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AUTUMN COURSE

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VARIATIONAL METHODS IN ANALYSIS AND MATHEMATICAL PHYSICS

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OPTIMAL CONTROL OF CONTINUOUS CASTING

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OPTIMAL CONTROL OF CONTINUOUS CASTING

The optimal control problem of a continuous casting process is characterized by the following points :

- the state equation is associated with a free-boundary problem (two phase Stefan problem) ;
- the dimension of an installation is very large. So very efficient algorithms for the simulation and the optimization are necessary.

Two lectures are devoted to this problem :

Lecture 1 : 1) Physical system and its formulation as an optimal control problem.

2) Theoritical point of view.

Lecture 2 : 1) Numerical algorithms, pratical implementation and numerical results.

2) Some open problems.

This work has been done at INRIA in collaboration with IRSID (French Research Institute of Steel Making). For details we refer to J. HENRY - M. LARRECQ - J. PETEGNIEFF - C. SAGUEZ [4] , C. SAGUEZ [5] .

I - THE CONTINUOUS CASTING PROBLEM.

I-1) The physical installation.

The principle is to cast the steel in a mold, the bottom of which is constitued by the solid part of the steel. A general scheme of a such installation is given Figure I.1. We distinguish two parts :

- the mold (approximately 70 cm long), in which the liquid steel is introduced by a nozzle. In general the mold is in cupper and the steel is cooled by a system of flowing water . At the end of the mold, the thickness of the solid shell of steel must be sufficient to avoid break-out ;
- the second cooling part, in which the ingot is supported by rolls. The steel is cooled by a water-spray system constitued by spray-nozzles distribued on 6 (or 7) zones . Each zone can be regulated independently. This part is approximatily 20 m long.

At the end, when the steel is completely solid, the ingot is cutted by a cutting torch.

The principal products are slabs , blooms and billets.

I-2) The regulation problem.

The physical problem, we consider, is to find the best regulation of the water-spray system to maximize the speed of extraction of the steel, such that the following constraints are verified :

- Metallurgical contraints

- to assure a complete solidification at a given point of the installation,
- to avoid break-out at the end of the mold,
- to obtain an admissible temperature at the unbending point,

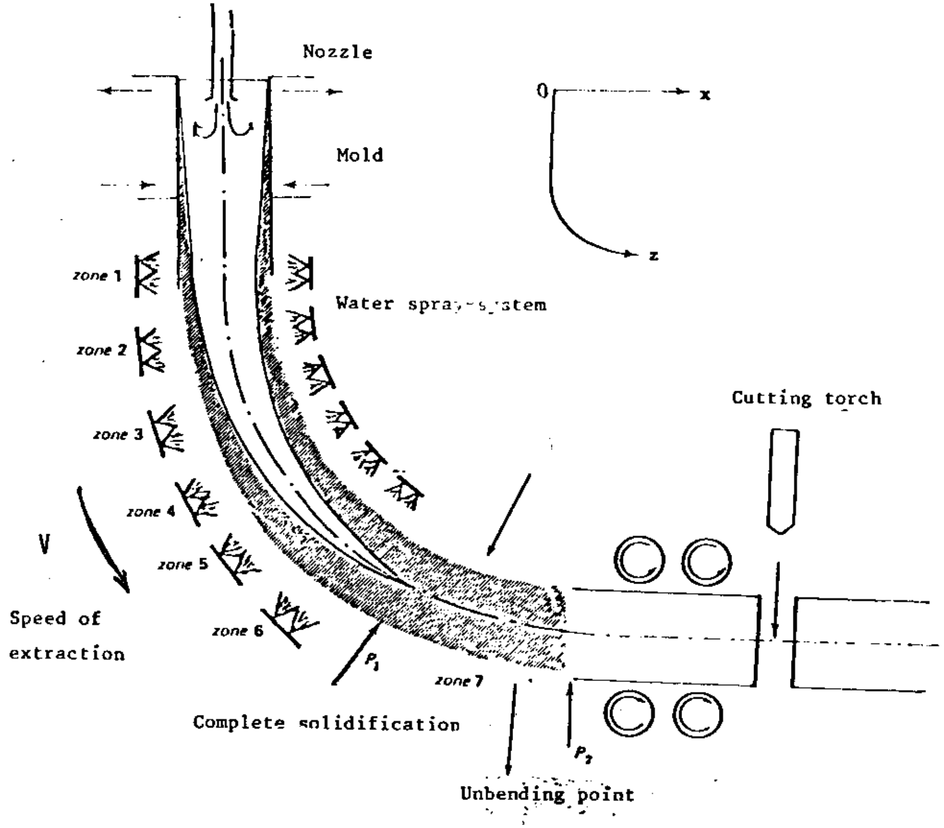


Figure 1.1. Continuous casting

- to avoid the formation of cracks,
- to limit the screep.

....

- Structural contraints

- limits of working of each spray-nozzle,
- usable quantity of water bounded.

I-3) The mathematical modelization.

We consider the system only from a thermal point of view (for example, we don't take into account mechanical problems as elasto-plasticity,...) and all metallurgical contraints are exprimed in tern of thermal constraints.

We present the problem in the stationary case (i.e. when the speed of extraction V is constant).

- The state equation

We have a classical solidification problem which is modelized by the following two phase Stefan problem (θ is the temperature of the steel) in the domain Ω (see figure 1.2.).

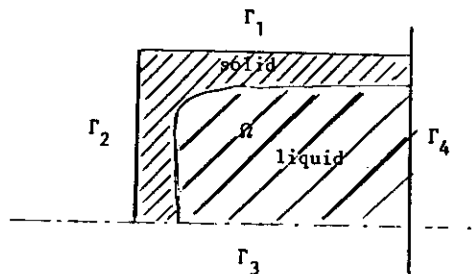


Figure 1.2.

(1.1) $$\rho V \frac{\partial H(\theta)}{\partial z} - \text{div}\,(\lambda(\theta)\ \text{grad}\ \theta) = 0 \quad \text{in} \quad \Omega \times]0,Z[$$

with ρ the density the steel

λ the conductivity

H the enthalpy of the system (see figure 1.3).

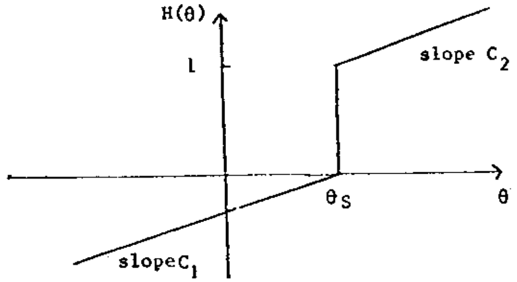


Figure 1.3. : Enthalpy graph

We have the boundary conditions :

- for the mold :

(1.2) $\lambda(\theta)\ \overrightarrow{\text{grad}\theta} . \vec{n} = g$

(g given along $\Gamma_1 U \Gamma_2$, $g \equiv 0$ along $\Gamma_3 U \Gamma_4$)

for the water spray system :

(1.3)
$$\begin{cases} \lambda(\theta)\ \overrightarrow{\text{grad}\theta} . \vec{n} + h\,(\theta - \theta_e) = 0 & \text{on } \Gamma_1 U \Gamma_2 \\ \lambda(\theta)\ \overrightarrow{\text{grad}\theta} . \vec{n} = 0 & \text{on } \Gamma_3 U \Gamma_4 \end{cases}$$

(h, the exhange coefficient between water and steel, is the control variable).

Finally we have the initial condition :

(1.4) $\theta(x,0) = \theta_0(x)$.

the contraints

Metallurgical contraints

The contraints,described in I-2,can be exprimed in term of thermal constraints as follows :

(1.5) $H(\theta(x,z)) \leq H(\theta_s) = L \quad \forall z \in [Z_1, Z]$

(L is the latent heat;Z_1 the point of complete solidification).

(1.6) $\theta(.,Z_2)_{|\Gamma_1 U \Gamma_2} \notin]\theta_1, \theta_2[$

(Z_2 is the unbending point. In fact, we consider successively $\theta(.,Z_2)_{|\Gamma_1 U \Gamma_2} \geq \theta_2$ and $\theta(.,Z_2)_{|\Gamma_1 U \Gamma_2} \leq \theta_1$)

(1.7) $CM \leq \dfrac{\theta(.,z) - \theta(.,z+\Delta z)}{\Delta z}_{|\Gamma_1 U \Gamma_2} \leq CC$

(whereΔz is given).

(1.8) $\theta(.,z)_{|\Gamma_1 U \Gamma_2} \leq \theta_M(z) \quad \forall z \geq Z_L$

(Z_L is the length of the mold)

- Structural contraints :

The water-spray system is divided in N_z zones (in general 6 or 7).

For each zone, the exhange coefficient is independant of z and we denote by $h_i \in L^2(\Gamma_1 U \Gamma_2)$ the coefficient for the zone i.

We have the following constraints :

(1.9) $h_{m_i} \leq h_i \leq h_{M_i} \qquad i = 1,\dots, N_z$

(1.10) $\sum_{i=1}^{Nz} \alpha_i\, h_i \leq D_M$

(D_M is the global quantity of water).

I-4) The optimal problem.

We transform the initial problem, to find the maximal speed V such that there exists an admissible h (i.e. h, such that the contraints (1.5)-(1.10) are verified), in the following one :

- the state equation is given by (1.1)-(1.4)
- the admissible set of control is :

$\mathcal{H}ad=\{h_i,\ i=1,\dots Nz\ |\ \{h_i\}\text{verify } (1.9)-(1.10)\}$

- the fonctional J is :

$J(h)=\gamma_1 J_1(h) + \gamma_2\ J_2(h)+\gamma_3 J_3(h) + \gamma_4 J_4(h)$

with $\gamma_i \in \mathbb{R}^+$ and $J_i(h)$ defined by :

(1.11) $$J_1(h) = \int_{Z_I}^{Z} \int_{\Omega} [(H(\theta(x,z))-L)^+]^2 dx\, dz$$

(1.12) $$J_2(h) = \int_{\Gamma_1 \cup \Gamma_2} [(\theta(x,Z_2) - \theta_2)^-]^2 d\Gamma$$

(if we consider the case $\theta(.,Z_2) \geq \theta_2$)

(1.13) $$J_3(h) = \int_{Z_L}^{Z-\Delta z} \int_{\Gamma_1 \cup \Gamma_2} \left\{\left[\frac{\theta(x,z) - \theta(x,z+\Delta z)}{\Delta z} - CC\right]^+\right\}^2 + [\{\frac{\theta(x,z) - \theta(x,z+\Delta z)}{\Delta z} - CM\}^+]^2\}\ d\Gamma\ dz$$

(1.14) $$J_4(h) = \int_{Z_L}^{Z} \int_{\Gamma_1 \cup \Gamma_2} [(\theta(.,z) - \theta_M(z))^+]^2\, d\Gamma\, dz$$

- The optimal control problem

(1.15) $$\begin{cases} \text{To find } \bar{h} \in \mathcal{H}_{ad} \text{ such that} \\ J(\bar{h}) \leq J(h) \quad \forall h \in \mathcal{H}_{ad} \end{cases}$$

Then, if $J(\bar{h})=0$, we see that $\bar{h}$ is an admissible h is the above sense. Now we study the optimal control problem (1.15)

Remark 1.1.

In this presentation, the formulation in term of enthalpy is considered. An other formulation, with the concept of variational inequality, could be used (see C. SAGUEZ [5])

□

Now to simplify the presentation for the theorical results and the numerical methods , we shall assume that $\lambda(\theta)= \lambda= 1$; $\rho=1$ and that $\gamma_2= \gamma_3= \gamma_4=0$; $\gamma_1=1$.

The numerical results, we shall present, are obtained for the general formulation. We shall indicate, the modifications, we have to do, to take into account the general case.

We consider the temperature at the bottom of the mold as initial data (we have no control in the mold). So the boundary conditions are (1.3).

II - THEORICAL POINT OF VIEW.

The principal goal of this chapter, is to resume some theoritical results (In these lectures, we insist principally on the modelization problems and the numerical methods).

II-1) The state equation.

The state equation can be written in the abstract form :

$$(2.1)\quad \begin{cases} \dfrac{\partial u}{\partial z} + A\theta \ni f \text{ p.p.}, \quad u(z) \in B\theta(z) \text{ p.p.} \\ u(o) = u_o \end{cases}$$

where - $(B\theta)(x) = H(\theta(x))$ and $B = \partial\Phi_B$

(Φ_B is a convex continuous function from $L^2(\Omega)$ in $\mathbb{R}$)

- $A = \partial\Phi_A$; $\Phi_A(\theta) = \frac{1}{2}\int_\Omega |\text{grad}.\theta|^2 d\Omega + \frac{1}{2}\int_{\Gamma_1 \cup \Gamma_2} |\theta|^2 d\Gamma$

(Φ_A is a convex continuous function from $H^1(\Omega)$ in $\mathbb{R}$)

- $(f,\phi)_{(H^1(\Omega))' - H^1(\Omega)} = \int_{\Gamma_1 \cup \Gamma_2} h\theta_e d\Gamma$

Using results of O. GRANGE - F. MIGNOT [2], it is easy to prove, in this case, the proposition :

Proposition 1 : if $f \in L^\infty(o,T;(H^1(\Omega))')$; $\frac{df}{dz} \in L^2(o,T;(H^1(\Omega))')$

$\theta_o \in H^1(\Omega)$; $u_o \in B\theta_o$

the problem (2.1) admits a solution (u,θ), with

$\theta \in L^\infty(o,T, H^1(\Omega))$; $u \in L^\infty(o,T; L^2(\Omega))$, $\frac{du}{dz} \in L^\infty(o,T; (H^1(\Omega))')$.

The demonstration use the following semi-discretized problem :

$$\begin{cases} (2.2)\quad \left(\dfrac{u^{n+1} - u^n}{k}, \phi\right) + (\text{grad } \theta^{n+1}, \text{grad } \phi) + \int_{\Gamma_1 \cup \Gamma_2} h^{n+1}(\theta^{n+1} - \theta_e^{n+1})\phi d\Gamma = o \quad \forall\phi \in H^1(\Omega) \\ (2.3)\quad u^{n+1} \in H(\theta^{n+1}) \\ (2.4)\quad u^o(x) = u_o(x) \in H(\theta_o) \ ; \ \theta^o(x) = \theta_o(x) \end{cases}$$

and results of duality to pass at the limit.

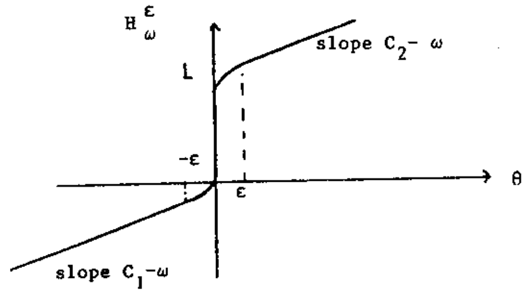
□

II-2) The optimal control problem.

Using the same techniques than for the proposition 1, we prove the existence of a solution for the optimal control problem (1.15).

The main problem is to obtain necessary optimatity conditions. Such conditions have been only demonstrated for the following regularized semi-discritezed problem :

$$\begin{cases} (2.5)\quad \left(\dfrac{w_\varepsilon^{n+1} - w_\varepsilon^n}{k}, \phi\right) + (\text{grad } \theta_\varepsilon^{n+1}, \phi) + \left(\omega \dfrac{\theta_\varepsilon^{n+1} - \theta_\varepsilon^n}{k}, \phi\right) + \\ \qquad \int_{\Gamma_1 \cup \Gamma_2} h^{n+1}(\theta_\varepsilon^{n+1} - \theta_\varepsilon^{n+1}) \phi d\Omega = o \\ \qquad n = o, \ldots, NZ-1 \\ (2.6)\quad w_\varepsilon^{n+1} \in H_\omega^\varepsilon(\theta_\varepsilon^{n+1}) \end{cases}$$



$(o < \omega < \text{Min } (C_1, C_2))$

Figure 2.1.

The G-differentiability of the solution with respect to $\{h^{n+1}\}$ is proved and if (r_i^n, y_i^n) denotes the G- derivative function of $(w_\varepsilon^n, \theta_\varepsilon^n)$ with respect to h^i in the direction s^i, we have

$$
\begin{cases}
\left(\dfrac{r_i^{n+1}-r_i}{k},\phi\right) + (\text{grad } y_i^{n+1}, \text{grad } \phi) + \dfrac{\omega}{k}(y_i^{n+1}-y_i^n,\phi) \\
\qquad + \int_{\Gamma_1 U \Gamma_2} h^{n+i} y_i \, \phi d\Gamma + \int_{\Gamma_1 U\Gamma_2} s^i \delta_i^{n+1} (\theta_\varepsilon^{n+1}-\theta_e^{n+1}) \, \phi d\Gamma \\
\qquad \forall \phi \in H^1(\Omega), \; n=o,\ldots, NZ-1 & (2.7) \\
r_i^{n+1} = H_\omega^{'\varepsilon,\mu}(\theta_\varepsilon^{n+1} + \mu w_\varepsilon^{n+1}) \quad (y_i^{n+1} + \mu r_i^{n+1}) & (2.8) \\
y_i^o = o \; , \; r_i^o = o & (2.9)
\end{cases}
$$

where $H_\omega^{'\varepsilon,\mu}$ is the derivative function of $H_\omega^{\varepsilon,\mu}$, Yosida approximation of H_ω^ω.

For the case of the functional :

$$J(h) = \sum_{i=1}^{NZ} ||(u^n-L)^-||^2_{L^2(\Omega)} \qquad (2.10)$$

(discretization of the fonctional $J_1(h)$).

We obtain the necessary optimality conditions :

Proposition 2 : $(w_\varepsilon^n, \theta_\varepsilon^n, h_\varepsilon^n)$ Solution of the regularized semi-discretized optimal control problem, verify the necessary optimality conditions :

State equation :

$$
\begin{cases}
\left(\dfrac{w_\varepsilon^{n+1} - w_\varepsilon^n}{k},\phi\right) + (\text{grad } \theta_\varepsilon^{n+1}, \text{grad } \phi) + \dfrac{\omega}{k}(\theta_\varepsilon^{n+1} - \theta_\varepsilon^n,\phi) \\
\qquad + \int_{\Gamma_1 U\Gamma_2} h_\varepsilon^{n+1} (\theta_\varepsilon^{n+1}-\theta_\varepsilon^{n+1}) \phi d\Gamma = o \quad \forall \phi \in H^1(\Omega) & (2.11) \\
w_\varepsilon^{n+1} \in H_\omega^\varepsilon (\theta_\varepsilon^{n+1}) \qquad n = o,\ldots, NZ-1 & (2.12) \\
\theta_\varepsilon^o = \theta_o \; ; \; w_\varepsilon^o = w_o \in H_\omega^\varepsilon(\theta_o) & (2.13)
\end{cases}
$$

Adjoint state equation :

$$
\begin{cases}
\dfrac{1}{k}(q_\varepsilon^n H_\omega^{'\varepsilon,\mu}(\theta_\varepsilon^{n+1}+\mu w_\varepsilon^{n+1}),\phi) + (\text{grad } p_\varepsilon^n, \text{grad}\phi) + \dfrac{\omega}{k}(p_\varepsilon^n - p_\varepsilon^{n+1},\phi) \\
\qquad + \int_{\Gamma_1 U\Gamma_2} h^{n+1} p_\varepsilon^{n+1} \phi d\Gamma = -2((u_\varepsilon^{n+1}-L)^-,\phi) \quad \forall \phi \in H^1(\Omega) & (2.14) \\
p_\varepsilon^n - p_\varepsilon^{n+1} = q_\varepsilon^n (1-\mu H_\omega^{'\varepsilon,\mu}(\theta_\varepsilon^{n+1}+\mu w_\varepsilon^{n+1})) \qquad n = o,\ldots, NZ-1 & (2.15) \\
p_\varepsilon^{NT} = o & (2.16)
\end{cases}
$$

Optimality conditions :

$$\sum_{n=1}^{NZ} \int_{\Gamma_1 U\Gamma_2} (\theta_\varepsilon^n - \theta_e^n) \, p_\varepsilon^{n-1} (s^n - h_\varepsilon^n) \, d\Gamma \leq o \quad \forall \{s^n\} \in \mathcal{H}ad. \qquad (2.17)$$

□

Remark 2.1. It is possible to prove that, when $\varepsilon \to o$, the solution $(w_\varepsilon^n, \theta_\varepsilon^n, h_\varepsilon^n)$ converges to (w^n, θ^n, h^n) solution of the semi-discretized problem.

□

III - NUMERICAL METHODS.

We present succesily :

- the algorithms to solve the state equation and to compute the optimal control,
- the discretization (in x) of the problem by the finite element methods.

III-1) Resolution of the state equation.

We have to solve the system (2.2)-(2.4). At each step of time, the problem is of this type :

To find (u,θ), such that :

$$(3.1)\quad \begin{cases} \frac{1}{k}(u,\phi) + (\text{grad}\ \theta, \text{grad}\ \phi) + \int_{\Gamma_1 \cup \Gamma_2} h(\theta-\theta_e)\,d\Gamma = (f,\phi) \quad \forall \phi \in H^1(\Omega) \\ u \in H(\theta) \end{cases}$$

As in the Chapter II, we introduce ω such that $o<\omega$ $\mathrm{Min}(c_1,c_2)$ and we transform (3.1) in :

$$(3.2)\quad \begin{cases} \frac{1}{k}(w,\phi) + (\text{grad}\ \theta, \text{grad}\ \phi) + \frac{\omega}{k}(\theta,\phi) + \int_{\Gamma_1 \cup \Gamma_2} h(\theta-\theta_e)\,d\Gamma = (f,\phi) \\ \qquad \forall \phi \in H^1(\Omega) \\ u \in H_\omega(\theta) = H(\theta) - \omega\theta. \end{cases}$$

Now we use the following equivalence :

$$w \in H_\omega(\theta) \Longleftrightarrow w = H_\omega^\mu(\theta+\mu w)\ \forall\ \mu>o.$$

Then we deduce the algorithm :

1) w^o given, $i = o$

2) $i = i + 1$
To compute θ^i solution of the P.D.E. :

$$(\text{grad}\ \theta^i, \text{grad}\ \phi) + \frac{\omega}{k}(\theta^i,\phi) + \int_{\Gamma_1 \cup \Gamma_2} h\,(\theta^i-\theta_e)\phi\,d\Gamma = (f + \frac{1}{k}w^{i-1},\phi) \quad \forall \phi \in H^1(\Omega)$$

3) $w^i = H_\omega^\mu(\theta^i+\mu w^{i-1})$

4) Test of convergence

if verified → END

if not, go to 2).

We have the result of convergence :

<u>Proposition 3</u> : <u>for</u> $\mu>\mu_o = \frac{1}{k}\ \frac{1}{2\,\mathrm{Min}(1,\frac{\omega}{k})}$, <u>we have</u>

$w^i \rightharpoonup w$ in $L^2(\Omega)$ weakly

$\theta \longrightarrow \theta$ in $H^1(\Omega)$ strongly.

For the demonstration we refer to C. SAGUEZ [5]. □

III-2) Algorithm of optimization.

In fact, we solve the regularized problem and to compute the optimal control, a gradient method with projection is used. The algorithm is classical, except for the projection of the control on $\mathcal{U}_{ad}$.

Suppose that we know the control $\{h_k^i\}$, and the gradient $\{G_k^i\}$ (i is the index of iteration ; k the index of zone). To compute $\{h_k^{i+1}\}$ we do the iterations :

$$h_k^{i+1,j} = \mathrm{Max}\,(h_{m_k},\ \mathrm{Min}\,(h_{M_k} - \rho_1^j\,(g_k^i - \lambda^j \alpha_k))$$

$$\lambda^{j+1} = \mathrm{Min}\,(o, \lambda^j - \rho_2^j\,(\Sigma_k \alpha_k h_k^{i+1,j} - D_M))$$

where ρ_1^j and ρ_2^j are positive constants.

This algorithm has been introduced in J. HENRY [3].

III-3) Approximation of the problem.

- The state equation has been approximated by finite elements methods.
- In each case, we determine explicitly the discretized adjoint state equation associated with the discretization of the problem. We don't detail here this point.

III-4) Numerical results.

We present, in one dimensional case, numerical results obtained for the continuous casting process of USINOR-Dunkerque.

i) Data of the problem.

The conductivity of the steel is given by :

T(oC)	λ(cal/cm/s/oC)	T(oC)	λ(cal/cm/s/oC)
550	0,091	1100	0,068
600	0,086	1150	0,070
650	0,081	1200	0,071
700	0,076	1250	0,072
750	0,071	1300	0,073
800	0,068	1350	0,075
850	0,065	1400	0,076
900	0,064	1450	0,077
950	0,065	1500	0,078
1000	0,066	1550	0,079
1050	0,067	1600	0,080

Conductivity of the steel

The enthalpy is given Figure 3.1, with the data :

$C_1 = 0{,}2$ cal/g°C ; $C_2 = 0{,}16$ cal/g°C ; $L = 63{,}44$ cal/g

$\theta_s = 1496$; $\theta_L = 1519$

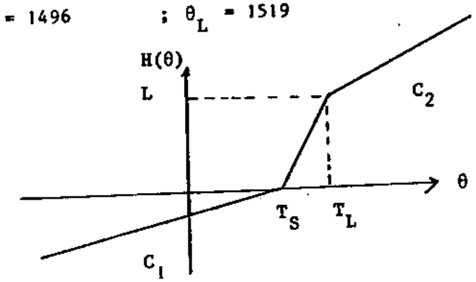


Figure 3.1.

Initial temperature $\theta_o(x) = 1535$°c

Density of the steel $\rho = 7$g/cm3

In the mold $g = 23$ cal/cm²/s

Dimensions of the installation

Lenght of the mold : 70 cm

Water-spray system : zone 1 27 cm
zone 2 32 cm
zone 3 187 cm
zone 4 150 cm
zone 5 347 cm
zone 6 412 cm
zone 7 400 cm

Lenght of the installation : 1650 cm.

Thickness of a slab : 21 cm.

Metallurgical contraints

Complete solidification at $Z_1 = 16$ m

At the unbending point $\theta_2 = 1000°c$

CM = $-2°c/s$ on 60 s ; CC = $1°c/s$ on 60 s

$\theta_M(x) = 1100°c$

Structural contraints

We have the data :

Zone	h_{iM}	h_{iM}	α_i
1	0,0225	0,0208	21022
2	0,0211	0,0135	2394
3	0,0168	0,0087	4454
4	0,0125	0,0076	2532
5	0,0086	0,0052	5329
6	0,0060	0,0044	5746
7	0,0055	0,0039	6088

$D_M = 130\ m^3/h$

Parameters of the functional

$\gamma_1 = 1$; $\gamma_2 = 1$; $\gamma_3 = 1000$ $\gamma_4 = 1$

Parameters of discretization

$\Delta z = 10$ cm ; $\Delta x = 0,35$ cm

ii) Results.

We present the exhange coefficient and the boundary temperature for the USINOR regulation (Figure 3.1) and for the computed optimal control (Figure 3.2). The conclusions are the following :

- With this control, we can cast with a speed V = 1,075 m/mn (to compare to the speed obtained at Dunkerque 0,9 m/mn), (all the metallurgical contraints are verified).

- We can use this method to determine an "optimal" strategy for the automatization of the process.

Actually two others software are developped

- The stationary case in two-dimensional case (the software is now experimented at IRSID),

- The evolutionary case, in one-dimensional case.

CONCLUSION.

- We have present an efficient method to control a free boundary system (two-phase Stefan problem). This method is applied to the control of a continuous casting process to developp a strategy of automatization.

- Many problems are always open

  . to obtain optimality conditions for the initial continuous problem,

  . to developp real time regulation,

  . to take into-account mechanical phenomena (thermo-elasto-plasticity,...),

  . to take into account the different components of the steel (control of the solidification of an alloy).

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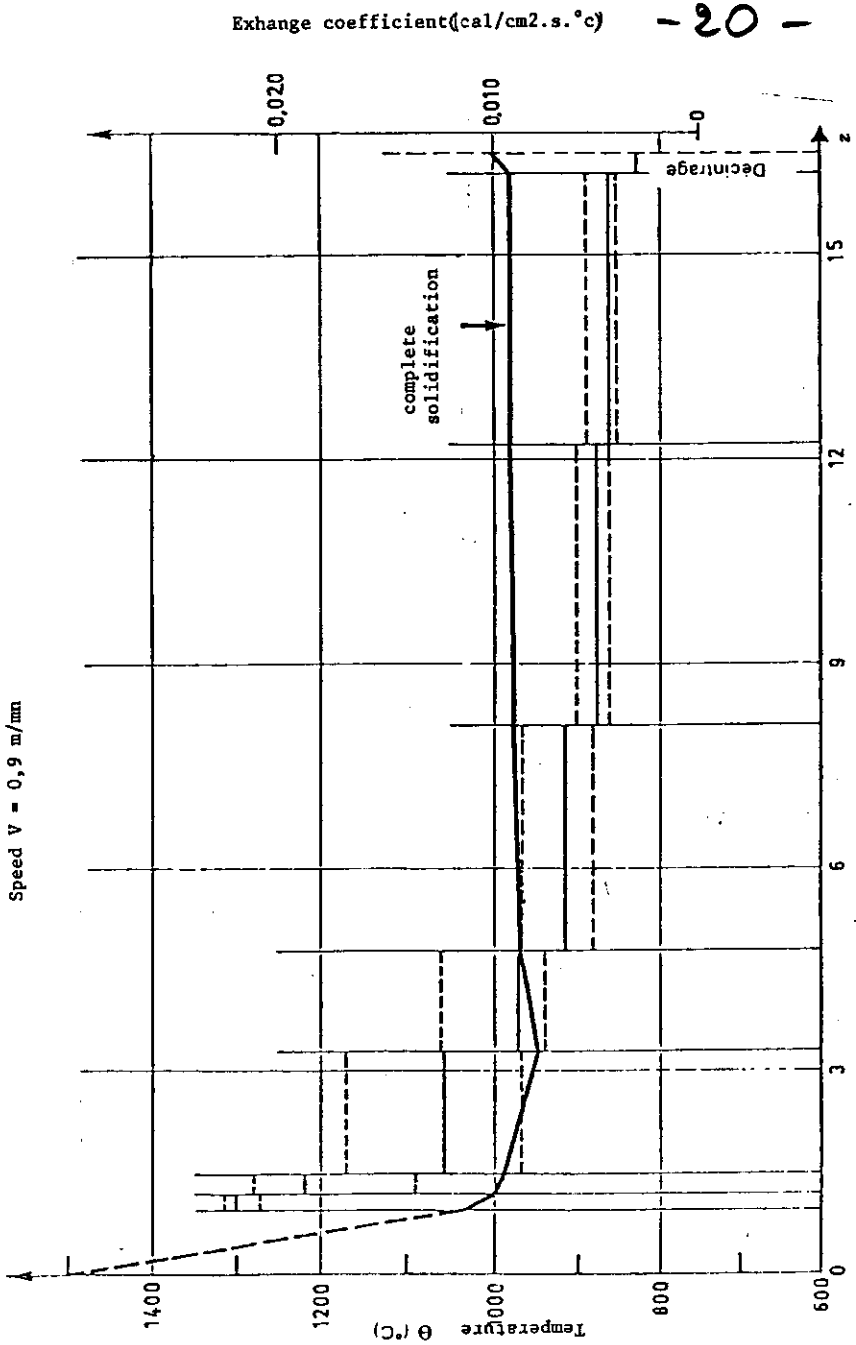
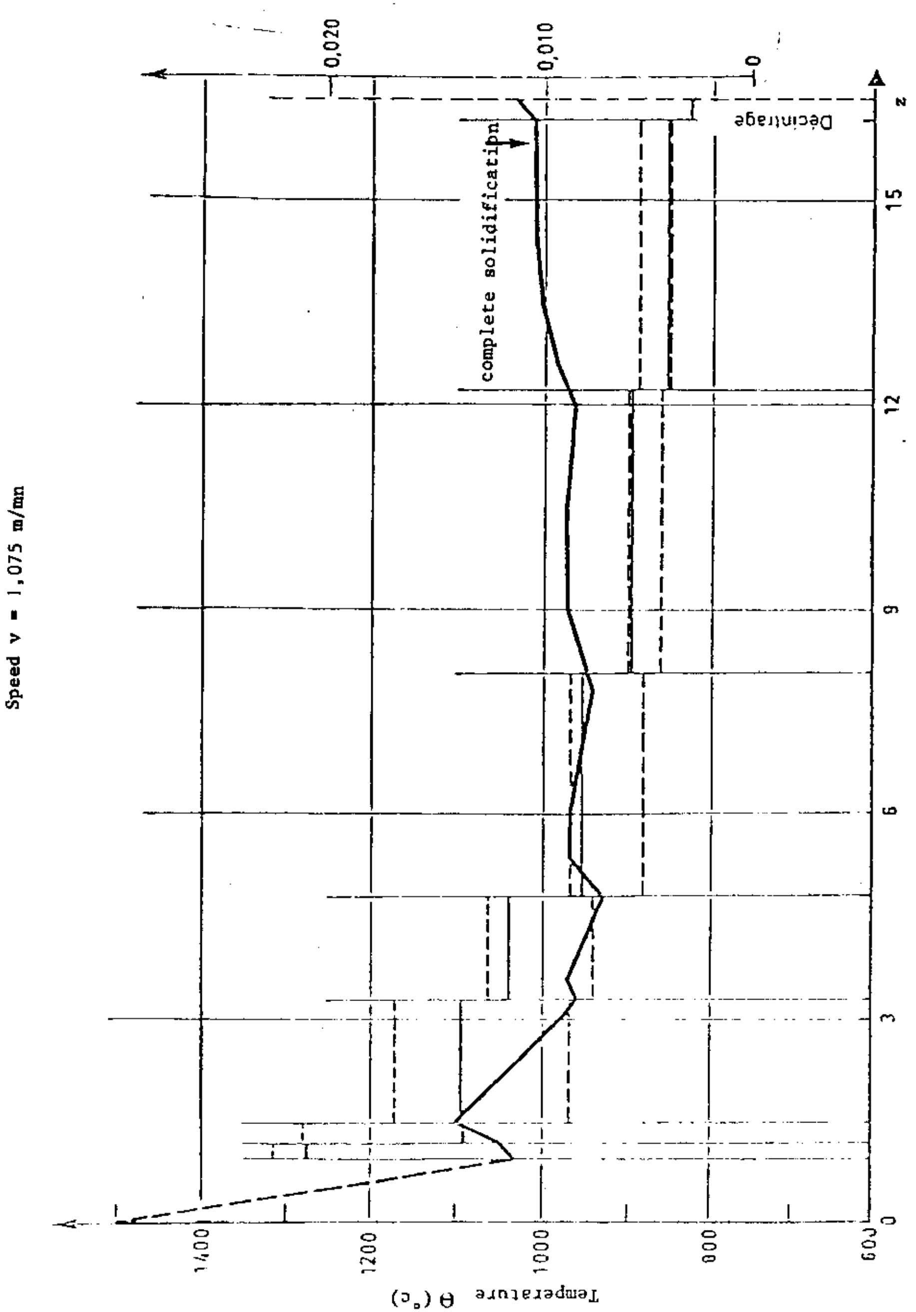


Figure 3.2 : USINOR - Regulation.

Speed V = 0,9 m/mn



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