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III

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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BASICS OF CFT, STRING COMPACTIFICATIONS, MIRROR SYMMETRY AND SPACETIME TOPOLOGY CHANGE

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SUMMARY OF LECTURE I

I) INTRODUCTION/MOTIVATION

II) N = 2 SUPERCONFORMAL ALGEBRA

• II.1) THE ALGEBRA

$$\{ T(z), G^{\pm}(z), J(z) \} \times \{ \bar{T}(\bar{z}), \bar{G}^{\pm}(\bar{z}), \bar{J}(\bar{z}) \}$$

- $[L_m, L_n] = (m-n) L_{m+n} + c/12 (m^3 - m) \delta_{m+n,0}$
- $[L_n, G^{\pm}_{m\pm a}] = (n/2 - (m \pm a)) G^{\pm}_{m+n\pm a}$
- $\{ G^{+}_{n+a}, G^{-}_{m-a} \} = 2L_{m+n} + (n+m+2a) J_{n+m} + c/3 [(n+a)^2 - 1/4] \delta_{m+n}$
- $[L_n, J_m] = -m J_{m+n}$
- $[J_n, G^{\pm}_{m\pm a}] = \pm G^{\pm}_{m+n\pm a}$
- $[J_m, J_n] = c/3 \cdot m \, \delta_{m+n,0}$

• II.2) HIGHEST WEIGHT REPRESENTATIONS

- $\phi(z)$ SUPERCONFORMAL PRIMARY FIELD

  $|\phi\rangle = \phi(0)|0\rangle$

- $L_0|\phi\rangle = h_\phi|\phi\rangle$ • $J_0|\phi\rangle = Q_\phi|\phi\rangle$
- $L_n|\phi\rangle = J_n|\phi\rangle = G^{\pm}_n|\phi\rangle = 0 \quad n > 0$
- USE $L_{-n}, J_{-n}, G^{\pm}_{-n}$ TO "RAISE" TO OTHER STATES

$$\bullet\ T(z)T(w) \sim \frac{c/2}{(z-w)^4} + \frac{2}{(z-w)^2}T(w) + \frac{1}{(z-w)}\partial_w T(w)$$

$$\bullet\ T(z)\,G^{\pm}(w) \sim \frac{3}{2}\cdot\frac{G^{\pm}(w)}{(z-w)^2} + \frac{\partial G^{\pm}(w)}{(z-w)} + \cdots$$

$$\bullet\ G^{+}(z)G^{-}(w) \sim \frac{2c}{3(z-w)^3} + \frac{2J(w)}{(z-w)^2} + \frac{2T(w)+\partial J(w)}{(z-w)} + \cdots$$

$$(G^{+}G^{+} \sim G^{-}G^{-} \sim \mathcal{O}(1))$$

$$\bullet\ J(z)G^{\pm}(w) \sim \pm G^{\pm}(w)/(z-w) + \cdots$$

$$\bullet\ J(z)J(w) \sim \frac{c/3}{(z-w)^2} + \cdots$$

RONG
AFA
ARNER

II.3) CHIRAL PRIMARY FIELDS

- (CHIRAL, CHIRAL) = (C,C) : $G^{+}_{-1/2}|\phi\rangle = \bar{G}^{+}_{-1/2}|\phi\rangle = 0$

  (ANTICHIRAL, CHIRAL) (a, c) : $G^{-}_{-1/2}|\phi\rangle = \bar{G}^{+}_{-1/2}|\phi\rangle = 0$

- **WE FOUND**:

  1) (C,C) $\Leftrightarrow$ $h_\phi = Q_\phi/2$; $\bar{h}_\phi = \bar{Q}_\phi/2$

     (a,c) $\Leftrightarrow$ $h_\phi = -Q_\phi/2$, $\bar{h}_\phi = \bar{Q}_\phi/2$

  2) $h, \bar{h} \leq c/6$ FOR SUCH STATES

  3) OPERATOR ALGEBRA : CLOSED NONSINGULAR FINITE RING

     (C,C) RING (a,C) RING

II.4) SPECTRAL FLOW

- PERIODIC / ANTIPERIODIC BOUNDARY CONDITIONS ON $G^{\pm}$ $\Rightarrow$ $G^{\pm}_{n\pm a}$, $a = 0$ "R", $a = 1/2$ "NS"

- ALGEBRA INDEP. OF a

- $U_{1/2} = e^{-i\sqrt{c/3}\,\frac{1}{2}\phi}$ "SPECTRAL FLOW"

- CHIRAL PRIMARY : $G^{+}_{-1/2}|\phi\rangle = 0 \longrightarrow |\hat{\phi}\rangle : G^{\pm}_{0}|\hat{\phi}\rangle = 0$

  "RAMOND GROUND STATE"

0.3) CONFORMAL FIELD THEORY & STRING THEORY

• RECALL: MOTIVATED CFT DISCUSSION BY REALIZING (CLASSICAL) CONFORMAL SYMMETRY OF STRING ACTION

$$\bullet S = \frac{1}{4\pi\alpha'}\int d\sigma d\tau \sqrt{h}\, h^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu G^M_{\mu\nu}(x)$$

• KEY POINTS:

1) FOR GENERAL $G^M_{\mu\nu}$ CLASSICAL CONFORMAL SYMMETRY IS VIOLATED FAR MORE SEVERLY THAN VIA CENTRAL CHARGE C. (WILL RETURN). FOR NOW: TAKE $G^M_{\mu\nu} = \eta_{\mu\nu}$ (FLAT).

2) THE VALUE OF C IN THIS CASE IS # X's. (WILL PRESENTLY SEE).

3) FOR STRINGS: NEED $C = \bar{C} = 0$

4) GHOSTS FOR GAUGE FIXING REPARAMETRIZATION INVARIANCE CONTRIBUTE $C_g = \bar{C}_g = -26$
$\Rightarrow$ NEED <u>26</u> X's.

5) SUPERSYMMETRIC CASE: $C_{sg} = \bar{C}_{sg} = +11 \Rightarrow$
$C_{g\,tot} = \bar{C}_{g\,tot} = -15$. THUS NEED <u>10</u>$(\underset{1}{X} + \underset{1/2}{\Psi}) \Rightarrow c = 15.$

More General Formulation of Strings

- **Original: Bosonic**

$$S = \int \partial X^{\mu} \bar{\partial} X^{\nu} \eta^{26}_{\mu\nu} \qquad \mu,\nu = 1,\dots,26$$

$$= \underline{\int \partial X^{\mu} \bar{\partial} X^{\nu} \eta^{4}_{\mu\nu}} + \underline{\int \partial X^{\mu'} \bar{\partial} X^{\nu'} \delta^{22}_{\mu'\nu'}}$$

- **General:** $S = \int \partial X^{\mu} \bar{\partial} X^{\nu} \eta^{4}_{\mu\nu}$ + "Internal" CFT with $c = \bar{c} = 22$

- **Original Superstring**

$$S = \int \partial X^{\mu} \bar{\partial} X^{\nu} \eta^{10}_{\mu\nu} + \psi^{\mu} \partial \psi^{\nu} \eta^{10}_{\mu\nu} + \lambda^{\mu} \bar{\partial} \lambda^{\nu} \eta_{\mu\nu}$$

$$\mu,\nu = 1,\dots,10$$

- **General:**

$$S = \int \left(\partial X^{\mu} \bar{\partial} X^{\nu} \eta^{4}_{\mu\nu} + \psi^{\mu} \partial \psi^{\nu} \eta^{4}_{\mu\nu} + \lambda^{\mu} \bar{\partial} \lambda^{\nu} \eta^{4}_{\mu\nu} \right)$$

+ "Internal" $c = \bar{c} = 9$ CFT
with $N=2$ World Sheet Supersymmetry

- **STRING THEORIES/MODELS/VACUA**

	LEFT	RIGHT
GHOSTS	$c=-15$	$\bar{c}=-15$
SPACETIME 4-D	X^μ ψ^μ $c=6$	X^μ λ^μ $\bar{c}=6$
"INTERNAL"	$N=2$ $c=9$	$N=2$ $\bar{c}=9$

- MODULAR INVARIANCE
- HETEROTIC CONVERSION
- "FREEDOM OF CHOICE" for "INTERNAL": EACH YIELDS DIFFERENT LOW E PHYSICS.

III) FAMILIES OF N=2 THEORIES

III.1) MARGINAL OPERATORS

- Φ WEIGHTS $(h, \bar{h})$
  - IRRELEVANT IF $h+\bar{h} > 2$
  - RELEVANT IF $h+\bar{h} < 2$
  - MARGINAL IF $h+\bar{h} = 2$

$$S_{CFT} \rightarrow S_{CFT} + \epsilon \int d^2 z \, \Phi(z,\bar{z}) \xrightarrow{\text{RG FLOW}} S'_{CFT}$$

- IRRELEVANT $\Rightarrow$ $S'_{CFT} = S_{CFT}$
- RELEVANT $\Rightarrow$ $S'_{CFT} \neq S_{CFT}$
- MARGINAL $\Rightarrow$ $S'_{CFT} \neq S_{CFT}$ BUT IS CONTINUOUSLY CONNECTED TO S_{CFT} ($\epsilon \to 0$).

- Φ IS TRULY MARGINAL IF IT STAYS MARGINAL IN PERTURBED THEORY

$\rightarrow$ TRULY MARGINAL OPERATORS GENERATE A *FAMILY* OF CFT'S ALL CONTINUOUSLY RELATED TO ONE ANOTHER.

SIMPLE EXAMPLE

$N=0$ $\quad C=1$ CFT : BOSON ON CIRCLE OF RADIUS R

$$S_R = \int d^2z\, \partial X \bar{\partial} X \qquad X \sim X + 2\pi R$$

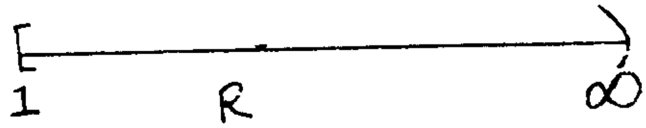
- CONSIDER $(1,1)$ OPERATOR : $\mathcal{O} = \partial X \bar{\partial} X$
- DEFORM S_R:

$$S_R \to S_R + \epsilon \int d^2z\, \mathcal{O}(z,\bar{z})$$

$$= (1+\varepsilon)\int d^2z\, \partial X \bar{\partial} X \quad . \text{ LET } \tilde{X} = \sqrt{1+\varepsilon}\, X \Rightarrow$$

$$= \int d^2z\, \partial \tilde{X} \bar{\partial} \tilde{X} \quad \text{WITH } \tilde{X} \sim \tilde{X} + 2\pi R(\sqrt{1+\varepsilon})$$

- SO : MARGINAL OPERATOR $\mathcal{O}$ CHANGES RADIUS OF TARGET SPACE CIRCLE.
- FAMILY OF CFT's GENERATED : BOSON ON CIRCLE OF ARBITRARY RADIUS
- IN FACT : $S_R \cong S_{1/R}$
- MODULI SPACE :



EXAMPLE: FREE BOSON

- $S = \int \partial X \bar{\partial} X$. EOM $\Rightarrow X(z,\bar{z}) = \chi(z) + \chi(\bar{z})$.

- PROPAGATOR: $\langle \chi(z)\chi(\omega)\rangle = -\log(z-\omega)$; $\langle \chi(\bar{z})\chi(\bar{\omega})\rangle = -\log(\bar{z}-\bar{\omega})$
- OPERATOR PRODUCT: $\partial\chi(z)\cdot\partial\chi(\omega) = -\frac{1}{(z-\omega)^2} + \cdots$
- $T(\omega) = -\frac{1}{2} :\partial\chi(z)\partial\chi(\omega): \quad \Rightarrow$

a) $$T(\omega)\,\partial\chi(z) = -\tfrac{1}{2} :\partial\chi(z)\partial\chi(z): \partial\chi(\omega)$$
$$= -\tfrac{1}{2}\cdot 2\cdot \partial\chi(z)\langle\partial\chi(z)\partial\chi(\omega)\rangle + \cdots$$
$$= \partial\chi(z)\cdot\frac{1}{(z-\omega)^2} + \cdots$$
$$= \left(\partial\chi(\omega) + (z-\omega)\partial^2\chi(z)\right) 1/(z-\omega)^2 + \cdots$$
$$= \frac{1\cdot\partial\chi(\omega)}{(z-\omega)^2} + \frac{1}{(z-\omega)}\partial(\partial\chi(z)) + \cdots$$

b) $$T(\omega)T(z) = -\tfrac{1}{2}:(\partial\chi(\omega)\partial\chi(\omega)): -\tfrac{1}{2}:(\partial\chi(z)\partial\chi(z)):$$
$$= \cdots = \frac{\boxed{1}/2}{(\omega-z)^4} + \frac{2}{(\omega-z)^2}T(z) + \frac{1}{(\omega-z)}\partial T(z) + \cdots$$
$$\Rightarrow C = 1.$$

$\Rightarrow$ FOR d BOSONS $\Rightarrow C = d$.

N=2 SCFT

• TRULY MARGINAL OPERATORS INCLUDE

i) $\phi \in (c,c)$ WITH $h=\bar{h}=1/2,\ Q=\bar{Q}=1$

$G^+(\omega)\,\phi(z,\bar{z}) \sim \mathcal{O}(1)\ ;\ \ \bar{G}^+(\bar{\omega})\,\phi(z,\bar{z}) \sim \mathcal{O}(1)$

$$\hat{\phi}(z,\bar{z}) = (G^-_{-1/2}\,\phi)(z,\bar{z}) = \oint G^-(\omega)\,\phi(z,\bar{z}) \left\{ \begin{array}{l} h=1,\ Q=0 \\ \bar{h}=1/2,\ \bar{Q}=1 \end{array} \right.$$

$$\Phi_{(c,c)} = (\bar{G}^-_{-1/2}\,\hat{\phi})(z,\bar{z}) = \oint \bar{G}^-(\bar{\omega})\,\hat{\phi}(z,\bar{z}) \left\{ \begin{array}{ll} h=1 & Q=0 \\ \bar{h}=1 & \bar{Q}=0 \end{array} \right.$$

• BY SCWI, $\Phi_{(c,c)}$ IS TRULY MARGINAL

ii) $\phi \in (a,c)$ WITH $h=\bar{h}=1/2,\ Q=-1,\ \bar{Q}=1$

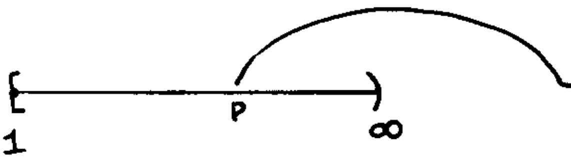
$$\Phi_{(a,c)} = \oint \bar{G}^-(\bar{\omega}) \oint G^+(\omega)\ \phi(z,\bar{z}) \left\{ \begin{array}{l} h=\bar{h}=1 \\ Q=\bar{Q}=0 \end{array} \right.$$

• BY SCWI, $\Phi_{(a,c)}$ IS TRULY MARGINAL

• WE WILL FOCUS ON THESE TRULY MARGINAL OP'S AND THEIR LOWER COMPONENT SUPERPARTNERS IN (C,C), (a,c) RINGS.

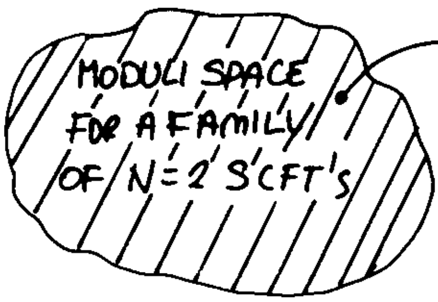
III.2) Moduli Spaces: I

- For boson on circle:



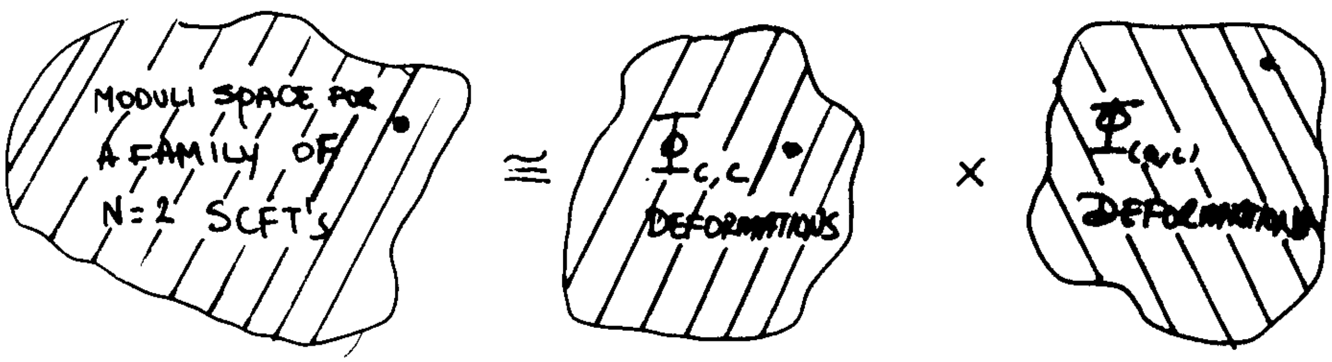
→ Each point corresponds to a CFT: $S_{R(p)}$

- More generally: Multidimensional parameter space; "moduli space"



- Each point corresponds to 1 N=2 SCFT
- Can deform any such SCFT to that corresponding to a different point by using the truly marginal ops.

- For N=2 SCFT: At least locally CFT metric (Zamoldchikov) is block diagonal between $\Phi_{c,c}$ & $\Phi_{a,c}$:



- GOAL: FULLY UNDERSTAND THIS PICTURE, ESPECIALLY WHEN WE ARE STARTING FROM AN INITIAL $N=2$ SCFT BUILT AS A NLσM.

- TO START: IF WE HAVE AN $N=2$ SCFT, REALIZED AS NLσM, CAN WE GIVE GEOMETRIC INTERPRETATION TO $\Phi_{(c,c)}$, $\Phi_{(a,c)}$ MARGINAL ops?

- ALREADY DESCRIBED:

$$(c,c) \text{ FIELDS} \longleftrightarrow \text{HARMONIC } (2,1) \text{ FORMS ON } M$$
$$(a,c) \text{ FIELDS} \longleftrightarrow \text{HARMONIC } (1,1) \text{ FORMS ON } M$$

- IN PARTICULAR: (TO LOWEST ORDER)

$$(a,c) \leftrightarrow h_{i\bar{j}} \psi^i \lambda^{\bar{j}}$$

$$\Phi_{(a,c)} \sim G^{+}_{-1/2} \bar{G}^{-}_{-1/2} (h_{i\bar{j}} \psi^i \lambda^{\bar{j}})$$

$$\sim h_{i\bar{j}} \partial X^i \bar{\partial} X^{\bar{j}}$$

r, s MEASURE U(1) CHARGE

- THESE OPERATORS DEFORM "SIZE" OF M: (WHILE KEEPING IT RICCI FLAT!)

(r,s) (a,c)

(r,s) (c,c) J = KÄHLER FORM ON $M = i g_{i\bar{j}} dX^i dX^{\bar{j}}$

24
24.1
24.2

- (C,C) ~ HARMONIC (2,1) FORMS

$$\sim \quad h_{ij,\bar{k}} \Omega^{ijk} \psi^{\bar{k}} \lambda_k$$

$$= \quad h_{\bar{k}}{}^{k} \psi^{\bar{k}} \lambda_k \qquad\qquad h_{\bar{k}}^{k} \in H^1(M,T)$$

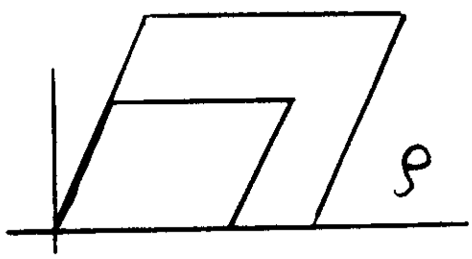
$$\Phi_{(c,c)} \sim G^-_{-1/2} \bar{G}^-_{-1/2} \left(h_{\bar{k}}{}^{k} \psi^{\bar{k}} \lambda_k \right)$$

$$\sim h_{\bar{k}}{}^{k} \partial X^{\bar{k}} \bar{\partial} X^{\bar{k}'} g_{k\bar{k}'}$$

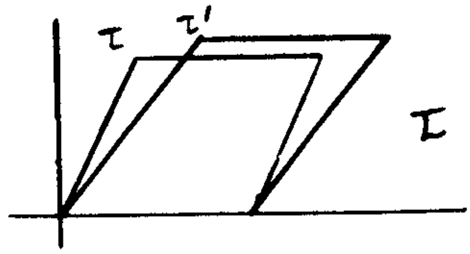
$$\sim h_{\bar{k}\bar{k}'} \partial X^{\bar{k}} \bar{\partial} X^{\bar{k}'}$$

- THESE OPERATORS DEFORM METRIC BUT SPOIL ITS (1,1) TYPE STRUCTURE (KEEPING IT RICCI FLAT). THEY ARE MOST APPROPRIATELY INTERPRETED BY PERFORMING A NONHOL. CHANGE OF COORDS TO PUT METRIC BACK IN (1,1) FORM — i.e. THESE OPERATORS CHANGE COMPLEX ST. OF M.

- $\Phi_{(a,c)}$: DEFORM KÄHLER ST. OF M ("SIZE")
 $\# = h^{1,1} = \dim H^{1,1}_{\bar{\partial}}(M)$



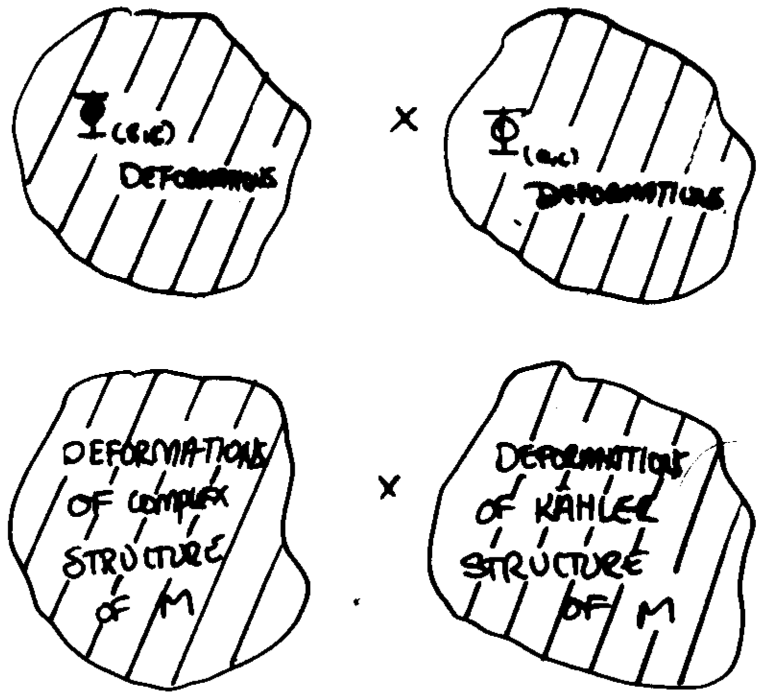
$\Phi_{(c,c)}$: DEFORM COMPLEX ST. OF M ("SHAPE")
 $\# = h^{2,1} = \dim H^{2,1}_{\bar{\partial}}(M)$



Pictorially: Moduli Space

N=2 SCFT (generally) :

Geometric interpretation on N=2 NLσM :



- This geometric interpretation is deficient. We will return to this

- Note: (As a prelude) Asymmetry

  $\Phi_{(c,c)}$, $\Phi_{(a,c)}$ differ by sign of $U(1)$ charge

  Complex St, Kähler St differ significantly.

IV) Interrelations between various $N=2$ SCFT's

- NLσM on CY target
- Landau-Ginsburg (L-G) Models
- Minimal Models (MM)

Relations?

ZAMOLODCHIKOV
VW
WITTEN
⋮

A) L-G/MM

Consider LG theory with single chiral superfield Ψ:

$$S = \int d^2z\, d^4\theta\, K(\Psi,\bar{\Psi}) + \left(\int d^2z\, d^2\theta\, W(\Psi) + c.c.\right)$$

Take $W(\Psi) = \Psi^{p+2}$. LVW calculate central charge c_p at IR fixed point:

$$c_p = 3p/p+2$$

…STR, MARTINEC
…ENKAR;
…G, VAFA,
…RNER;
…RTINEC;
…ITTEN

B) L-G / CY σ-MODEL

- CONSTRUCTING (SOME) CY MANIFOLDS:

  HYPERSURFACES IN $WCP^4_{(\omega_1,\ldots,\omega_5)}$:

- $WCP^4_{(\omega_1,\ldots,\omega_5)}$: $[z_1,z_2,z_3,z_4,z_5] \sim [\lambda^{\omega_1} z_1, \lambda^{\omega_2} z_2, \ldots, \lambda^{\omega_5} z_5]$

- $\dim_{\mathbb{C}} WCP^4 = 4$: PLACE 1 CONSTRAINT TO GET A 3 COMPLEX DIM SPACE
  (CENTRAL CHARGE = 9)

- EQN: $\sum_i z_i^{p_i+2} = 0$ SUCH THAT:

  - MAKES SENSE UNDER ALL $z_i \to \lambda^{\omega_i} z_i$
    i.e. $\omega_i (p_i+2)$ ALL EQUAL

  - $d = \omega_i (p_i+2) = \sum \omega_i$

    (THIS ENSURES M IS CY)

- e.g. $CP^4_{(1,1,1,1,1)}$: $\sum z_i^5 = 0$ "QUINTIC" CY.

• RECALL: $c<3$ $N=2$ THEORIES ARE THE MINIMAL MODELS!

$\Rightarrow$ L-G FOR $W = \Psi^{p+2} \cong$ LEVEL p $N=2$ M-M

(MORE PRECISELY: A-INVARIANT LEVEL p M-M)

• FOR LATER USE NOTE:

$$MM_{p_1} \otimes MM_{p_2} \otimes \cdots \otimes MM_{p_r} \cong$$

IR FIXED POINT OF $\left\{ \int d^2z\, d^4\theta\, K(\Psi_i, \bar{\Psi}_i) + \int d^2z\, d^2\theta \left(\sum_i \Psi_i^{p_i+2} \right) + c.c. \right\}$

• CONSIDER NOW:

L-G THEORY: 5 CHIRAL SUPERFIELDS $(\Psi_1,\dots,\Psi_5)$; $W(\Psi_1,\dots,\Psi_5) = \sum \Psi_i^{P_i+2}$

WITH P_i's AS FOR C-Y

• WHAT IS CENTRAL CHARGE?

$$C = \sum_{i=1}^{5} 3P_i / P_i+2 \qquad (P_i+2)\omega_i = d \Rightarrow$$

$$= \sum 3\,(d/\omega_i - 2)/(d/\omega_i) = 3\cdot\Sigma(1 - 2\omega_i/d)$$

$$= 3\cdot 5 - 3\cdot 2\cdot \Sigma\omega_i/d = 3\cdot 5 - 3\cdot 2 = \boxed{9}$$

GEOMETRICAL CY CONDITION $\leftrightarrow$ CFT CONDITION

COULD IT BE THAT:

• NLσM: $\int d^2z\, g_{\mu\bar\nu}\, \partial X^\mu \bar\partial X^{\bar\nu} + \dots$ ON $M = \sum z_i^{P_i+2} = 0$ IN $WCP^4_{(\omega_1,\dots,\omega_5)}$

• LG: $\int d^2z\, d^4\theta\, K(\Psi^i; \Psi^{\bar i}) + \left(\int d^2z\, d^2\theta\, (\Sigma \Psi_i^{P_i+2}) + c.c.\right)$

IN IR LIMIT ARE ISOMORPHIC CFT's?

VW; ARTINEC; ITTEN

• YES!

• ARGUMENT #1 (GVW)

CONSIDER LG THEORY

$$\int K(\psi^i, \psi^{\bar{i}})\, d^2z\, d^4\theta + \left(\int d^2z\, d^2\theta \sum \psi_i^{p_i+2} + c.c.\right)$$

• RECALL : W NOT RENORMALIZED ALONG RG FLOWS
K IS IRRELEVANT OPERATOR
$(d^4\theta$ picks out wt $(h+1, \bar{h}+1)$ component
IF h = dim of lowest component of K
$h, \bar{h} \geq 0 \Rightarrow h+1+\bar{h}+1 \geq 2)$

• SO : DEFORM $K \to \varepsilon K$: SHOULD YIELD SAME FIXED POINT THEORY

• IN FACT : BE BOLD : $\varepsilon \to 0$

$\Psi = \phi_\psi + \theta\psi + \theta\bar{\theta}\Psi_1$

• $\int [D\psi_1] \cdots [D\psi_5]\, e^{i\left(\int d^2z\, d^2\theta \sum \psi_i^{p_i+2} + c.c.\right)}$

• CHANGE VARIABLES : $\xi_1 = \psi_1^{p_1+2}$, $\xi_i^{p_i+2} = \psi_i^{p_i+2} / \psi_1^{p_1+2}$ $i = 2,3,4,5$
$(\psi_1 \neq 0)$

• By direct calculation:

$$[\mathcal{D}\psi_1]\cdots[\mathcal{D}\psi_5] = [\mathcal{D}\xi_1]\cdots[\mathcal{D}\xi_5] \quad (\text{Use } d = \sum \omega_i)$$

• Theory $\Rightarrow \int [\mathcal{D}\xi_1]\cdots[\mathcal{D}\xi_5]\, e^{i\left(\int d^2z\, d^2\theta\, \xi_1 (1+\sum \xi_i^{p_i+2}) + c.c.\right)}$

• Consider factor:

$$\int [\mathcal{D}\xi_{1,F}][\mathcal{D}\xi_2]\cdots[\mathcal{D}\xi_{5,}]\, e^{i\left(\int d^2z\, \xi_{1,F} (1+\sum \xi_{i,s}^{p_i+2}) + c.c.\right)}$$

F-comp scalar comp

$$\propto \delta\left(1+\sum \xi_{i,s}^{p_i+2}\right)$$

• $\Rightarrow$ Scalar part of L-G superfields live on

$$1+\sum \xi_{i,s}^{p_i+2} = 0$$

• But, CY: $\sum z_i^{p_i+2} = 0 \Rightarrow [z_1,\ldots,z_5] \sim [\lambda^{\omega_1} z_1,\ldots,\lambda^{\omega_5} z_5]$

with $\lambda^{\omega_1} = z_1^{-1}$

$$\Rightarrow \quad 1+\sum y_i^{p_i+2} = 0 \quad \left(y_i^{p_i+2} = z_i^{p_i+2}/z_1^{p_1+2}\right)$$

So: LG fields actually live on CY!

REMARKS

1) NONLINEAR CHANGE OF VARIABLES NOT 1:1.
TO REMEDY:
IDENTIFY $(\Psi_1, \dots, \Psi_5) \sim (\alpha^{\omega_1} \Psi_1, \dots, \alpha^{\omega_5} \Psi_5)$
$\alpha^d = 1$.

SO: REALLY: LG ORBIFOLD W/ $\cong$ CY σ-MODEL W=0

2) SPECIFYING EQN $\sum Z_i^{P_i+2} = 0$ FIXES COMPLEX STRUCTURE OF CY. WHAT ABOUT KÄHLER STRUCTURE? (VAFA) (ENHANCED SYMMETRIC POINT)

LG ORBIFOLD (W) $\cong$ CY σ-MODEL
W=0 AT A SPECIFIC KÄHLER STRUCTURE

3) ARGUMENT INVOLVES "UNCOMFORTABLE" MANIPULATIONS → WITTEN HAS GIVEN A MORE POWERFUL ARGUMENT.

B) CY/LG CONNECTION, AGAIN (WITTEN)

- CONSIDER A 2-D $N=2$ $U(1)$ GAUGE THEORY:
  (DO CASE OF QUINTIC TO BE CONCRETE) (NOT CONF. INV.!)
- FIELDS: CHIRAL SUPERFIELDS CHARGE

CHIRAL SUPERFIELDS	CHARGE
P	-5
S_1, S_2, S_3, S_4, S_5	1

- ACTION: $S = S_{KIN} + S_W + S_{GAUGE} + S_{FAYET\text{-}ILLI} \left\{-r\int d^2y\, D\right\}$

- SUPERPOTENTIAL: $W(P, S_1, \ldots, S_5) = P \cdot G(S_1, \ldots, S_5)$

$$G(S_1, \ldots, S_5) = \sum_i S_i^5 \quad (P \cdot G \text{ is } U(1) \text{ INV})$$

- BOSONIC POTENTIAL:

$$U = |G(s_i)|^2 + |p|^2 \sum_i |\partial G/\partial s_i|^2 + \frac{1}{2e^2} D^2 +$$

$$2|\sigma|^2 \left(\sum |s_i|^2 + 5^2 |p|^2\right)$$

$$D = -e^2 \left(\sum \bar{s}_i s_i - 5\bar{p}p - r\right)$$

(SMALL LETTERS = SCALAR COMPONENTS OF CORRESPONDING SUPERFIELD;
σ ARISES FROM GAUGE FIELD (IN 4-D $\to$ 2-D).)

• **NOW**: CONSIDER VACUUM CONFIGURATIONS FOR EITHER $r \gg 0$ or $r \ll 0$:

i) $r \gg 0$: $D = 0 \Rightarrow S_i$ NOT ALL ZERO
$\Rightarrow |\partial G/\partial S_i|^2 \neq 0 \Rightarrow p = 0 \Rightarrow \sum S_i \bar{S}_i = r$.
$\Rightarrow \sigma = 0. \Rightarrow G(S_i) = 0$
(p, σ, GAUGE FIELD MASSIVE).

• SO: $(S_1, \ldots, S_5)$ WITH $\sum S_i^5 = 0$ AND $\sum S_i \bar{S}_i = r$.
• FURTHERMORE: $U(1)$ INV $\Rightarrow (S_1, \ldots, S_5) \sim (e^{i\theta} S_1, \ldots, e^{i\theta} S_5)$

• NOTE: IN $\mathbb{CP}^4$: $(z_1, \ldots, z_5) \sim (\lambda z_1, \ldots, \lambda z_5)$. CAN CHOOSE $|\lambda|$ S.T. $\sum \bar{z}_i z_i = r$. THEN STILL HAVE $(z_1, \ldots, z_5) \sim e^{i\theta}(z_1, \ldots, z_5)$
r SETS "SIZE"
• THUS: $r \gg 0 \Rightarrow$ NLσM ON QUINTIC IN $\mathbb{CP}^4$
(IN IR LIMIT $\rightarrow$ <u>N=2 SCFT ON QUINTIC</u>)

ii) $r \ll 0$:
$D = 0 \Rightarrow p \neq 0$. $p \neq 0 \Rightarrow S_i$ ALL $= 0$ $(|\partial G/\partial S_i|^2 = 0)$
$\Rightarrow p = \sqrt{-r/5} \quad \Rightarrow \sigma = 0$.

• S_i ARE MASSLESS FLUCTUATIONS ABOUT UNIQUE VACUUM STATE, (ALL OTHER FIELDS MASSIVE), W DEGENERATE. $\Rightarrow$ <u>L-G THEORY</u>

• $p \neq 0 \Rightarrow U(1) \rightarrow \mathbb{Z}_5 \Rightarrow$ LG WITH $\mathbb{Z}_5$ GAUGE GROUP
• i.e. <u>LG ORBIFOLD</u>.

- THUS:
- AS $N=2$ NON CONF. THEORIES: $S(r) \begin{cases} \xrightarrow{r \gg 0} \text{NON-CONFORMAL CY } \sigma\text{-MODEL ON } G=0 \\ \xrightarrow{r < 0} \text{NON-CONF. LG ORBIFOLD, SUPERPOT} = G \end{cases}$

REMARKS

1) TAKE IR LIMIT OF ABOVE: RECOVER CONFORMAL LG/CY CORRESPONDENCE.
   - MORE RIGOROUS
   - $\tilde{r}$ CORRESPONDS TO (COEFF OF) MARGINAL OPERATOR ($\tilde{r}$ = IMAGE OF r AFTER IR LIMIT)