UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL ATOMIC ENERGY AGENCY
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
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THIRD AUTUMN WORKSHOP ON MATHEMATICAL ECOLOGY

(14 October - 1 November 1996)

"Models of Agricultural Systems"

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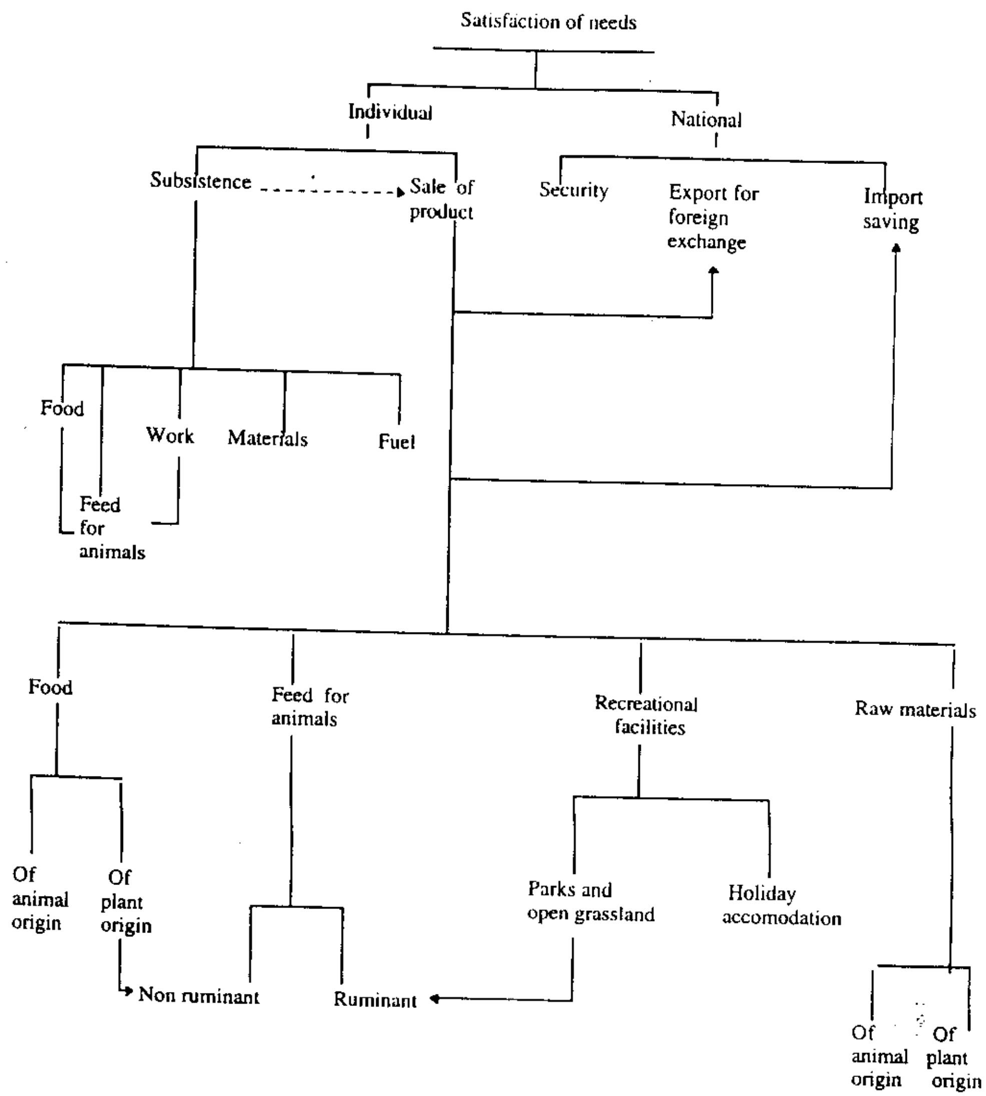
These are preliminary lecture notes, intended only for distribution to participants.

Third Autumn Workshop on Mathematical Ecology
14 October - 1 November 1996. Trieste

P. Racsko: Models of Agricultural Systems



Questions of long range agroecological modelling

Objectives: Economically efficient agriculture
Minimum of environmental damage
Reservation of natural equilibrium

Spatial structuring:
Small vs. large units
Natural vs. artificial boundaries

Structuring in time:
seconds → 100 yrs.

Functional structuring:
trophic networks – populations

Procedural structuring:
hydrology, atmosphere
biology, geology, etc.

PLANT GROWTH MODELS, VEGETATIVE PHASE
COMMON APPROACH:

$$X_{t+1}^{i} = f_t(X_t^1, X_t^2, \ldots, X_t^n) \cdot S_t^1 \cdot S_t^2 \ldots$$

- X_{t+1}^{i}: **biomass of part** i
- $f_t(\ldots)$: **empirical growth function at optimal cond.** ← **very hard to measure, to verify and to transfer to other places. impossible to use at non-standard cond.**
- $S_t^1 \cdot S_t^2 \ldots$: **stress factors** $0 \le S_t^j \le 1$ ← **not always true.**

Self organizing principle - <u>Principle of maximal productivity</u>

Growth equations:

$$x_i(t+1) = \{x_i(t) + e_i(t)y[x(t), V(t)]\}(1 - w_i)$$

where

$x_i(t)$ - **biomass of leaves, stem & roots**

$V(t) = (v_1(t) \ldots v_n(t))$ - **environmental cond.**

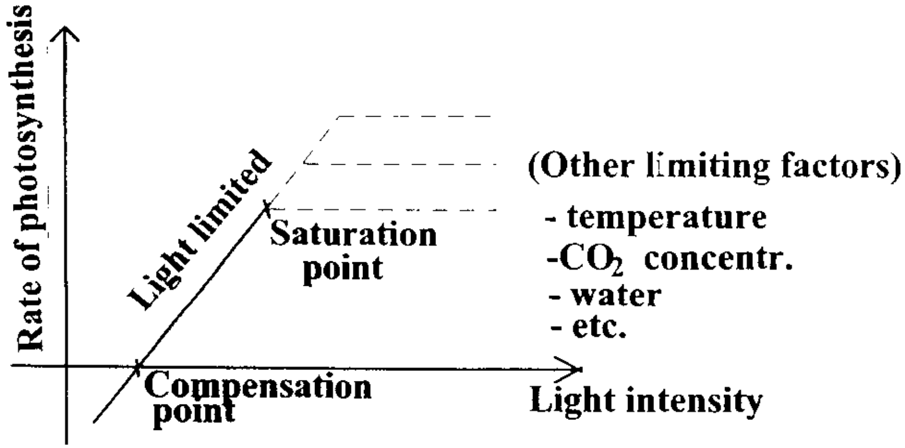
$e_i(t)$ - **allocation rate of the new assimilation**

$y(t)$ - **qty of new assimilates produced from** t **to** $t+1$

$w_i(t)$ - **loss rate**

$$\sum e_i(t) = 1 \qquad 0 \le e_i \le 1$$

Light intensity and photosythesis



Empirical formula for leaf photosynthesis:

$$P_l = \frac{\alpha I_l P_{max}}{\alpha I_l + P_{max}}$$

where P_{max} **- the value of** P **at saturating high level,**

α **-constant** (dP/dI **at** $I_l = 0$), **known as the photosynthetic efficiency.**

Response to CO2 level: $P_{max} = \tau C_a$

where τ **- CO2 conductance.**

Typical parameter values:

$\tau = 0.002$ $\mathbf{ms^{-1}}$ $C_a = 0.0006$ $\mathbf{kg\,CO_2m^{-3}}$

$P_{max} = 1.2 \times 10^{-6}\mathbf{kgCO_2m^{-2}s^{-1}}$

$\alpha = 13 \times 10^{-9}\mathbf{kgCO_2(JPAR)^{-1}}$

↑ **Photosynthetically active radiation**

Light interception on a horizontal plane

Monsi-Saeki equation: $I = I_0 e^{-kL}$

where: I_0 - light flux intensity above canopy

I- light intensity within canopy

L- leaf area index (leaf area/unit ground area)

k- constant (extinction coefficient)

on the leaf surface:

$$I_l = \left(\frac{k}{1-m}\right) I = \left(\frac{k I_0}{1-m}\right) e^{-kL}$$

where m - transmission coefficient of the leaf

Crop photosynthesis

$$P_c = \int_0^L \frac{\alpha I_l P_{max}}{\alpha I_l + P_{max}}\, dl$$

Substitution of the Monsi-Saeki equation gives:

$$P_c = \int_0^L \frac{k I_o \alpha P_{max} e^{-kL}}{k I_o \alpha e^{-kL} + P_{max}(1-m)}\, dL$$

or

$$P = -\frac{P_{max}}{k}\left\{\ln\left[k \cdot I_o e^{-kL} + P_{max}(1-m)\right]\right\}_o^L$$

The initial slope of photosynthesis:

$$\frac{dP_c}{dI_o}(I = 0) = \frac{\alpha\left(1 - e^{-kL}\right)}{1-m}$$

and a light saturation: $P_c(I_o \to \infty) = P_{max} L$

Table 8.4 Calculated maximum daily rates of net crop photosynthesis

Item	Process	Value
Total radiant energy available to crop	Total radiant energy	1674 x 10^8 J ha^{-1} day^{-1}
	50% in visible sector (photosynthetically active)	837 x 10^8 J ha^{-1} day^{-1}
Radiant energy losses to crop	Albedo 11% Not intercepted 11% Total lost 22% Total utilised by crop canopy 78%	652 x 10^8 J ha^{-1} day^{-1}
Efficiency of gross photosynthesis	C4 system Max. efficiency of white light conversion = 22% at low light intensities (Chapter 6)	143 x 10^8 J ha^{-1} day^{-1}
	Estimated loss of efficiency to allow for leaves receiving light at high intensity, less 30%	100 x 10^8 J ha^{-1} day^{-1}
	Equivalent DM = $\frac{100 \times 10^8}{167 \times 10^8}$	0.60 t ha^{-1} day^{-1}
	C3 system Less a further estimated 33% to allow for photorespiration	0.40 t ha^{-1} day^{-1}
Efficiency of net photosynthesis	C4 system Less 33% whole crop respiration*	0.40 t ha^{-1} day^{-1}
	C3 system Less 33% whole crop respiration*	0.27 t ha^{-1} day^{-1}
DM formed per 100 x 10^8 J ha^{-1} day^{-1} total incident radiant energy	C4 system C3 system	0.024 t ha^{-1} day^{-1} 0.016 t ha^{-1} day^{-1}
Total incident radiant energy required to produce 1 t DM J ha^{-1}	C4 system C3 system	4185 x 10^8 6194 x 10^8
Proportion of total incident radiant energy appearing as chemical energy in crop (%)	C3 system C4 system	4.0 2.7

* Arbitrary value - varies with temperature

Table 7.1 Growth rates and energy conversion of closed crop canopies in different climatic regions (after Cooper, 1975)

		Crop growth rate (gm^{-2} day^{-1} dry matter)	Total daily light energy incidence ($MJ\ m^{-2}$)	Conversion of light energy[a] (%)
Temperate				
C3 species:				
Tall fescue	UK	43	22.00	7.0
Ryegrass	UK	28	19.83	5.0
	Netherlands	20	(18.80)	(3.8)
Potato	Netherlands	23	(16.70)	(4.9)
Sugar beet	UK	31	12.30	8.6
	Netherlands	21	(14.60)	(5.0)
Kale	UK	21	15.98	4.4
Barley	UK	23	20.25	3.6
	Netherlands	18	(18.80)	(3.3)
Wheat	Netherlands	18	(18.80)	(3.3)
C4 species:				
Maize	Netherlands	17	(14.60)	(4.1)
	New Zealand	29	(18.80)	(5.5)
	Iowa, USA	28	(22.60)	(4.4)
Sub-tropical				
C3 species:				
Alfalfa	Calif., USA	23	(28.40)	(2.9)
Potato	Calif., USA	37	(28.40)	(4.6)
Cotton	Georgia, USA	27	(23.00)	(4.1)
Rice	N.S.W., Australia	23	(27.20)	2.7
C4 species				
Sorghum	Calif.,USA	51	28.87	6.0
Maize	Calif., USA	52	30.79	5.8
Tropical				
C3 species:				
Cassava	Tanzania	17	(18.80)	(3.3)
	Malaysia	18	(16.70)	(4.1)
Oil palm	Malaysia	11	15.90	3.0
Rice	Philippines	27	(16.70)	(5.8)
C4 species:				
Pennisetum typhoides	N.T., Australia	54	21.34	8.6
Pennisetum purpureum	El Salvador	39	(16.70)	(8.4)
Sugar cane	Hawaii, USA	37	16.87	7.6
Maize	Thailand	31	(20.90)	(5.3)

[a] Assuming 50% of total radiation is photosynthetically active light. Values in brackets estimated from radiation data in Black (1956), and using conversion factor of 1 g dry matter=17.8 kJ of fixed energy.

Table 8.6 Some examples of total radiant energy (RE) conversion from actual data

Crop or ecosystem	Location	Growth period (days)	Photo-synthesis system	Total RE ($J \times 10^8$ $ha^{-1}\ yr^{-1}$)	'Available' R ($J \times 10^8\ ha^{-1}$)	Production DM ($t\ ha^{-1}\ yr^{-1}$)	Production Conversion of RE into DM (%)
Climax - Tropical rain forest	Ivory Coast	365	C3	611 101	611 101	11.5	0.32[a]
Beech - 46 yr old	Denmark	180	C3			12.2	0.62[a]
86 yr old						10.1	0.52
Pine forest	UK	365	C3	345 680	320 723	37.3	1.95[a]
Deciduous forest	UK	180	C3	345 680	285 231	18.3	1.07[a]
Mass algate culture	Japan	365	C3	413 600	413 600	53.1	2.15[b]
Sugar cane - March	Hawaii	365	C4	676 236	676 236	78.7	1.95[b]
August	Hawaii	365	C4	676 236	676 236	40.3	1.00[c]
Elephant grass	2 Porto Rico	365	C4	676 000	676 000	110.6	2.66[b]
Mean of 3 high-yield reports	1 El Salvador						
More normal values	1 El Salvador	365	C4	676 000	676 000	24.6	0.59[c]
Mean of 4 tropical grasses (high N)	Uganda	365	C4	611 680	611 680	25.4	0.69[b]
Maize (2 crops)	Uganda	135			135 734	19.10	
		+	C4	611 680			2.35[b]
		135			135 734	19.10	
Maize (1 crop)	Kenya (uplands)	240	C4	-	274 016	22.5	1.37[b]
Soya beans (2 crops)	Uganda	135	C3	611 680	135 734	7.67	0.95[c]
		+			+		
		135			135 734		
Perennial ryegrass (mean of 6 cvs)	UK	365	C3	310 404	208 680	23.9	1.43[b]
Rice	Japan	180	C3	413 600	146 414	16.9	1.93[b]
Winter wheat	Holland	319	C3	403 213	189 227	14.7	1.30[c]
Spring barley	UK	152	C3	345 680	132 211	11.8	1.49[c]

[a] = Natural ecosystem investigations [b] = Data from experiments [c] = Very good commercial yield

Table 8. 7 Comparative study of total radiant energy (RE) conversion efficiencies: different crops at different centres

A

Data class	Kale UK (A)	Kale UK (B)	Kale Italy (A)	Kale Italy (B)	Sugar beet UK (A)	Sugar beet UK (B)	Maize:Caragua UK (A)	Maize:Caragua UK (B)	Maize:Caragua Italy (A)	Maize:Caragua Italy (B)	Maize: Inra 200 UK (A)	Maize: Inra 200 UK (B)	Maize: Inra 200 Italy (A)	Maize: Inra 200 Italy (B)
Lenght of trial[a]	2	2	-	2	2	1	2	2	2	2	2	2	1	2
Y,DM (t ha^{-1})	18.92	21.18	-	22.23	21.02	22.70	9.31	19.10	45.91	32.37	8.65	11.89	24.76	25.70
Total RE conversion(%)	1.79	1.61	-	1.12	1.97	1.79	0.88	1.45	3.12	1.85	0.73	0.95	1.74	1.33
'Weighted' RE conversion (%)	2.98	2.68	-	1.87	3.28	2.98	1.47	2.42	5.20	3.08	1.22	2.03	2.90	2.22

[a] Data from 1 yr or mean of 2 yrs.

Centres: UK (A) Leeds Univ. Farm, (B) Cawood Exp. Stn.
Italy (A) Turin, (B) Rome.

B

	Mean 'weighted' RE conversion (%) Total	Cool areas	Warm areas
C3 crops: Kale	2.51		
Sugar beet	3.13		
Total	2.82	2.98	1.87
C4 crops: Maize: Caragua	3.04		
Maize: Inra 200	2.09		
Total	2.57	1.79	3.35

Adapted from Gibbon et al. (1970).

Partitioning between plant organs

$$\Delta C_{net} = \Delta W_{c,shoot} + \Delta W_{c,root} + \Delta W_{c,inflorescence} + \Delta W_{c,leaves}$$

Dynamic partitioning

- **fixed rates between organs**
- **Michaelis - Menten equation for different organs (1 and 2)**

$$\text{Growth rate}_1 = \frac{a_1 S}{B_1 + S}$$

$$\text{Growth rate}_2 = \frac{a_2 S}{B_2 + S}$$

- **functional approach**
- **hypothesis:**
  there exists a functional equilibrum between the size and activity of organs. Thornley: $\frac{\Delta W_{shoot}/W_{shoot}}{\Delta W_{root}/W_{root}} = \text{const} \times \left(\frac{N}{C}\right)^q$

- **mechanistic approach**

  -transport and conversion processes

<u>whole plant level</u> **(overview by (L. Gross 1988)**

Optimization of fitness measures by distribution between the plant's organs

<u>fitness measures:</u>

- **empirical dry weight proportion (Thornley & France 1982)**
- **substrate (C/N) concentration stay constant in various parts**
- **goal seeking methods (constant ratios)**
- **timing of vegetative vs. reproductive growth (Cohen, 1971, etc.)**

Hypothesis:

The new photosynthate in every time step is distributed between the leaves, stem & roots in such a way that the plant could produce the maximum quantity of new photosynthate during the next time period provided the environmental conditions do not change.

Formally, $e_1(t), e_2(t), e_3(t)$ **is the solution of the following problem of mathematical programming:**

$$y(x(t+1), V(t)) \rightarrow \max$$

$$\sum_{i=1}^{3} e_i(t) = 1; \qquad 0 \le e_i(t) \le 1$$

Risk assessment with crop models

The model:

$$Y = f(W, X, T) \quad \text{— prod. per unit area}$$

$W = \{w_i(t)\}$ - the multidimensional weather process

$X = \{x_j\}$ - production site parameters (static)

$T = \{T_k\}$ - management practices, technology (human input)

W - is dynamic

X - is static (yearly basis)

T - the set $\{T_k\}$ (quasi-static, slow rate of change)

$$Y_i = f(W, X, T_i) = f_i(W) \qquad i = 1, \dots i_N$$

Y_i is a random variable, the stochasticity is caused by the weather.

Let us suppose Y_i has the following distribution function:

$$F_{Y_i}(x) = P(Y_i < x) = P(f_i(W) < x)$$

$$i = 1 \dots i_N, \quad x \in \mathbb{R}.$$

Decision concept:

Def. The management practice T_i is better, than T_j $(T_i \succ^{p} T_j)$ at a risk level p $(0 \le p \le 1)$ (reliability level $1-p$) if Y_i and Y_j have $1-p$-th order percentile $x_p^{(i)}$, and $x_p^{(j)}$ respectively, and

$$x_p^{(i)} > x_p^{(j)}$$

Consequences:

$$P(x_p^{(i)} \le Y_i) \ge 1-p \; ; \quad P(Y_i \ge Y_j) \ge 1-p$$

A game-theoretical interpretation:

player I ≡ NATURE

player II ≡ decision maker

Step 1.

player I selects Θ from Ω.

Step 2.

player II selects d from D.

Loss of player II: $\mathcal{F}(\Theta, d(\xi))$

player I: ?

Example

Crop-production growth model

$$Y_t = L_t + \lambda_t \xi_t$$

where Y_t – yield (kg/ha)

L_t – deterministic function (logistic curve)

$\lambda_t \equiv 1$ for maize

$\lambda_t \equiv \sqrt{a+bt}$ for wheat

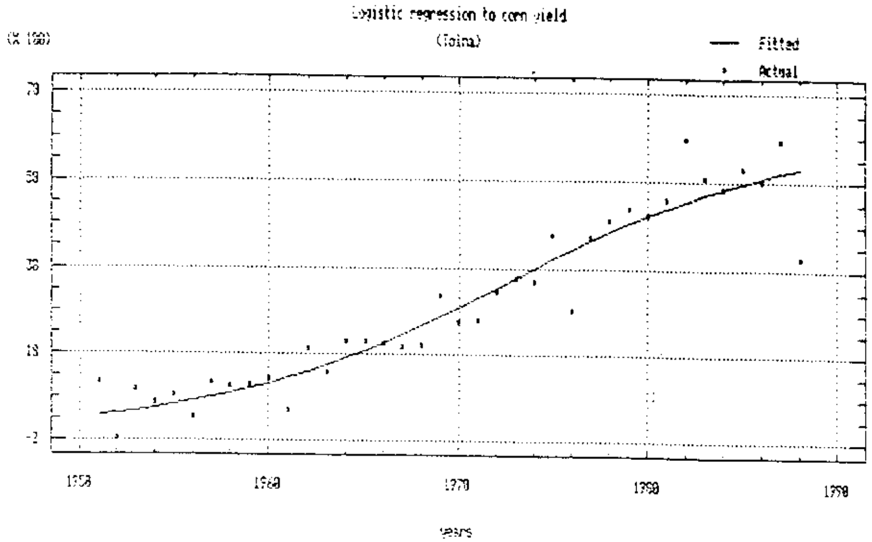
a, and b constants

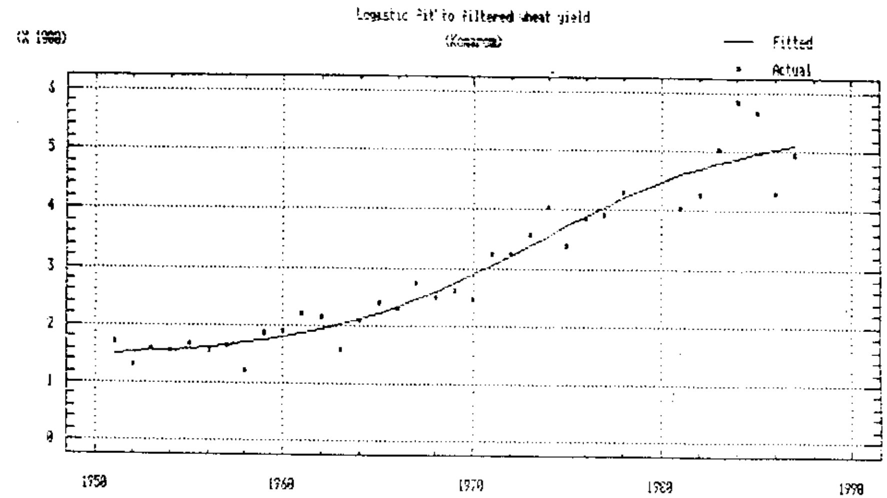
ξ_t – random variable

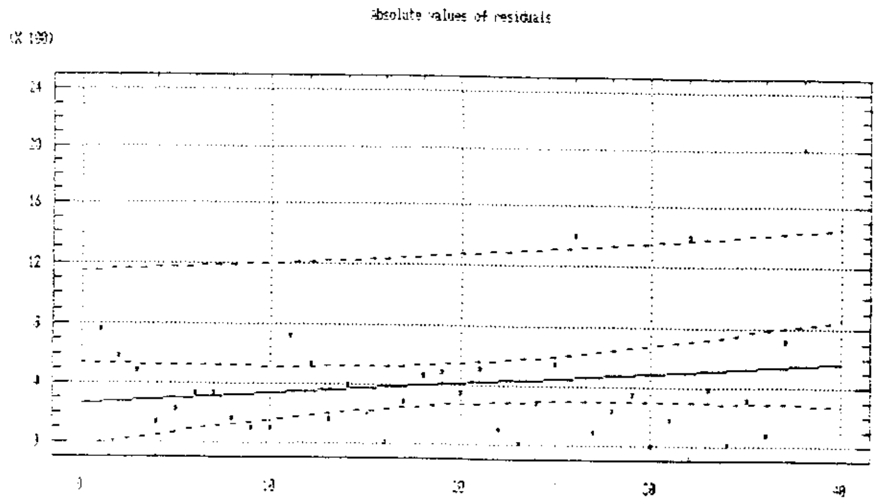
The following hypothesis was tested and accepted:

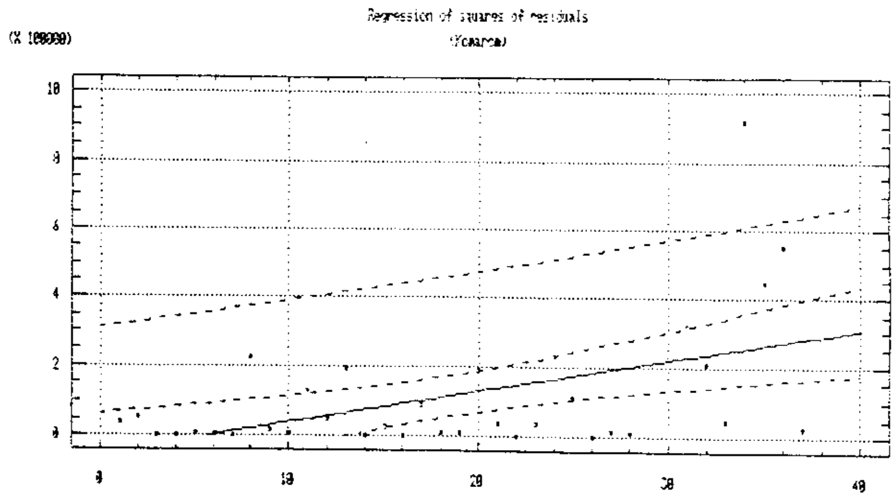
$$\xi_t = \xi \sim N(m, \sigma)$$

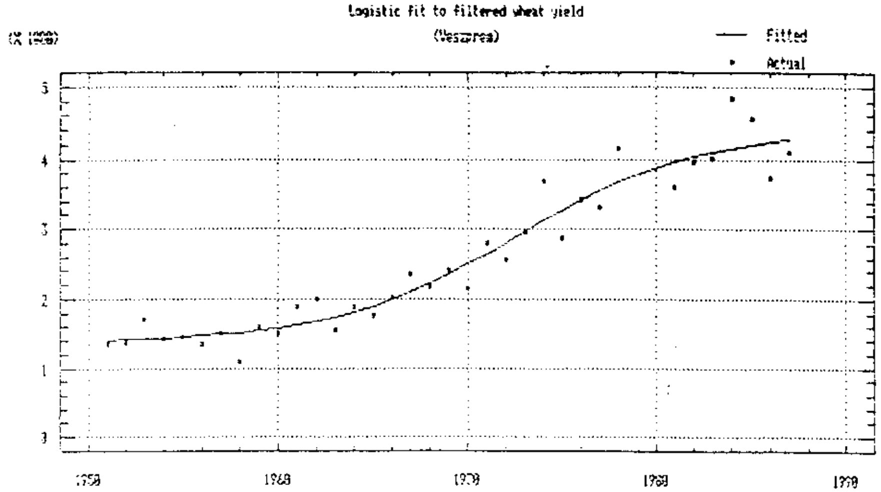
m and σ are site-dependent.


Logistic regression to corn yield
(Iowa)
(X 100)
Fitted
Actual
1950
1960
1970
1980
1990
years


Logistic fit to filtered wheat yield
(Komarom)
(X 1000)
Fitted
Actual
1950
1960
1970
1980
1990


Absolute values of residuals
(X 100)


Regression of squares of residuals
(Komarom)
(X 100000)


Logistic fit to filtered wheat yield
(Veszprem)
(X 1000)
Fitted
Actual
5
4
3
2
1
0
1950
1960
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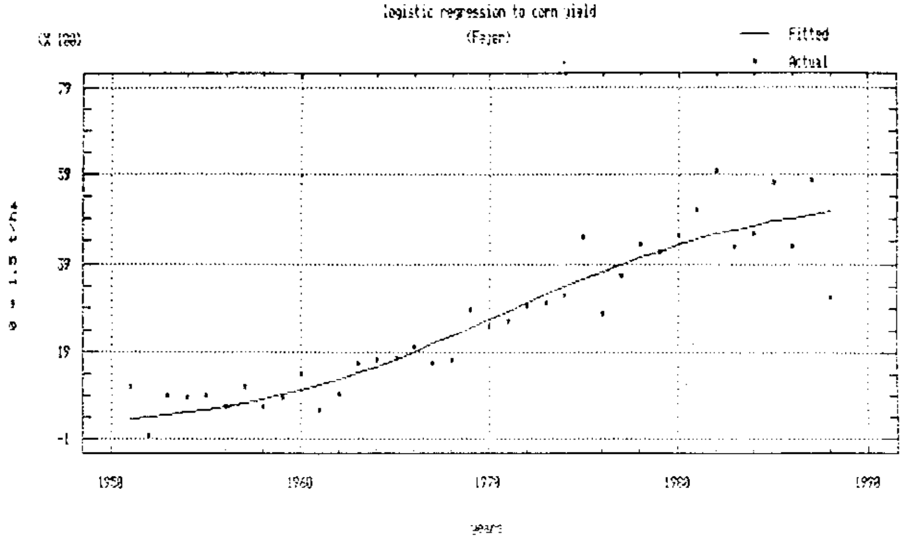
1980
1990


logistic regression to corn yield
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Actual
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1950
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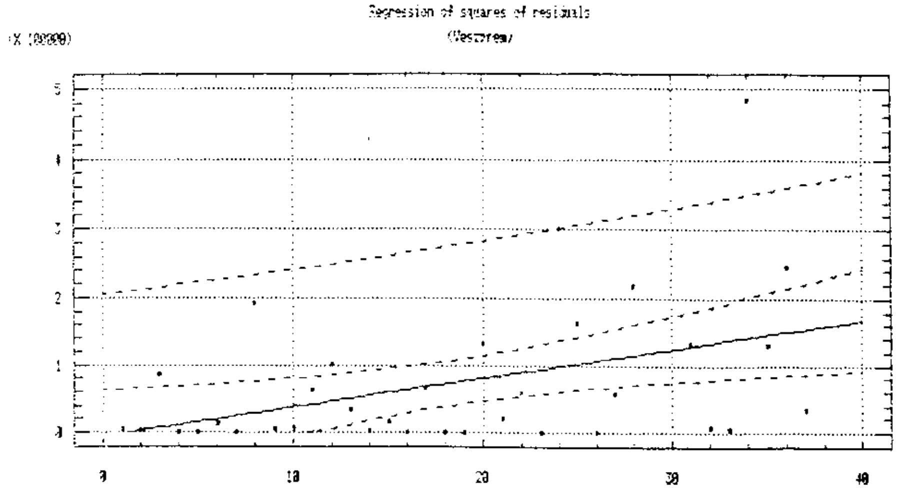
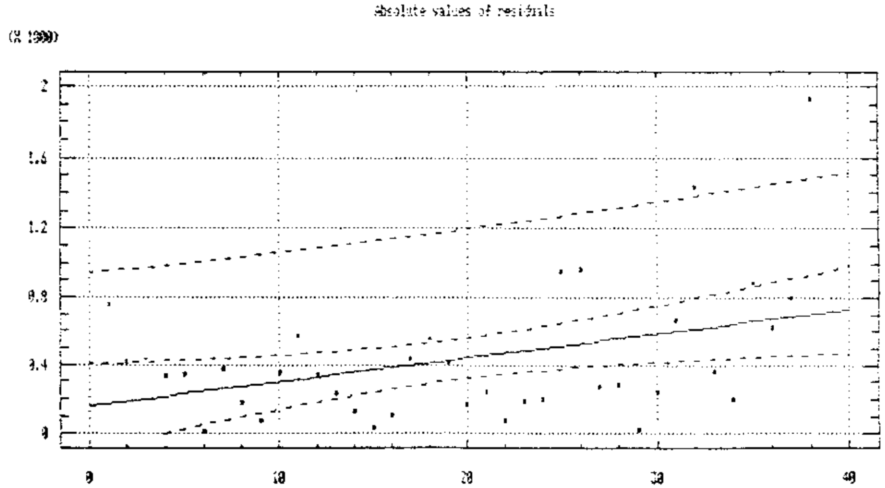

Regression of squares of residuals
(Veszprem)
(X 100000)
5
4
3
2
1
0
0
10
20
30
40


Absolute values of residuals
(X 1000)
2
1.6
1.2
0.8
0.4
0
0
10
20
30
40

A maximum likelihood method was used for the estimation of (m, σ). The estimated values are $(\hat{m}, \hat{\sigma})$

The decision here is:

application of the maximum likelihood method to obtain $(\hat{m}, \hat{\sigma})$, and acceptance of $(\hat{m}, \hat{\sigma})$ for (m, σ).

The risk is connected to the fact, that another parameter-pair is true.

How to get an optimal risk of type A ?

1st concept:

Let $d_1 \neq d_2 \in D$.

d_1 is better than d_2 if

1.) $r(\theta, d_1) \leq r(\theta, d_2)$ for each θ

2.) there exists a $\theta^* \in \Omega$, that $r(\theta^*, d_1) < r(\theta^*, d_2)$

Problem:

Partial ordering

Solution (if there exists):

A "good" subset of decisions.

2nd concept:

The optimal decision d^* is determined by

$$\min_{d} \{ \max_{\theta \in \Omega} r(\theta, d) \} = \max_{\theta \in \Omega} r(\theta, d^*)$$

Problem:

too pessimistic

3., The Bayes-solution

restriction: The distribution of the parameters Θ must be known àpriori.

Let $q(\Theta)$ be the known density of Θ

The function

$$\bar{r}(q,d) = \sum_{i=1}^{k} r(\Theta_i, d)\, q(\Theta_i) =$$

$$= \sum_{t=1}^{\infty} \sum_{i=1}^{k} F(\Theta_i, d(\xi_t))\, p(\xi_t | \Theta_i)\, q(\Theta_i)$$

is called the average risk.

If there exists a $d^* \in D$ such that

$$\bar{r}(q, d^*) \leq \bar{r}(q, d) \quad \text{for any } d \in D,$$

then d^* is the Bayes-solution.

(If D is "big enough", there exists a Bayes-solution)

Type B risk

A tipical description of the crop yield/unit area:

$$Y = f(X, W, T)$$

where

X – parameters of the production site
- soil quality
- geographical position
- etc

W – dynamic weather parameters
- precipitation
- daily avg, min, max temperature
- soil radiation
- humidity of the atmosphere

T – farm management
- application of fertilizers
- " of pesticides
- tillage technology
- etc.

X changes slowly → constant

W changes stochastically

T control variable

If $T \in \{T_1, T_2, \dots T_m\}$ then

$$Y_i = f(X, W, T) = f_i(W); \quad i = 1, \dots m$$

As W is a random variable, the distribution function

$$F_i(x) = P(Y_i < x) = P(f_i(W) < x)$$

can also be determined.

The decision is – which T_i to apply?

Let $d(i)$ denote the decision to apply T_i.

$D = \{d(i) \ i = 1, \dots n\}$.

Def #1.

The risk of decision d_1 is less, than the risk of decision d_2 at the level x, iff

$$P(Y_{d_1} > x) > P(Y_{d_2} > x)$$

Def #2.

The decision d_1 is better, than the decision d_2 at a risk level p, iff for the p-th order percentiles of Y hold

$$X_p(d_1) > X_p(d_2)$$

If $L^1_x(Y, d) = \ell_x(Y, d) * (-1) * I(Y > x \mid d)$ is the loss function for def #1

and

$L^2_p(Y, d) = L^1_{X_p}(Y, d)$ for def #2

Taking the average

$$\rho_P(P,d) = \int_\Omega L_P^{*}\, dP$$

over the distribution of the yield supposing decision d, $\rho_P(P,d)$ gives a measure of d.

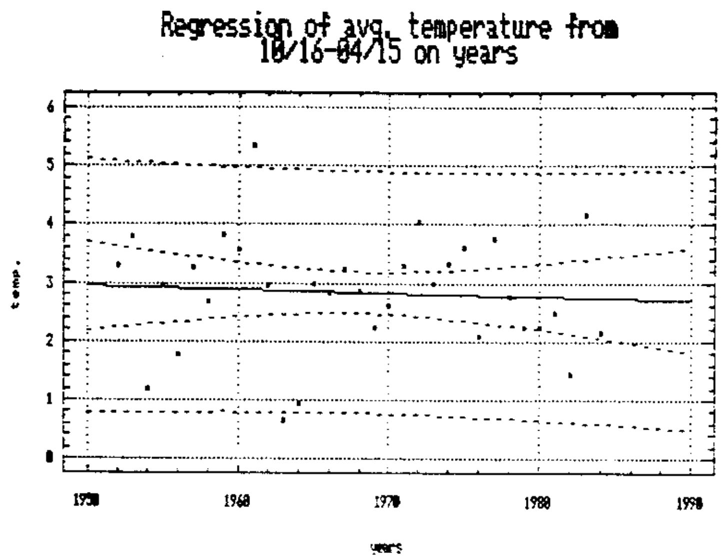
If $\rho^*(P) = \min_{d \in D} (\rho(P,d))$ holds

for a decision $d^* \in D$, then d^* is a Bayes-decision, and $\rho^*(P)$ is a Bayes-risk.

PROBLEMS:

① P

② L



Regression Analysis - Linear model: Y = a+bX

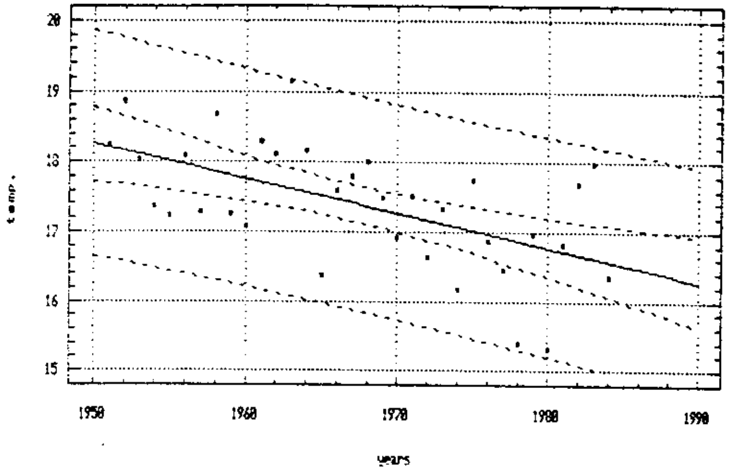
Dependent variable: avg.temperature 10/16-04/15 Independent variable: years

Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	14.895	35.9014	0.414886	.68108
Slope	-6.12801E-3	0.0182424	-0.335922	.73919

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	.1123570	1	.1123570	.112843	.73919
Error	30.866416	31	.995691		
Total (Corr.)	30.978773	32			

Regression of average temperature between 04/16 and 10/15 on years



Regression Analysis - Linear model: Y = a+bX

Dependent variable: avg.temperature 04/16-10/15   Independent variable: years

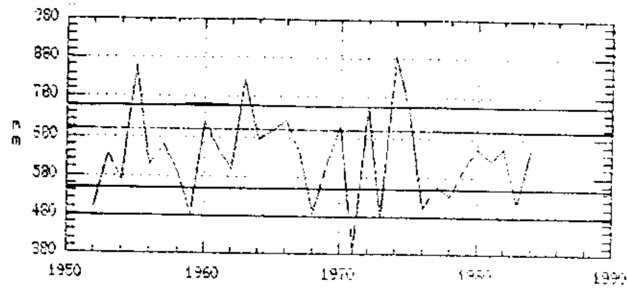
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	115.231	25.69	4.48543	.00009
Slope	-0.0497288	0.013057	-3.80859	.00060

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	8.092741	1	8.092741	14.50534	.00060
Error	17.853263	32	.557914		
Total (Corr.)	25.946003	33			

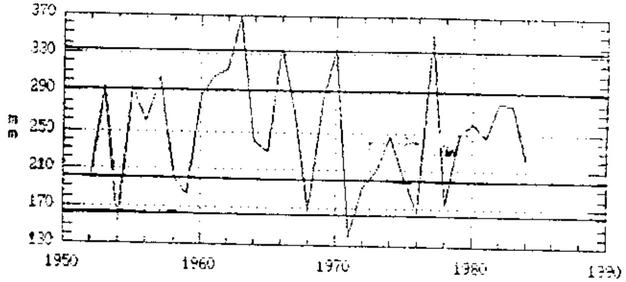
Correlation Coefficient = -0.558486   R-squared = 31.19 percent
Stnd. Error of Est. = 0.746937

Precipitation between 10.16 - 10.15. (Iregszemcse)



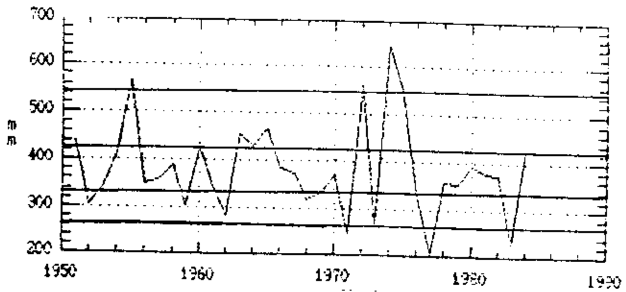
average=629 mm, std=113mm

Precipitation between 10.16 - 04.15. (Iregszemcse)



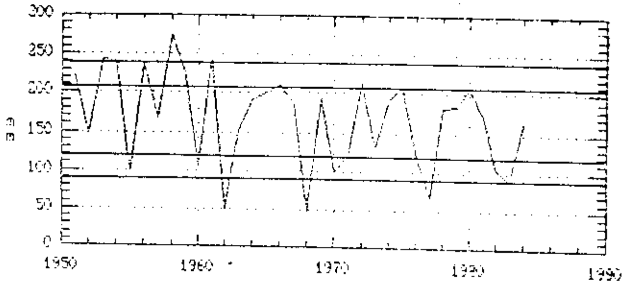
average=249 mm, std=60mm

Precipitation between 04.16 - 10.15. (Iregszemcse)



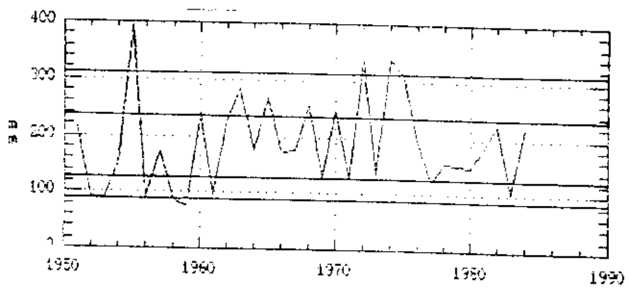
average=381 mm, std=97 mm

Precipitation between 04.16 - 06.30. (Iregszemcse)



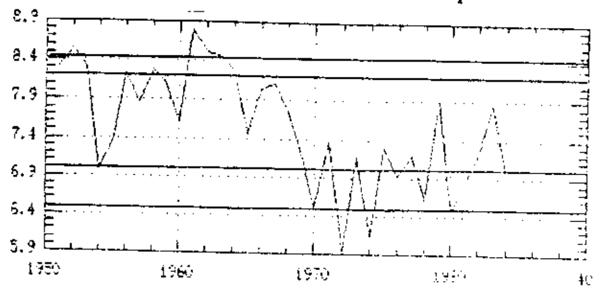
average=167 mm, std=60mm

Precipitation between 07.01 - 09.30. (Iregszemcse)



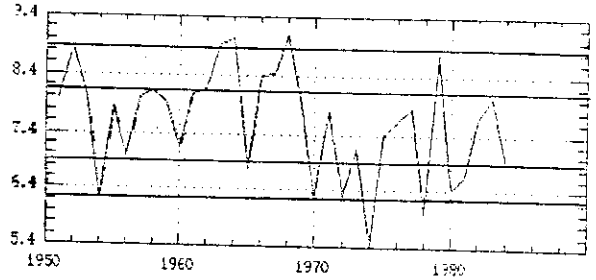
average=188 mm, std=81mm

Number of solar hours 04.16.-10.15. Kompolt



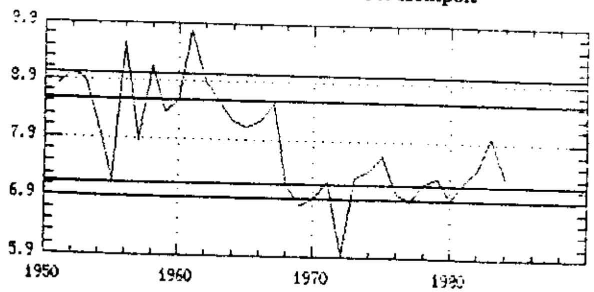
average=7.55hours, std= 0.75hours

Number of solar hours 04.16.-06.30. Kompolt



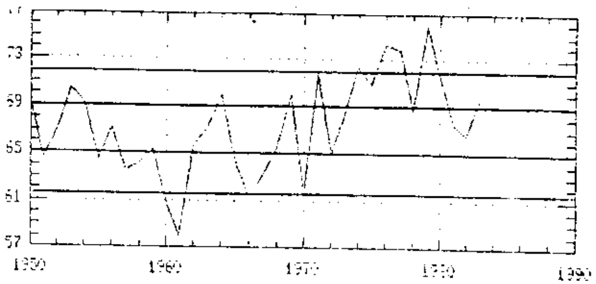
average=7.59hours, std= 0.94hours

Number of solar hours 07.01.-09.30. Kompolt



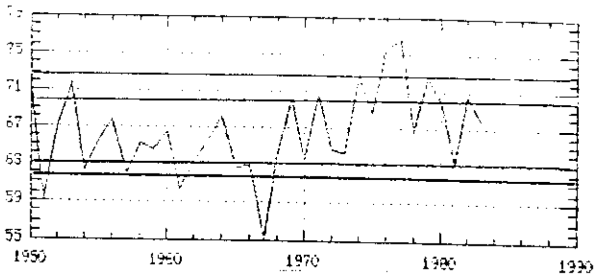
average=7.86 hours, std= 0.89hours

Daily avg. humidity between 04.16-10.15. Kompolt



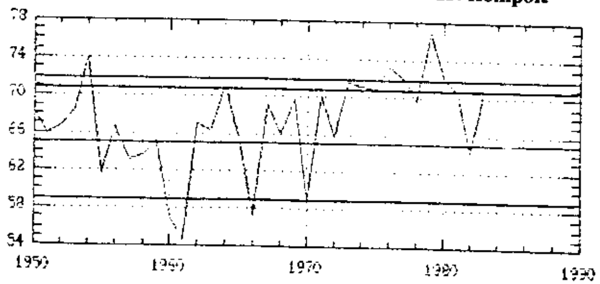
average= 67%, std=4.1%

Daily avg. humidity between 04.16-06.30. Kompolt



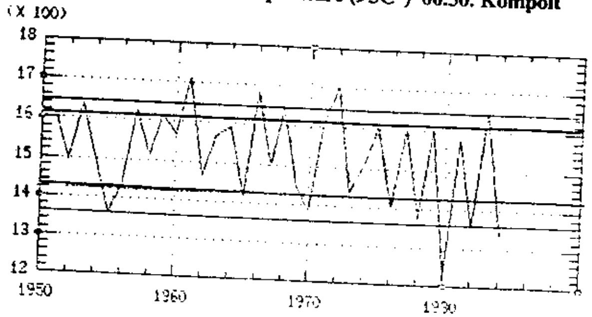
average= 66%, std=4.7%

Daily avg. humidity between 07.16-10.15. Kompolt



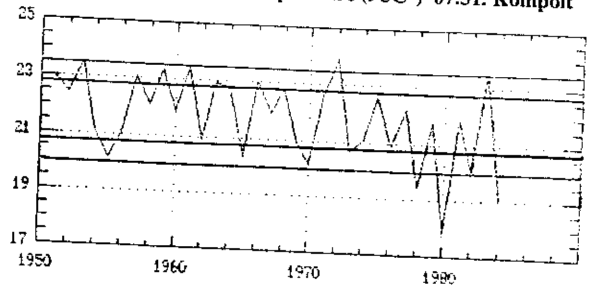
average= 67%, std=4.1%

Sum of effective temperature (>5C°) 06.30. Kompolt



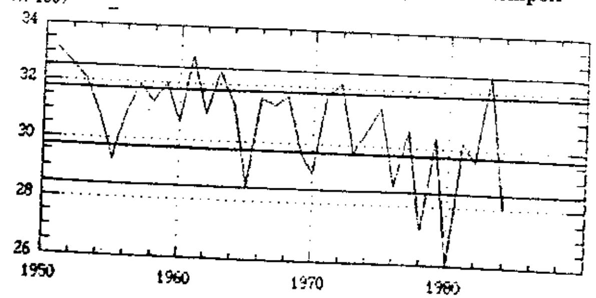
average=1552C° std=120C°

Sum of effective temperature (>5C°) 07.31. Kompolt

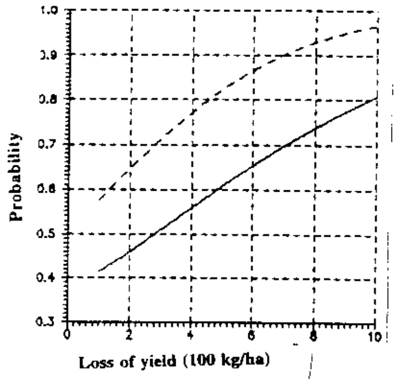


average=2169C° std=143C°

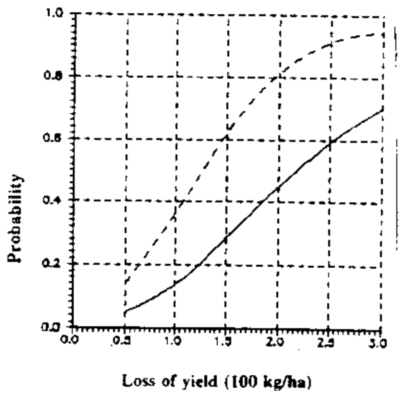
Sum of effective temperature (>5C°) 09.15. Kompolt



average=3065C° std=166C°

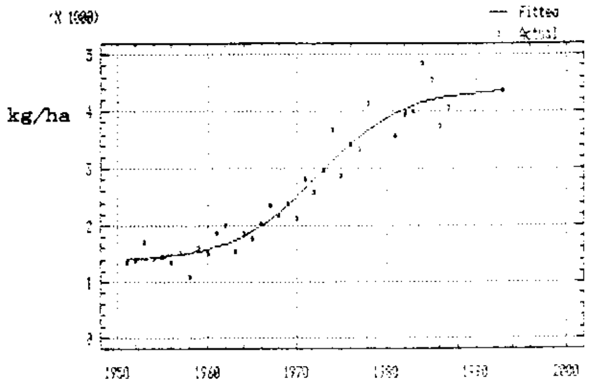


Probability of negative error of the prediction for the next year

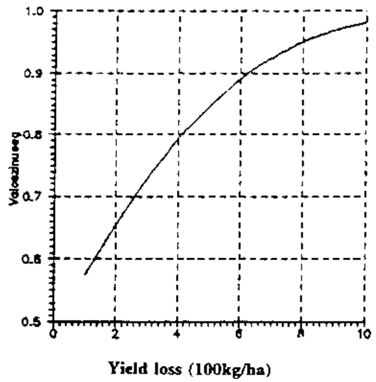


Probability of negative error of the next five years prediction

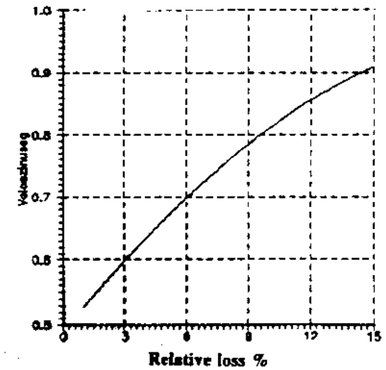
Wheat production (I)

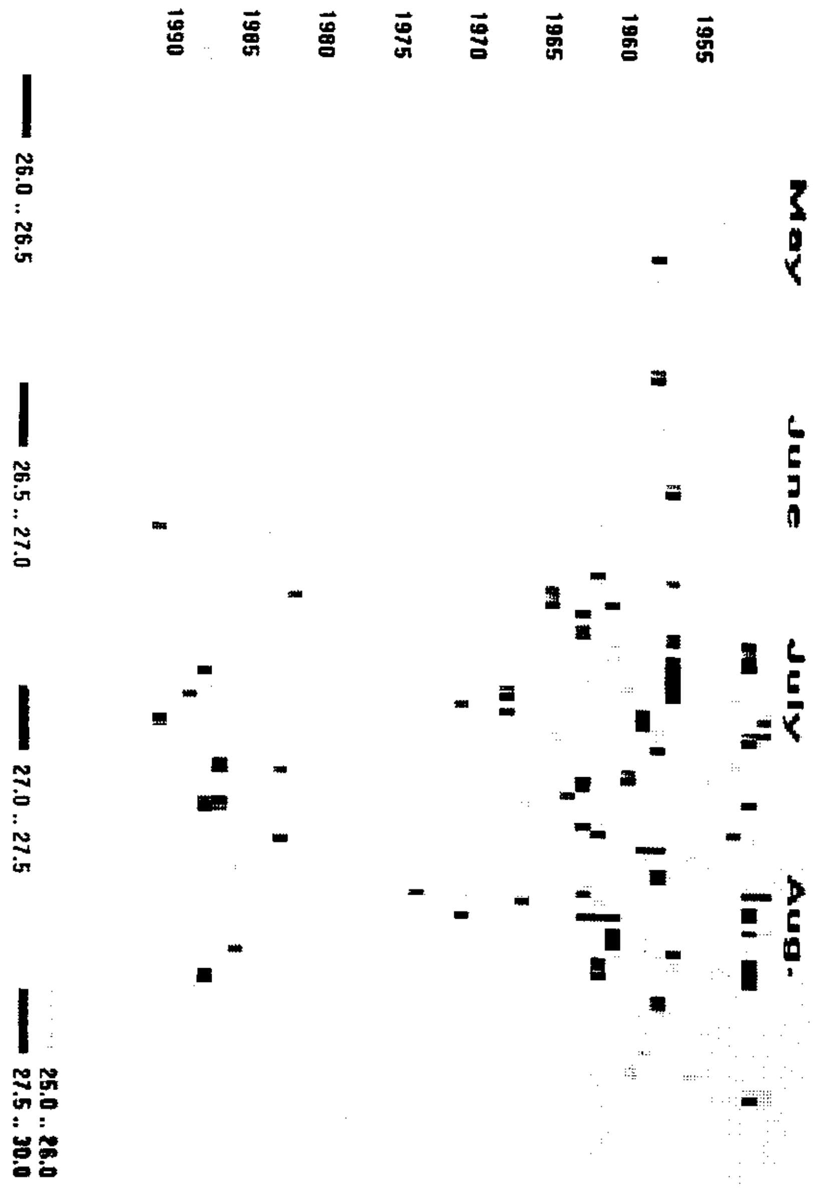


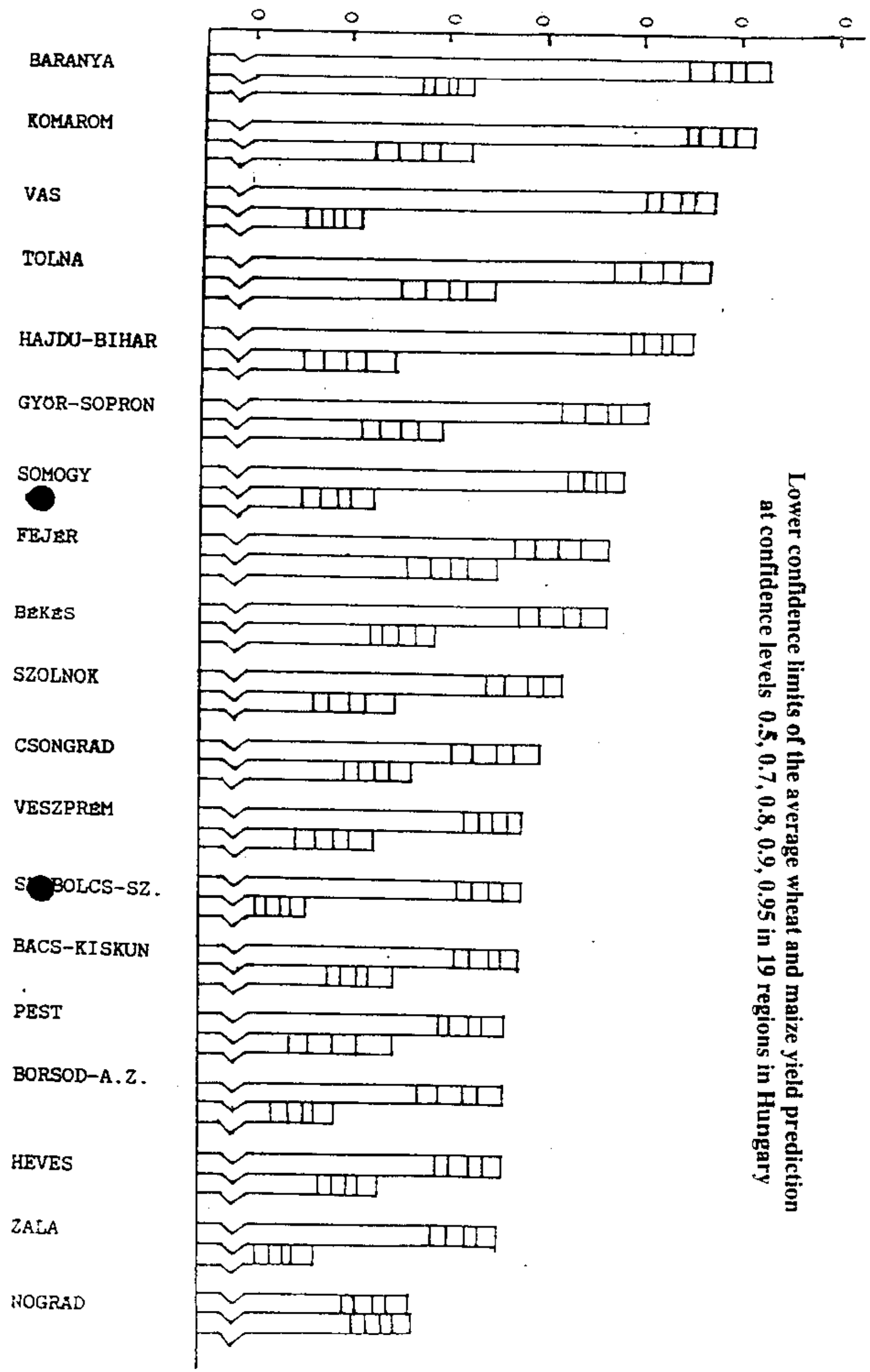
year	estimated yield (kg/ha)
1989	4320
1990	4336
1991	4349
1992	4360
1993	4369



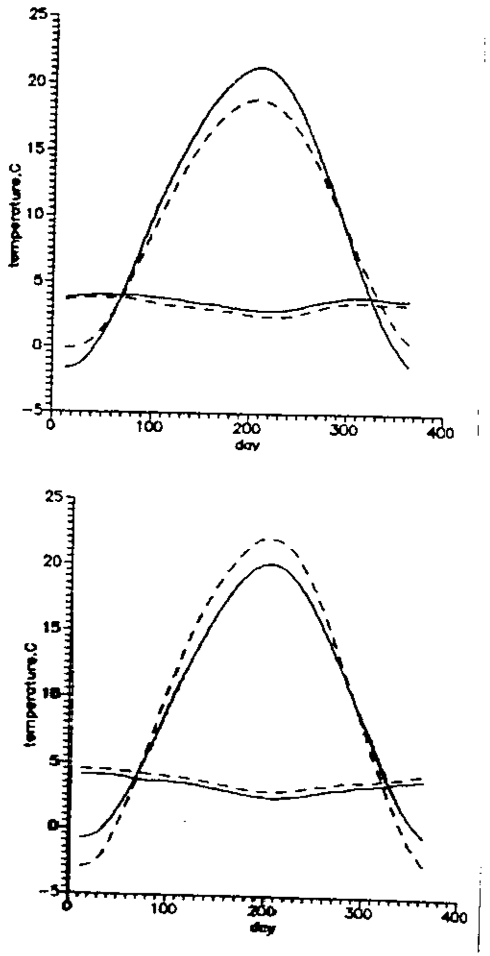
Probability of negative prediction error







Lower confidence limits of the average wheat and maize yield prediction at confidence levels 0.5, 0.7, 0.8, 0.9, 0.95 in 19 regions in Hungary



[redacted]

Fourier approximations of daily average temperature and std. deviation in Kompolt for the 1st day of series of any length (solid line) and for any other day of series (dashed line) (a) wet series, (b) dry series.