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SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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LARGE SCALE STRUCTURE: THEORY VERSUS OBSERVATIONS

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Large Scale Structure: Theory vs. Observations

R. Schaefer

Outline

- Theoretical (Quick Review) Descriptions of Structure
- Observational Constraints
- Cosmological Uncertainties

Robert Schaefer

Description of Structure

Two point density correlation function

$$1 + \xi(|\vec{R}|) \equiv \frac{\langle \rho(\vec{x})\,\rho(\vec{x}+\vec{R}) \rangle_k}{(\langle \rho \rangle)^2}$$ ensemble average

$\xi(|\vec{R}|)$ represents P_r of finding a similar density a distance $|\vec{R}|$ away

Fourier Transform of $\xi(|\vec{R}|)$ is the Power Spectrum $P(k)$

$$\xi(|\vec{R}|) \equiv \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot\vec{R}}\, P(k)$$

$\xi(0)$ is the $(\text{rms})^2$ mass fluctuations

$$\xi(0) = \left(\frac{\delta M}{M}\right)^2(0) = \int \frac{d^3k}{(2\pi)^3}\, P(k)$$

$$\xi(0) = \int \frac{dk}{k}\, \frac{k^3}{2\pi^2}\, P(k)$$

Contribution to $(\text{mass fluctuation})^2$ per unit log interval in k is

$$\frac{k^3}{2\pi^2}\, P(k)$$

This quantity specifies the mean square mass or density fluctuation on the length scale k^{-1}

A more relevant quantity for comparing to observations is the filtered density contrast

- filtering removes the "noise" like ★'s, HII, ★, even individual galaxies. for a look at the large scale pattern

Let us consider a gaussian (Top hat) filter on scale R_f

$$\frac{1}{(2\pi R_f^2)^{3/2}}\, e^{-\frac{r^2}{2R_f^2}} \qquad (\text{top hat})\ \Theta(R_f - R)$$

A Gaussian (Top Hat) Filter encloses a mass

$$M = (2\pi)^{3/2} R_f^3 \langle \rho \rangle \qquad M = \frac{4\pi}{3} R_f^3 \langle \rho \rangle$$

So if we study objects of Mass M, we should use a filtering radius

$$R_f = \frac{1}{\sqrt{2\pi}}\left[\frac{M}{\langle\rho\rangle}\right]^{1/3} \quad \text{or} \quad \left[\frac{3}{4\pi}\frac{M}{\langle\rho\rangle}\right]^{1/3}$$

(Gaussian) (top hat)

$$\langle\rho_0\rangle = 1.876 \times 10^{-29} (\Omega h^2) \frac{g}{cm^3} \quad \left(= \frac{3H_0^2}{8\pi G}\right)$$

$$= 2.771 \times 10^{11} (\Omega h^2)\ M_\odot / Mpc^3$$

$$M_\odot = 1.989 \times 10^{33}\ g$$

$$1\ Mpc = 3.08568 \times 10^{24}\ cm$$

Some Objects of Study

Object	Mass	R_f (gaussian)	R_f (top hat)
galaxy	$\sim(10^{11}-10^{12})h^{-1}M_\odot$	$\sim(0.3-0.6)h^{-1}Mpc$	$\sim(0.4-0.9)h^{-1}M$,
cluster	$\sim 10^{15} h^{-1} M_\odot$	$6h^{-1}Mpc$	$10h^{-1}Mpc$
"great attractor"	$\cdot 10^{17} h^{-1} M_\odot$	$30h^{-1}Mpc$	$44\ h^{-1}Mpc$

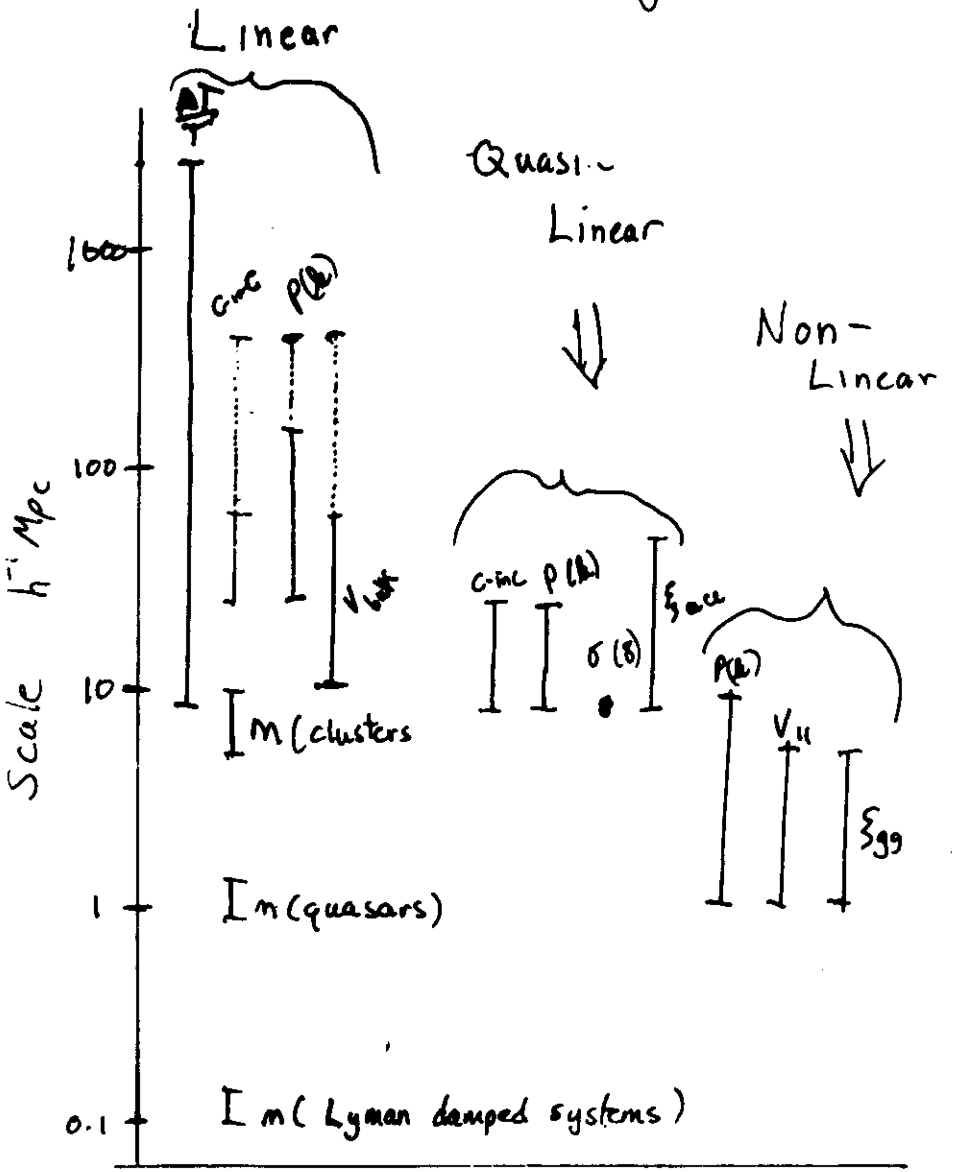
Theoretical Techniques for Comparing Models to Observations are Determined by the amplitude of $\left.\frac{\delta M}{M}\right|_{rms}$

A) $\rightarrow$ $\left.\frac{\delta M}{M}\right|_{rms} \ll 1$ Linear Perturbation Theory

C) $0.1 \lesssim \left.\frac{\delta M}{M}\right|_{rms} \lesssim 1$ non-Linear treatment or approximate non-linear (2nd order pert. theory)

B) $1 \lesssim \left.\frac{\delta M}{M}\right|_{rms}$ non-linear methods

Constraints on Large Scale Stucture

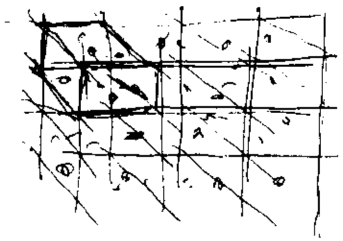


put aside

Note: there are constraints ~~[illegible]~~ which can be calculated using linear theory only.

Useful for efficient testing of different theories

Counts-in-cells



$$\left(\frac{\delta N_s}{N_s}\right)^2 = \sum_{i=1}^{N_{cell}} \frac{\left(N_i - \langle N\rangle\right)^2}{\langle N\rangle^2} \frac{1}{N_{cell}}$$

$$\frac{\delta N}{N} \propto \frac{\delta M}{M}$$

If mass traces light $\frac{\delta N}{N} = \frac{\delta M}{M}$

If galaxies are "biased" tracers

$$\frac{\delta N}{N} = b \frac{\delta M}{M}$$

Usual meaning: structures form first at highest peaks first. If long wavelength fluctuations present → leads to galaxies more clustered than the mass.

Counts-in-Cells

$$\left(\frac{\delta N}{N}\right)^2(\ell) = \frac{1}{N_{cell}} \sum_{i=1}^{N_{cell}} \frac{(N_i - \langle N \rangle)^2}{\langle N \rangle^2}$$

Linear (constant b) biasing seems to be valid only on scales where the fluctuations are linear.

If measured in redshift space (cz/H = radial coordinate), infall velocities seem to further enhance clustering (Kaiser)

$$\left(\frac{\delta N}{N}\right)^2 = b^2 \left(1 + \frac{2}{3b} + \frac{1}{5b^2}\right)\left(\frac{\delta M}{M}\right)^2$$

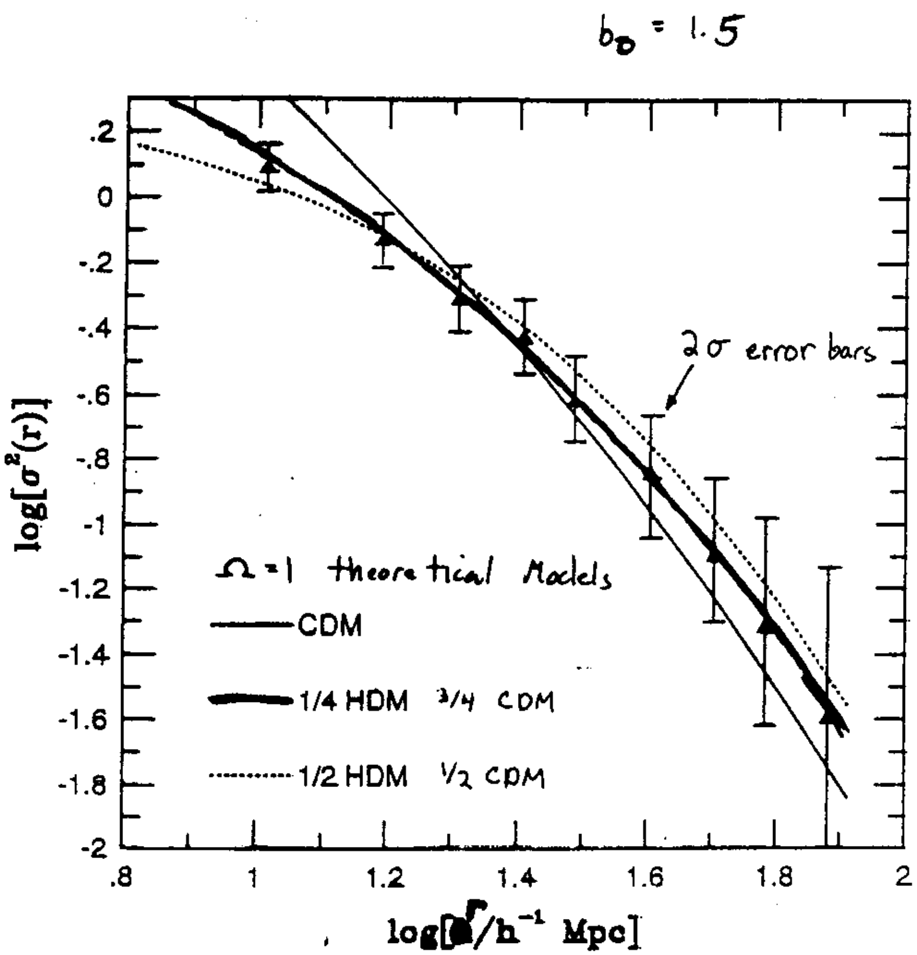
(in linear theory).

Counts-in-cells data exist for many different classes of objects: APM galaxies, IRAS galaxies, Abell Clusters, Quasars, etc.

Uncertainty in value of b allows some theoretical "wiggle" room.

- show data

APM - Stromlo Counts-in-Cells Variance (adj. Norm.)



Data from Loveday, et al., 1992

Power Spectra $P(k)$

The Power spectrum is obtained by Fourier transforming the density autocorrelation obtained in a large 3-D catalogue of galaxies.

Many estimates of $P(k)$ now available

IRAS (QDOT + Berkeley)
APM (Baugh & Efstathiou)
etc. → see Peacock + Dodds

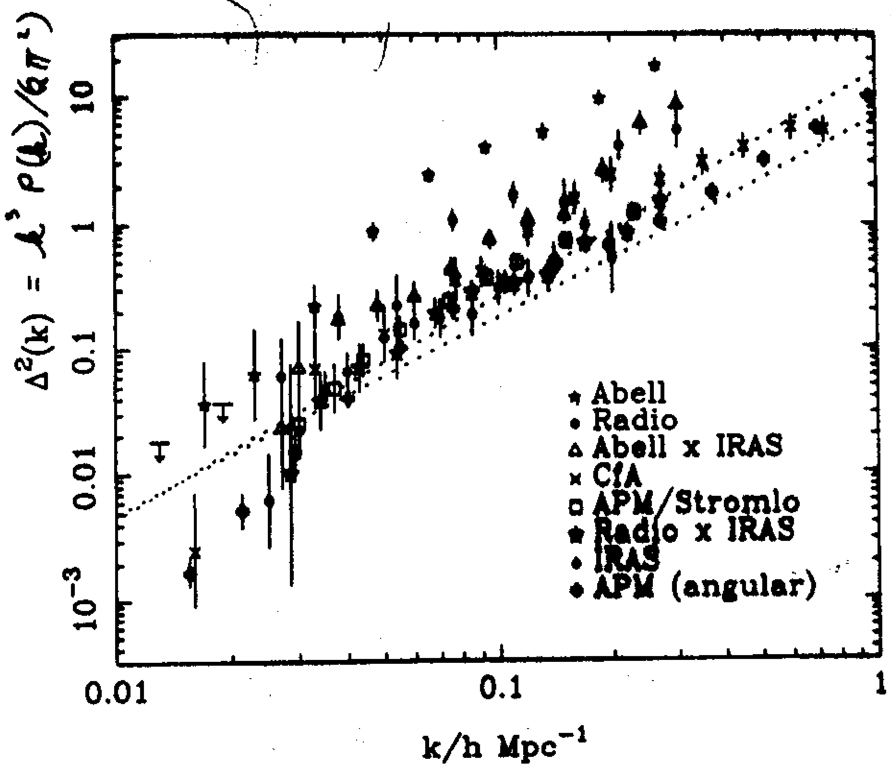
the shape of the spectrum is generally consistent among different data sets, still uncertainties

→ Power spectrum of galaxy distribution subject to the same "caveats" about biasing

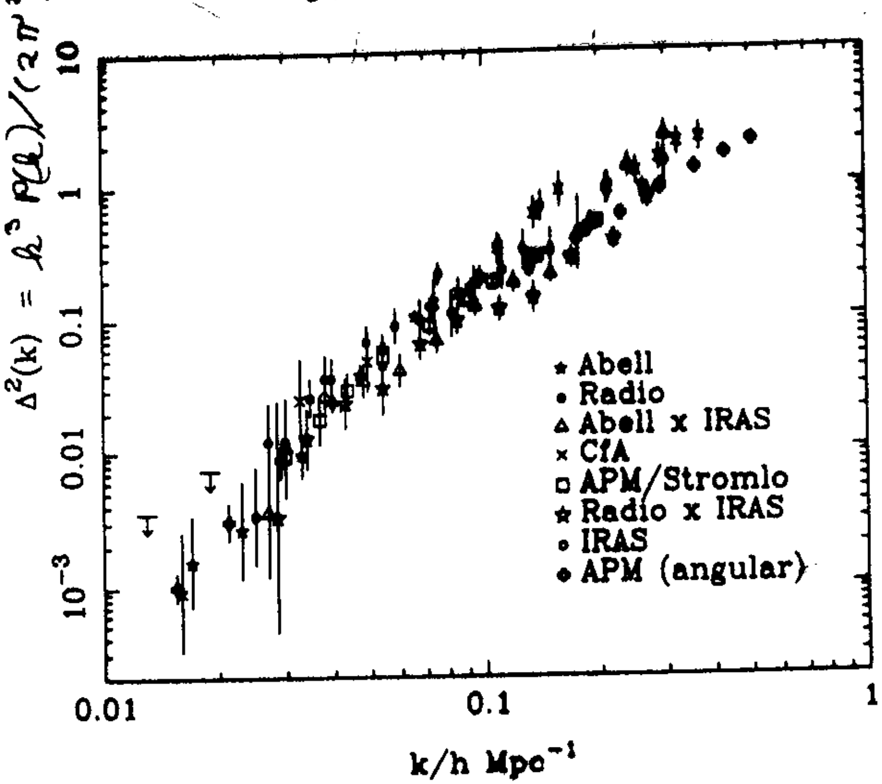
$$P_g(k) = b^2\left(1+\tfrac{2}{3b}+\tfrac{1}{6b^2}\right)P_M(k)$$

warning: distance errors can lead to correlated errors (systematic errors) in $P_g(k)$ estimates.

many $P_g(k)$ are determined using data in the non-linear regime.



bias "corrected"



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Initially $P(k, T_i) = (2\pi)^3 \Delta_H^2 k^n \left(\frac{a}{H(T_i)}\right)^4$,

today:

$$P(k, t_0) = (2\pi)^3 \Delta_H^2 k^n \left(\frac{a}{H_0}\right)^4 T^2(k, z=0)$$

Δ_H = amplitude of density perturbation when λ = Horizon size (at T)

T = "transfer function" describes growth of scale k

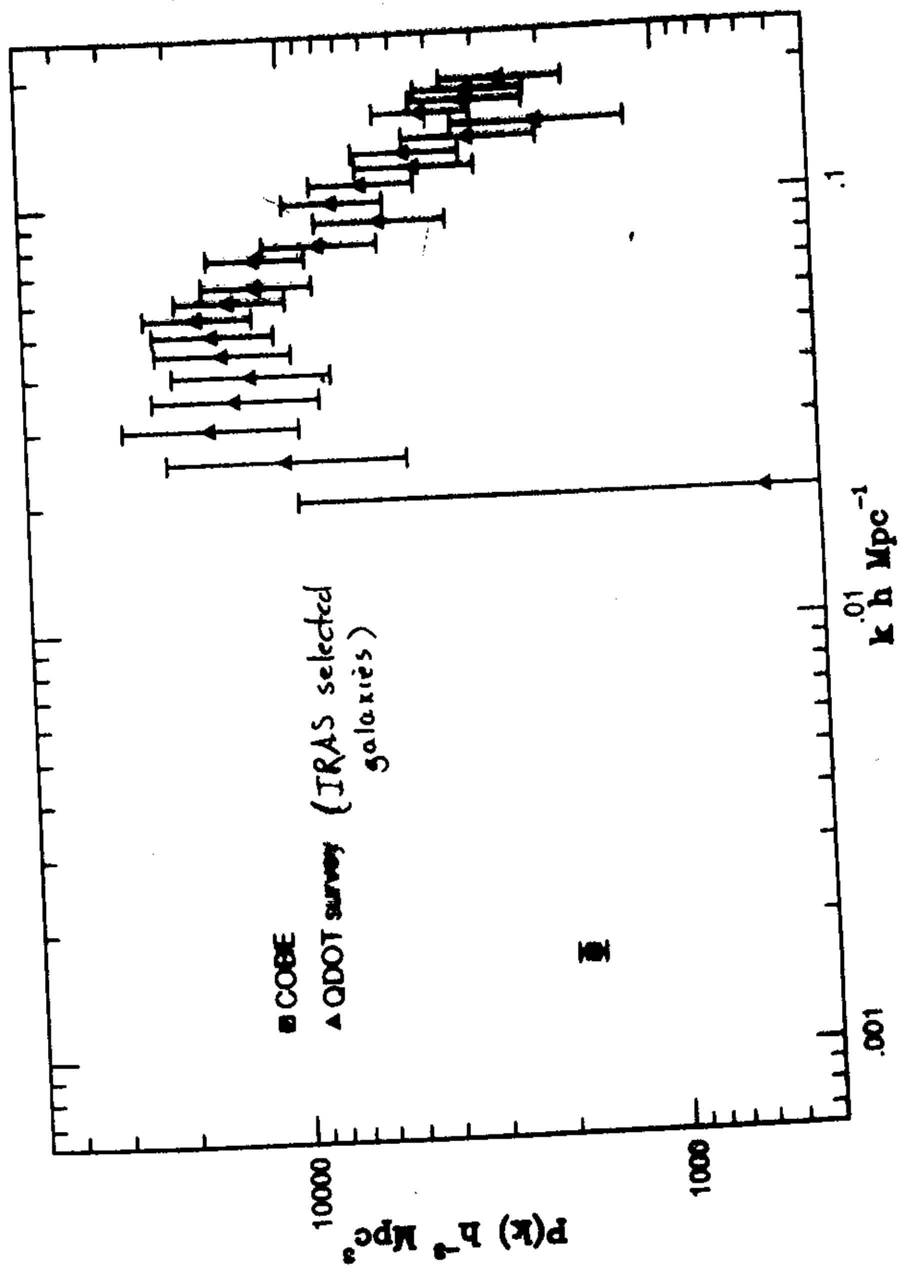
n = spectral index

H_0 = Hubble constant

Warning different authors put factors of 2's + π's in different places

P(k) IRAS

P(k) Peacock + Dodds



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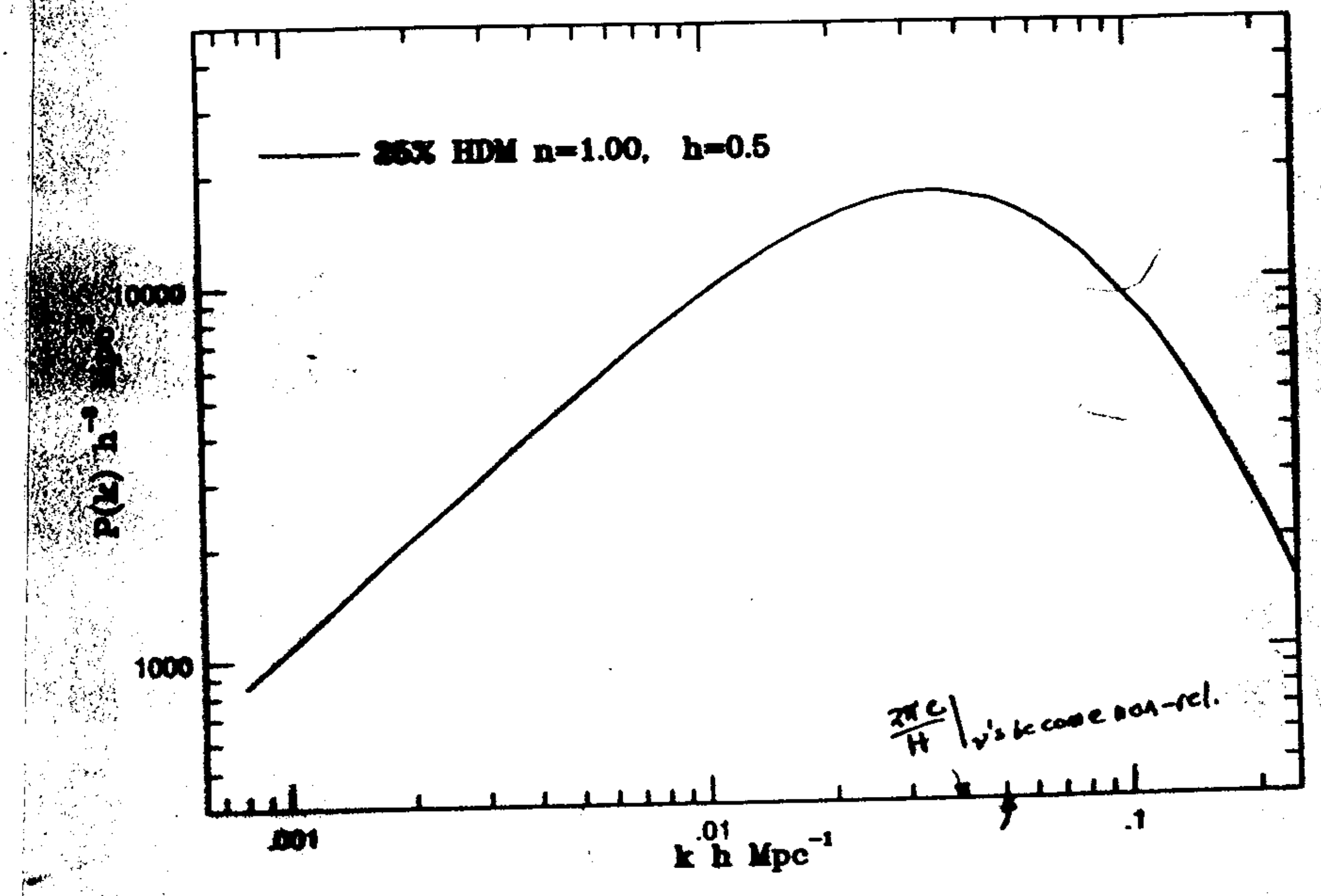
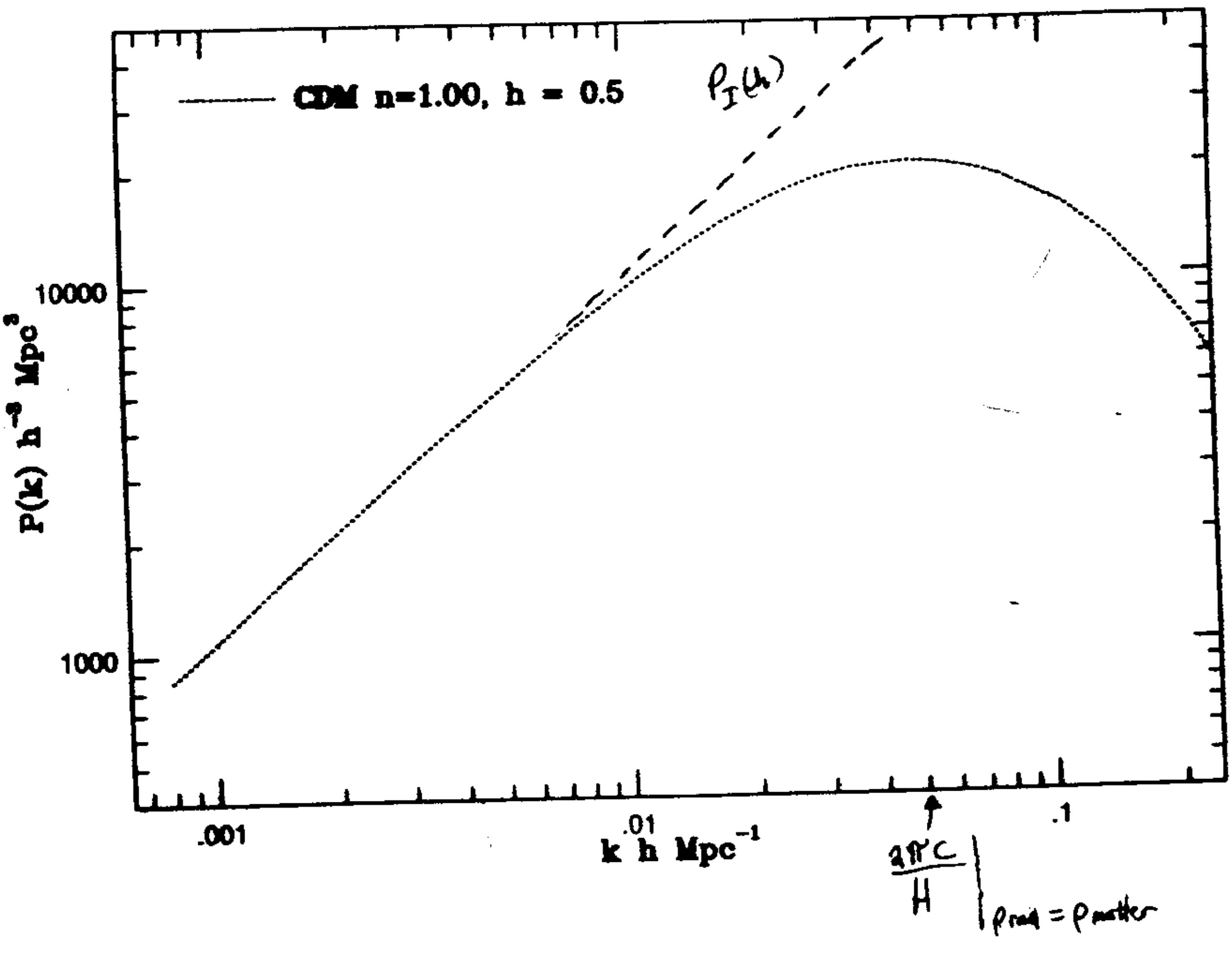


26% HDM n=1.00, h=0.5
P(k) h^-3 Mpc^3
10000
1000
.001
.01
.1
k h Mpc^-1
2πc/H | ν's become non-rel.



CDM n=1.00, h = 0.5
P_I(k)
P(k) h^-3 Mpc^3
10000
1000
.001
.01
.1
k h Mpc^-1
2πc/H | ρ_rad = ρ_matter

Bulk Velocities

In linear perturbation theory, the velocity of bulk galactic flows are linearly related to the amplitude of density fluctuations

$$\cancel{\frac{d}{dt}\frac{\delta\rho}{\rho} \nabla \cdot \vec{v}} \qquad \text{(continuity equation)}$$

$$\frac{d}{dt}\left(\frac{\delta\rho}{\rho}\right) = \frac{\nabla \cdot \vec{v}\rho}{\rho}$$

in Fourier space continuity equation is

$$\left(\frac{\dot{\delta\rho}}{\rho}\right) = - k\, v \left(\frac{\delta\rho}{\rho}\right) \qquad \left(\frac{\dot{\delta\rho}}{\rho} = \nabla \cdot (\vec{v}\rho)\right)$$

M.O.

$$V^2_{bulk}(R) = H_0^{\,2} \int \frac{d^3k}{(2\pi)^3} \frac{P(k)}{k^2} W^2(k,R)$$

vs.

$$\left(\frac{\delta M}{M}\right)^2 = \int \frac{d^3k}{(2\pi)^3} P(k)\, W^2(k,R)$$

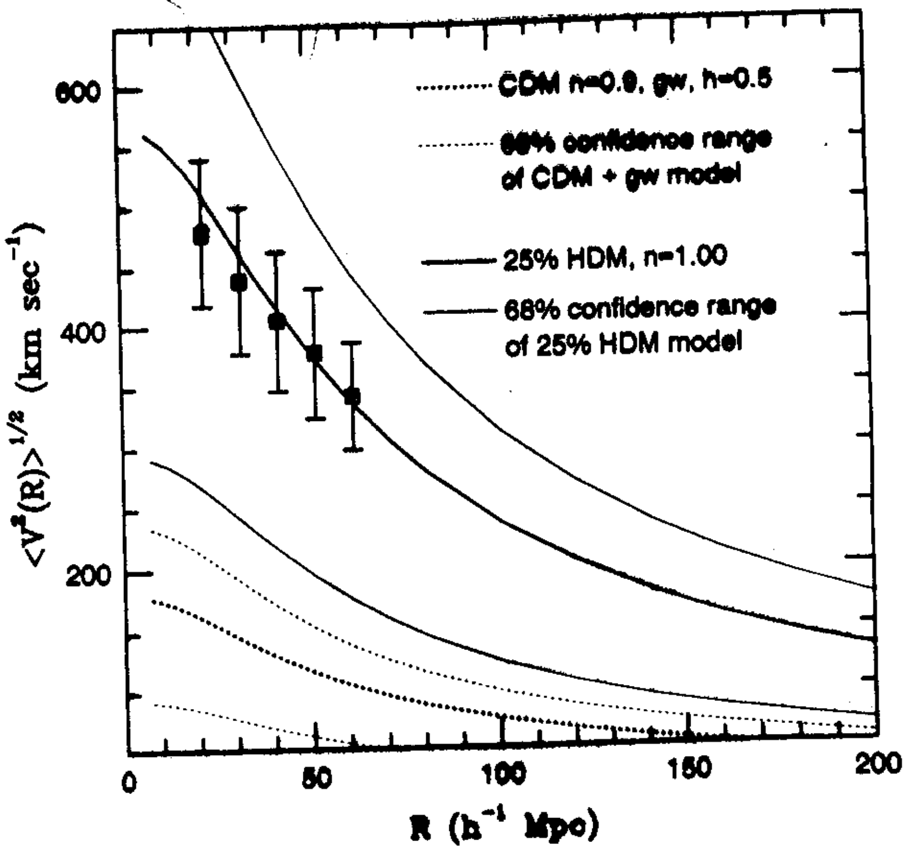
$V_{bulk}(R)$ probes larger scales ~~than~~ part of $P(k)$ than $\frac{\delta M}{M}$

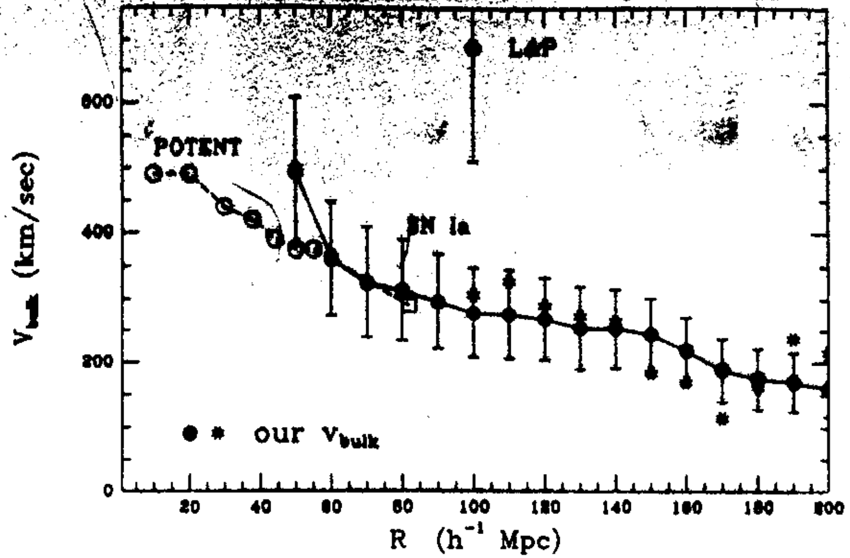
because of weighting in integral

Note: V depends on Mass, ⇒ no funny bias factors

Velocity field more difficult to construct than $\frac{\delta M}{M}$ — limited statistics

■ POTENT velocities Dekel 1994





Branchini & Plionis

Number densities

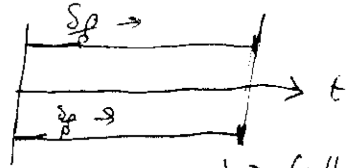
Press - Schecter : Formula for gaussian fluctuation

$$\Omega_{\text{collapsed}}(M > M(R), z) = \frac{2}{\sqrt{\pi}} \int_{\left[\frac{\delta_c}{\sqrt{2}\,\sigma(R,z)}\right]}^{\infty} dx \; e^{-x^2}$$

$$\left[\sigma(R,z) = \left[\left(\frac{\delta M}{M}\right)^2 (R,z)\right]^{1/2}\right]$$

$$= \operatorname{erfc}\left(\frac{\delta_c}{\sqrt{2}\,\sigma(R,z)}\right)$$

δ_c is some critical value of a linear perturbation amplitude which corresponds to a formed object



full theory

If use spherical collapse approximation

$\delta_c = 1.68$

$\frac{\delta\rho}{\rho}(\text{full}) = 178$

Press - Schecter (theory or Phenomenology ?)

compare to simulations

fit with $\delta_c = 1.7 \pm 0.1$ (top hat)
$\delta_c = 1.5 \pm 0.1$ (Gaussian)

Number of clusters

$$n(M,z)\,dM = \frac{\rho}{M}\,dM\,\frac{d}{dM}\,\Omega(M>M(R),z)$$

$$n(M,z) = -\sqrt{\frac{2}{\pi}}\,\frac{\rho}{M}\,\frac{\delta_c}{\sigma^2(R,z)}\,\frac{d\sigma}{dM}\,\exp\left(-\frac{\delta_c^2}{2\sigma^2(R,z)}\right)$$

For a class of objects with Mass $> M_c$

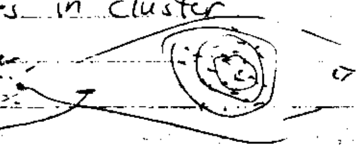
$$N(M > M_c, z) = \int_{M_c}^{\infty} dM\, n(M,z)$$

Applications [first need to know masses of objects

1) Clusters

1st need to know masses of objects

a) Virial Theorem $\langle T\rangle = \frac{1}{2}\langle V\rangle$
velocities of galaxies in cluster
$\Rightarrow \langle v\rangle \Rightarrow M_{cluster}$



b) X-Ray
(isothermal)

$\langle T_{gas}\rangle$ related to $\langle V\rangle$
assume hydrostatic equilibrium

c) Lens...

Xray & Virial M in good agreement $\Rightarrow \Omega = 0.2$
Lense M slightly higher
(up to 3x)

Large D.M. Haloes ?

Simulations of CDM say No

Mixture of CDM + HDM small increase in mass at large distances (>1.5 Mpc) due to addition of hot component but $< 50\%$

– see Primack et al.

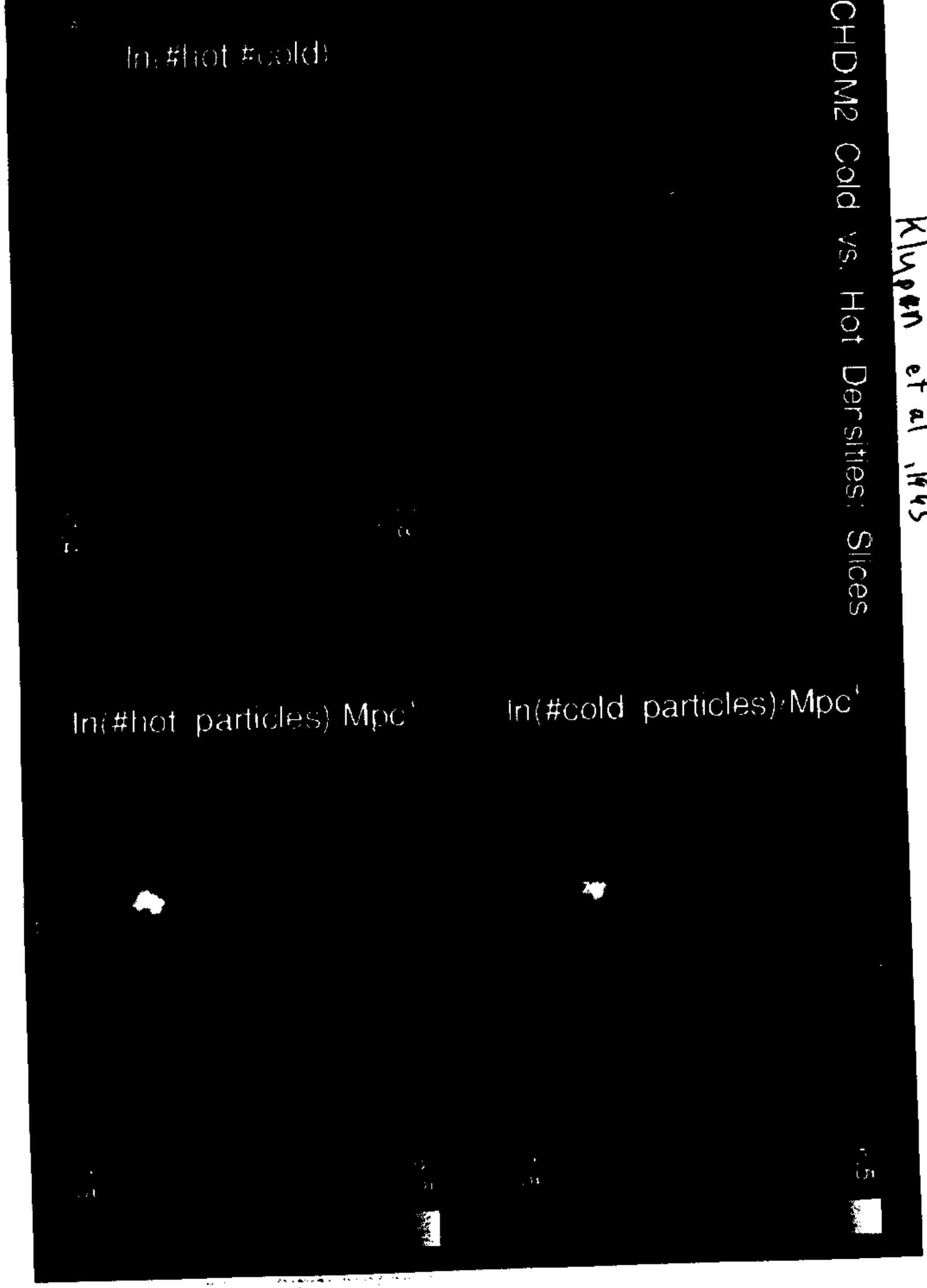
Cluster constraint turned into $\sigma(8h^{-1}\,\mathrm{Mpc})$

linear theory σ

$$\sigma(8h^{-1}\,\mathrm{Mpc}) = 0.62^{+0.24}_{-0.18}$$ Liddle et al 199?

Best constraint on mass fluctuations in Linear perturbation theory

warning: Modeling of clusters is still at a simple Level this constraint may change...



Klypin et al 1995

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Damped Lyman α Systems

clouds of neutral H with D.M. Halos

produce absorption lines in quasar spectra

looking along many lines of sight & counting how many absorption lines are seen

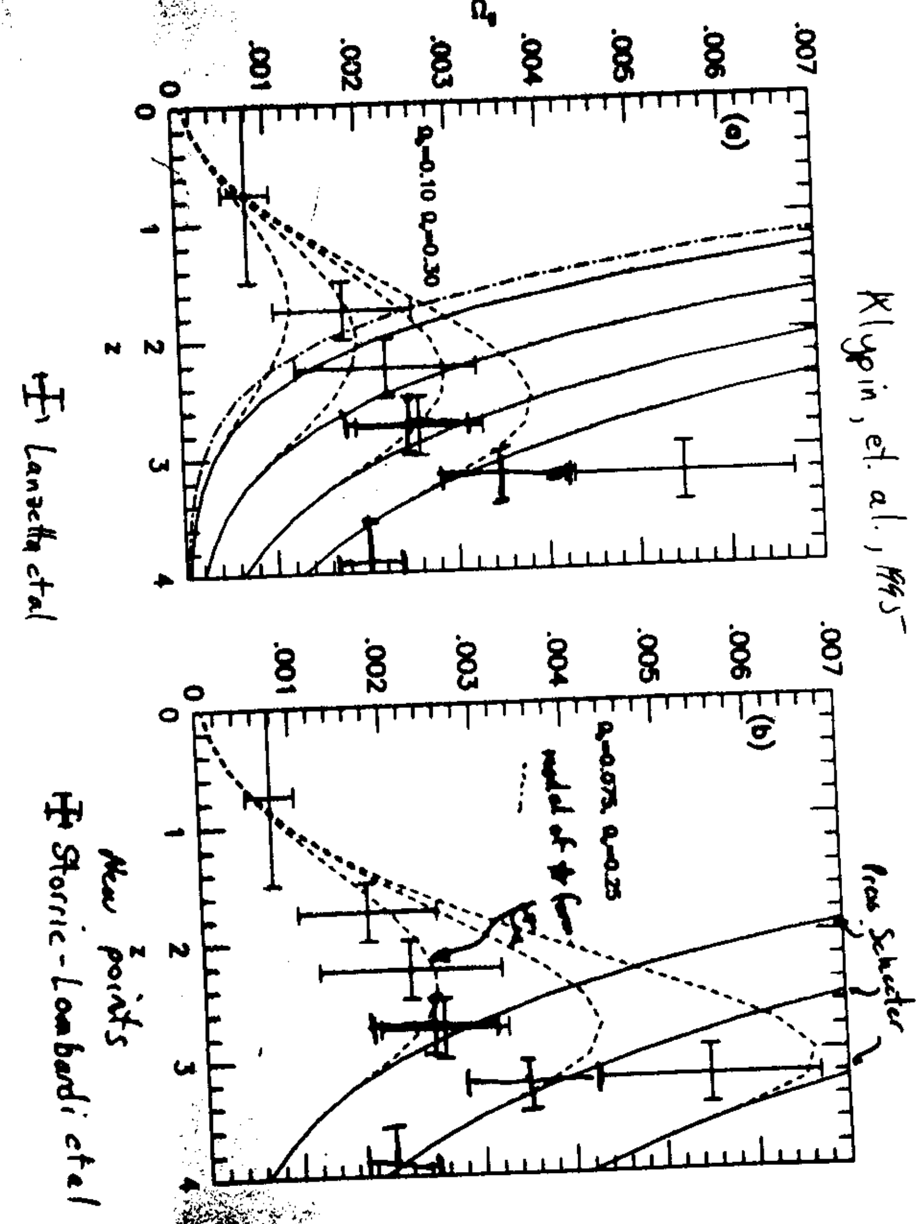
$\Rightarrow \Omega_{H\,gas} \Rightarrow \Omega_{DM\,collapsed}$

Lanzetta et al 1995   Storrie-Lombardi et al.

$\sigma(0.13\, h^{-1} Mpc) \gtrsim 0.7$ at $z=4$

• $\frac{N_H}{cloud} \gtrsim 10^{20} / cm^2$

assume roughly spherical object



Temperature Fluctuations

schematically in cold matter universe ~~with adiabatic fluctuations~~

$$\frac{\Delta T}{T}(\hat{e}) \approx \left[\frac{1}{3}\,\psi(\hat{e})\Big|_{r=r_{ls}} + \frac{1}{4}\,\Delta_{cr}(\hat{e})\Big|_{r=r_{ls}} + i\vec{k}\cdot\hat{e}\,d_H\,V_r(\hat{e})\right]_{r_{ls}}$$

$$\times\, e^{i\vec{k}\cdot\hat{e}\,d_H}$$

Sachs-Wolfe

$\Delta_{cr} = \frac{\delta\rho_r}{\rho_r}$ "Intrinsic $\frac{\Delta T}{T}$"

'Doppler'

d_H = distance to last scat. surf.

Note most of "Doppler Peak" comes from here

estimate sizes of terms

$$\nabla^2 \psi = 4\pi G\rho\,\Delta$$

$$\psi \sim \ell^2\, 4\pi G\rho\,\Delta$$

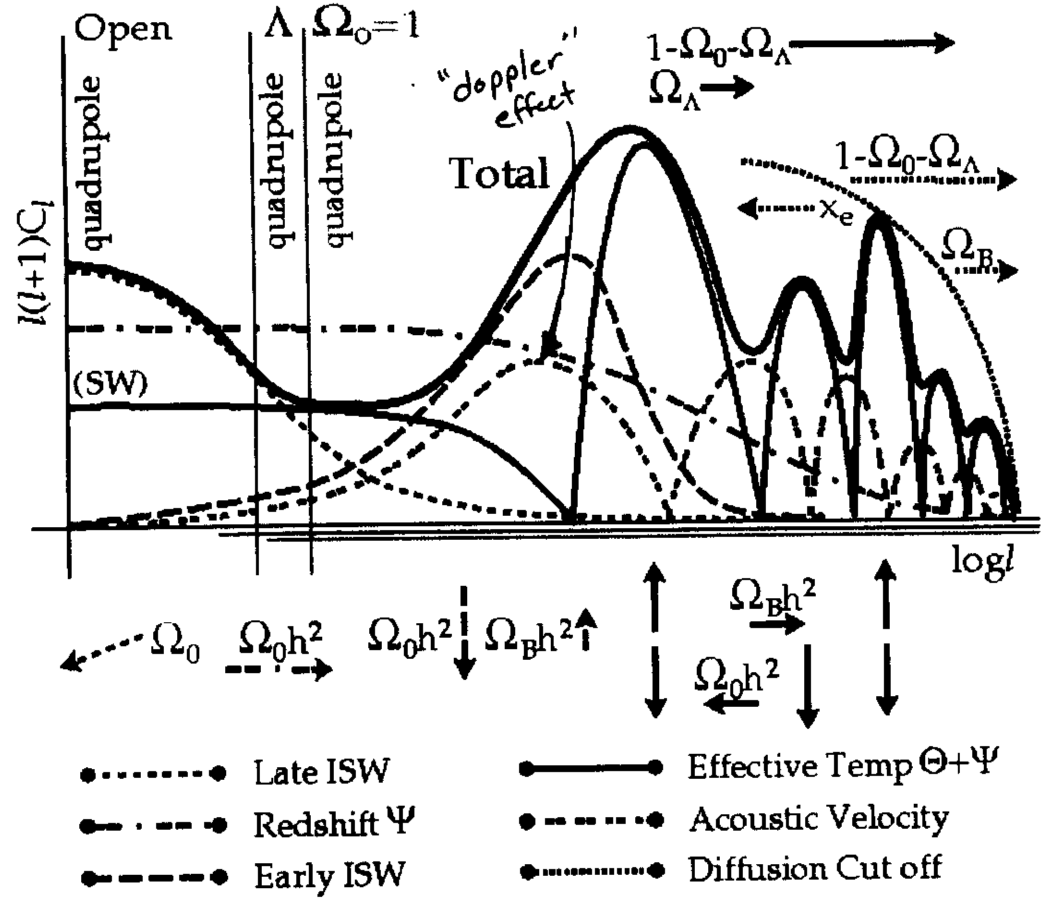
$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho = \frac{1}{\lambda_H^2} = \frac{1}{(\text{Hubble Radius})^2}$$

so

$$\psi \sim \frac{3}{2}\,\frac{\ell^2}{\lambda_H^2}\,\Delta$$

$\ell \gg \lambda_H$ $\quad\psi \gg \Delta, V_r, \Delta_r$ $\quad$ {adiabatic

$\ell \ll$ $\quad\psi \ll \Delta, V_r, \Delta_r$ $\quad$ {



Hu, Sugiyama, & Silk (1995)

Just "Sachs-Wolfe" term

$$\left\langle \frac{\delta T}{T}(\hat{e}_1)\,\frac{\delta T}{T}(\hat{e}_2)\right\rangle = \sum_\ell Q_\ell^2 P_\ell(\hat{e}_1\cdot\hat{e}_2)$$

P_ℓ most sensitive to $\theta = \frac{180^\circ}{\ell}$

for low ℓ, adiabatic: $n=1$

$$Q_\ell^2 = 8\pi\,\Delta_H^2\,\frac{(2\ell+1)}{\ell(\ell+1)}$$

Same Δ_H as in Power spectrum

$n\neq1$ $\quad Q_\ell^2 = 4\pi^{3/2}\Delta_H^2(2\ell+1)(d_H)^{1-n}\frac{\Gamma[\frac{3}{2}-\frac{n}{2}]}{\Gamma[2-\frac{n}{2}]}\frac{\Gamma(\ell+n/2-1/2)}{\Gamma(\ell-n/2+\frac{5}{2})}$

Approximate norm to COBE
Bunn, Scott, + White

$d_H = \frac{2c}{H_0}\left[1-\frac{1}{\sqrt{1+z_{ls}}}\right] \approx \frac{2c}{H_0}$ $\quad z_{ls}\approx1100$

$$Q_2^2 = \left(\frac{19.9 \pm 1.5}{2.726\times10^6}\right)^2 \exp[2(0.69)(1-n)]$$

$$= [5.33 \pm 0.03]\times10^{-11}\exp[1.38(1-n)]$$

$$\Rightarrow Q_2 = [7.30 \pm 0.55]\times10^{-6}\exp[(0.69)(1-n)]$$

for $n=1$

$\Rightarrow \Delta_H = 1.6 \pm 0.1 \times 10^{-6}$ $\quad$ H-Z.

Warning

not true for: 1) isocurvature

$\Delta_r >> \psi$ even at large scales

2) Many different COBE norms around

$6.2\times10^{-6} < Q_2 < 7.7\times10^{-6}$

3) Tensor fluctuations

$$(Q_\ell^S)^2 + (Q_\ell^T)^2 = Q_\ell^2$$

$\downarrow$

Δ_H

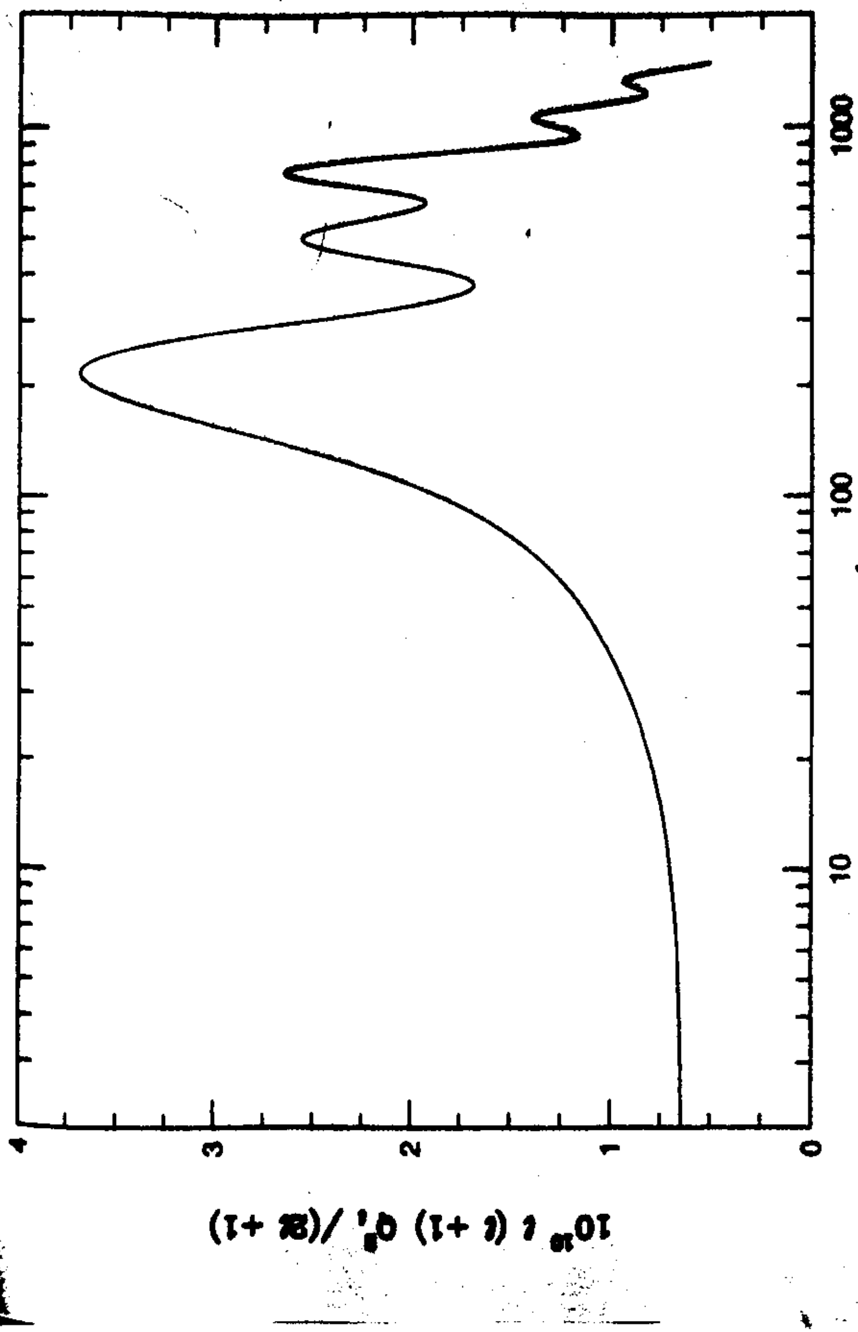


Figure 5. Multipole moments (CDM)

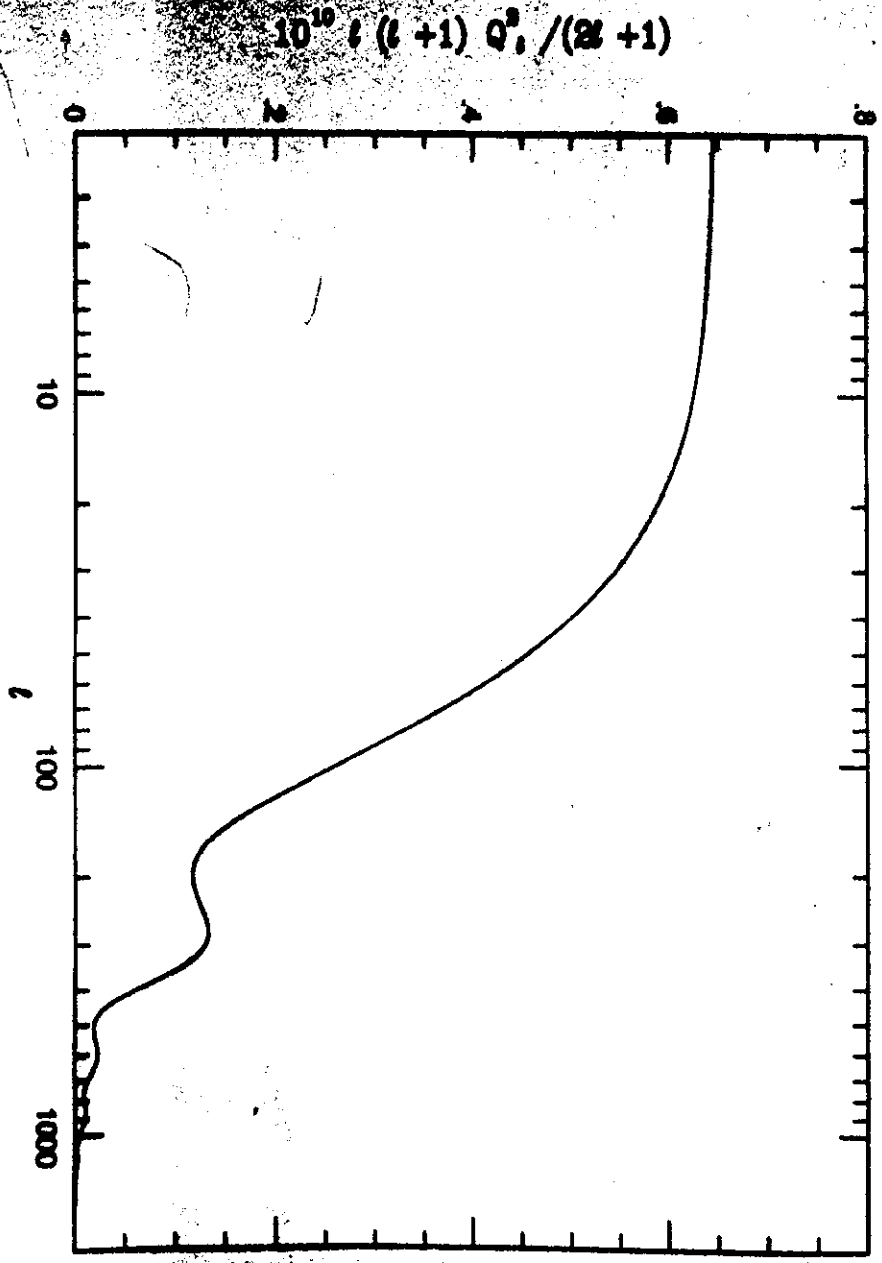
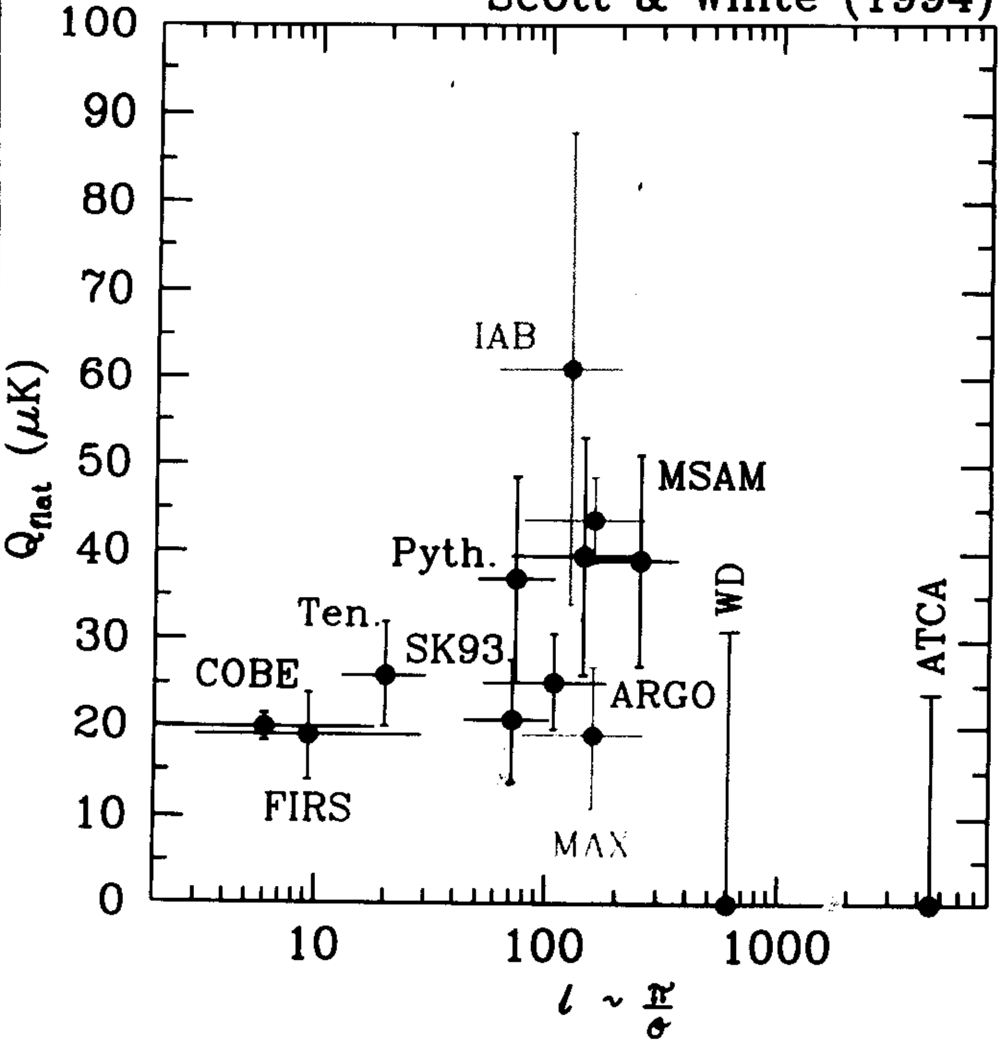


Figure 5. Multipole moments (CDM)

Figure 6. Multipole moments (HDM)

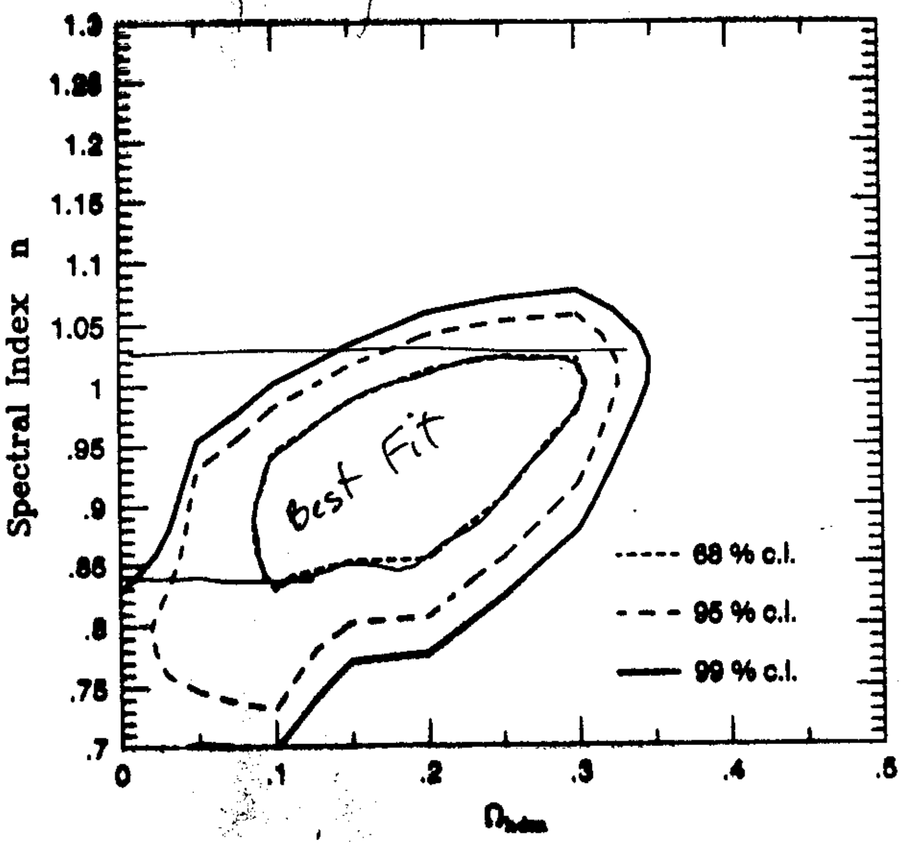
Still some systematic Errors?

Scott & White (1994)



1 Flavor of Neutrino is massive

contours in models h=0.5, no gravity waves, Q = 20 μ K



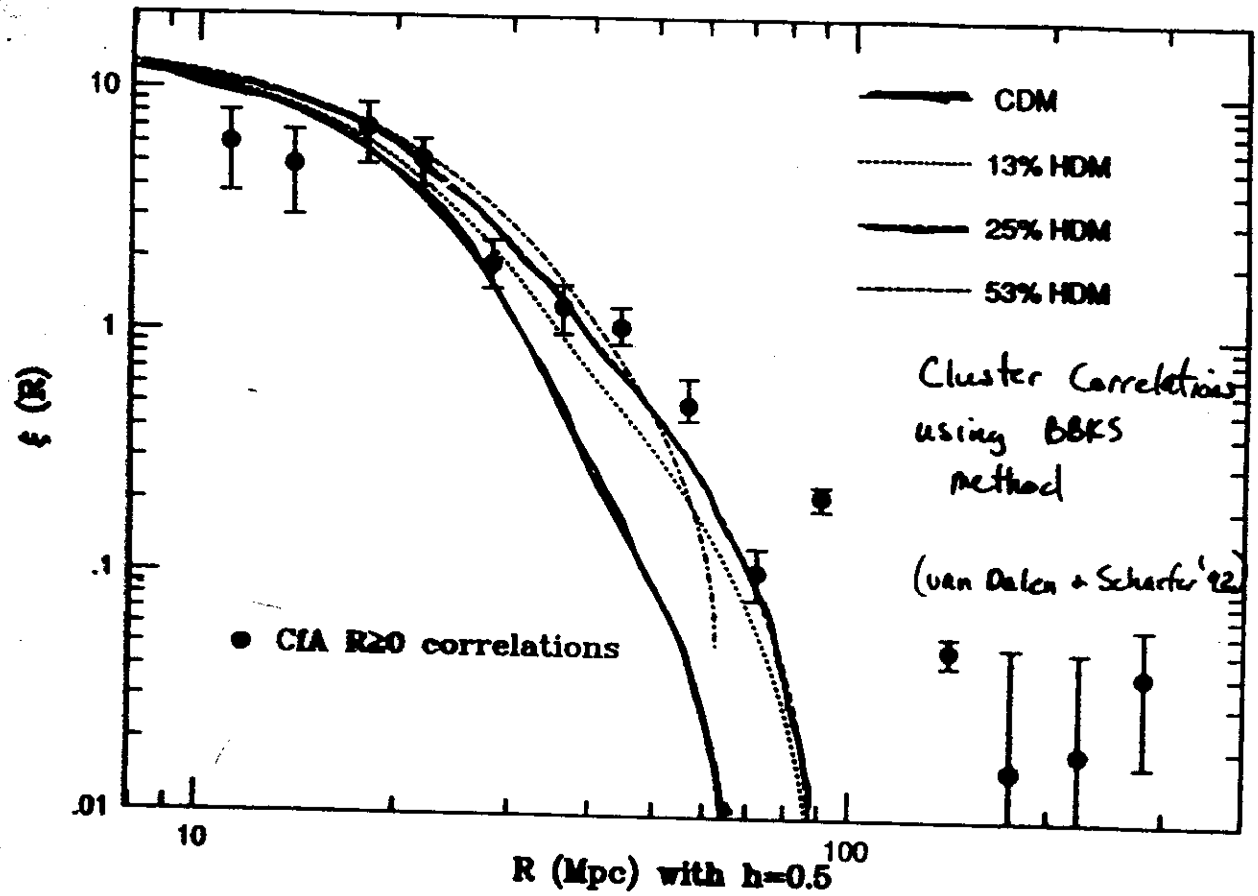
Quasi-Linear Methods

2^{nd} order perturbation theory $\quad \frac{\delta\rho}{\rho} = 0.2$

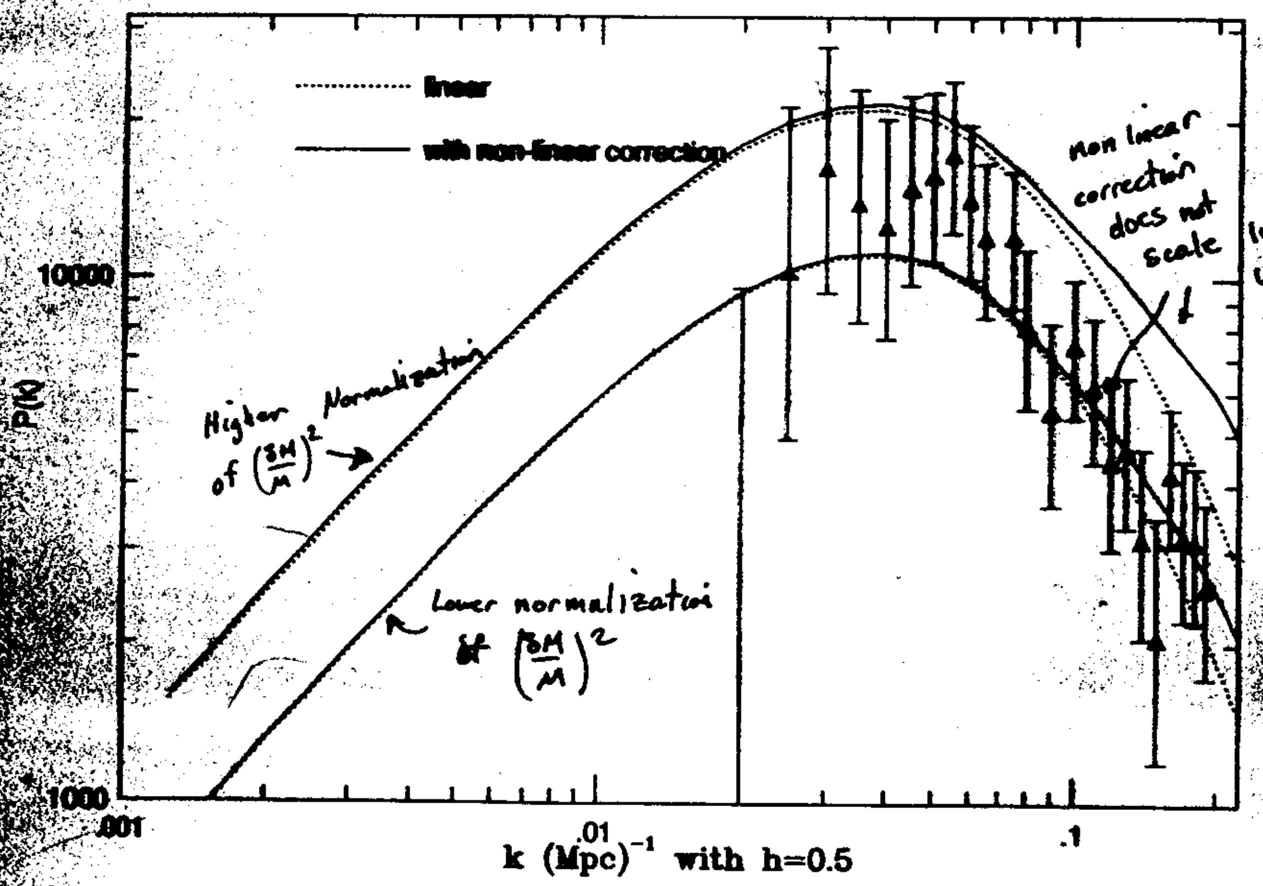
$\left(\frac{\delta\rho}{\rho}\right)^2 \sim 20\% \frac{\delta\rho}{\rho}$

other approximations

Zeldovich
constant potential



Power spectrum 30% HDM 3 Mpc/h smoothing.



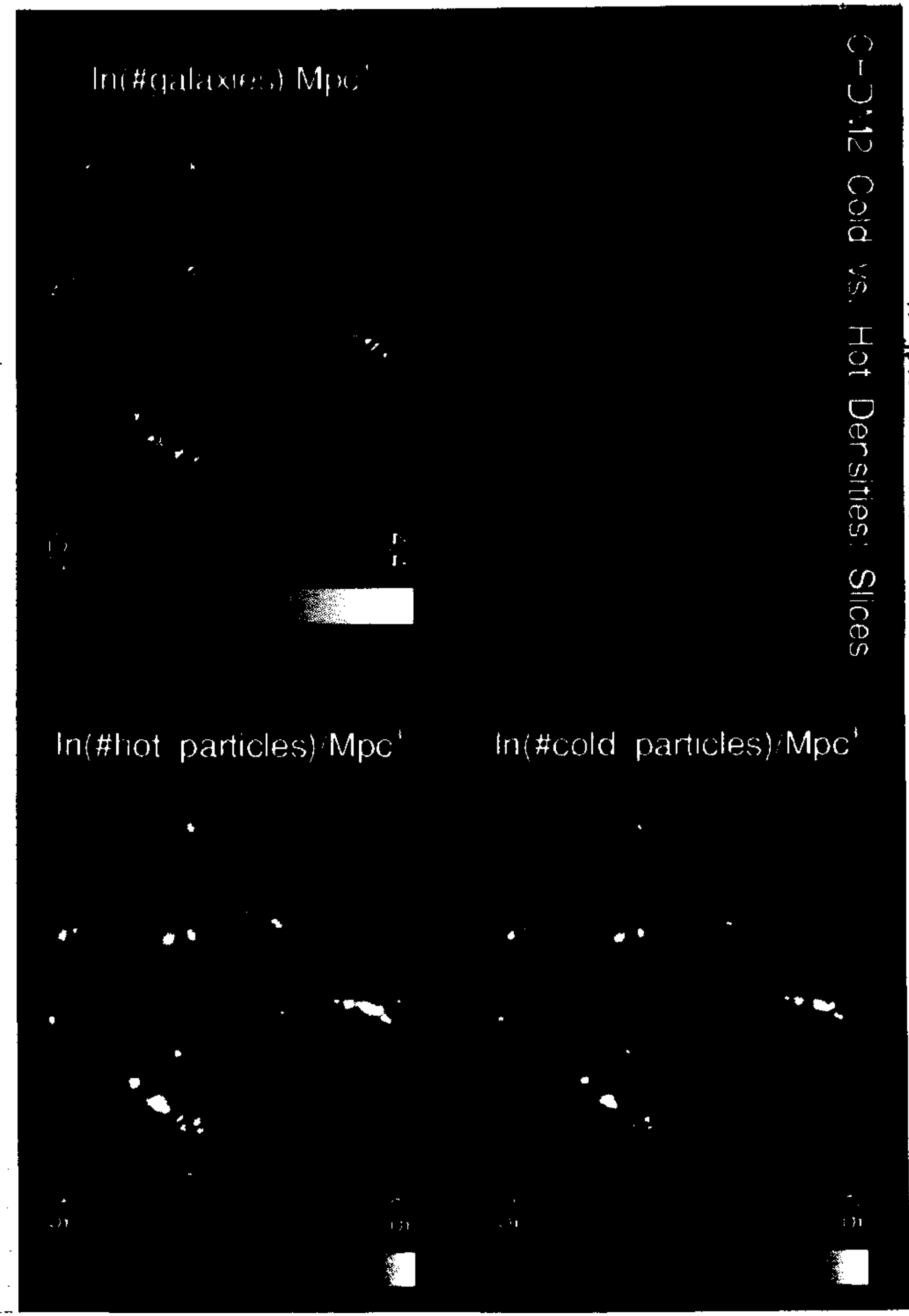
Non-Linear Methods

Usually N-Body techniques

Take a large cube with periodic BC.

- implement initial perturbations as a random realization of $P(k)$
- typically using $(256)^3$ particles ≈ 16 million in a 60^3 Mpc box $\Rightarrow m = 3\times10^{?} M_\odot$!
- let them act under the force of gravity (with expanding space)
- look for "condensates" to form
- mimic observations of structure

$v_{\parallel} = ?$



Klypin et al 1995

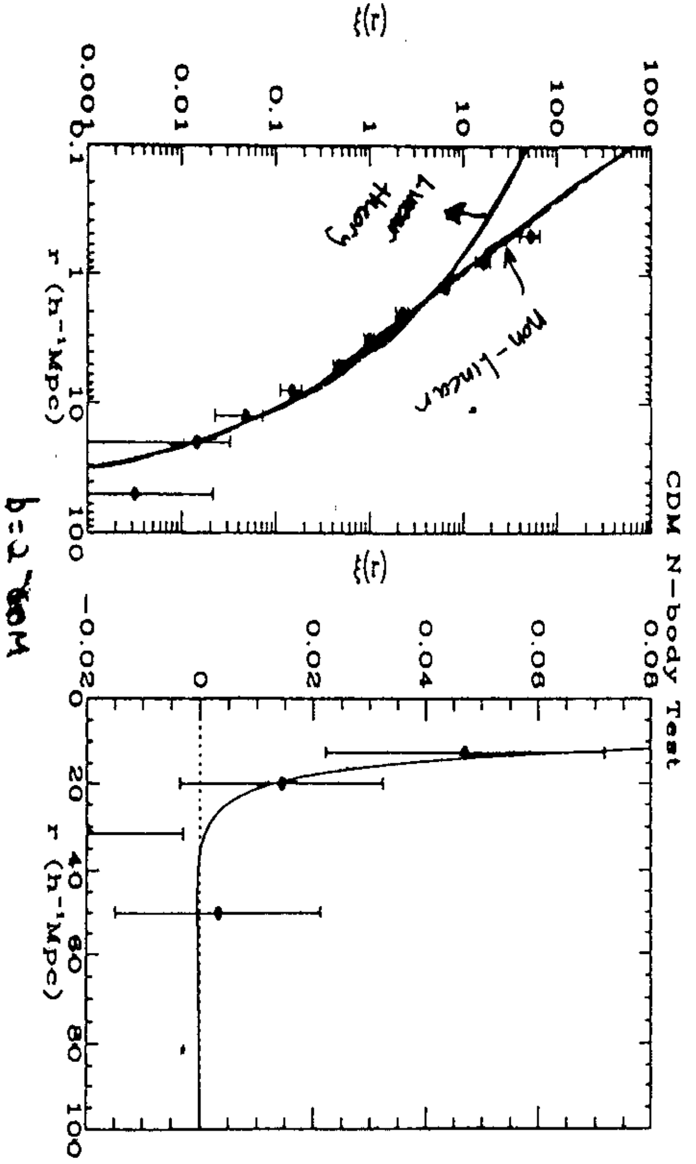


Figure 2. Test of method using volume-limited subsamples; samples were volume-limited to 6000 km s^{-1}. Dots represent the mean of ten mock *IRAS* catalogs drawn from a CDM *N*-body simulation. Error bars represent the standard deviation of the mean over the ten catalogs. The solid line is the linear theory prediction while the dashed line is the full nonlinear $\xi(r)$.

Conclusions

With all these tests - why don't we know what model(s) fit the data?

1) Data is contaminated with unknown systematic errors ↔ current observational work aimed partly at eliminating these.

2) uncertainties in knowledge of basic cosmological parameters.
H_0, Ω_0, Dark matter composition
Λ?