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"Large-Scale Condensation"

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LARGE-SCALE CONDENSATION

Large-scale condensation is one of the most important physical processes in atmospheric circulation which is included in the Numerical prediction models. The atmosphere is considered not to attain supersaturation and this condition allows the large scale condensation heating (or **stable heating**) to be computed from the time rate of change of saturation specific humidity. Stable heating is calculated if the following three conditions are all satisified at the same time:

(a) The atmosphere must be saturated or nearly-saturated, $q/q_S \cong 1$
where q = specific humidity in a unit atmospheric column
and
qs = saturated specific humidity.

(b) There is upward vertical velocity in the atmosphere, $\omega < 0$.
(c) The atmosphere must be absolutely stable,

$(i)\ \frac{\partial \theta}{\partial p} < 0 \qquad (ii)\ \frac{\partial \theta_e}{\partial p} < 0$

If the above conditions exist, then the **stable heating rate** is defined by the expression

$$H = -L\frac{\partial q_s}{\partial t} \qquad (1)$$

where L is the latent heat of condensation. In areas of ascending motion, the stable heating is approximated by the expression

$$H \cong -L\frac{\partial q_s}{\partial p}\omega \qquad (2)$$

Note that if the above three conditions are not fulfilled, then the stable heating, **H = 0**. It is also worthy noting that the term $\partial q_S/\partial p$ represents the **slope** of the specific humidity along the **reference moist adiabat** that passes through the point given by the parameters P, T, q_S at which point the stable heating is computed. At this reference point (P, T, q_S)) the **moist static energy** is given by the relation

$$E = C_pT + gz + Lq_S \qquad (3)$$

The conservation of the moist static energy along the moist adiabat can be expressed by considering the derivative of the equation (3) with respect to pressure i.e.

$$0 = c_p \frac{\partial T}{\partial p} - \frac{RT}{p}(1+0.61q_s) + L\frac{\partial q_s}{\partial p} \tag{4}$$

And by making the term $\frac{\partial T}{\partial p}$ the subject in equation (4) we get the lapse rate

$$\frac{\partial T}{\partial p} = \frac{RT}{c_p p}(1+0.61q_s) - \frac{L}{c_p}\frac{\partial q_s}{\partial p} \tag{5}$$

Calculation of the SATURATION SPECIFIC HUMIDITY q_s is approximated from TETEN'S formula given by

$$q_s = \frac{0.622e_s}{p - 0.378e_s} \tag{6}$$

Normally in the atmosphere, $p >> e_s$ so that (6) becomes

$$q_s = \frac{0.622e_s}{p} \tag{7}$$

The vapour pressure e_e is obtained from the relation

$$e_s = 6.11\exp\left[\frac{a(T-273.16)}{T-b}\right] \tag{8}$$

where the constants a and b are defined in terms of saturation over water $[a = 17.26,\ b = 35.86]$ and over ice $[\mathrm{a} = 21.87,\ \mathrm{b} = 7.66]$.

To get $\frac{\partial q_s}{\partial p}$, we differentiate equation (7) with respect to pressure i.e.

$$\frac{\partial q_s}{\partial p} = \frac{-0.622}{p^2}e_s + \frac{0.622}{p}\frac{\partial e_s}{\partial p} \tag{9}$$

In equation (8), let $E^* = \exp\left[\frac{a(T-273.16)}{(T-b)}\right]$ so that

$$e_s = 6.11E^* \tag{10}$$

Then substituting for e_s in equation (9) we obtain

$$\frac{\partial q_s}{\partial p} = -0.622 \times 6.11\left[\frac{E^*}{p} - \frac{\partial E^*}{\partial p}\right] \tag{11}$$

But $$\frac{\partial E^*}{\partial p} = \frac{\partial E^*}{\partial T} \times \frac{\partial T}{\partial p} = E^*\left[\frac{a}{T-b} - \frac{a(T-273.16)}{(T-b)^2}\right]\frac{\partial T}{\partial p} \tag{12}$$

Substitute for $\frac{\partial T}{\partial p}$ from (5) to get the final expression for $\frac{\partial q_s}{\partial p}$ given by

$$\frac{\partial q_s}{\partial p} = -\frac{0.622 \times 6.11}{p} \exp\left[\frac{a(T-273.16)}{(T-b)}\right]$$
$$\times\left[\frac{1}{p} - \frac{a-}{(T-b)} - \frac{a(T-273.16)}{(T-b)^2}\right]$$
$$\times\left[\frac{\mathrm{RT}}{p}(1+0.61q_s) - \frac{L}{C_p}\frac{\partial q_s}{\partial p}\right] \qquad (13)$$

One can solve for $\frac{\partial q_s}{\partial p}$ from equation (13) to obtain the relation

$$\frac{\partial q_s}{\partial p} = -\frac{c_1 c_2}{1+(L/c_p)c_1 c_3} \qquad (14)$$

where $c_1 = -\frac{0.622 \times 6.11}{p} \exp\left[\frac{a(T-273.16)}{T-b}\right]$ (15)

$$c_3 = \left[\frac{a}{T-b} - \frac{a(T-273.16)}{(T-b)^2}\right] \qquad (16)$$

and

$$c_2 = \frac{1}{p} c_3 \left[\frac{RT}{c_p T}(1+0.61q_s)\right] \qquad (17)$$

STABLE RAINFALL RATE

The total stable rainfall rate is obtained as

$$R = \frac{1}{g}\int_0^{p_s} \frac{H}{L} dp \qquad (18)$$

where H is computed from (2) as discussed above.

The stable heating rate is normally expressed in units of °Cday^{-1} and the stable rainfall rate in mmday^{-1}. For this reason, the stable heating rate is denoted by

$$\frac{H}{c_p} = \frac{L\omega}{c_p}\frac{\partial q_s}{\partial p} \qquad (19)$$

Equation (19) has the units of $°Cs^{-1}$. Therefore, since a day has 86400 seconds, the equation

$$\frac{86400H}{c_p} = \frac{86400}{c_p} L\omega\frac{\partial q_s}{\partial p} \qquad (20)$$

will have the units of $°Cday^{-1}$.

Similarly , the expression

$$R = -\frac{86400}{g}\int_0^{p_s} \omega\frac{\partial q_s}{\partial p}dp \qquad (21)$$

gives the TOTAL STABLE RAINFALL RATE in $mmday^{-1}$ when ω is expressed in mb / s, g in m / s^2 and q_s in gm / gm.