perturbative string theory useful?

  (A maybe answer)