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"Lateral Bounday Conditions in Regional Models"

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Please note: These are preliminary notes intended for internal distribution only.

LATERAL BOUNDARY CONDITIONS IN REGIONAL MODELS

Regional, (mesoscale or limited-area, LAM) models are applied both operationally and for research purposes on a sub-area within the global or hemispheric domain. They are useful tools for detailed forecast of weather features which include the meso-α and meso-β with space scales of 20-2000km corresponding to time scales of 2hrs to 3 days, respectively.

These systems include features like regional monsoons, fronts, low - level mesoscale jets, orographic effects, surface convergence zones, sea/breeze circulations, tropical cyclones, etc.

Because of its finite spatial extent, the limited -area model domain is enclosed within perimeter boundaries on the lateral sides, on its top and bottom. These boundaries act like 'side walls' around the limited area domain.

The TOP and LATERAL boundaries have no physical meaning. But it is required, for computational purposes, to specify values at every point on the boundary. These specified values on the boundary are referred to as the BOUNDARY CONDITIONS.

IMPORTANCE OF LATERAL BOUNDARY CONDITIONS IN REGIONAL MODELS

(1.) Errors due to SHORT-SCALE WAVES found in the INITIAL CONDITIONS within the regional mesoscale domain can be removed by the advection into the domain from well-defined Lateral Boundaries at the INFLOW points (Errico and Baumhofner, 1987).

(2.) The Lateral Boundary formulation can act to damp the errors in the solution at the BOUNDARY ZONE and in time remove the errors in the interior as they propagate away into the boundary region (Sashegyi and Madala, 1994).

(3.) If the scale of the evolving disturbance is large compared to the mesoscale domain, the lateral Boundary Conditions act to constrain the solution and to reduce the error (Vukicevic and Errico, 1990).

Boundary Conditions for a limited-area model are usually provided by a forecast from a global model. Therefore, effective representation of the forcing by the large scale circulation on the limited area model domain is via the lateral boundaries.

(4.) Serious errors in the Boundary Conditions obtained from coarse resolution large-scale global forecasts will adversely affect the accuracy of the fine regional forecast, or contaminate it outrightly in a very short time (Dell'Osso, 1983).

Example

Rapid Evolution of a synoptic system; like the passage of a Front or a Tropical Cyclone through the mesoscale domain (Pielke, 1984).

TYPES OF ERRORS

The errors are the outward propagating SHORT-SCALE WAVE features, that is:

(i) Gravity Wave Oscillations (inertial - buoyancy waves);

(ii) Advective Waves.

These are generated by unbalanced initial conditions in the model interior and by computational schemes.

Eliminating or minimizing the reflection of the gravity and the Advective waves from the boundary back into the interior of the limited-area model domain is done through suppression of the amplitudes, wavelengths and phase velocities of these short -scale wave perturbations.

COMMONLY USED METHODS OF SPECIFYING LATERAL BOUNDARY CONDITIONS

The type of boundary conditions depend on the feature one wants to study. Before the various aspects of the lateral boundary conditions, it is deemed expedient to first define the terms 'OPEN' and 'CLOSED' boundary conditions.

OPEN LATERAL BOUNDARY CONDITIONS

These boundary conditions allows mesoscale perturbations to pass into and out of the limited area model domain without inhibition.
The open lateral boundary is sometimes referred to as EXTENDED GRID SYSTEM. It can be conveniently applied to 2-D features usually moving zonally like in the study of the:

Sea/Land Breeze Circulation
Effect of Mountains on Sea/Land Breeze Circulation
Effect of ITCZ on Sea/Land Breeze Circulation.

With this system, you extend your domain on all sides using coarse grid so that lateral boundaries are far away from the area of interest. This structure prevents energy reflections from going back into the domain. It is also easy to compute.

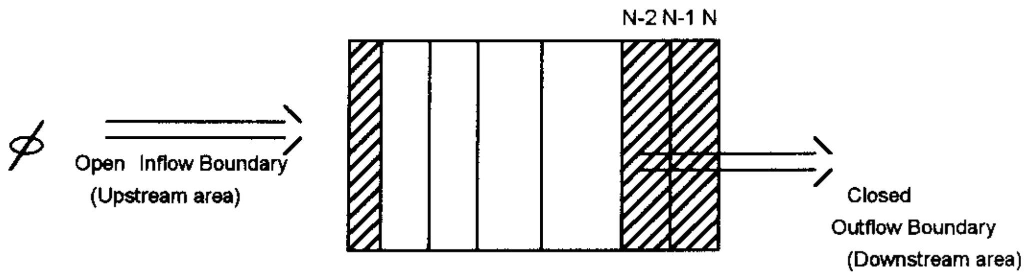
CLOSED LATERAL BOUNDARY CONDITIONS

Mesoscale perturbations are not allowed to enter or exit the model domain. This means that mesoscale perturbations $u' = v' = w' = o$. But the large-scale flow

$U \neq 0$, $V \neq 0$, $W \neq 0$.
That is, the boundary is only closed to the mesoscale perturbations but not to the large-scale flow.

VARIOUS TREATMENT OF LATERAL BOUNDARY CONDITIONS

(1.) Constant Inflow, Gradient Outflow Conditions



Philosophy

(a) The air entering the limited area model domain is assumed to be unaffected by the downstream mesoscale perturbations to the flow, so that the dependent variables remain unchanged at the INFLOW boundary points.

(b) The air exiting the model domain at the OUTFLOW boundary points is made to have the same value as found one grid point (N-1) just before the perimeter boundary (N).

$$\frac{\partial \phi}{\partial \chi} \triangleq \frac{\Delta \phi}{\Delta \chi} = \frac{\phi(N-1) - \phi(N)}{\Delta \chi} = 0$$

Since $\phi(N) = \phi(N-1)$
Where $\phi(N)$ is the value of ϕ at the outflow boundary.
ϕ is any of the dependent variables (u, v, ω, p, θ, q, Φ)

Disadvantage

This procedure is not effective in handling mesoscale disturbances which propagate upstream and simultaneously handle correctly the downstream boundary as advection wave propagations move information at a finite speed from the last interior grid point (N-1) to the boundary (N). Pielke (1984).

(2.) Radiative Boundary Conditions

Philosophy

The philosophy behind this procedure is to modify the value of the prognostic variables (u,v,p,q,theta,phi) at the Outflow Boundary so as to minimize the reflections of the outward propagating mesoscale perturbations back into the limited-area model domain.

The lateral boundary condition schemes may be discussed by considering the linearized advection equation

$$\frac{\partial \bar{\phi}}{\partial t} + c\frac{\partial \bar{\phi}}{\partial x} = 0$$

where ϕ is any one of the propagating prognostic variables (u,v,w,p,q, θ, ϕ).

(i) Orklanski (1976) Radiative Scheme

This scheme is given by the expression

$$c = -\frac{\partial u}{\partial t} \bigg/ \frac{\partial u}{\partial x}$$

evaluated at the last internal grid point (N-1) from the boundary (N).

The requirement is that

$$0 \le c \le \frac{\Delta x}{\Delta t}$$

(ii) The variations to Orklanski's scheme are those due to Klemp and Lilly (1978);Klemp and Wilhelmson (1978a).

(3.) SPONGE BOUNDARY CONDITIONS

Philosophy

This method uses enhanced explicit viscosity smoothing near the lateral boundaries to damp Advective and gravity wave disturbances as they move towards the periphery of the model domain.

Some of the spatial smoothers are also referred to as Low Pass Filters because they only remove the shortest waves (mesoscale) but leave longer waves relatively unaffected.

(i) Diffusive Damping Scheme

For this scheme to be used a marginal Diffusion Zone is prescribed close to the lateral boundary where an explicit diffusion formulation is applied (e.g. Deavens, 1974) by changing the phase and amplitude of the short-scale waves.
In this Diffusion Zone the linearized advection equation will take the following form:

$$\frac{\partial u}{\partial t}+C\frac{\partial u}{\partial x}=\frac{\partial}{\partial x}\left(v_H\frac{\partial u}{\partial x}\right)$$

where the horizontal eddy exchange coefficient $\nu=\nu(x)$ assumed appreciable values only within this marginal Diffusion Zone located at $x=o+,L_-$. For instance ν_H may be computed from

$$v_H=\alpha(\Delta x)^2\left[\frac{1}{2}\left(\frac{\partial\bar{v}}{\partial x}+\frac{\partial\bar{u}}{\partial y}\right)^2+\left(\frac{\partial\bar{u}}{\partial x}\right)^2+\left(\frac{\partial\bar{v}}{\partial y}\right)^2\right]$$

The coefficient α is arbitrarily adjusted until $2\Delta x$ wavelenghts do not appear to degrade the solutions significantly.

(ii) Tendency Modification Scheme

In this scheme, the time tendencies of the dependent prognostic variables are modified in the prescribed marginal zone. The weighted average is set to a maximum at the boundary and gradually decreases to zero at the interior extremity of the marginal zone.
Thus, with this scheme the linearized Advective equation will take the following form in the marginal zone:

$$\frac{\partial u}{\partial t}+c\frac{\partial u}{\partial x}=-\gamma\frac{\partial}{\partial t}(u-U)$$

where U=U (x,t) is the externally specified boundary value.

Because of the accumulation of 'error energy' in the boundary zone, there is need to use a spatial smoother with this scheme (Davies, 1983). For explicit smoothing, an operation is added to the prognostic equations which generate smoothed variables from the original dependent variables. The ideal smoother is the one that eliminates wavelengths smaller than $4\Delta\chi$ at each time step but leaves the longer wavelengths unaffected. (Pielke, 1984). Such smoothers are referred to as low_pass Filters and are said to be selective since they damp only short-scale waves.

An example of a selective filter is that due to Pepper et al.(1979) and is of the form:

$$(1-\delta)\phi^{*}_{i+1}+2(1+\delta)\phi^{*}_{i}+(1-\delta)\phi^{*}_{i-1}=\phi_{i+1}+2\phi_{i}+\phi_{i-1}$$

where ϕ^* = dependent variable to be smoothed

ϕ = the smoothed dependent variable

δ = an arbitrarily chosen weighting parameter for the smoothed values.

The above filter eliminates $2\Delta\chi$ waves at each application and its smoothing of longer waves is dependent on the values of δ.

(iii) <u>Boundary Relaxation Scheme</u>

In this scheme, the prognostic variables are themselves subjected to a forcing in a prescribed marginal zone that constrains then to relax towards the exteranally specified field on a time scale that varies with distance from the boundary (Davies, 1983).

In this scheme, the linearized Advective equation will take the following form in the marginal relaxation-zone:

$$\frac{\partial u}{\partial t}+c\frac{\partial u}{\partial x}=-K(u-U)$$

The term on the right hand side determines the degree to which the LAM solution u relaxes towards the prescribed value U from the global forecast or analysis.

K is called a RELAXATION COEFFICIENT. It is used to linearly combine the boundary values with the interior solution values of the LAM Domain.

K is maximum at the model boundary and decreases towards the interior extremity of the marginal relaxation-zone.

As the short-scale wave error field is advected into the boundary zone, its amplitude is reduced due to the relaxation damping. However errors in the boundary values are free to propagate into the interior.

When synoptic forcing through the boundary is crucial to the evolution of mesoscale phenomena in the LAM domain, this scheme is the most appropriate to use .
If the domain size is small, the frequency of the boundary updating must be increased as well as the number of points of the marginal relaxation-zone (Dell'Osso, 1983).

But the increased filtering cannot be applied abruptly at some selected distance from the lateral boundary because erroneous reflections back into the model domain will result (Pielke, 1984).

(4.) <u>PERIODIC BOUNDARY CONDITIONS</u>

These conditions are similarly referred to as CYCLIC BOUNDARY CONDITIONS. For this conditions, the values of the dependent variables at one boundary of the model domain are assumed identically equal to the values at the other end.

$$\phi(x_0) = \phi(x_\Delta)$$

It is of considerable value in comparing a numerical model with an exact analytical solution.
But realistic mesoscale simulations generally do not allow the perturbations to the basic flow to be re-introduced into the Inflow area of the model once they have exited the Outflow boundary, like is the case with this boundary condition(Pielke, 1984).

(5.) <u>NESTED GRID SYSTEM</u>
With this procedure, the limited_area domain is nested within a coarser grid. There must be a ratio between coarse mesh grid and the nested fine mesh grid.

<u>Example</u>

<u>Coarse Grid</u> : <u>Fine Grid</u>
2:1
or 3:1
or 4:1

The LAM uses the coarse grid data for initialization and as boundary conditions which are updated with time.

One could even mesh three grids, for instance around a tropical cyclone moving. The fine mesh catches the mesoscale phenomena whereas the coarse grid catches the synoptic features.

There are two types of interaction with nested grids:

(1.) A ONE-WAY INTERACTION, where the energy is directed from the coarse to the fine mesh only.

(2.) A TWO-WAY INTERACTION, where energy from the coarse grid is reflected back by the fine mesh. This is difficult to implement, but it is very useful in balancing the model results.

The 'SPONGE' (e.g Perkey and Kreitzberg, 1976) and 'RELAXATION' (e.g Davies, 1976) are frequently used in nested models. In Perkey and Kreitzberg Scheme, model computed tendencies are damped at each time step to specify boundary tendencies in a boundary zone. In the Davies Scheme computed model variables themselves are relaxed at each time step towards the externally specified boundary values in the boundary or marginal zone.

SUMMARY OF THE LATERAL BOUNDARY CONDITIONS

The necessary test to apply to any selected lateral boundary conditions is to:

(1.) Enlarge the model domain progressively (e.g by adding grid points) until successive enlargements have no appreciable effect on the solutions within the region of interest.

(2.) The 'RADIATIVE' boundary condition seems to be the form that permits the least expansion of the model domain since dependent variables are defined at the boundary points. Several radiative boundary schemes have been constructed. But Orklanski's(1976) scheme has been used by various researchers(e.g Clark 1979; Hack and Schubert, 1981) and is therefore a good scheme for any modeller to begin with.

(3.) The sensitivity of the mesoscale model domain to grid volume averaged accelerations also indicates that the model domain must have a reasonable spatial extent in the horizontal dimension. Although large scale fields are input through these boundaries, the sensitivity of the forecast results to slight changes in pressure and velocity makes it difficult to input observational data with a suitable degree of accuracy (Anthes and Werner, 1978).
Thus, very small domain sizes would result in mesoscale circulations that are significantly influenced by the large-scale field through the lateral boundaries.

TOP BOUNDARY CONDITIONS

The upper atmosphere acts both as ABSORBER and REFLECTOR of the Energy propagated from down below.

If a model upper boundary is placed at a finite height level, it should allow for transmission of energy through that level whose value corresponds to that which in the real atmosphere would be absorbed in the layers above it (Davies, 1983).

The top boundary condition can be removed as far a possible from the region of significant mesoscale disturbance (ideally, where p=o so that ρ=o).

But there is a Stable Stratified Layer in the Lower Stratosphere/ upper troposphere. This layer inhibits vertical advection and tend to generate circulations that have larger horizontal than vertical scales (Pielke, 1972). Increased stratification causes shallower circulations to develop and makes hydrostatic assumption more applicable.

Only through vertical propagation of wave motion can information from near the earth's surface be propagated upward through this stable layer.

Using this special characteristic of the earth's atmosphere, modellers have placed the tops of their model domains at the following levels:
(1.) Deep within the stratosphere (Klemp and Lilly, 1978)
(2.) At the tropopause (Mahrer and Pielke, 1975)
(3.) Within the stable layer of the troposphere (Estoque 1961)

The following top boundary conditions have been applied:
(I) A RIGID TOP (Estoque, 1961)
(II) IMPERVIOUS MATERIAL SURFACE (Pielke, 1974a)
(III) ABSORBING LAYER (Anthes and Werner, 1978; Klemp and Lilly, 1978)

A RIGID TOP BOUNDARY CONDITION

This condition is supposed to eliminate rapidly moving External Gravity Waves from the solution thereby permitting longer time steps to be used in the explicit finite difference schemes.

The vertical wind velocity is set to zero at the top of the model (i.e. $\omega = 0$) and the pressure in the model is set to account for mesoscale perturbations at this level. This condition constrains the solutions arbitrarily, since these solutions do not necessarily approach zero at the prescribed rigid level.

IMPERVIOUS MATERIAL SURFACE

This 'material surface' S is placed at the tropopause level on a constant Theta surface which is defined above the model top level Zt. S is therefore a function of THETA (an isentropic surface). i.e. S = S (θ).

Assume incompressible fluid in the equation of conservation of mass so that

$$\frac{\partial \bar{u}}{\partial x}+\frac{\partial \bar{v}}{\partial y}+\frac{\partial \bar{w}}{\partial z}=0 \quad From \quad where \frac{\partial \bar{w}}{\partial z}=-\left(\frac{\partial \bar{u}}{\partial x}+\frac{\partial \bar{v}}{\partial y}\right).$$

Integrate the above equation between Z_t (the highest fixed rigid level) and the material surface S = S (θ) where θ = constant.

Then

$$\varpi_s = \varpi_t - \int_{z_t}^{s_\theta}\left(\frac{\partial \bar{u}}{\partial x}+\frac{\partial \bar{v}}{\partial y}\right) \; dz.$$

Hence the material surface moves in response to the divergence below and is considered to be more realistic representation of conditions at the top of the model.

ABSORBING LAYER

This uses multiple levels to represent the top of the model to prevent the downward reflection of the vertically propagating internal gravity waves (Klemp and Lilly, 1978) This layer is placed above the main portion of the model domain. In this region horizontal filtering is increased from the base of the absorbing layer to the top of the model to prevent wave energy from being reflected downward erroneously.

The schemes used in the absorbing layer are closely related to the 'SPONGE' and the 'RELAXATION' schemes.

To prevent reflections caused by smoothing, the Fielter(e.g 'SPONGE' boundary condition) must be increased gradually.
The depth of the absorbing layer must be greater than the vertical wavelength of the disturbance.

SUMMARY OF THE TOP BOUNDARY CONDITIONS

(1.) If the vertically propagating internal gravity wave energy is equal to or more important than the Advective properties in a mesoscale model, then an absorbing layer may be used.

(2.) If Advective properties are dorminant, then a 'material surface top placed along an isentropic surface at the tropopause, and well above the region where Advective changes are significant, is recommended.

(3.) The use of a rigid top in a mesoscale model is inappropriate unless if the advection effects are dominant and the depth of the model is much greater than the mesoscale disturbance so that perturbations that reach that level will have no impact to any of the dependent variables.

BOTTOM BOUNDARY CONDITIONS

In a mesoscale atmospheric model, the bottom is the only boundary with a physical meaning.
Differential gradients of the dependent variables along the bottom boundary generate many mesoscale circulations, which may be categorized as:

(1.) Terrain-induced mesoscale systems triggered by topographic inhomogenuity and differential heating at the land-water interface;

(2.) Synoptically- induced mesoscale systems triggered by initial conditions.

The following components of the BOTTOM BOUNDARY need to be considered:

(a) WATER BODIES
Ocean currents/ upwelling, etc.

(b) LAND SURFACES
Bare soils
Vegetated areas

(c) <u>SOIL MOISTURE AND SOIL TEMPERATURE</u>

(D) <u>ANTHROPOGENIC SOURCES OF SURFACE HEATING</u>

Man-induced land-scape changes

One also needs to specify the following at the Bottom Boundary:

(I) Wind Velocity .

(II) Fluxes of heat, momentum and moisture.

(III) Temperature and heat budget at the surface which include net radiation,thermal diffusivity/heat storage, albedo.etc.

(IV) Moisture and moisture budget at the surface which include vaporation, precipitation, desertification, etc.

(V) Aerosol specification for different chemical constituents

(VI) Surface pressure.