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"Economic Theory and Exhaustible Resources"

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These are preliminary lecture notes, intended only for distribution to participants.

from P. Dasgupta and G. Heal: Economic Theory and Exhaustible Resources (1979)

Strictly from a *formal* point of view our example suggests that, so long as all costs in running an institution are nil, a tax equilibrium and a competitive equilibrium with markets for externalities are equivalent.[4] And yet one should not accept this suggestion without investigating whether the result is robust. Again, from a formal point of view we noted that, since individuals transact in named goods in markets for externalities, the number of commodities purchased and sold by an individual is considerably larger at a Lindahl equilibrium than the number of commodities purchased and sold at a tax equilibrium. To be precise, in our example individual i participated in $2(N-1)$ externality markets $\{g_{ih}(h \neq i)$ and $g_{hi}(h \neq i)\}$ each of which was assumed inoperative at the tax equilibrium. At the tax equilibrium each individual assumed the quantities of the public good purchased by the others as given. They were not quantities over which he had any control. Our general discussion of the existence of a competitive equilibrium in Chapter 2 should now draw our attention to the following point: namely that, if the existence of an equilibrium is to be assured, each individual's utility function will have to be quasi-concave in the space of those commodities whose quantities the individual can control. In our example, of course, the utility function of the representative individual was strictly quasi-concave (in fact strictly concave) whether or not named goods were introduced. But intuition suggests that it may be simple to construct examples where the convexity assumptions fail to be satisfied if named goods are introduced but where they are satisfied if named goods are not introduced. In such cases it is possible that, while a tax equilibrium exists, a Lindahl equilibrium does not.[5]

In fact it emerges that such possibilities can arise rather generally when externalities are of a detrimental kind, like noise or pollution. They arise in an interesting form in the case of free access to a common property known widely as the problem of the common. We touched on this when discussing the problem in establishing property rights for neighbouring oil men. The implication on the rate of depletion of a common oil pool we defer to a later chapter when we have introduced time into our analysis. In the following section we analyse the problem of the common in some detail in

[4] For general propositions along these lines see Foley (1970) and Starrett (1972). We emphasize the fact that such an equivalence depends crucially on institutional costs being at least comparable (in our example, strictly zero).

[5] For a general discussion of this point see Starrett and Zeckhauser (1971).

those situations where the intertemporal aspects of the problem can be ignored, without our losing sight of the essential structure of the problem.

4. Common Property Resource, or the Problem of the 'Common'

(i) *The problem:* Imagine a body of water, such as a lake or the open seas. It will be supposed that the body of water derives its value from the marine life it sustains. While the open seas may appear at first sight to be too large in surface area for the common's problem to be relevant, the recent Icelandic fishing dispute should remind us that there are various kinds of aquatic life, like cod and haddock, that are relatively non-migratory in character and are thus localized in different parts of the seas. We are concerned here with a specific species of aquatic life, localized in a specific part of the open sea. In other words, we are concentrating our attention on a specific fishing ground. It will be supposed, as is actually the case, that nobody owns the fishing ground in the open sea. Consequently everyone has an equal right to fish. There is thus free access to the fishing ground. Our interest in focusing attention on aquatic life in discussing the problem of the common lies in the fact that while such species are self-renewable if the size of their population is large enough, the chance of a given species surviving is severely reduced if the population size gets below a certain threshold level. This threshold level varies greatly from species to species and depends as well on the environment in which the species exists. What is known as the 'biotic potential' of many kinds of fish is so large that the threshold level associated with it may be 'very small'. On the other hand, for land animals the threshold level is often 'large'. The analysis that follows pertains not only to fishing from a common fishing ground but as well to hunting or trapping for animals from a common ground. The main point is that given a positive threshold level, the rate of total catch is crucially important in judging whether a particular species is endangered.

Suppose there to be N fishing firms ($i = 1, \ldots, N$) assumed identical. We shall regard fishing as a production activity in which the catch is the output and labour and fishing equipment are the variable inputs. To simplify, we shall aggregate these variable inputs into one and regard this single input as 'vessels', assumed perfectly divisible. If S is the size of the fishing ground and if there are X vessels on the fishing ground, the total catch, Y, is assumed to satisfy the constraint $Y \leqslant H(X, S)$ where H is a production function with constant returns to scale in the two factors (X and S) with positive but diminishing returns to each factor. Suppose also that $H(0, S)=0$. Now, the fishing ground is fixed in size. Denote the size by $\bar{S}$. Because of constant returns to scale we note that $H(X, \bar{S})= \bar{S}H(X/\bar{S}, 1)$. Since $\bar{S}$ is constant for our problem we may as well normalize and set $S=1$. Now write $H(X, 1)=F(X)$. By assumption $F(0)=0$, $F'(X)>0$, $F''(X)<0$, and furthermore, $F(X)$ is bounded above (presumably by the size of the total stock of fish). Notice that these assumptions imply that

$$\frac{F(X)}{X} > F'(X) \quad \text{and that} \quad \lim_{x \to \infty} \frac{F(X)}{X} = 0.$$

(see Diagram 3.1 below).

The assumption of diminishing returns (i.e. $F''(X)<0$) is crucial to the exercise and it reflects the fact this is a fixed area of the sea which the particular species under consideration inhabits. There is in effect a crowding of vessels. Given that X is the number of vessels on the sea, $F(X)/X$ is the average catch per vessel when the vessels are efficiently manned. If the ith firm owns x_i of these vessels it will be supposed, for simplicity, that its catch is $x_iF(X)/X$. But $X=\sum_{i=1}^{N} x_i$. It follows that, since the average product $F(X)/X$ is a diminishing function of X, the catch accruing to i is dependent

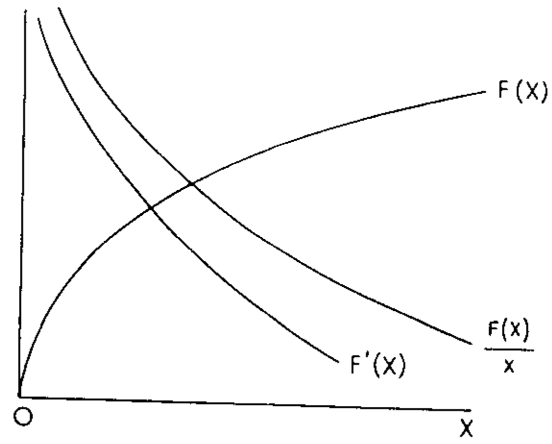


Diagram 3.1

not only on i's input x_i, but as well on the number of vessels introduced by the other firms. Given the diminishing returns it is also the case that the externality that others impose on one is of a detrimental nature; that is, unlike the earlier example, we are in the realm of external diseconomies. Write

$$X_{N-i} = \sum_{\substack{j=1 \\ j \neq i}}^{N} x_j$$

and denote by y_i the ith firm's catch of fish. We have therefore supposed that

$$y_i \leqslant \frac{x_i F(X_{N-i} + x_i)}{X_{N-i} + x_i}. \tag{3.25}$$

Assume that the markets for both boats and the catch are perfectly competitive. Choosing the catch as our numeraire good, let p be the rental value of a vessel. Firms are profit maximizing and our first task is to compute the market equilibria. Given that firms are identical the computation is simple enough. As in our earlier example let firm i suppose that each of the other firms will introduce $\hat{x}$ vessels. It follows that i wants to choose x_i with a view to maximizing

$$\frac{x_i F\{(N-1)\hat{x} + x_i\}}{(N-1)\hat{x} + x_i} - px_i. \tag{3.26}$$

It follows immediately that it will choose x_i (provided $x_i>0$) at the solution of this equation

$$\frac{(N-1)\hat{x}F\{(N-1)\hat{x} + x_i\}}{\{(N-1)\hat{x} + x_i\}^2} + \frac{x_i F'\{(N-1)\hat{x} + x_i\}}{(N-1)\hat{x} + x_i} = p. \tag{3.27}$$

At this market equilibrium with free access to the fishing ground, condition 3.27 will hold for all i, and by symmetry $x_i=\hat{x}$ for all i. Thus the equilibrium number of boats, $\hat{x}$, per firm is the solution of the equation

$$\frac{F(Nx)}{Nx} - \frac{1}{N}\left\{\frac{F(Nx)}{Nx} - F'(Nx)\right\} = p. \tag{3.28}$$

In other words at this market equilibrium the total number of vessels, X, on the sea is the solution to the equation

$$3.29 \qquad \frac{F(X)}{X} - \frac{1}{N}\left\{\frac{F(X)}{X} - F'(X)\right\} = p.$$

Equation 3.28 (or, equivalently, equation 3.29) is the fundamental result of the problem of the common, and the question arises whether a positive value of X satisfies 3.29. It is simple to see that the answer is 'yes' if we were to assume in addition the innocuous condition $F'(0) > p$ (for if not, then it would not be worth anyone's while undertaking to catch the species). The question arises whether the allocation at the market equilibrium as implied by 3.28 is Pareto efficient for these N firms. Again, the answer, as in the previous example, is 'no'. To obtain the symmetric Pareto-efficient allocation for these N firms, one needs to choose x so as to maximize the total net profit

$$3.30 \qquad F(Nx) - pNx,$$

and then dividing this profit equally between them. In other words, the N firms by their joint collusive action 'internalize' the externalities that have resulted as a consequence of the fact that the fishing ground is an unpaid factor in production. Maximizing 3.30 readily yields the condition that x ought to be the solution of the equation

$$3.31 \qquad F'(Nx) = p,$$

or equivalently,

$$3.32 \qquad F'(X) = p.$$

Equation 3.32 is, of course, widely familiar and reflects the efficiency condition that the marginal product of vessels ought to equal their rental price. Denote by $\tilde{x}$ the solution to 3.31, and by $\tilde{X}$ the solution to 3.32. That is, $\tilde{X} = N\tilde{x}$. One can confirm as well that the allocation implied by $\tilde{x}$ is in the core for these N firms, for no coalition of firms can guarantee a profit level as high as that implied at the Pareto efficient outcome with symmetric division of the total profit. If it attempts to block the allocation, the complementary coalition of firms can introduce a 'large' number of vessels (since there is no *legal* limit on the number of vessels a firm can introduce) and thereby push the average product of vessels to a level equal to p.

Denote by $\hat{X}$ the solution of equation 3.29, and let $\hat{X} = N\hat{x}$. Using equation 3.29 one has that

$$3.33 \qquad p - F'(\hat{X}) = \frac{(N-1)}{N}\left\{\frac{F(\hat{X})}{\hat{X}} - F'(\hat{X})\right\} > 0.$$

Using 3.32 and 3.33 it follows that

$$\tilde{X} < \hat{X} \quad (\text{or } \tilde{x} < \hat{x}).$$

It follows that at the free access market equilibrium there are too many vessels and, therefore, too large a catch; in the sense that each firm's profit could be raised if the firms were to undertake a joint decision to *reduce* their fishing activity and procure a *smaller* catch. Notice as well that at the market equilibrium each firm makes a positive profit. To see this one notes from equation 3.29 that

$$3.34 \qquad \frac{F(\hat{X})}{\hat{X}} - p > 0$$

and indeed that at the equilibrium each firm's net profit, $\hat{\pi}$, is

$$3.35 \qquad \hat{\pi} = \frac{1}{N^2}\{F(\hat{X}) - \hat{X}F'(\hat{X})\} > 0.$$

and thus that the average product strictly exceeds the rental price of a vessel (see Diagram 3.2).

It will be noticed that, contrary to what is often claimed, the problem of 'the common' and the resulting sub-optimality of the market equilibrium are *not* formally identical to an N-person version of the prisoner's dilemma game. The key feature of the prisoner's dilemma game, or so we noted, is not only that its unique Nash equilibrium is Pareto inefficient, but also that the Nash equilibrium is characterized by *dominant* strategies on the part of each agent. It is a simple matter to confirm in the foregoing formulation that firms do not possess dominant strategies in the common's problem. Indeed, $\hat{x}$ is the profit maximizing number of vessels for the representative firm only when the remaining firms introduce $(N-1)\hat{x}$ vessels. It is only for pedagogic reasons that in section 2 of the previous chapter we illustrated the prisoner's dilemma game by means of an artificially restricted problem of the common. Nevertheless, the above formulation implies that it is in the interest of each firm

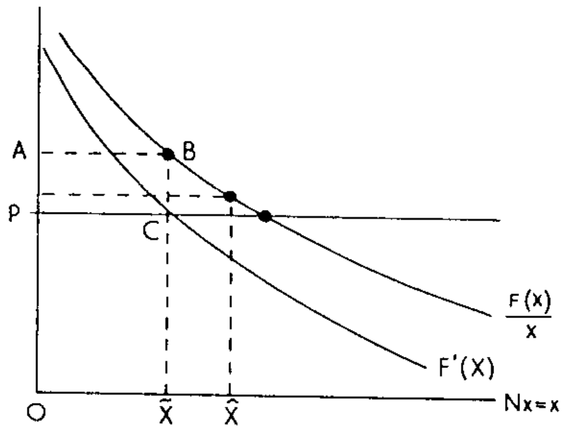


Diagram 3.2
(unlimited number of firms)

to come to an agreement to restrict its input to level $\tilde{x}$; indeed this allocation is in the core while the market equilibrium allocation is not. But it is not in the firms' interests to do so in the absence of a collective agreement. While for this simple symmetric model it may seem that an agreement to introduce only $\tilde{x}$ vessels can easily be reached, whether or not it will be complied with will depend on whether or not the agreement can be *enforced* easily. Enforcement may be costly to administer. Plainly its costliness will vary from context to context.

The fact that the market equilibrium allocation is not in the core and in particular that there is excessive fishing at the equilibrium may appear a bit paradoxical since we have not introduced any monopolistic elements in the market for boats and catch. But it is not difficult to see why the result comes out the way it does. The introduction of an extra boat by a firm alters its catch, of course, but it also inflicts a diseconomy on the other firms in the sense that their catch is reduced. Granted that if N is large this external diseconomy on each of these other firms will be negligible. But the *sum* of these negligible quantities need by no means be negligible. We demonstrate this by deriving the special result that follows when N is arbitrarily large.

Given that p is a constant we may regard the total number of vessels $\hat{X}$ given by the market equilibrium condition 3.29 as simply a function of N. The question arises about the functional form of $\hat{X}$.

Notice first that if $N=1$ then from equations 3.29 and 3.32 we have $\hat{x}=\tilde{x}$ and consequently there is no problem of the common. The problem that we have been discussing arises, as our arguments have shown, only when $N>1$. Thus write the LHS of equation 3.29 as $G(X, N)$. That is

$$G(\hat{X}, N) \equiv \frac{F(\hat{X})}{\hat{X}} - \frac{1}{N}\left\{\frac{F(\hat{X})}{\hat{X}} - F'(\hat{X})\right\} = p.$$

To keep the argument simple regard N ($\geqslant 1$) as a continuous variable. From equation 3.29 it is immediate that

$$\frac{\partial \hat{X}}{\partial N} = -\frac{\dfrac{\partial G}{\partial N}}{\dfrac{\partial G}{\partial \hat{X}}}.$$

Routine calculation now yields that

$$\frac{\partial \hat{X}}{\partial N} > 0.$$

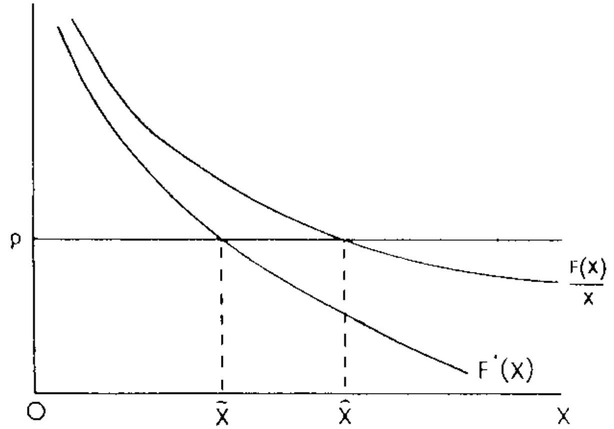
In other words, $\hat{X}$ is monotonically increasing in $N (N \geqslant 1)$. But it must be bounded above, since, if not, then given that by our assumptions regarding F one has

$$\lim_{\substack{X\to\infty \\ N\to\infty}} G(\hat{X}, N) = 0,$$

equation 3.29 would be violated. It follows that $\hat{X}$ tends to a finite limit as $N \to \infty$, and thus that at the market equilibrium each firm introduces an 'infinitesimal' number of vessels. But at this limit, equation 3.29 reduces to

3.36 $$\frac{F(X)}{X} = p,$$

or, in other words, that in the large numbers case the *average product* of vessels is equated to their rental price at the free access equilibrium (see Diagram 3.3). It follows that in the case of large numbers the profits are diluted to zero at the free access equilibrium.



(unlimited number of firms)

Diagram 3.3

The consequences of such over-exploitation of aquatic life at the free access equilibrium can be serious, and possibly disastrous. For suppose that the total population size of such a species is Z, and suppose $\underline{Z}(0 < \underline{Z} < Z)$ is the threshold population size such that below this size the entire species is seriously threatened with extinction. It is entirely possible that

3.37 $$F(N\tilde{x}) < Z - \underline{Z} < F(N\hat{x})$$

Granted that the arguments leading to 3.37 have relied on firms being singularly myopic in concerning themselves solely with one period's profit. One might wish to argue that firms will not over-exploit if the species is threatened with extinction, since this would cut into future profits. Quite apart from the question of whether firms are as far-sighted as some would like to believe, the introduction of time will not necessarily alter the essentials of the argument. Restricting its catch in order to husband the resource will not really help a firm's future profits much if the competing firms decide to make a killing at one go and virtually wipe out the species. Thus if each firm were to fear that the total catch of all the competing firms will bring the population size below the threshold $\underline{Z}$ it may find it most profitable to join in the rampage and get as much in the first period as possible, and this despite the firm having taken future

possibilities into account.[6] In other words such fear could be self-fulfilling, leading to a market equilibrium in which the species is over-exploited in the first period.

In our example we have not imputed any value to the species other than the value that the catch fetches in the market. To put it loosely we have imputed only an economic value to the species. An environmentalist's reaction to our example could be that this merely proves the profit motive of firms to be incompatible with the environmentalist's goals. But inequality 3.37 indicates that it is not quite as simple as that. It is in the interest of each firm to agree to restrict output if that is the only way to get the other firms to do likewise. Profits would be higher by this means. If 3.37 holds, maximization of total profits leads to a catch that is not inconsistent with the environmentalist's goals. In our example if condition 3.37 holds, the guilty party is not the profit motive *per se*. Rather, it is the economic and legal environment in which the profit motive is allowed free play.

So far we have interpreted the inefficiency of the market equilibrium *with free access* as being due to the externalities that each firm inflicts on the others in their production activities. An equivalent way of interpreting this inefficiency is to recognize that the open sea is an asset that is not owned by anyone in particular. Consequently no rental is charged to the different firms for the right to fish. Under common property the sea is a free good for the individual firm. Indeed it is for this reason that we have been referring to the resulting market equilibrium as a free access equilibrium.

(ii) *Competitive markets for named vessels:* Now our earlier discussion of the problems of establishing competitive markets for externalities would suggest that so long as the fishing ground is to remain common property it is doubtful that competitive markets for 'named' *vessels* would develop. Specifically, points 1 and 2 (see pp. 48–50) would seem very relevant for such doubts. But there is in fact an additional reason why one would not wish to rely on the appearance of competitive markets for named vessels. For so long as the fishing ground remains a common property recall that the ith firm's production possibilities are represented in 3.25. Recall also that an obvious route in such a situation would be to have competitive markets for 'named' vessels. Thus write

[6] In fact total exhaustion of a potentially renewable resource can rather easily arise even if firms are assumed not to fear that their competitors will be involved in wholesale rampage. In Chapter 5 we shall explore such possibilities.

$$x_{ij} = x_j \quad \text{for } i, j = 1, \ldots, N.$$

Consequently 3.25 can be expressed as

$$y_i \leqslant \frac{x_{ii} F\left(\sum_{j=1}^{N} x_{ij}\right)}{\sum_{j=1}^{N} x_{ij}}.$$

The idea then is that firm i can enter into a transaction with firm j supplying i with x_{ij} which is an input in i's production y_i. There are in effect N inputs (x_{ij}, $j=1, \ldots, N$) in the ith firm's production function. The critical question now is whether the production

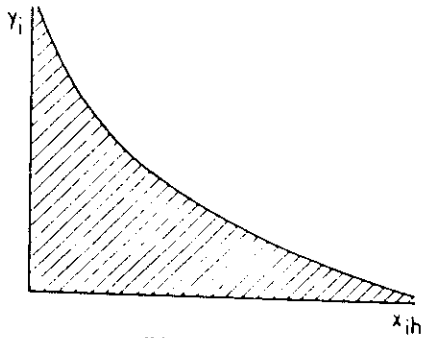


Diagram 3.4

possibilities open to i satisfy the convexity conditions that we assumed in the description of the private ownership economy. It is easy to verify that the answer is 'no'. For consider some $h(\neq i)$ and hold all x_{ij} fixed with $j \neq h$. Then one confirms readily enough that y_i as a function solely of x_{ih} is a declining one (see Diagram 3.4 below). With our assumptions regarding F the curve does not cut the horizontal axis but approaches it gradually. In any event the underlying region is non-convex and one suspects that a Lindahl equilibrium may not exist. That this may indeed happen can easily be argued. Recall that, at a Lindahl equilibrium that we are envisaging here, there will be a price p_{ih} that i must 'pay' h for h supplying each unit of the named commodity x_{ih}. Can p_{ih} be negative

(the case where h actually pays i for the right to introduce a unit of x_h)? Not so. For if it were, then i could set $x_{ii}=0$, close down production and demand an unlimited quantity of x_{ih}. This would yield an unlimited profit to i even though i is not actually undertaking any fishing activity. But at a positive price that h is required to pay i, firm h will hardly be willing to supply an unlimited quantity of x_{ih}. Demand and supply of x_{ih} will not match when $p_{ih} < 0$ and so p_{ih} cannot be negative at equilibrium. Can p_{ih} equal zero at equilibrium? Again the answer will generally be 'no', for if $p_{ih}=0$ then i will not demand any positive amount of x_{ih} (since x_{ih} is detrimental to i's production). But with no payments required, firm h will generally wish to introduce a positive quantity of x_h ($=x_{ih}$) at its profit maximizing production plan. Again, the supply and demand for the named good x_{ih} will generally not match. The argument is strengthened further if we were to assume that $p_{ih} > 0$. The source of the problem lies in that so long as a resource is common property the production possibility set of the representative firm is non-convex in the space of named commodities.

(iii) *Optimum regulations:* Now even though a competitive equilibrium with markets for 'named' vessels is a most unlikely outcome, there are a number of other avenues that we might wish to explore towards achieving an efficient allocation. Analytically, the most direct avenue would be to parcel the fishing ground into N 'plots' of equal size and allow each firm to have proprietary rights over one and only one such plot. By this means the fishing ground ceases to be common property and, as we shall see, so does the non-convexity vanish. We have already noted that for certain resources, such as oil underground, this is technically simply not possible. Our present case of fishing from a body of water raises similar problems, because while by this scheme firm i will not be allowed to fish on firm j's plot (that is, not without paying a competitive rent to j) it will presumably be able to utilize techniques to entice the fish under j's plot to drift into its own. Even so, for the sake of argument let us suppose that private property rights to the plots can costlessly be established and enforced in the catchment area. Recall that we began this section by supposing that efficient catch, Y, is a function of the total number of vessels, X, and the size of the catchment area, S, and that there are constant returns to scale. Thus $Y = H(X, S)$. But the total catchment area is fixed in size at $\bar{S}$. Now if the fishing ground is parcelled out and *if it is costless to protect one's property rights*, firm i faces production possibilities given by $y_i \leqslant H(x_i, \bar{S}/N)$.

But given that H is a function with constant returns to scale one has

$$H\left(x_i, \frac{\bar{S}}{N}\right) = \frac{\bar{S}}{N} H\left(\frac{Nx_i}{\bar{S}}, 1\right) \equiv \frac{\bar{S}}{N} F\left(\frac{Nx_i}{\bar{S}}\right).$$

As before, we may as well normalize and set $\bar{S}=1$. It follows that i is concerned with choosing x_i with a view to maximizing its own net profit, $1/N.F(Nx_i)-px_i$. Consequently x_i will be chosen at that level for which

$$F'(Nx_i) = p, \qquad i = 1, \ldots, N. \tag{3.38}$$

But conditions 3.38 and 3.31 are identical. In other words, the allocation implied by 3.38 is Pareto efficient. Thus if symmetric property rights are established to what was originally a common property fishing ground each firm will introduce precisely $\hat{x}$ vessels, and each firm will capture a profit equal to $F(N\hat{x})/N-p\hat{x}$ which, in turn, will emerge as the competitive rent per plot on the fishing ground. In other words, by assigning property rights on what was originally a common property resource (i.e. the fishing ground) the problem takes on a conventional form.

But given that for many cases it is simply too costly (if not impossible) to devise and enforce private property rights on certain resources, the foregoing avenue is not really a universal way out of the problem. (Imagine the difficulties in enforcing each citizen's right to a clean air-space directly over his private property.) Consequently one is encouraged to look elsewhere.

In the case of the open seas, where the body of water (and consequently, the aquatic life under it), is not owned by anyone, the firms (countries) may agree jointly to impose on themselves a quantity control (that is, a quota system), limit themselves to $\hat{x}$ vessels per firm (and hence $F(N\hat{x})/N$ units of catch per firm), and introduce a policing system to ensure that no individual firm cheats. This is often termed the *pure quota scheme*. If, on the other hand, the body of water is a lake located within a well-defined national boundary, there are at least two other schemes that the government might wish to consider in ensuring an efficient outcome. The idea, in each case, is for the government to take charge of the common property resource and to introduce regulations aimed at the attainment of allocative efficiency.

The first, which is often called the *pure licensing scheme*, consists in the government issuing a fixed number, $\bar{X}$, of licences for the total number of vessels that are allowed to be introduced into the catchment area. A market for these licences is then allowed to be developed among the firms. If, as we have been supposing, N is large, it is plausible that the market for these licences is more or less competitive. Let us suppose that this is so. Denote by $\bar{p}$ the competitive price of a licence when $\bar{X}$ is the total number of licences issued. It follows that each firm now faces a rental price $p+\bar{p}$ per vessel. What is done with the government revenue generated by the issue of these licences is a distributional question that we do not go into at this stage. For our actual example, we might, to fix our ideas, wish to consider the firms jointly issuing the total number of licences, $\bar{X}$, allowing a competitive market to develop for them; and then dividing the resulting revenue equally among themselves. We now construct the market equilibrium. If the ith firm were to assume that each of the other firms will introduce $\bar{x}$ vessels, its profit will be defined by

$$\frac{x_i F\{(N-1)\bar{x}+x_i\}}{(N-1)\bar{x}+x_i} - (p+\bar{p})x_i.$$

Consequently x_i would be chosen so as to satisfy the condition

$$\frac{(N-1)\bar{x}F\{(N-1)\bar{x}+x_i\}}{\{(N-1)\bar{x}+x_i\}^2} + \frac{x_i F'\{(N-1)\bar{x}+x_i\}}{(N-1)\bar{x}+x_i} = p+\bar{p}. \tag{3.39}$$

If condition 3.39 is to lead to an equilibrium one must have $x_i=\bar{x}$, given that firms are identical. It follows that 3.39 reduces to

$$\frac{(N-1)\bar{x}F(N\bar{x})}{(N\bar{x})^2} + \frac{F'(N\bar{x})}{N} = p+\bar{p}, \tag{3.40}$$

where $N\bar{x}=\bar{X}$, the number of licences issued. But presumably the government issues precisely $\hat{X}$ $(=N\hat{x})$ licences, since it is concerned with sustaining the Pareto-efficient allocation. Recall equation 3.31. It follows that 3.40 reduces to

$$\frac{(N-1)\hat{x}F(N\hat{x})}{(N\hat{x})^2} + \frac{F'(N\hat{x})}{N} = F'(N\hat{x}) + p$$

and consequently

3.41 $$\tilde{p} = \frac{(N-1)}{N}\left\{\frac{F(N\tilde{x})}{N\tilde{x}} - F'(N\tilde{x})\right\} > 0$$

where $\tilde{p}$ denotes the equilibrium price of a licence when $\tilde{X}$ licences are issued in all. It follows that if $\tilde{X}$ is the total number of licences issued by the government (regulatory agency) the equilibrium price, $\tilde{p}$, of a licence will be given by 3.41 and faced with this price each firm will find it most profitable to introduce precisely $\tilde{x}$ vessels if it is supposed that the other firms will purchase $(N-1)\tilde{x}$ licences in all. In other words, the government's problem consists solely in the choice of the total number of licences it issues.

An alternative regulatory device, often called the *pure tax scheme*, is in some sense a mirror image of the pure licensing scheme. It is in fact the Pigouvian tax notion discussed in section 3 in the context of external economies. The idea here is that the government (regulatory agency) imposes a tax per vessel (that is, a licensing fee per vessel) introduced by each firm. As in the case of the pure licensing scheme we do not concern ourselves at this stage with what is done with the tax revenue. As before, we might like to suppose that the firms impose a specific tax on themselves and divide the resulting revenue equally among themselves. If this is so and if we can show that there exists a tax equilibrium that is efficient in the sense that each firm finds it most profitable to limit itself to $\tilde{x}$ boats, when it assumes that each of the other firms will limit itself to $\tilde{x}$ boats, then the pure tax scheme and the pure licensing scheme envisaged earlier would be identical.

Denote by t the specific tax imposed on each vessel. If the ith firm were to assume that each of the other firms will introduce $\bar{x}$ vessels, its profit will be defined by

$$\frac{x_i F\{(N-1)\bar{x} + x_i\}}{(N-1)\bar{x} + x_i} - (p+t)x_i.$$

Consequently x_i would be chosen so as to satisfy the condition

$$\frac{(N-1)\bar{x}F\{(N-1)\bar{x} + x_i\}}{\{(N-1)\bar{x} + x_i\}^2} + \frac{x_i\{F'(N-1)\bar{x} + x_i\}}{(N-1)\bar{x} + x_i} = p + t.$$

If this is to lead to an equilibrium one must have $x_i = \bar{x}$ given that firms are identical. It follows that

3.42 $$\frac{(N-1)\bar{x}F(N\bar{x})}{(N\bar{x})^2} + \frac{F'(N\bar{x})}{N} = p + t.$$

But we want to choose t so as to ensure that $x_i = \tilde{x}$ is a possible equilibrium value. In other words, if each firm supposes that each of the others will introduce $\tilde{x}$ boats then $\tilde{x}$ would be its profit maximizing input level. Towards this set t at $\tilde{t}$, where

3.43 $$\tilde{t} = \frac{(N-1)}{N}\left\{\frac{F(N\tilde{x})}{N\tilde{x}} - F'(N\tilde{x})\right\}.$$

Using 3.43 in the RHS of 3.42 and setting $\bar{x} = \tilde{x}$ one obtains the condition

$$F'(N\tilde{x}) = p$$

which is precisely what is desired. Comparing equations 3.41 and 3.43 one notes that $\tilde{p} = \tilde{t}$ and, therefore that at the optimum for our problem the competitive price of the licences is equal to the Pigouvian tax per vessel that the government selects. This might suggest that the pure licensing scheme and the pure tax scheme are identical. For our present problem the results obtained by the two schemes are the same. Nevertheless, the schemes are different in spirit. We have noted that in the pure licensing scheme the government dictates the number of licences permitted (that is, the total number of vessels allowed) and the price system developed for this fixed number of licences allocates these licences among the N firms. In the pure tax scheme the government does not dictate directly the total number of vessels allowed on the fishing ground. Profit maximizing firms decide on how many vessels to introduce as a response to the licence fee introduced by the government on each vessel. While for our present problem the pure quota scheme, the pure licensing scheme and the pure tax scheme emerged as being identical in impact at the optimum, we shall note in Chapter 13 that this is not always so.

Notice that at the tax equilibrium the total tax revenue, R, is

3.44 $$R = N\tilde{x}\tilde{t} \equiv \frac{(N-1)}{N}\{F(N\tilde{x}) - N\tilde{x}F'(N\tilde{x})\}$$

$$\equiv \frac{N-1}{N}\{F(\hat{X}) - \hat{X}F'(\hat{X})\}.$$

For large N one has from equation 3.44 that

3.45 $$R \simeq \tilde{R} \equiv F(N\tilde{x}) - N\tilde{x}F'(N\tilde{x}) \equiv F(\tilde{X}) - \tilde{X}F'(\tilde{X}),$$

where $\tilde{R}$ is defined in equation 3.45 as the rent that ought to be imputed to the fishing ground. That is, $\tilde{R}$ is the area of the rectangle ABCp in Diagram 3.2. Indeed $\tilde{R}$ would have emerged as the competitive rent of the fishing ground had it not been a common property. Since $R<\tilde{R}$ when N is finite, it is the case that firms would make a positive profit at the tax equilibrium even if the entire tax revenue were expropriated by the government and not returned to the firms on a lump sum basis. The question arises whether firms are better or worse off at the free access equilibrium than they are at the tax equilibrium if the entire tax revenue is expropriated from them by the government. Rather surprisingly, perhaps, it is easy to show that they are unambiguously better off at the free access equilibrium. To see this, note that at the tax equilibrium if the entire tax revenue is expropriated, the total net profit (expressed as $N\tilde{\pi}$) made by the N firms as a whole is

$$N\tilde{\pi} = F(\tilde{X}) - R - p\tilde{X}.$$

Using equations 3.43 and 3.44 in this expression for $N\tilde{\pi}$ one has

$$N\tilde{\pi} = \frac{1}{N}\{F(\tilde{X}) - \tilde{X}F'(\tilde{X})\}$$

and thus that profit per firm is

3.46 $$\tilde{\pi} = \frac{1}{N^2}\{F(\tilde{X}) - \tilde{X}F'(\tilde{X})\}.$$

Comparing equations 3.35 and 3.46 and noting that $\tilde{x}<\hat{x}$ one sees readily enough that $\hat{\pi}>\tilde{\pi}$.

One can see the nature of the corrective specific tax 3.43 more heuristically as follows. Recall that the catch obtained by the ith firm is represented by 3.25. Now the marginal loss in catch imposed on i by the addition of an extra boat by some other firm is $\partial y_i/\partial X_{N-i}$, which routine calculation shows to be

3.47 $$\frac{\partial y_i}{\partial X_{N-i}} = \frac{x_i}{X}\left\{F'(X) - \frac{F(X)}{X}\right\}.$$

The corrective tax that needs to be imposed on a given firm is plainly the *sum* of all the marginal losses in catch that are sustained by all the other firms when this firm adds an extra boat to its active fleet.[7] Thus if the ith firm adds an extra boat the total marginal loss, M_L, to all firms is obtained from 3.47 as

3.48 $$M_L = \frac{X_{N-i}}{X}\left\{F'(X) - \frac{F(X)}{X}\right\}.$$

Since $M_L<0$ it is indeed a loss that is being inflicted. When $X=N\tilde{x}$ a glance at 3.43 and 3.48 shows that $t=|M_L|$, where $|M_L|$ is the absolute value of the loss.

Now the marginal benefit, M_B, to the ith firm when it introduces an extra boat is plainly $(\partial y_i/\partial x_i - p)$, which from 3.25 yields

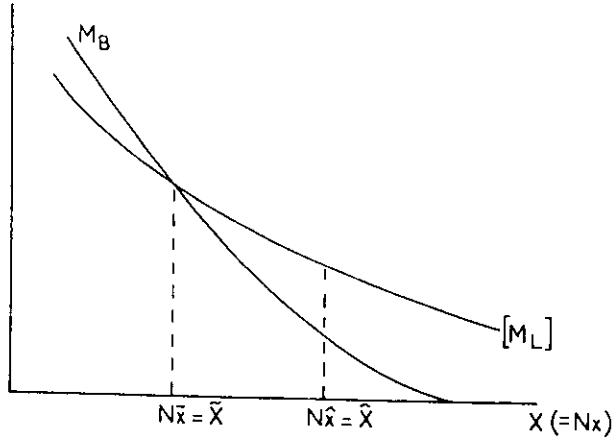


Diagram 3.5

[7] Compare this with our example of external *economies*. There we noted that the corrective *subsidy* that was needed to be imposed on the purchase of a unit of the public good by an individual was the sum of the marginal benefits enjoyed by all the other individuals as a consequence.

$$M_B = \frac{F(X) + x_i F'(X)}{X} - \frac{x_i F(X)}{X^2} - p. \tag{3.49}$$

Examining the difference between M_B and $|M_L|$ from 3.48 and 3.49 yields

$$M_B - |M_L| = F'(X) - p. \tag{3.50}$$

Given the assumptions that have been made about F there is a unique value ($N\tilde{x}$) of X at which expression 3.50 is zero (see Diagrams

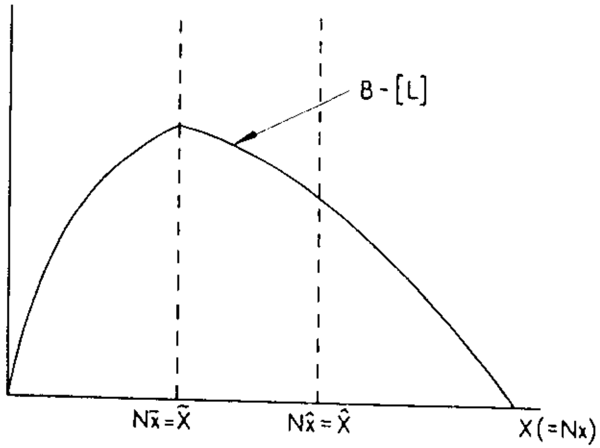


Diagram 3.6

3.5 and 3.6). The tax system is designed to locate this value of X.

The link between designing a tax system that will force profit maximizing firms to operate at what is collectively the desired level of operation (i.e. $N\tilde{x}$), and conducting a social cost–benefit analysis to locate this desired level and thereby to impose a pure licensing system, can also be brought out by this example. Social cost–benefit analysis will signal the need for expanding the level of fishing activity, X, whenever $M_B > |M_L|$ or equivalently, whenever

the *total* net benefit curve $B - L$ is increasing.[8] To be precise, if $M_B > |M_L|$ at a given level of activity then a marginal project ΔX (when $\Delta X > 0$) will show positive net social benefits, since $(M_B - |M_L|)\Delta X > 0$. It will be worth undertaking. One would wish to go on accepting such marginal projects until $M_B = |M_L|$. Likewise, if at a level of activity one finds $M_B < |M_L|$ then a marginal project ΔX will be judged desirable only when $\Delta X < 0$, since only for such a 'project' will net social benefit $(M_B - |M_L|)\Delta X$ be positive. In this case as well one would wish to continue to choose such 'projects' until $M_B = |M_L|$.

5. Some Examples of Common Property Resources

We have analysed the problem of the common in the context of individuals hunting for aquatic life from a body of water that is not owned by anyone in society or, to put it equivalently, when the fishing ground is a common property resource. Several other examples would seem to fit the general texture of the arguments that we have developed from this specific context. We have already mentioned that a similar problem occurs when oil men drill for oil from a common underground reservoir and when the rule of capture prevails. The case of hunting or trapping for animals in a common ground is another example. On most occasions the pollution of the atmosphere is also such an instance. The atmosphere's capacity to absorb pollution is, while large, clearly finite. But if individuals are not charged for disposing of pollutants into the atmosphere they enjoy the benefit of a free service, namely the service that the atmosphere performs in absorbing pollutants. Our formal analysis has suggested that this benefit can in fact be largely a mixed blessing in that there is a tendency for the market equilibrium to sustain too much pollution in a sense that can be made precise. In those situations where the individuals can be regarded as roughly identically placed (such as a community of motorists emitting noxious

[8] We are calling this social cost–benefit analysis even though benefits and costs are being measured in terms of the profits of the different firms. There is no consumer surplus to be taken into account in the exercise since the market for the catch has been assumed perfectly competitive. The term social cost–benefit analysis could, however, be misleading for this exercise if one wants to impute a value to the aquatic life itself on environmental grounds. We are here restricting attention to the economic value of such animals or rather to the value of the catch. In effect then we are discussing the optimum social management of a common property resource when the resource is socially valued solely in terms of the value of the catch.

fumes from their cars and at the same time preferring clean air to polluted air), our formal analysis of the common's problem will carry over directly. The problem of the common arises as well in the case of individuals drawing water from a common underground reservoir. Under what is known as the 'riparian' doctrine (which is similar to the rule of capture in the case of oil extraction), each owner of a parcel of land is allowed to extract as much water as he desires without regard to its effect on the owners of neighbouring parcels. Thus the doctrine provides no protection to a well-owner from the lowering of the water table caused by his neighbour's action. If in addition, an excessive drawing of water causes salt water intrusion, the ground water basin may well be destroyed. In this event we are confronted with a problem exactly akin to the problem of common property fishing. The grazing of cattle or sheep in a common land has similar features as well, and the free access equilibrium would imply an over-grazing of such property. To the extent that such over-grazing deteriorates the quality of the land (as has happened, for example, in some of the arid grasslands in the Middle East) the curtailment of the total size of the herds grazed is not merely in the economic interests of the grazer but also in the interest of the environmentalist. The problem is analogous to those that our example has already brought out. But our formal analysis, while suggestive, has nevertheless been limited in scope. We have analysed the problem in an entirely static context. This limitation prevents one from describing sharply the fact that common property resources are often eroded gradually over time. In certain cases the total erosion can happen in a matter of a few decades, as in the case of the American bison in its natural habitat, but others take a longer time. The Negev Desert was not created overnight. It is only when we introduce time explicitly into the analysis that we shall be able to analyse the general question of husbanding a potentially renewable resource. We shall go into this in Chapter 5.

The fact that common property resources tend to be exploited at an excessive rate has been recognized for a long time. It would appear that historically property rights for the common property resource were often difficult to establish for a variety of reasons. Here we have emphasized the purely technical difficulties that can often arise, as in the case of oil or water in an underground reservoir, or in the case of a hunting resource that migrates over large areas of land, or indeed as in the case of the atmosphere which is in a constant state of diffusion and movement. But technical difficulty

is of course only one possible reason. Custom can play its part. Often the notion of private property rights may be alien to the society in question, particularly so when the resource is vital to its members. Thus, for example, in the medieval manorial economy in England the domestic animals were privately owned but the land the animals grazed was a common property resource. On occasion the problem can be a direct political one as, to take an example, when different nations are involved in exploiting a resource from international waters.

Generally speaking, when the problem of over-exploitation of a common property resource has been recognized and when the resource has continued to remain a common property, attempts have been made to regulate its exploitation via *quantity* controls. Thus, for example, in the United States after years of excessive drilling in the states of Oklahoma, Texas, California and New Mexico, the Conally 'Hot Oil' Act of 1935 was a culmination of two decades of attempts at regulating production of oil via controls on the number of wells drilled, their spatial distances, and even on the quantities extracted.[9] Often, when agricultural land is communally exploited, elaborate quantity controls are devised to prevent over-utilization. Thus, for example, in medieval England the manor would regulate the grazing of domestic animals in the common land by limiting the number of animals grazing, as well as the duration of grazing.[10] More recently, in the United States the Taylor 'Grazing' Act of 1934 was designed precisely to 'stop injury to the public grazing lands by preventing over-grazing' of the open grangelands of the Cascade Range and the Sierra Nevada Mountains.[11] But on occasion the problem is sighted a bit too late. The American whooping crane is now extinct. It is also possible that the bowhead and the right whales have suffered a similar fate.

The problems of devising and enforcing such regulations are plainly acute when nations vie with one another for a common property resource. The 'Cod War' between Iceland and Great Britain is still unresolved. And it can hardly be said that the International Whaling Commission has been spectacularly successful. Established in 1946 with seventeen member countries, the Commission's task is to protect whale species by setting maximum animal catch limits,

[9] For a thorough discussion of petroleum regulations in the United States, see McDonald (1971).

[10] On this, see Lipson (1949), p. 72.

[11] See Foss (1960), p. 61.

designating areas closed for hunting, and setting minimum population size limits below which the species are considered endangered. But the Commission has neither inspection nor enforcement authority. Consequently ignoring the Commission's recommendations appears to have been the single systematic policy followed by some of the member countries.[12] So much so that despite the concern expressed by many over the years, the blue whale species was pretty close to extinction by the early 1960s. A main problem of arriving at an agreed upon regulation (as opposed to enforcing it) is that unlike the formal model that we have studied, 'firms' exploiting a common property resource are generally not identical. For Iceland cod fishing is a major industry; for Great Britain, it is not. Devising a fair regulation in such instances is understandably difficult. Then again, problems arise whenever there are disagreements about technological possibilities. Iceland argues that excessive catches in the past have brought the cod population off its territorial waters down to a dangerously low level. Great Britain does not believe the gloomy statistics.[13] Regulations are difficult to arrive at when the factual basis on which the regulations are to be built is not agreed upon.

These are, of course, among the more spectacular examples. Regulations are, understandably, easier to introduce if the 'firms' exploiting the common property resource do not form a powerful block. Big-game hunters may find a government's assessment of the threshold population size a of given species too high. But if they do not form a powerful political lobby, strong regulations can in fact be enacted. And they often are.

Our simple model describing N identical firms exploiting a common property resource suggests that there are several ways of meeting the problem of over-utilization at a free access equilibrium. We noted that quantity restriction (i.e. specifying the input level $\tilde{x}$ per firm) is identical in impact to introducing an optimal tax on each unit of the input introduced (which in turn is identical in impact to the pure licensing scheme), so long as the tax revenue is distributed evenly among the N firms.[14] A third scheme that we have explored would be to legislate private property rights to the

[12] On this, see McVay (1966).

[13] On this, see Shapley (1972).

[14] We shall note later that this equivalence between a quantity mode of regulation and a price mode of regulation does not hold if the central authority's knowledge of technological and economic possibilities is imperfect and when the regulation is plausibly circumscribed. See Chapter 13.

resource. This last, as we have noted, is simply not feasible if the resource is oil or water underground.

But it is certainly feasible with arable or grazing land. When this scheme is introduced the resource ceases to be common property and the problem is solved at one stroke.[15] It would appear that even in early societies land was as often as not privately owned; and even in those where there was free access to land there were often elaborate regulations co-ordinating their utilization by the various members of society.[16] But while it is true that a free access equilibrium is allocationally inefficient, it should by no means be thought that the introduction of property rights on what was originally a common property resource is necessarily a move towards raising welfare. We have seen that when N is finite each firm makes a positive net profit at the free access equilibrium. Introducing the tax that we analysed would certainly be in the interests of efficiency. But we have noted that if not a penny of the tax revenue is paid back to the firms, they are all worse off even though they still make positive profits. Then again, if what was originally a communal property is suddenly expropriated by an 'outsider' who proceeds to exact the full rent from the property accruing to it at the profit maximizing level of activity $N\tilde{x}$, each of the firms is yet worse off since in the other two cases, they were making a positive profit at least and now they are not. This expropriation of a common property resource, while blessed at the altar of efficiency, can have disastrous distributional consequences. The point that is being raised is analytically, of course, a trivial one, but it has been argued by some that it is nevertheless historically rather important. Thus, for example, during the fifteenth and sixteenth centuries the enclosure movement swept the English countryside, thereby putting an end to communal arable land. In its wake it seems were left literally thousands of impoverished peasants whose simple means of livelihood were

[15] These differing schemes, for our model, are identical so long, of course, as the costs involved in sustaining these schemes are equal. In our example we have supposed that such costs are nil. But one should bear in mind that there are administrative costs in establishing and enforcing taxes. At the same time there are costs of policing private property rights (e.g. fences that separate one's grazing ground from one's neighbour's). When contemplating alternative *institutional* systems such cost considerations will presumably matter. For an excellent discussion of such matters, see Dales (1968).

[16] For example in Greco-Roman times land was usually privately owned by a few landlords (see Finley (1973)). We are by no means suggesting that private ownership of land is prompted by recognition of the problem of the common; simply that where land is communally owned regulations toward its utilization often seem to appear.

wrecked and who had consequently to search for industrial employment.

'Communal property . . . was an old Teutonic institution which lived on under cover of feudalism. We have seen how the forcible usurpation of this, generally accompanied by the turning of arable into pasture land, begins at the end of the fifteenth and extends into the sixteenth century. But, at that time, the process was carried on by means of individual acts of violence against which legislation, for a hundred and fifty years, fought in vain . . . The parliamentary form of the robbery is that of Acts for enclosures of commons, in other words, decrees by which the landlords grant themselves the people's land as private property, decrees of expropriation of the people.'[17]

A remarkable feature of the problem of common property resource is the variety of examples that one can rather readily construct in exemplifying it.[18] While the general nature of the problem appears in each such example, one's reaction to the various means of coping with it no doubt depends on the example in question. If the tax proceeds from big-game hunting are expropriated entirely by the government, the resulting distributional impact will not usually stick in one's throat. Not so, presumably, if the 'firms' happen to be individuals eking out an existence from a common property resource. While we have focused attention only on the inefficiency involved in the exploitation of a common property resource, the distributional consequences of alternative mechanisms of removing this inefficiency should certainly be borne in mind when examining any particular case.

6. Asymmetrical Externalities and the Multiplicity of Tax Equilibria

We have analysed the problem of the common at some length because of the importance of the problem and also because of the simplicity of its underlying structure. A distinguishing feature of our

[17] Marx (1961), p. 724. The Marxian thesis regarding the distributional impact of the enclosure movement of the fifteenth and sixteenth centuries has been systematically challenged over the years (see, for example, Kerridge (1969)). For a recent revival of the thesis see the interesting paper by Cohen and Weitzman (1975).

[18] For illuminating early discussion of the problem of the common, see Gordon (1954) and Milliman (1956). Gordon emphasized the common fishery's problem and Milliman those of common water resources. For a popular and dramatic statement of the problem see Hardin (1968).

model of common property resource is the symmetric nature of the externalities. An implication of this assumption of symmetry is that the tax equilibrium is unique. To state this another way, the symmetry assumption implies that the marginal benefit curve (M_B) cuts the marginal loss curve ($|M_L|$) at a single point (see Diagram 3.5) or, equivalently, that the net benefit curve ($B-|L|$) as a function of the total number of vessels (X) is single peaked (see Diagram 3.6). This is an amiable property for the problem to have. We noted that marginal social cost–benefit analysis allows one to locate the number of vessels $\hat{X}$ that ought ideally to be utilized.

Unhappily a great many examples of external diseconomies do not have this simple structure. Consequently it is entirely possible in such cases for there to be a multiplicity of tax equilibria. In other words it is possible in such cases that the net benefit curve has multiple peaks, some of which are merely locally the greatest in value and not globally so.[19] Marginal social cost–benefit analysis in such cases can be treacherous since, depending on the level of activity from which social cost–benefit analysis is begun, the analysis may quite easily lead to a mere local optimum and miss out the global one without anybody being the wiser. We shall illustrate such a possibility by means of an example. The example will also enable us to discuss a number of further issues that are relevant in discussing the theory of externalities.

Suppose that industry α consists of N identical firms, ($i = 1, \ldots, N$), all located upstream of a river. Firm i utilizes two variable inputs, l_i and x_i, to produce a homogeneous product, y_i. For simplicity we take it that production possibilities open to i are represented by

$$3.51 \qquad y_i \leqslant l_i^a x_i^b, \qquad a, b > 0 \quad \text{and} \quad a + b < 1; \qquad i = 1, \ldots, N.$$

The production process, however, consists as well in the creation of effluent, e_i, which to be specific, is a transformed product of the input, x_i. This effluent, it is supposed, can only be deposited in the river. A detailed account of production possibilities would have us take into the fact that the quantity of effluent can often be controlled (say, by breaking it down into relatively harmless molecules)

[19] Stating the point in yet another way; the motivation for devising a tax system is to allow the economy to find the optimum of the net benefit curve (assuming, of course, that the government has a clear assessment of how to aggregate individual benefits and costs into social benefits and costs). Formally speaking the taxes are computed from the first order conditions pertaining to the maximum of the net benefit curve. It follows that every local maximum and every local minimum can be established as a tax equilibrium.

with the help of further resources. Here we shall wish to keep the analysis simple. Consequently, we ignore such possibilities of treating the waste and suppose simply that e_i is proportional to x_i, and, in particular that

3.52 $$e_i = \mu x_i, \qquad \mu > 0,^{20} \qquad i = 1, \ldots, N.$$

Industry β consists of M identical firms ($j = 1, \ldots, M$), all located downstream of this river. For simplicity we suppose that firm j utilizes a variable input, v_j, and a fixed quantity of water to produce a homogeneous product, z_j. But while the quantity of water required is fixed (because, say, the plant size is fixed), its usefulness depends on the quality of the water and, in particular, it is supposed that the less contaminated the water, the more productive it is. Again, a detailed treatment of production possibilities would have us take into account the fact that the contaminated water can often be purified (at least up to a point) by j with the help of further resources. Once again, we shall wish to keep j's production possibilities in a simple form. Thus we ignore such possibilities. Write $E = \sum_{i=1}^{N} e_i$, for the total quantity of effluent in the river. Then we take it that production possibilities open to j can be represented by the form

3.53 $$\left.\begin{aligned} & z_j \leqslant \frac{k(v_j)}{1 + h(E)}, \qquad j = 1, \ldots, M \\ & \text{where} \\ & k(0) = 0, \quad k'(v_j) > 0, \quad k''(v_j) < 0, \\ & h(0) = 0, \quad \text{and} \quad h'(E) > 0. \end{aligned}\right\}$$ [21]

As in the earlier sections of this chapter we shall be interested in the notion of an equilibrium outcome for these two industries.

[20] To give only an example of how one may wish to capture the fact that treatment of the waste is possible by i one could suppose that i can utilize a further resource z_i to control the waste via the production constraint

$$e_i \geqslant \frac{\mu x_i}{1 + g(z_i)},$$

where $g(0) = 0$ and $g'(z_i) > 0$. In 3.52 it is supposed that $g(z_i) = 0$ for all z_i.

[21] A simple example of how one may wish to capture the fact that j is capable of purifying the river water for its own use would be as follows. Denote by m_j the level of pollution of the water actually used by j in its production of the final good. Let s_j denote a variable input in its water purification plant. Then we could suppose that

$$m_j \geqslant \frac{h(E)}{1 + r(s_j)} \quad \text{and} \quad z_j \leqslant \frac{k(v_j)}{1 + m_j}$$

where $r(0) = 0$ and $r'(s_j) > 0$. In 3.53 we are supposing that such purification possibilities do not exist and, therefore, that $r(s_j) = 0$ for all s_j.

Again, as in our earlier examples let us cushion these two industries from the outside world by supposing that they trade with the rest of the world at fixed prices. Thus denote by p_y, p_l and p_x the prices of the single output and the two variable inputs involved in industry α, and by p_z and p_v the prices of the single output and the single variable input involved in industry β.

An important feature in which our present example differs from the example of the 'common' is the asymmetrical nature of the interaction between the two industries. There is a temptation to say that so long as $\sum_{i=1}^{N} e_i > 0$ industry α imposes an externality on industry β and not the other way around, and common parlance would describe the interaction in precisely such a manner. We shall note presently that the matter is somewhat more ambiguous than common parlance would suggest and that it depends critically on the precise specification of property rights. Even so, it is plain that a distinguishing feature of the present example is the asymmetry in the interaction.

If both N and M are reasonably large it may seem plausible to contemplate once again the notion of a market equilibrium which we introduced in section 1. We shall compute such equilibria in what follows. Given that firms are profit maximizing each firm will produce efficiently. Consider firm i in industry α. Let A (a constant) denote the cost that i has to bear on its fixed capital. Net profit for i can then be denoted as $(p_y y_i - p_l l_i - p_x x_i - A)$, which, on using 3.51 can be expressed as

3.54 $$p_y l_i^a x_i^b - p_l l_i - p_x x_i - A.$$

Firm i is concerned with maximizing 3.54 by choosing l_i and x_i. The critical question pertains to the *admissible* set of values of these two variables. Now so far as l_i is concerned, presumably it can take any non-negative value that i chooses. But what of x_i? For note 3.52. The admissible range of values for x_i will plainly depend on the law pertaining to the amount of pollution that i is allowed to deposit into the river. For example, if industry β has a right to *pure* river water then *in the absence of any negotiations* between firm i and industry β, x_i will have to be set at zero. Now as we are considering the market equilibrium in the sense that we have defined it we take it by definition that there are no negotiations. The equilibrium that we are studying is a non-co-operative one. It is, therefore, informative to suppose that i chooses l_i only, and to express i's maximized profit, π_α^i as a function of the quantity of pollution, e_i $(=\mu x_i)$ that it deposits into the river. This procedure will enable us to describe the set of market equilibria as a function of the precise pollution rights specified by law. Thus write

$$\pi_\alpha^i(e_i) = \max_{l_i}\left(\frac{p_y l_i^a e_i^b}{\mu^b} - p_l l_i - p_x \frac{e_i}{\mu} - A\right). \tag{3.55}$$

Given that by assumption $a+b<1$ (see 3.51) it is simple to check that $\pi_\alpha^i(e_i)$ is a concave function. In Diagram 3.7 a typical functional form of π_α^i is presented. The diagram also contains the resulting form of the marginal profit function $d\pi_\alpha^i(e_i)/de_i$.

Consider now firm j in industry β. If we were to take it that the total level of pollution in the river is E, its net profit $(p_z z_j - p_v v_j)$, can, on using 3.53 be denoted as

$$p_z \frac{k(v_j)}{1+h(E)} - p_v v_j. \tag{3.56}$$

(We assume, for simplicity, that j incurs no fixed cost.) Firm j is concerned with maximizing 3.56 by a judicious choice of v_j. Denoting by π_β^j the maximized value of profit we have, by definition

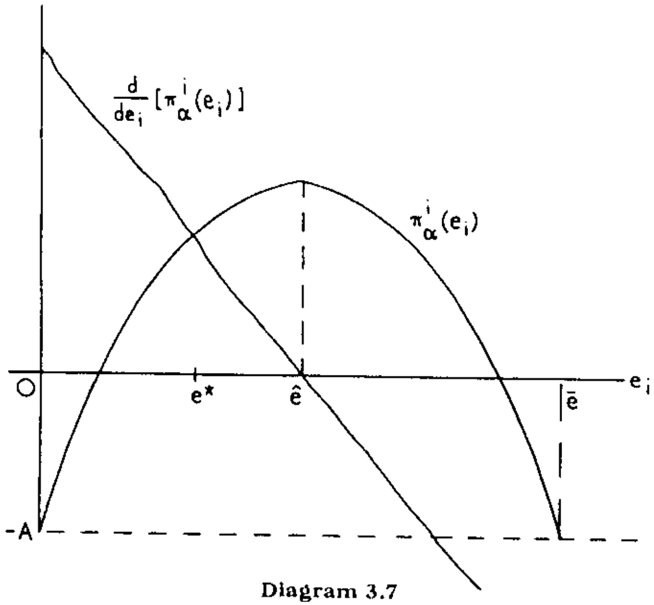


Diagram 3.7

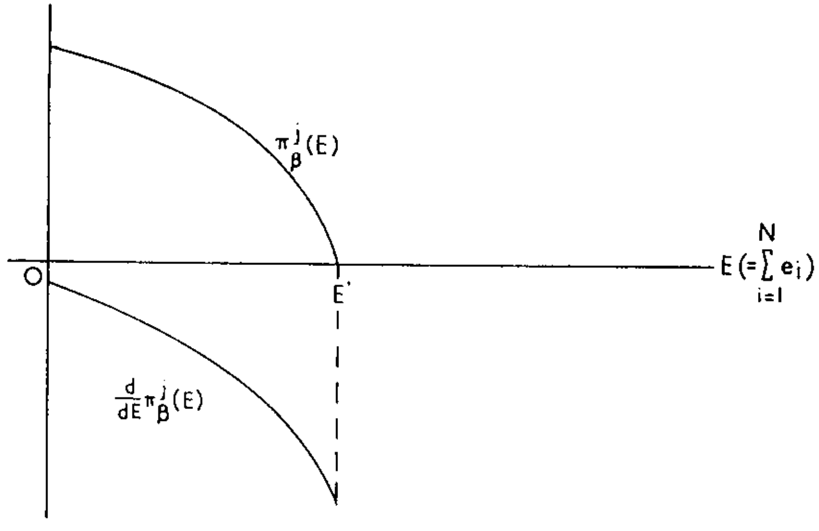


Diagram 3.8

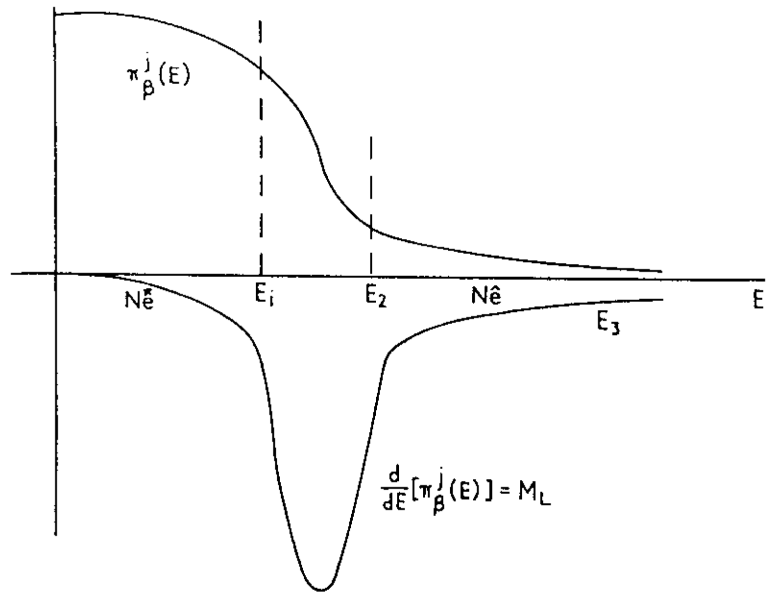


Diagram 3.9

3.57 $$\pi_\beta^j(E) \equiv \max_{v_j} \left\{ \frac{p_z k(v_j)}{1 + h(E)} - p_v v_j \right\}.$$

It is immediate from the RHS of 3.57 that $\pi_\beta^j(E)$ is a declining function of E. But it can never be negative, since j always has the option of setting $v_j = 0$ and thereby closing down its operation. In Diagrams 3.8 and 3.9 two plausible forms of π_β^j are presented. The diagrams also depict the corresponding shapes of $d\pi_\beta^j(E)/dE$, the marginal impact of E on j's maximized profit level.

In Diagram 3.8 it is supposed that up to the pollution level E', π_β^j is concave and that for all $E > E'$ one has $\pi_\beta^j = 0$. Diagram 3.9

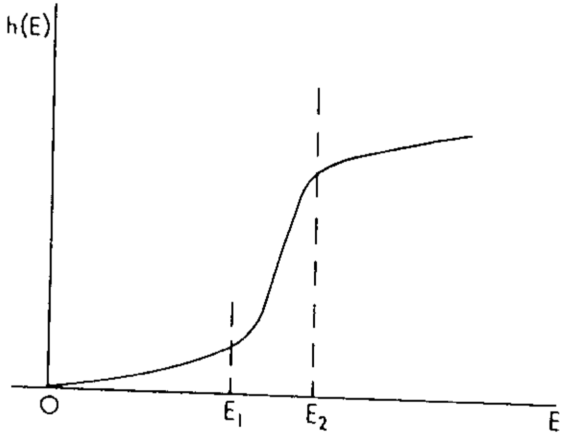


Diagram 3.10

portrays a more interesting situation. Recall 3.53. Apart from asserting that $h(0) = 0$ and $h'(E) > 0$ we have left the functional form of h unspecified. In many situations it is plausible to suppose that $h(E)$ has the shape depicted in Diagram 3.10. The situation captured by Diagram 3.10 is one where up to a total pollution level E_1 the water impurity does not affect production possibilities in industry β by much. In other words, up to E_1, $h'(E)$ is rather 'low' (though positive). Over the range E_1 to E_2 the water impurity begins to affect production possibilities in industry β quite seriously. That is to say $h'(E)$ is 'large'. Beyond E_2 it is supposed that h more or less flattens out. Most of the damage has already been done. Now if $h(E)$ has the form depicted in Diagram 3.10, then it is immediate from 3.57

that $\pi_\beta^j(E)$ will have the form drawn in Diagram 3.9. Up to a level of pollution E_1 maximum profit, π_β, declines very slowly. It declines dramatically over the range E_1 to E_2 during which the marginal impact of E is great. Beyond E_2 maximum profit declines slowly, possibly to zero for large enough values of E (say for $E \geq E_3$). In either case (i.e. Diagrams 3.8 or 3.9) π_β^j is a non-concave function if the entire non-negative range of E is contemplated. In what follows we shall, to be specific, take it that $\pi_\beta^j(E)$ has the functional form depicted in Diagram 3.9.

We have described the profit functions of the representative firms in the two industries and it remains to characterize the market equilibria. As one might be inclined to guess, the structure of the market equilibria depends critically on the specification of pollution rights. Suppose, to take an extreme example, that in this economy the law recognizes pollutor's rights but not the rights of the pollutees. That is to say, suppose that industry β has no legal *right* to pure water. Recall that we are attempting to describe an equilibrium that is characterized by an absence of negotiations between firms. Since by law firm i can pollute as much as it likes without penalty it will come to pollute up to the level $\hat{e}$ (see Diagram 3.7) at which $\pi_\alpha^i(e_i)$ is maximized. Total pollution in the river will therefore be $N\hat{e}$. Turning to industry β, so long as $N\hat{e} < E_3$ (see Diagram 3.9) firm j will find it profitable to undertake production. The interesting situation is one where this is indeed so. Thus in the case where the pollutor has the right to pollute indefinitely the market equilibrium is characterized by a total pollution level $N\hat{e}$. Total profit for the two industries taken together at the equilibrium can be read off from Diagrams 3.7 and 3.9 and can be expressed as

3.58 $$\pi(\hat{e}) \equiv N\pi_\alpha^i(\hat{e}) + M\pi_\beta^j(N\hat{e}).$$

We shall presently compare $\pi(\hat{e})$ with maximum total profit for the two industries taken together; that is, with a Pareto-efficient allocation. But for the moment consider an altered legal structure, one where there are some rights for the pollutees as well. It is supposed that the law recognizes that it must not empower the pollutee with the right to pure river water (i.e. $E = 0$) since in the absence of negotiations (that is, at a market equilibrium), industry α will be forced to close down and absorb a total loss NA. Consequently the law empowers the pollutee with only partial rights, in that it allows firm i in the industry to pollute only up to a level e^*

(>0) with impunity. In other words, e^* is the *benchmark* level of pollution per firm in industry α.[22] Now if $0 < e^* < \hat{e}$ (see Diagram 3.7) and if, as we are supposing, there are no negotiations between firms, then i will deposit precisely e^* units of pollution, since π^i_α is increasing at e^*.[23] It follows that at a market equilibrium total pollution will be at the level Ne^*. Consequently total profit for the industries taken together is

$$\pi(e^*) = N\pi^i_\alpha(e^*) + M\pi^j_\beta(Ne^*) \tag{3.59}$$

(see Diagrams 3.7 and 3.9). Notice that $\pi^i_\alpha(e^*) \neq \pi^i_\alpha(\hat{e})$, $\pi_\beta(e^*) \neq \pi^j_\beta(\hat{e})$, and in general that $\pi(e^*) \neq \pi(\hat{e})$. Notice also that the lower the benchmark level of pollution, e^*, the more the equilibrium distribution favours industry β. We conclude that at a market equilibrium *both* the total profit for the two industries taken together, as well as the distribution of this profit between the two industries, depend on the specification of pollution rights.[24]

In what follows we shall be concerned, for simplicity, with the size of total profits (sum of producers' surpluses). In other words, we shall be concerned with Pareto-efficient allocations. It is rather plain that a market equilibrium will be Pareto *inefficient* unless, by fluke or design, e^* has been chosen so as to support a Pareto-efficient outcome. So the first question to ask is whether a Lindahl equilibrium can exist for this problem. In fact it is rather easy to check that it does not. For recall that a Lindahl equilibrium will consist of competitive markets for named commodities, e_{ji}, where in equilibrium

$$e_i = e_{ji}, \qquad i = 1, \ldots, N \quad \text{and} \quad j = 1, \ldots M. \tag{3.60}$$

To say that firm i in industry has been empowered by law with a benchmark level of pollution, e^*, is a way of saying that i has an initial endowment, e^*, of pollution rights. Denote by p_{ji} the price that firm i in industry α has to pay firm j in industry β for a unit of pollution that i deposits into the river. Thus, in fact, if i deposits

[22] We are supposing that it is costless to monitor the quantity of effluent generated by *each* firm in industry α.

[23] If $e^* > \hat{e}$ then i will deposit precisely $\hat{e}$, since π^i_α is a declining function beyond $\hat{e}$.

[24] This feature is, of course, true as well in the case of the problem of the common. We did not emphasize it in the discussion of the common's problem, however, because the problem there is characterized by a simple and unambiguous set of property rights; namely that each firm has the right to exploit the common property to any extent it chooses. That is, the law ascribes full rights to the pollutor.

e_i ($=e_{ji}$) units, its net payment to j is $p_{ji}(e_i - e^*)$. Now it is plain that at an equilibrium p_{ji} cannot be positive. For with $p_{ji} > 0$ firm j will be encouraged to undertake little or no production activity but to earn all its profits by selling pollution rights to i; the more it sells, the higher its profits. But i will hardly wish to purchase an unlimited quantity of pollution rights, most certainly not beyond the level $\hat{e}$ at which $\pi^i_\alpha = -A$ (see Diagram 3.7), and actually at a level less than $\hat{e}$. Demand and supply of e_{ji} will not match, and consequently an equilibrium cannot be sustained with $p_{ji} > 0$. Notice next that p_{ji} cannot be zero in equilibrium since in this case there is in fact no cash transfer between i and j. Consequently j will demand that $e_{ji} = 0$ while i would wish to supply e_{ji} at a level $\hat{e}$. The argument is reinforced if $p_{ji} < 0$. One concludes that a Lindahl equilibrium for our problem does not exist. The source of the problem, as one would be inclined to guess, is that in the space of named commodities firm j's production possibility set is non-convex. This can be readily confirmed by considering 3.53. Holding constant v_j and all except one named commodity e_{ji} one can describe production possibilities as circumscribed by the curve in Diagram 3.11. The set is, of course, non-convex.

The question that arises next pertains, as in our earlier examples, to the kinds of regulatory measure that will ensure that a market equilibrium sustains an efficient allocation. Now it can be checked that given our characterization of production possibilities for the

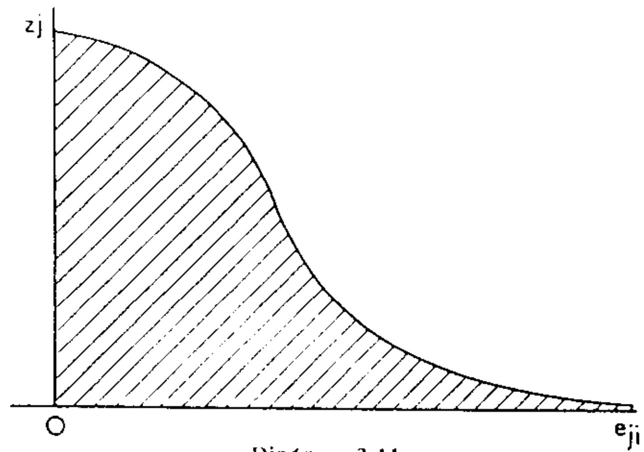


Diagram 3.11

two industries the pure quota rule, the pure licensing scheme and the pure pollution tax scheme will all in principle work for this problem. To illustrate this, consider for example the pure tax scheme. The regulatory agency, would wish to impose a specific tax, t, per unit of effluent, e_i ($\geqslant 0$) that i deposits in the river. *Given* t, i's profit will read as

$$\frac{p_y l_i^a e_i^b}{\mu^b} - p_l l_i - \left(\frac{p_x + \mu t}{\mu}\right) e_i - A,$$

and consequently i will be concerned with selecting l_i and e_i optimally, *with no constraint on either variable*. The regulator's problem consists in choosing the correct value of t, where, by a correct value, we mean one that sustains a Pareto-efficient outcome. This is precisely what was aimed at in the previous examples. However, a source of worry for this example is that unlike the problem of the common there may be computational difficulties in *locating* an efficient regulatory measure. This is a point we raised at the very beginning of this section and it is time to elaborate on it. In order to do this it will be useful to simplify the example and suppose that $N = M$. Given that firms in a given industry are identical, one is concerned with those allocations in which firms in a given industry behave identically. Recall Diagram 3.9. Denote by

$$M_L \equiv \frac{d}{dE}\{\pi_\beta^j(E)\} < 0,$$

the marginal loss to firm j due to an increase in river pollution and, therefore, by $NM_L(E)$ the *sum* of the marginal losses to the firms in industry β (see Diagram 3.9). Denote by $N|M_L(E)|$ the absolute value of this loss. Using Diagram 3.9 the general shape of this can easily be protrayed, as in Diagram 3.12.

The benefit to industry α due to extra pollution is also simple to describe. As firms are identically treated let each firm pollute at a level e. Total pollution is $E = Ne$. Write

$$B(E) = \sum_{i=1}^{N} \pi_\alpha^i(e) = N\pi_\alpha(e) = N\pi_\alpha\left(\frac{E}{N}\right)$$

for total profit for industry α when the total pollution level is Ne, and by

$$M_B = B'(E) = \frac{d}{dE}\left\{N\pi_\alpha\left(\frac{E}{N}\right)\right\}$$

the marginal profit to industry α. From Diagram 3.7 it is readily checked that this last is monotonically decreasing and equals zero at the level of pollution $N\hat{e}$.

In Diagram 3.12 the points E_4, E_5 and E_6 denote three levels of pollution, E, at which the curves M_B and $N|M_L|$ intersect. In

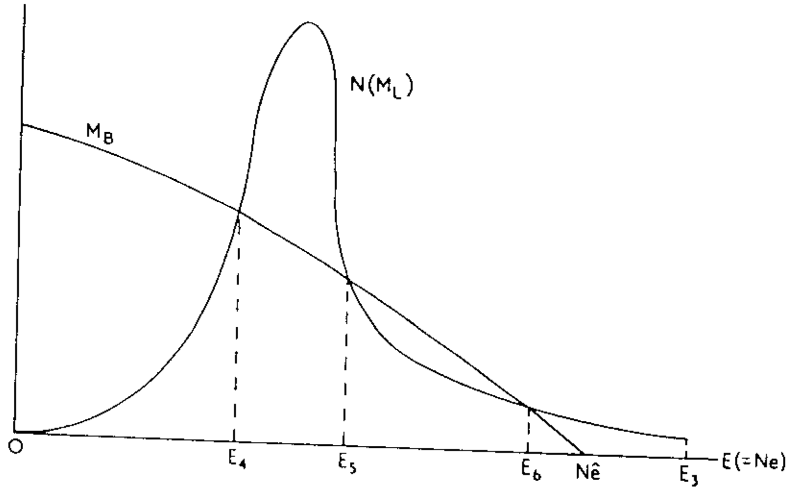


Diagram 3.12

Diagram 3.13 they denote the corresponding 'turning' points of the resulting net benefit curve $B - N|L|$.[25] Plainly E_4, being the global maximum argument of the function $(B - N|L|)$, denotes the socially efficient level of pollution in the river, while E_5 is a local minimum

[25] We are assuming that net social benefit is merely the algebraic sum of the net profits of the two industries. Notice that in Diagram 3.13 we have drawn the curve so that $B - N|L| < 0$ at $E = 0$. Thus it is supposed that the fixed cost, NA, for industry α is 'large'. In such a situation it would be silly to clamour for a zero pollution level. One might wish to argue that industry α should not have been allowed to be located upstream in the first place. But without knowing the nature of industry α one can make no such claims. Perhaps transport costs for the inputs and outputs of industry are low at this geographical location.

and E_6 is a local maximum without being globally so. Each one of them is a tax equilibrium but, of course, E_4 is the desirable one.[26] Now suppose that in the absence of any social management of the problem the market equilibrium has resulted in a level of pollution, E, that is in excess of E_5 (because, say, the law ascribes rights to the pollutor so that the market equilibrium level of pollution is $N\hat{e}$ $(> E_5)$). If now an attempt is made to manage the level of pollution via a sequence of marginal cost–benefit analyses, then the system will eventually find itself a resting place at E_6, since each move in the sequence is made with a view of climbing the 'local hill' whose

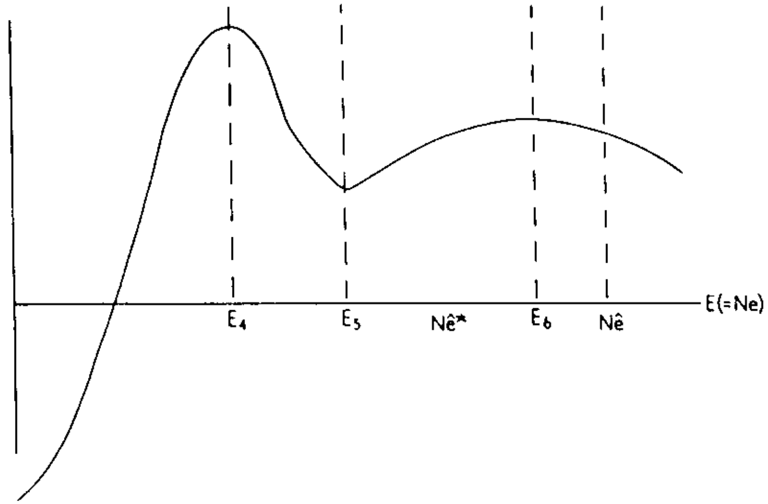


Diagram 3.13

peak is at E_6. It follows that social management via marginal social benefit cost analysis will lead the economy to the local maximum E_6 which, while plainly better than the market equilibrium level of pollution, is still far removed from E_4. There is consequently a definite sense in which there is over-pollution at E_6, but one would typically not know that this is so. The reason is that at E_6 whether

[26] Actually the point $E = 0$ is a tax equilibrium as well where industry α is taxed at such a prohibitive rate for the effluent it discharges that it is forced to close down entirely.

at the margin one increases or decreases the level of pollution there is a drop in net benefit. The problem really concerns the availability of global information. If the government really knew the technological and economic possibilities in their entirety, so that the entire net benefit curve $B - N|L|$ was known, there would be no problem. The authorities would merely glance at the curve, pick out E_4, and legislate it, or equivalently, choose that rate of taxation on industry which would support E_4. The crux of the problem lies in that typically the net benefit schedule over the entire non-negative range of the level of pollution is not known to the government. The regulator picks up the problem at the market equilibrium point. Typically it contemplates marginal moves (it obtains only local information about $B - N|L|$ from the firms) and continues supporting these moves so long as net benefit $B - N|L|$ is increasing at each move. If the market equilibrium level of pollution happens to lie to the left of the point E_5 (say because the benchmark level of pollution e^* is small) there is no problem, since social management via cost–benefit analysis will unerringly take the system to E_4. But as we have seen, if the market equilibrium level of pollution E happens to lie to the right of E_5 this procedure merely takes the system to E_6 which, while superior to the market equilibrium, still supports too much pollution.

The upshot of this discussion is that Pigouvian taxes for socially managing externalities are not necessarily reliable. There may be multiple tax equilibria. Likewise, a simple 'gradient process' for locating the optimal level of pollution will not necessarily work because the net benefit curve can on occasion have multiple peaks. The source of the problem that has arisen here lies in a sense in the fact that external diseconomies, like pollution, often imply non-convexities in technological possibilities. But we do emphasize that this is so only in a sense. For recall that in the problem of the common, while individual firms inflict diseconomies on one another, the symmetric nature of such infliction resulted in the net benefit curve being single peaked (see Diagram 3.6). For external diseconomies in general the case that we have just analysed is more likely to be the rule than an exception. The problem of pollution control is a difficult one to solve.[27]

[27] Recently some attempts have been made to devise search procedures that will locate the global optimum even in the face of non-convexities in technological possibilities (see in particular Heal (1973)).

7. Conclusions

The examples analysed in this chapter appear to suggest the following conclusions.[28]

(1) In the presence of externalities a market equilibrium may well be Pareto inefficient.

(2) External diseconomies in production often imply non-convexities in the space of named commodities, and this *can* ensure that a competitive equilibrium with markets for externalities simply does not exist.

(3) A competitive equilibrium with markets for externalities, when it exists, is Pareto efficient.

(4) If a competitive equilibrium with markets for externalities exists, so does a tax equilibrium exist. Moreover, every competitive equilibrium allocation with markets for externalities can as well be established as an appropriate tax equilibrium.

(5) There are cases (e.g. the problem of the common) where even though a competitive equilibrium with markets for externalities may not exist, there exists a unique tax equilibrium which is Pareto efficient.

(6) Even ignoring income effects, both the size of total net benefits and its distribution among different agents at a market equilibrium depend on the specification of legal rights for generating externalities.

(7) Even ignoring income effects, different schemes designed to produce the optimum levels of externalities have different distributional implications.

(8) In general when production possibility sets are non-convex in the space of named commodities a tax equilibrium is not unique, and each local minimum and each local maximum of the net benefit function will be supported by a tax equilibrium. In such situations the government will need to conduct *global* cost–benefit analysis to compute the optimal allocation and thus calculate the tax structure that will support it.

(9) If there is a presumption that competitive markets for externalities will not be established, the production of external economies ought to be subsidized and the production of external diseconomies ought to be taxed.

(10) An implication of (9) is the presumption that at a market equilibrium with externalities there is an under-production of external economies and an over-production of external diseconomies.

[28] We emphasize that with the exception of 3, 4, 6, 7 and 9 the conclusions are all qualified by an existential quantifier. And yet such qualified conclusions — which our examples support — are worth stating explicitly because they indicate the general tendencies for 'plausible' economies with externalities. One can certainly construct examples where these general tendencies are violated. For example, 10 is surely not an implication of 9 because, for example, income effects may be 'perverse'. But even in the absence of income effects 10 can be untrue without further qualifications (see, for example, Buchanan and Kafoglis (1963) and Diamond and Mirrlees (1973)). For general arguments leading to conclusions 2 and 5 see Foley (1970), Starrett (1971, 1973) and Bergstrom (1974).

Notes on Chapter 3

The literature on externalities is simply huge. It would be almost impossible to give even a reasonably complete bibliography for it. Our account of the problem of externalities in general and Pigouvian taxes in particular is based on the writings of Pigou (1932), Meade (1952), Samuelson (1954), Musgrave (1954), Arrow (1969), Foley (1970) and Starrett (1972, 1974). For a more detailed exposition of the subject see Maler (1974), Meade (1973), and Baumol and Oates (1975). An excellent treatment of common property problems is in Dales (1972) and Christy Jr. and Scott (1972). The terminology 'named' goods is taken from Hahn (1971) who used it in a different context.

The problem of the common was brought to the attention of non-economists in a widely cited article by Hardin (1968). The article is vigorously written and is characterized as much by loose analyses as by incorrect conclusions.

So far as we know our *presentation* of the problem of the common is new. The literature usually deals with the case when there is *free entry* to the common property by potential firms (see, for example, Gordon (1954). Firms are assumed to continue to enter so long as there are positive profits to be made. In equilibrium, therefore, firms make no profit, and the average product of vessels is equal to the rental price. We have, instead, kept the number of firms fixed at N and have investigated the notion of an equilibrium when the common property resource (i.e. the fishing ground) is free to each of them. Stating it another way, in the conventional treatment of the problem the number of firms exploiting the common property resource is endogenous to the analysis. In our presentation it is given exogenously. Given a fixed number of firms, N, we have shown that firms make a positive profit at the market equilibrium, but that the size of the profit is small if N is large. Positive profit for each firm is implied at the market equilibrium for our problem so long as in deciding how many vessels to introduce, each firm i takes into account the effect of its fleet of vessels, x_i, on the average product of vessels on the fishing ground (equation 3.27). Suppose instead, that even though N is finite each firm i *pretends* that the average product $F(X)/X$ is independent of the number of vessels, x_i, that it introduces. Instead of equation 3.28 the symmetric market equilibrium condition will then read as $F(Nx)/Nx = p$ and, therefore, that in equilibrium there are no profits to be made by firms. This last is a sensible notion of an equilibrium so long as it is sensible for each firm to suppose that it cannot influence the average product of vessels. But presumably this will be a reasonable position for a firm to take so long as N is large. It is for this reason that we have analysed the more general condition of equilibrium as embodied in equation 3.28 and have obtained the zero profit case as a limiting one when N tends to infinity.

As we have remarked, there are a number of ways one can present the problem of the common. For a full general equilibrium treatment of the problem (i.e. where the input price—in our case, p—is determined within the system), see Cohen and Weitzman (1975). Unlike Cohen and Weitzman, who are concerned with modelling an entire economy in which the single fixed factor (land) is communally owned, we have been concerned with a common property resource that appears in only one sector of the economy. A partial equilibrium approach with the number of firms given exogenously seems best suited for this purpose. It enables us to handle the diverse examples discussed in section 5.

An important question that we have not touched on in our discussion of optimal regulation of externalities is the one regarding incentives designed to make agents reveal their true preferences regarding the supply of externalities. We have tacitly supposed that the true preferences are revealed. For firms this is not an absurd assumption, given that the externalities are of a technological nature. Their effects can, in principle, be determined. But the assumption that the government can elicit the true preferences of consumers (e.g. the example in section 3) needs justification. We have not provided one. For a discussion of this range of questions see, for example, the recent contributions of Groves and Ledyard (1977), Green and Laffont (1977) and Maskin (1977).

Our discussion of externalities—though not our definition of the concept—has focused attention on what is usually, but somewhat misleadingly, called non-pecuniary externalities. A sharp distinction between pecuniary and non-pecuniary externalities is difficult to make. For discussions on this point see Scitovsky (1954) and Starrett (1974).