INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

SMR.858 - 48

II & III

**SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY**

**12 June - 28 July 1995**

# 4-DIMENSIONAL SUPERSTRING MODELS

J. Lykken
Theoretical Physics Department
Fermi National Accelerator Laboratory
60510 Batavia
U.S.A.

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MAIN BUILDING STRADA COSTIERA, 11 TEL. 2240111 TELEFAX 224163 TELEX 460392 ADRIATICO GUEST HOUSE VIA GRIGNANO, 9 TEL.224241 TELEFAX 224531 TELEX 460449
MICROPROCESSOR LAB. VIA BEIRUT, 31 TEL. 224471 TELEFAX 224600 TELEX 460392 GALILEO GUEST HOUSE VIA BEIRUT, 7 TEL.2240311 TELEFAX 2240310 TELEX 460392

# Lecture # 2

## Supersymmetry and Currents on the World-Sheet:

The spectrum of the perturbative closed bosonic string contains a tachyon. How do we get rid of it?

- Spacetime supersymmetry will automatically imply no tachyons (not allowed by Dirac eqn)

A necessary but not sufficient condition for spacetime SUSY is to implement world-sheet SUSY

$N=1$ world-sheet SUSY:

$$X^\mu \Rightarrow X^\mu, \psi^\mu$$

$\psi^\mu(z,\bar{z})$ are D Majorana world-sheet fermions (note $\mu$ is a spacetime vector index)

Action becomes:

$$\int d^2\sigma \left[ \partial_\alpha X^\mu \partial^\alpha X_\mu - i \bar{\psi}^\mu \gamma^\alpha \partial_\alpha \psi_\mu \right]$$

2D gamma matrices

This gauge-fixed action has global world-sheet SUSY; the ungauge-fixed action has local world-sheet SUSY.

The $\psi^\mu$ contribute to $T(z)$, $\bar{T}(\bar{z})$ and thus to the anomaly:

$$c = D + \frac{D}{2} = \frac{3D}{2}$$

In addition, local world-sheet SUSY implies new constraint operators (the superpartners of $T(z)$, $\bar{T}(\bar{z})$)

$T_F(z)$, $\bar{T}_F(\bar{z})$; altogether Virasoro $\Rightarrow$ $N=1$ superconformal algebra

$$T_F(z) \sim \partial_z X^\mu \psi_\mu$$
$$\bar{T}_F(\bar{z}) \sim \partial_{\bar{z}} X^\mu \bar{\psi}_\mu$$

where we have broken up

$$\psi^\mu(z,\bar{z}) = \begin{pmatrix} \psi^\mu(z) \\ \bar{\psi}^\mu(\bar{z}) \end{pmatrix} \begin{matrix} \leftarrow \text{left-moving Majorana-Weyl spinor} \\ \nwarrow \text{right-moving Majorana-Weyl spinor} \end{matrix}$$

M-W spinors exist in 2D, unlike 4D!

Boundary Conditions: We chose periodic b.c. for $X^\mu$

For e.g. $\psi^\mu(z)$ we may choose *either*

- periodic (Ramond or R)
- antiperiodic (Neveu-Schwarz or NS)

R, NS is better notation because notion of periodic vs antiperiodic can be interchanged by a conformal mapping

What do R, NS b.c.'s mean from point of view of 2D conformal field theory?

- There is a unique cft vacuum $|0\rangle$ which is the Fock vacuum of the NS modes of $\psi^\mu(z)$, $\bar\psi^\mu(\bar z)$.

There is another set of local operators called <u>twist fields</u> $\sigma(z)$, $\mu(z)$ ( $\bar\sigma(\bar z)$, $\bar\mu(\bar z)$ ) 4D altogether

$\left.\begin{array}{l}\sigma(0)|0\rangle \\ \mu(0)|0\rangle\end{array}\right\}$ are the doubly degenerate Ramond vacuum

So: NS modes of $\psi^\mu(z)$ acting on $|0\rangle$ make NS states
R modes of $\psi^\mu(z)$ acting on $\sigma(0)|0\rangle$ or $\mu(0)|0\rangle$ make R states

Since (by Lorentz invariance) there is always a <u>product</u> of D twist fields (on for each $\mu$) acting together to make physical states (or no twist fields at all) we call this <u>product</u> of twist fields a

"spin field" $S_\alpha(z)$

Check Lorentz invariance: in light-cone gauge $\psi^\mu$ has $D-2$ transverse components.

- If D is even these pair up as $\frac{1}{2}(D-2)$ Weyl fermions
- This implies the total R vacuum degeneracy is $2^{\frac{D-2}{2}}$
- So the index $\alpha$ in $S_\alpha(z)$ takes $2^{\frac{D-2}{2}}$ values
  $\equiv$ spacetime <u>spinor</u> index

e.g. in 4 dim. $S_\alpha$ takes 2 values (helicity + or −)

The Weyl Anomaly: Because we have increased the local symmetries on the world-sheet, the Fadeev-Popov ghost structure also changes.

Ghosts contribute $c = -15$

$\therefore$ To cancel Weyl anomaly using <u>only</u> $X^\mu$, $\psi^\mu$, + ghosts we need $\frac{3D}{2} - 15 = 0$

$\Rightarrow$ $D = 10$ is "critical dimension" of <u>the superstring</u>

Also, the constant "a" in the mass-shell condition changes $(L_0 - 1)|phys\rangle = 0$

becomes $(L_0 - \frac{1}{2})|phys\rangle = 0$

Notice there is still the danger of a tachyon!

Superstrings always have world-sheet SUSY (or they wouldn't be consistent) but they may or may not have spacetime SUSY

Now, in 1st-quantized formalism

spacetime SUSY $\equiv$ massless spin 3/2 particle (gravitino) in physical spectrum.

We know the form of a vertex operator which creates a gravitino:

$$V_\alpha^\mu(z,\bar{z}) \sim S_\alpha(z)\,\Sigma(z)\,\partial_{\bar{z}} X^\mu e^{ip\cdot X}$$

- where I have dropped ghost field dependence
- $\Sigma(z)$ is an analytic local operator with some known conformal field theory properties

The existence of $\Sigma(z)$ in the 2D conformal field theory
$\Rightarrow$ there is also an abelian current $J(z)$

<u>and</u> the local $N=1$ superconformal symmetry extends to global $N=2$ superconformal symmetry:

$$T(z),\ T_F(z) \longrightarrow T(z),\ J(z),\ T_F^+(z),\ T_F^-(z)$$

There are no gauge bosons associated with $J(z)$, but there is a Ward identity and a conserved charge in all Green fns.

- Because the spacetime SUSY generator carries a nonzero value of this charge, this implies an <u>R-symmetry</u> in the low-energy effective field theory

This property is general, ie.

world-sheet current $\Leftrightarrow$ symmetry in low-energy effective theory

GSO Projections: If there is a physical gravitino vertex op. then tree-level modular invariance requires that it has a <u>local</u> operator product expansion w.r.t. all other physical vertex ops.

- Thus spacetime SUSY with modular invariance implies a <u>further</u> restriction on physical states besides the $N=1$ superconformal constraints.

- These modular invariance constraints all called the "GSO projection"

The (gravitino, graviton) vertex ops are mutually local <u>but</u> the (gravitino, tachyon) vertex ops are <u>not</u>

So the GSO projection removes the tachyon as a physical state but keeps the graviton

We will be more general from now on and use the term "GSO projection" to refer to <u>any</u> additional constraints on the spectrum of physical states which arise from modular invariance rather than local world-sheet constraints.

## The 10dim Heterotic Superstring:

"heterotic" is a trick: the Majorana $\Psi^\mu(z,\bar{z})$ split up into separate left-moving and right-moving Majorana-Weyl fermions $\Psi^\mu(z)$, $\bar{\Psi}^\mu(\bar{z})$

Thus we can e.g. implement $N=1$ superconformal invariance <u>just</u> on the right-movers

$$X^\mu(z,\bar{z}) \qquad \underset{\substack{\uparrow \\ \text{M-W}}}{\bar{\Psi}^\mu(\bar{z})} \qquad \mu = 0, \dots 9$$

This cancels the right-moving contribution to the anomaly, but for the left-moving part we need additional central charge $\quad 26 - 10 = 16$

- The simplest choice of new fields for complete anomaly cancellation is 32 left-moving Majorana-Weyl free fermions

$$\lambda^i(z) \qquad i = 1, \dots 32$$

The (gauge-fixed) action of the 10-dim. heterotic superstring is thus

$$\int d^2z \left[ \partial_z X^\mu \partial_{\bar{z}} X_\mu - 2i \bar{\Psi}^\mu(\bar{z}) \partial_{\bar{z}} \bar{\Psi}_\mu(\bar{z}) - 2i \lambda^i(z) \partial_z \lambda^i(z) \right]$$

The $\lambda^i(z)$ do not carry a space time index,
"internal" conformal field theory

What is the physical effect?

$$:\lambda^i(z)\,\lambda^j(z): \quad \text{for any } i \neq j \text{ is a world-sheet current}$$

These $\frac{N(N-1)}{2}$ currents define an operator product current algebra or "Kac-Moody" algebra:

$$J^A(z_1)\,J^B(z_2) = \frac{k\,\delta^{AB}}{(z_1-z_2)^2} + \frac{i\,f^{ABC}J^C(z_2)}{(z_1-z_2)} + \ldots$$

- where $f^{ABC}$ are structure constants of $SO(32)$
- $k$ is the "Kac-Moody level"
  $k=1$ in this case
- as we will see later, by a different implementation of the b.c.'s of the $\lambda$'s, we can also obtain $E_8 \times E_8$ rather than $SO(32)$

These currents make gauge bosons:

$$V^{\mu A}(z,\bar z) \sim \frac{1}{\sqrt{k}}\, J^A(z)\,\Psi^\mu(\bar z)\, e^{ip\cdot X}$$

note normalization

The massless spectrum of the 10D heterotic superstring is:

graviton, dilaton, antisymm. tensor
gauge bosons of $SO(32)$
or $E_8 \times E_8$
+ spacetime superpartners

## The 4-dimensional heterotic superstring!

Why not use more free fermions and construct a 4-dim heterotic superstring?

We take $\quad X^\mu(z,\bar z) \quad \Psi^\mu(\bar z) \quad \mu = 0, \dots 3$

$\lambda^i(z) \quad i = 1, \dots 32 \quad$ as before

same ghost structure

Now add 6 left-moving + 9 right-moving free Weyl fermions

$$\chi^a(z), \quad \bar\chi^\alpha(\bar z)$$

$$a = 1 \dots 6$$
$$\alpha = 1 \dots 9$$

to cancel the Weyl anomaly

(If we take the b.c.'s of all the Weyl fermions to be aligned) this is indeed a modular invariant 4-dim heterotic superstring.

However it is not a "new" superstring —

it is the same heterotic string expanded around a different background or "vacuum"

$$M^4 \times T^6 \quad \text{instead of} \quad M^{10}$$

$M^4$: 4-dim Minkowski spacetime

$T^6$: 6-dim torus

$M^{10}$: 10-dim Minkowski spacetime

How do we see this? **Bosonization:**

Modular invariance required that we pair M-W fermions into Weyl fermions. The cft of a free Weyl fermion in 2-dim is equivalent to the left or right moving modes of a free boson compactified on a circle of a certain radius.

$\boxed{\text{Moduli:}}$ there are massless scalars in the spectrum of this 4-dim theory called moduli

Turning on a vev of a modulus gives a new modular invariant superstring vacuum continuously connected to the original one.

- In our case the moduli vev's are just the radii of the torus
- In general, we have some complicated moduli space of continuously connected superstring vacua.
- Free fermion constructions like our 4-dim heterotic string always sample some special point in a moduli space.
- Moduli are a big problem in superstring phenomenology, since their vev's are not determined by string perturbation theory.
- The dilaton is a modulus; some superstring vacua e.g. Calabi-Yau compactifications, have hundreds of moduli fields.

# 4-Dimensional Superstring Models

## Lectures #2 and #3

J. Lykken
Fermilab

# Lecture # 3

## Superstring Phenomenology

My definition of
"Quasi-realistic" 4-dim. heterotic superstring models:

1. $N=1$ spacetime SUSY
2. 3 "light" (massless and chiral) generations of fermions
3. gauge group
   $$SU(3)_c \times SU(2)_L \times U(1)_Y \times G_{hidden} \times G_{horizontal}$$
   or larger group like $SU(5)$ + appropriate Higgs to break it

There are ~12 known quasi-realistic models constructed using various methods:

| # | method | |
|---|---|---|
| 0 | torus compactification | ($N=4$ SUSY) |
| 1-2 | Calabi-Yau | $E_8 \to E_6 \to SU(3)\times SU(3)\times SU(3)$ |
| ~6 | abelian orbifolds | $SU(3)\times SU(2)\times U(1)\times$ hidden |
| ~4 | Free fermions | $SU(5)\times U(1)$<br>$SU(3)\times SU(2)\times U(1)$<br>$SU(5)$ |
| 1 | Gepner model | $[SU(3)]^3$ |

- This is not a large number of models
- None of them are fully realistic

## Gauge-coupling Unification

parameters of string theory:

$\alpha'$ (length)$^2$

$$\frac{g_s^2}{4\pi} = e^{2\phi_0} \qquad \phi_0 = \text{dilaton vev}$$

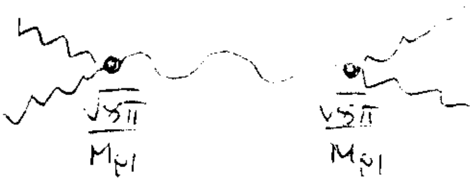
String tree-level 4-point (on-shell) gauge boson scattering:

field theory | string theory

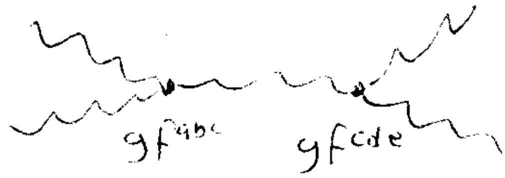
$$\frac{g^2}{4\pi} = \frac{g_s^2}{4\pi}(\alpha')^3\frac{1}{V_6}$$

$V_6$ is some function of moduli vevs
$\cong$ volume of compactified 6-dims

Better argument: take a ratio.




graviton exchange $\sim k^2$




gauge boson exchange $\sim k f^{abc} f^{cde}$

$$\frac{16\pi}{\alpha' M_{pl}^2} = k g^2$$

True for every group, so e.g. for $SU(3)_c \times SU(2)_L \times U(1)_Y$

$$k_3 g_3^2 = k_2 g_2^2 = k_1 g_1^2 = \frac{16\pi}{\alpha' M_{pl}^2}$$

at a running scale (usually defined by minimizing threshold effects)

$$M_U \simeq 5 \times 10^{17}\ \text{GeV}$$

- we have something like gauge coupling unification (depending on whether $k$'s $= 1$ or not) independent of GUT's

- Since $\frac{g^2}{4\pi}$ must be $\frac{1}{25} - \frac{1}{40}$ to fit RG running from low energies, the string scale is $\sim 5 \times 10^{17}$ GeV not $10^{19}$ GeV.

- This still doesn't fit usual RG running of MSSM where $M_U = 10^{16}$ GeV

$\boxed{\text{This is a problem!}}$

$\boxed{\text{Possible solutions:}}$

1. Threshold corrections are very large
   - doesn't seem natural in string theory
   - danger that, if true, string pert. theory is not reliable.

2. There is a GUT and adjoint Higgs gets vev $\sim 10^{16}$ GeV
   - don't know how to generate this vev

3. Intermediate scales to change running
   - ugly, but possible

4. Not all of the $k$'s $= 1$ "hidden" unification
   - difficult because $k$'s = positive integers for nonabelian groups

# Supersymmetry Breaking

SUSY must be broken (at a scale $\sim 1$ TeV) to agree with the real world. For a superstring model, there are 2 possibilities

1. The effective supergravity field theory below the string scale breaks SUSY spontaneously or dynamically.

2. String nonperturbative effects break SUSY

– or both of these!

Effective supergravity field theory:
action depends on 3 functions of chiral superfields

$K(\phi, \phi^*)$ Kahler potential

$W(\phi)$ superpotential (cubic + higher order terms because this is an effective action)

$f_{AB}(\phi)$ gauge kinetic function

SUSY breaking means giving a vev to either the F-term of a chiral superfield

$$\Phi = \varphi + \theta\psi + \theta\theta F$$

or the D-term of a vector superfield

– but D-term SUSY breaking appears not to happen in string theory

# Hidden sector SUSY breaking

Superstring models (almost) always have a hidden sector
Suppose the hidden gauge interactions get strong
(via RG running) at some scale $M$

Then the hidden gauginos $\lambda$ form a condensate:

$$\langle \lambda\lambda \rangle \sim M^3$$

This does not break SUSY because $\lambda\lambda$ is not an F term

But suppose there are hidden sector chiral matter multiplets also
Then these fermions may condense also

$$\langle \bar{\chi}\chi \rangle \sim M^3$$

and may break SUSY.

In the visible sector we would see this SUSY-breaking
only via gravitational effects – the effective scale
is set by the gravitino mass:

$$m_{3/2} \sim \frac{\langle \lambda\lambda \rangle}{M_{Pl}^2} \sim \frac{M^3}{M_{Pl}^2} \sim 1 \text{ TeV if}$$

$$M \sim 10^{13}\text{–}10^{14} \text{ GeV}$$

Possible – but difficult to make models that actually work

# Dilaton SUSY breaking

String theory predicts new particles called moduli that form chiral supermultiplets.
E.g. for the dilaton:

$$S = e^{-2\phi} + ia + \theta(\text{dilatino} + \text{axino}) + \theta\theta F_S$$

$\phi$ = dilaton
$a$ = axion

Since we know $\langle\phi\rangle \neq 0$ perhaps $\langle F_S \rangle \neq 0$ breaks SUSY.

Assume this happens (due to nonperturbative effects)

Then we get model-independent predictions! for low-energy SUSY breaking, because dependence of $K$, $W$, and $f_{AB}$ on $S$ is the same in all string models:

$$K \sim -\log(S+S^*) + \ldots$$

$W$ is $S$ independent

$$f_{AB} \sim k\,\delta_{AB} S + \ldots$$

String tree-level predictions:

$$m_{\text{gaugino}} = \sqrt{3}\, m_{3/2}$$

$$m^2_{\text{scalars}} = \delta_{ij}\, m^2_{3/2}$$

$$A_{ijk} = -\sqrt{3}\, m_{3/2}\, y_{ijk}$$

↖ Yukawas

2 problems with this.

1. At string loop level, or if $\langle F \rangle \neq 0$ for other moduli, you get FCNC problems due to nonuniversal scalar masses.

2. String nonperturbative corrections to (at least) $K$ are probably not small, so result is suspect.

## Global symmetries, proton decay, light fermion masses

- In string theory all global continuous symms of the effective low energy field theory are accidental i.e. not in the underlying string model and broken by higher order terms

- Since this applies to Baryon number, we must always worry about proton decay in string models

- However quasi-realistic string models have lots of discrete symms, which can suppress proton decay

- These discrete symms also tend to severely reduce the allowed Yukawa couplings for light fermions, as do gauged horizontal symmetries

## Example: $Z_3$ orbifold quasi-realistic model

gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y \times SO(10)_{hidden} \times U(1)_{horiz}$

$U(1)_{horiz}$ breaks spontaneously at a scale $O(\frac{g^2}{4\pi} m_{string})$

- Conservation of $q_{horiz}$ forbids all couplings of form

$$QLd^c, \quad u^c d^c d^c, \quad LLe^c$$

which could produce rapid proton decay

- It also forbids all down-type Yukawa couplings

$$Qd^c H$$

or even $Qd^c H S$ $\qquad$ $S$ = moduli vev or function of them

So there is too much restrictive flavor symmetry